Neutrino Masses in the MSSM

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A supersymmetric standard model

Find the most general Lagrangian which is

- invariant under Lorentz, $SU(3)_C \times SU(2)_L \times U(1)_Y$
  and supersymmetry transformations; renormalizable

- minimal in particle content

Neither lepton number (L) nor baryon number (B) are conserved
Two options:

- constrain lagrangian parameters

- impose a further discrete symmetry when constructing the lagrangian

Superpotential

The most general superpotential is given by

$$\mathcal{W} = Y_E L H_1 E + Y_D H_1 Q D^c + Y_U Q H_2 U^c - \mu H_1 H_2$$

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$$+ \frac{1}{2} \lambda L L E^c + \lambda' L Q D^c - \kappa L H_2$$

$$+ \frac{1}{2} \lambda'' U^c D^c D^c$$

Ibanez & Ross (1991) show that there are two preferred discrete symmetries, R-parity and a Z_3 allowing $\not{L}$.

R-parity

Under R-parity

- Standard model particles (including scalar Higgs bosons) are even

- corresonding superpartners are odd

If R-parity is imposed when constructing the Lagrangian

- Terms in the Lagrangian which violate either L or B are excluded

- The lightest supersymmetric particle (LSP) is stable

- Sneutrino fields do not acquire a non-zero vacuum expectation values; R-parity is not violated spontaneously

R-parity conserving minimal supersymmetric standard model (Rpc-MSSM)

Neutral scalar particle content

- neutral, complex scalar components of two higgs chiral supermultiplets, h_2^0 and h_1^0

- neutral, complex scalar components of lepton supermultiplets, $\tilde{\nu}_{Li}$

If R_P conservation is imposed on the Lagrangian there are no bilinear terms which cause mixing between fields with different R_P.

No mixing between Higgs bosons and sneutrinos.

The neutral scalars decouple into CP-even and CP-odd real scalar eigenstates
The Higgs bosons

$$\{h_2^0, h_1^0\} \longrightarrow \{h^0, H^0, A^0, G^0\}$$

and the sneutrinos

$$\tilde{\nu}_{Li} \longrightarrow \{\tilde{\nu}_{+\,i}, \tilde{\nu}_{-\,i}\}$$

The scalar potential can be minismised to obtain values for the vacuum expectation values (vevs) of h_2^0 and h_1^0.

L violating minimal supersymmetric standard model ($\not{L}$-MSSM)

Without imposing the conservation of L on the Lagrangian, bilinear terms exist between (and hence, mixing occurs between)

- charged gauginos, charged higgsinos and charged leptons

- neutral gauginos, neutral higgsinos and neutrinos

- Higgs bosons, sneutrinos

- ...

giving rise to some attractive features.

B. C. Allanach, A. Dedes and H. K. Dreiner, Phys. Rev. D **69**, 115002 (2004)

Neutrino Masses

The mixing between neutrinos and neutral gauginos/higgsinos produces one tree-level, 'see-saw' suppressed, neutrino mass.

$$
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & m_\nu^{\text{tree}}
\end{pmatrix}
$$

where

$$
m_\nu^{\text{tree}} = \left| \frac{v_d^2 \left(M_1 g_2^2 + M_2 g^2 \right)}{4 \text{Det}[M_{\chi^0}]} \right| \left(|\kappa_1|^2 + |\kappa_2|^2 + |\kappa_3|^2 \right)
$$

The neutral scalar bosons all mix

$$\{h_2^0, h_1^0, \tilde{\nu}_{Li}\} \longrightarrow \{h^0, H^0, A^0, G^0, \tilde{\nu}_{+\,i}, \tilde{\nu}_{-\,i}\}$$

In general

- 10×10 mixing matrix of real, scalar fields.

- Solve minimisation conditions to find 5 complex vevs.

- Not clear which phases can be rotated away and which are physical; explicit CP-violation?

- Not clear if vevs are real; spontaneous CP-violation?

Vanishing sneutrino vev basis

Notice that if R_P is not imposed, h_1^0 and $\tilde{\nu}_{Li}$ carry the same quantum numbers, even before symmetry breaking.

