

# Top-Pair Associated Higgs Production at the ILC

Cailin Farrell

Measurement  
Motivation  
ILC at 500 GeV

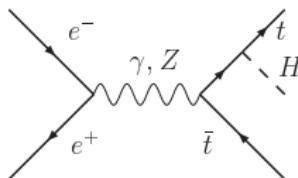
vNRQCD

NLL QCD

Polarization

NLL EW

Conclusion



Cailin Farrell

ECM, Universitat de Barcelona

[hep-ph/0604166], [hep-ph/0504220], . . .

Durham, 22-26.9.2008

# Outline

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- Physical motivation
- Measurement at the ILC at 500 GeV
- Nonrelativistic QCD
- NLL QCD effects
- NLL electroweak effects

# Measurement

## Motivation

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## Open questions:

- Generation of mass?
- Electroweak symmetry breaking?

In the SM: Higgs mechanism

	predicted?	LHC	$\frac{\Delta x}{x}$
Mass	—	✓	1%
Gauge couplings	$m_w$	✓	5-30%
Self-couplings	$\frac{m_h^2}{v^{(2)}}$	—	—
Yukawa couplings	$\frac{m_f}{v}$	top, $\tau$	30-50%

- Couplings discriminate between models
- Need ILC to measure them

# Measurement

## Motivation

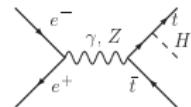
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Here:

Top Yukawa coupling at ILC

- How?

- Measurement of  $\sigma(e^+e^- \rightarrow t\bar{t}H)$



- Known: Born CS  
One-loop CS

[Gaemers, Gounaris, Djouadi, Kalinowski ]

[Dawson, Reina, Belanger, Dittmaier, Denner,  
Roth, Weber . . . ]

New: NLL CS in vNRQCD

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# Measurement

## Experimental Precision

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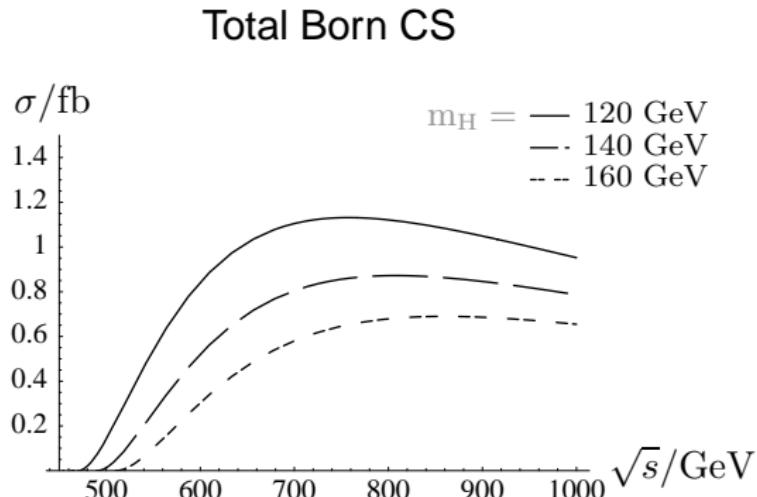
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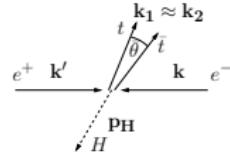
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### Dedicated Studies:

- At 800 GeV:  $\delta Y_t / Y_t \approx 5 - 10\%$  [Juste,Gay]
- At 500 GeV:  $\approx 25\%$  [Juste]  
 $2m_t + m_H \geq 475 \text{ GeV} \Rightarrow \text{Phase space is small}$

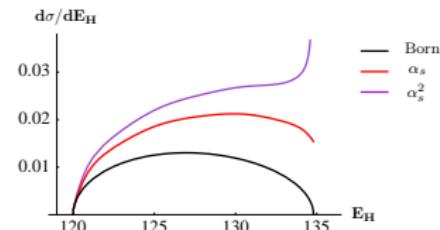
## Difficulties for $v \ll 1$ :



- Coulomb singularities  $\sim (\alpha_s/v)^n$

$$\text{Diagram} + \frac{\alpha_s}{v} + \left(\frac{\alpha_s}{v}\right)^2 + \dots$$

- $(\alpha_s \log v)^n$  singularities:  $\log \frac{E}{p}$ ,  $\log \frac{p}{m}$
- Fixed-order theory breaks down



At 500 GeV:  $v$  is always small  
Phase space is non-relativistic

⇒ Effective theory

$$\frac{d\sigma}{dE_H} \sim v \sum \left( \frac{\alpha_s}{v} \right)^n (\alpha_s \ln v)^n (1 (LL) + \# \alpha_s (NLL))$$

## Difficulties for $v \ll 1$ :

- Coulomb singularities  $\sim (\alpha_s/v)^n$

$$\text{Diagram series: } 1 + \dots + \frac{\alpha_s}{v} + \dots + \left(\frac{\alpha_s}{v}\right)^2 + \dots$$

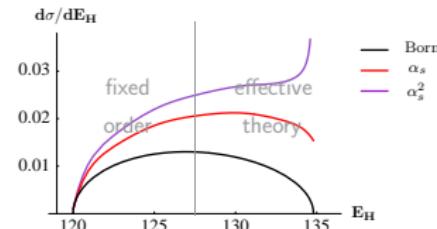
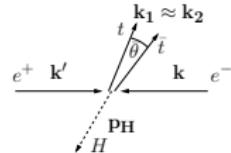
The diagram shows a series of Feynman diagrams representing the expansion of a loop diagram. The first term is a single vertex. The second term is a loop with one external line. The third term is a loop with two external lines. The fourth term is a loop with three external lines. This represents the expansion of the loop contribution to the cross-section in terms of the coupling constant alpha\_s and the velocity v.

- $(\alpha_s \log v)^n$  singularities:  $\log \frac{E}{p}$ ,  $\log \frac{p}{m}$
- Fixed-order theory breaks down

At 500 GeV:  $v$  is always small  
Phase space is non-relativistic

$\Rightarrow$  Effective theory

$$\frac{d\sigma}{dE_H} \sim v \sum \left( \frac{\alpha_s}{v} \right)^n (\alpha_s \ln v)^n (1 (LL) + \# \alpha_s (NLL))$$



# vNRQCD (velocity Non-Relativistic QCD)

[Luke, Manohar, Rothstein, Stewart, Hoang]

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- Power Counting:  $\alpha_s \sim v$

- Hierarchy of relevant scales:

$$m \gg mv \sim \vec{p} \gg mv^2 \sim E_{kin} \sim i\Gamma_t \gg \Lambda_{QCD}$$

- Infrared regulator  $\Gamma_t$  suppressing hadronization effects

- At LL:  $v \approx \sqrt{\frac{E+i\Gamma_t}{m}}$

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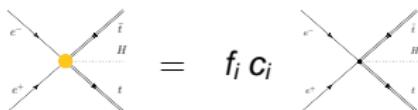
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- Lagrangian includes the fields of all resonant degrees of freedom → Schrödinger equation in the CMS
- Effective creation and annihilation currents:

$$\mathbf{O}_p = f_0 c_0(\mu, \sqrt{s}, m_H) (\psi_{\vec{p}}^\dagger \tilde{\chi}_{-\vec{p}}) + f_1 c_1(\mu, \sqrt{s}, m_H) (\psi_{\vec{p}}^\dagger \vec{\sigma} \tilde{\chi}_{-\vec{p}})$$



- Wilson coefficients contain the non-resonant contributions
  - RGE running:  $c(\mu) = c(m_t) \cdot u(m_t, \mu)$

# NLL QCD Effects

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Cross section  $\sigma(e^+e^- \rightarrow t\bar{t}H)$

$$\frac{d\sigma}{dE_H} (E_H \approx E_H^{\max}) \sim \left[ f_0^2 c_0^2(\mu, \sqrt{s}, m_t, m_H) + f_1^2 c_1^2(\mu, \sqrt{s}, m_t, m_H) \right] \text{Im } G_{\text{Coulomb}}^{\text{NLL}}$$

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**NLL QCD**

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# NLL QCD Effects

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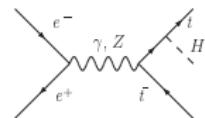
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$f_{0,1}^2$ : electroweak information in the endpoint

$$\sim \left( \frac{d\sigma_{0,1}}{dE_H} \right)_{\text{Born}} \text{ for } E_H \rightarrow E_{H,\max}$$



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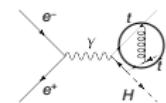
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$c_{0,1}^2$ : hard QCD corrections  
in the endpoint



[Denner, Dittmaier, Roth Weber]

$f_{0,1}^2$ : electroweak information in the endpoint

$$\sim \left( \frac{d\sigma_{0,1}}{dE_H} \right)_{\text{Born}} \text{ for } E_H \rightarrow E_{H,\max}$$

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Known:

- Renormalization group running of  $c_{0,1}$
- $G_{\text{Coulomb}}^{\text{NLL}}$