Define $\tilde{\nu}_{L\alpha} = (h_1^0, \tilde{\nu}_{Li})$ $\qquad$ $\mu_\alpha = (\mu, \kappa_i)$
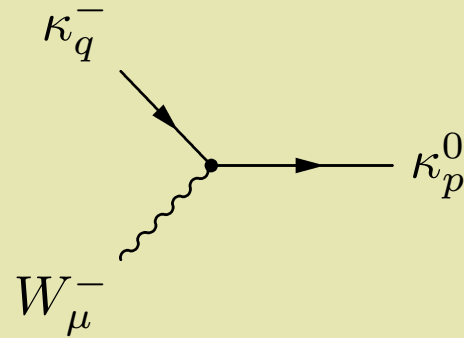
Solving the minimisation conditions defines a direction in this $(h_1^0, \tilde{\nu}_{Li})$ space.

In the interaction basis, we are free to define any direction in $(h_1^0, \tilde{\nu}_{Li})$ to be the Higgs.

Had the basis been rotated such that the Higgs points in this direction, a basis would have been chosen such that the sneutrino vevs are zero.

- We have studied the scalar and fermion sector without making any assumptions concerning the CP or family structure of the theory, finding a basis suitable for calculation.

- A method for initialising parameters has been found, such that the correct values are generated for lepton/quark masses and CKM/PMNS matrices.

- FORTRAN code has been developed to produce the tree-level mass spectrum, mixing matrices and Feynman rules.

- The renormalisation of the theory has been studied, one-loop corrections to the neutralino-neutrino masses have been derived and implemented within the code.
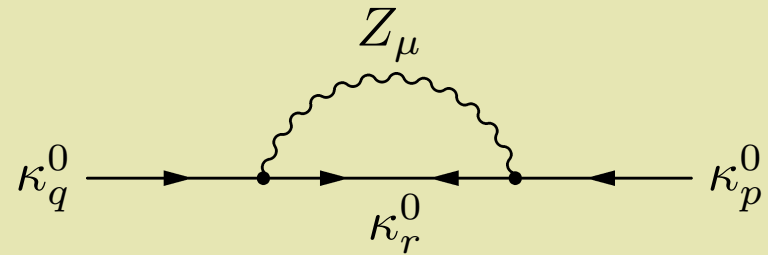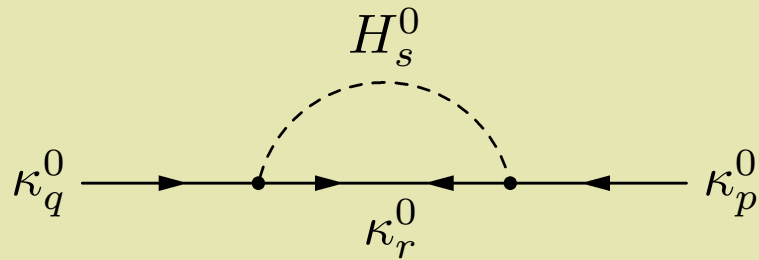
A. Dedes, SR, J. Rosiek and M. Schmidt-Sommerfeld, Phys. Lett. B **627** (2005) 161

A. Dedes, SR and J. Rosiek, in preparation

Vertex

$$\left\{ \frac{ie}{\sqrt{2}s_W} Z^*_{N(4+\alpha)p} Z^*_{-(2+\alpha)q} + \frac{ie}{s_W} Z^*_{N2p} Z^*_{-1q} \right\} \gamma^\mu P_L$$

Loop Diagrams

Other loops:
Neutralino/CP-odd Higgs
Chargino/charged Higgs
Quark/squark

Chargino/W-boson

Summary

- Our aim is to study the phenomenology of the most general $\not{L}$-MSSM

- The full set of Feynman rules have been derived

- The one-loop corrections to the neutrino masses have been calculated

- This is interesting to compare limits on λ/λ' with Tevatron/B-factory analyses

- Can now go on to study other processes connected with neutrino masses; neutrinoless double beta decay, lepton flavour violating events such as τ decays, . . .

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A. Dedes, SR and J. Rosiek, in preparation

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Neutrino-neutralino mass matrix

$$\mathcal{L} \supset -\frac{1}{2} \begin{pmatrix} -i\tilde{B}^0 & -i\tilde{W}^0 & \tilde{h}_2^0 & \tilde{h}_1^0 & \nu_i \end{pmatrix} \mathcal{M}_\mathcal{N} \begin{pmatrix} -i\tilde{B}^0 \\ -i\tilde{W}^0 \\ \tilde{h}_2^0 \\ \tilde{h}_1^0 \\ \nu_i \end{pmatrix}$$
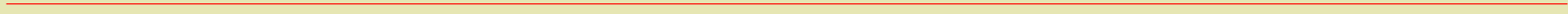
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$$\mathcal{M}_{\mathcal{N}} = \begin{pmatrix} M_1 & 0 & gv_u/\sqrt{2} & -gv_d/\sqrt{2} & -gv_j/\sqrt{2} \\ 0 & M_2 & -g_2 v_u/\sqrt{2} & g_2 v_d/\sqrt{2} & g_2 v_j/\sqrt{2} \\ gv_u/\sqrt{2} & -g_2 v_u/\sqrt{2} & 0 & \mu & \kappa_j \\ -gv_d/\sqrt{2} & g_2 v_d/\sqrt{2} & \mu & 0 & 0_j \\ -gv_i/\sqrt{2} & g_2 v_i/\sqrt{2} & \kappa_i & 0_i & 0_{ij} \end{pmatrix}$$

$$\langle h_2^0 \rangle = v_u \quad \langle h_1^0 \rangle = v_d \quad \langle \tilde{\nu}_{Li} \rangle = v_i$$

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A. S. Joshipura and M. Nowakowski, Phys. Rev. D **51**, 2421 (1995)

M. Nowakowski and A. Pilaftsis, Nucl. Phys. B **461**, 19 (1996)

$$\mathcal{M}_{\mathcal{N}} = \left(\begin{array}{cccc|c} M_1 & 0 & gv_u/\sqrt{2} & -gv_d/\sqrt{2} & -gv_j/\sqrt{2} \\ 0 & M_2 & -g_2 v_u/\sqrt{2} & g_2 v_d/\sqrt{2} & g_2 v_j/\sqrt{2} \\ gv_u/\sqrt{2} & -g_2 v_u/\sqrt{2} & 0 & \mu & \kappa_j \\ -gv_d/\sqrt{2} & g_2 v_d/\sqrt{2} & \mu & 0 & 0_j \\ \hline -gv_i/\sqrt{2} & g_2 v_i/\sqrt{2} & \kappa_i & 0_i & 0_{ij} \end{array} \right)$$

$$\mathcal{M}_{\mathcal{N}} = \left(\begin{array}{cc} M_{\tilde{\chi}\, 4\times 4} & m_{4\times 3} \\ m_{3\times 4}^{T} & 0_{3\times 3} \end{array} \right)$$

Suggestive of the seesaw mechanism

4 eigenvalues $\sim M_{\tilde{\chi}}$ 　　　　3 eigenvalues $\sim \dfrac{mm^T}{M_{\tilde{\chi}}}$

$$\mathcal{M}_{\mathcal{N}} = \begin{pmatrix} M_1 & 0 & gv_u/\sqrt{2} & -gv_d/\sqrt{2} & -gv_j/\sqrt{2} \\ 0 & M_2 & -g_2v_u/\sqrt{2} & g_2v_d/\sqrt{2} & g_2v_j/\sqrt{2} \\ gv_u/\sqrt{2} & -g_2v_u/\sqrt{2} & 0 & \mu & \kappa_j \\ -gv_d/\sqrt{2} & g_2v_d/\sqrt{2} & \mu & 0 & 0_j \\ \hline -gv_i/\sqrt{2} & g_2v_i/\sqrt{2} & \kappa_i & 0_i & 0_{ij} \end{pmatrix}$$

$$\mathcal{M}_{\mathcal{N}} = \begin{pmatrix} N_{4\times3} & n_{4\times4} \\ n'_{3\times3} & 0_{3\times4} \end{pmatrix}$$

The mass^2 given by eigenvalues of $\mathcal{M}_{\mathcal{N}}^{\dagger}\mathcal{M}_{\mathcal{N}}$

$$n = \begin{pmatrix} -gv_d/\sqrt{2} & -gv_1/\sqrt{2} & -gv_2/\sqrt{2} & -gv_3/\sqrt{2} \\ g_2v_d/\sqrt{2} & g_2v_1/\sqrt{2} & g_2v_2/\sqrt{2} & g_2v_3/\sqrt{2} \\ \mu & \kappa_1 & \kappa_2 & \kappa_3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

n has 3 linearly dependant rows, giving two zero eigenvalues with corresponding eigenvectors $\vec{e}_{1,2}$

$$\mathcal{M_N}\vec{q_i} = \begin{pmatrix} N_{4\times3} & n_{4\times4} \\ n'_{3\times3} & 0_{3\times4} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vec{e_i} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \vdots \\ \lambda_i\vec{e_i} \\ \vdots \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\vec{q}_{1,2}$ are zero eigenvectors of the full matrix.

The neutral scalar bosons all mix

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M. Bisset, O. C. W. Kong, C. Macesanu and L. H. Orr, Phys. Lett. B **430**, 274 (1998)

Y. Grossman and H. E. Haber, Phys. Rev. D **59**, 093008 (1999)

Finding the vanishing sneutrino vev basis

The neutral scalar potential takes the form

$$V_{\text{neutral}} = \left(\mathcal{M}^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\beta} + m^2_2 h^{0*}_2 h^0_2 - (b_\alpha \tilde{\nu}_{L\alpha} h^0_2 + \text{H.c}) + \frac{1}{8}(g^2 + g^2_2)[h^{0*}_2 h^0_2 - \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\alpha}]^2$$

where

$$\left(\mathcal{M}^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} \equiv \left(m^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} + \mu^*_\alpha\mu_\beta, \qquad m^2_2 \equiv m^2_{H_2} + \mu^*_\alpha\mu_\alpha$$

A. Dedes, SR, J. Rosiek and M. Schmidt-Sommerfeld, hep-ph/0506209.

Finding the vanishing sneutrino vev basis

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Redefine the sneutrino fields

$$\tilde{\nu}_{L\alpha} = U_{\alpha\beta} \tilde{\nu}_{L\beta}', \quad \mathbf{U} = \mathbf{V} \operatorname{diag}(e^{i\phi_\alpha}) \mathbf{Z}, \quad \mathbf{V} \text{ unitary}, \mathbf{Z} \text{ orthogonal.}$$

$$V_{\text{neutral}} = \left[Z^T \left(\hat{\mathcal{M}'}_{\tilde{\mathcal{L}}}^2\right) Z \right]_{\alpha\beta} \tilde{\nu}_{L\alpha}'^* \tilde{\nu}_{L\beta}' + m_2^2\, h_2^{0*}\, h_2^0$$

$$- \left[(b'Z)_\alpha \tilde{\nu}_{L\alpha}' h_2^0 + \text{H.c} \right] + \frac{1}{8}(g^2 + g_2^2)\left(h_2^{0*} h_2^0 - \tilde{\nu}_{L\alpha}'^* \tilde{\nu}_{L\alpha}' \right)^2$$

Minimisation conditions

$$\left[Z^T \left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}} \right) Z \right]_{\alpha\beta} v_\beta - (b'Z)_\alpha v_u - \frac{1}{8}(g^2 + g_2^2)(v_u^2 - v_\gamma v_\gamma)\, v_\alpha = 0$$

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$$\left[Z^T \left(\hat{\mathcal{M}'}^2_{\tilde{\mathcal{L}}} \right) Z \right]_{00} v_0 + \left[Z^T \left(\hat{\mathcal{M}'}^2_{\tilde{\mathcal{L}}} \right) Z \right]_{0j} v_j - (b'Z)_0 v_u - \frac{1}{8}(g^2 + g_2^2)(v_u^2 - v_\gamma v_\gamma) v_0 = 0$$

Minimisation conditions

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$$\left[Z^T \left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}} \right) Z \right]_{i0} v_0 + \left[Z^T \left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}} \right) Z \right]_{ij} v_j - (b'Z)_i v_u - \frac{1}{8}(g^2 + g_2^2)(v_u^2 - v_\gamma v_\gamma) v_i = 0$$

$$Z_{\alpha 0} = \frac{b'_\alpha \tan \beta}{\left(\hat{\mathcal{M}}'^2_{\tilde{\mathcal{L}}}\right)_{\alpha\alpha} - \frac{1}{2}M_Z^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1}} \ .$$

However, Z is an orthogonal matrix and orthoginality must be imposed

$$\sum_{\alpha=0}^{3} Z_{\alpha 0} Z_{\alpha 0} = 1$$

giving multiple solutions for $\tan \beta = \frac{v_u}{v_0}$
Each $\tan \beta$ corresponds to a different extrema.
By considering all solutions for $\tan \beta$ can find the solution which corresponds to
the deepest minima.
This is the correct vanishing sneutrino vev basis.
The first column of Z is now fully defined (3 parameters)

Have the freedom to define 3×3 orthogonal sub-matrix of Z such that

$$\left[\mathbf{Z}^T \left(\hat{\mathcal{M}'}^2_{\tilde{\mathcal{L}}} \right) \mathbf{Z} \right]_{ij}$$

Scalar potential in vanishing sneutrino vev basis

$$V_{\text{neutral}} = (M_{\tilde{\text{L}}}^2)_{\alpha\beta}\, \tilde{\nu}_{L\alpha}^*\, \tilde{\nu}_{L\beta} + m_2^2\, h_2^{0*}\, h_2^0$$

$$- \left[B_\alpha\, \tilde{\nu}_{L\alpha}\, h_2^0 \ + \ \text{H.c} \right] + \frac{1}{8}(g^2 + g_2^2) \left(h_2^{0*} h_2^0 - \tilde{\nu}_{L\alpha}^* \tilde{\nu}_{L\alpha} \right)^2$$

where

$$(M_{\tilde{\text{L}}}^2)_{\alpha\beta} \ \equiv \ \left[Z^T \left(\hat{\mathcal{M}'}_{\tilde{\mathcal{L}}}^2 \right) Z \right]_{\alpha\beta} \quad \text{and} \quad B_\alpha \ \equiv \ (b'Z)_\alpha$$

Have now moved to a basis where

- sneutrino vevs are zero

- sneutrino masses diagonal, $(\hat{\mathcal{M}}_{\tilde{\nu}}^2)_i$

- all the parameters of the scalar potential real; can split into CP-even and CP-odd eigenstates

Parametrizing the neutral scalar mass matrices

$$M_A^2 = \frac{2\,B_0}{\sin 2\beta} \quad \text{and} \quad \tan\beta$$

where M_A^2 is the mass of the lightest CP-odd Higgs boson in the Rpc limit.

The Lagrangian contains the terms

$$\mathcal{L} \supset - \begin{pmatrix} x_2 & r_\gamma \end{pmatrix} \mathcal{M}^2_{\text{EVEN}} \begin{pmatrix} x_2 \\ r_\delta \end{pmatrix} - \begin{pmatrix} y_2 & t_\gamma \end{pmatrix} \mathcal{M}^2_{\text{ODD}} \begin{pmatrix} y_2 \\ t_\delta \end{pmatrix}$$

where

$$h_2^0 = x_2 + iy_2 \qquad \tilde{\nu}_{L\alpha} = r_\alpha + it_\alpha$$

CP-even Higgs boson mass matrix

$$\mathcal{M}^2_{\text{EVEN}} =$$

$$
\begin{pmatrix}
\cos^2\beta M_A^2 + \sin^2\beta M_Z^2 & -\frac{1}{2}\sin 2\beta(M_A^2 + M_Z^2) & -B_j \\
-\frac{1}{2}\sin 2\beta(M_A^2 + M_Z^2) & \sin^2\beta M_A^2 + \cos^2\beta M_Z^2 & B_j\tan\beta \\
-B_i & B_i\tan\beta & \left((\hat{\mathcal{M}}^2_{\tilde{\nu}})_i + \frac{1}{2}\cos 2\beta M_Z^2\right)\delta_{ij}
\end{pmatrix}
$$

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-B_i & B_i\tan\beta & \left((\hat{\mathcal{M}}^2_{\tilde{\nu}})_i + \frac{1}{2}\cos 2\beta M_Z^2\right)\delta_{ij}
\end{pmatrix}
$$

Noticing that the top-left 2×2 sub-matrix is identical to the Rpc case, for which the Higgs masses are given by

$$M_{h,H}^2 = \frac{1}{2}\left(M_Z^2 + M_A^2 \pm \sqrt{(M_Z^2 + M_A^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right)$$

Applying Courant-Fischer theorem, it can be seen that in Rpv case one eigenvalue which is smaller that M_Z^2 exists

CP-odd Higgs boson mass matrix

The CP-odd mass matrix reads
$$\mathcal{M}^2_{\text{ODD}} =$$
$$\begin{pmatrix} \cos^2 \beta M_A^2 + \xi \sin^2 \beta M_Z^2 & \frac{1}{2} \sin 2\beta (M_A^2 - \xi M_Z^2) & B_j \\ \frac{1}{2} \sin 2\beta (M_A^2 - \xi M_Z^2) & \sin^2 \beta M_A^2 + \xi \cos^2 \beta M_Z^2 & B_j \tan \beta \\ B_i & B_i \tan \beta & (\hat{\mathcal{M}}^2_{\tilde{\nu}})_i \delta_{ij} \end{pmatrix}$$

ξ is the gauge fixing parameter. we can project out without approximation the gauge dependent part, the would-be Goldstone mode, of the CP-odd scalar matrix as

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$$\mathcal{V}^{\text{T}}\mathcal{M}^2_{\text{ODD}}\mathcal{V} = \begin{pmatrix} \xi M_Z^2 & 0 & 0 \\ 0 & M_A^2 & \frac{B_j}{\cos\beta} \\ 0 & \frac{B_i}{\cos\beta} & (\hat{\mathcal{M}}^2_{\tilde{\nu}})_i\delta_{ij} \end{pmatrix}$$