New:

- Matching conditions  $f_{0,1}$ ,  $c_{0,1}(\mu = m_t)$
- Inclusion of  $e^+e^-$  polarization
- Formula for  $\sigma_{\text{tot}}$
- Top-decay effects
- Phase space matching

# NLL QCD Effects

## Differential Cross Section

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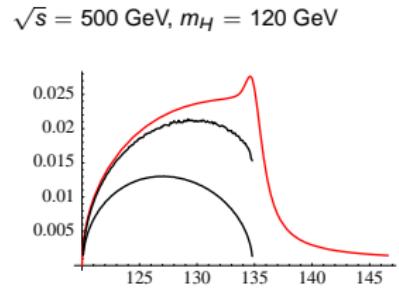
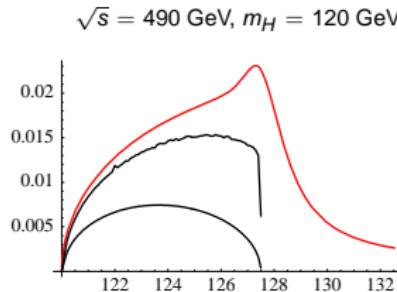
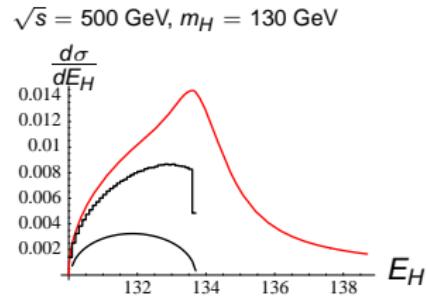
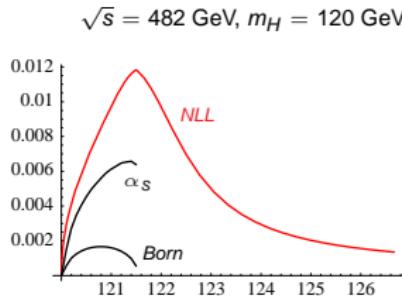
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⇒ Behavior far from threshold is well reproduced  
⇒ Increase of total cross section

# $e^+e^-$ Polarization

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Cross section depends on the helicity of  $e^+$  and  $e^-$ :



Total cross section:

$$\sigma_{pol} = \sigma_{unpol} [1 - P_- P_+ - A_{LR}(P_+ - P_-)]$$

$P_{\pm}$ :

degree of  $e^{\pm}$  polarization

$$\text{left-right asymmetry: } A_{LR} = \frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}$$

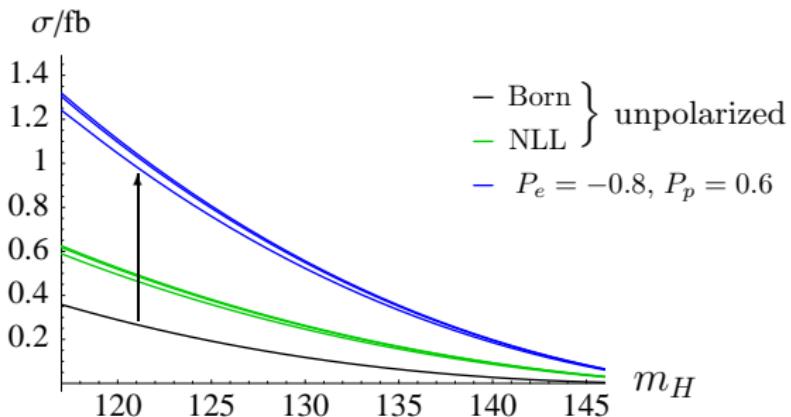
$\Rightarrow$  Increase of  $\sigma_{tot}$  by polarization possible

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Total Cross Section for  $\sqrt{s} = 500$  GeV,  $m_t = 175$  GeV



$\sim 400\%$  increase vs. unpolarized Born cross section  
 $\Rightarrow$  Decrease of statistical uncertainty by  $\sim 50\%$

# Electroweak effects

## Top Decay

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### Power Counting:

$$g \sim g' \sim v \sim \alpha_s$$

### Top decay:

At LL:

$$E \rightarrow E + i\Gamma_t$$

At NNLL: Imaginary Wilson coefficients

### Tops decay into $b - W$ -pairs

- Cutkosky: Identify cuts corresponding to  $bW$ -cuts
- At  $t\bar{t}$  threshold: Gram determinant often vanishes  
⇒ Use appropriate reduction method

[Denner,Dittmaier]

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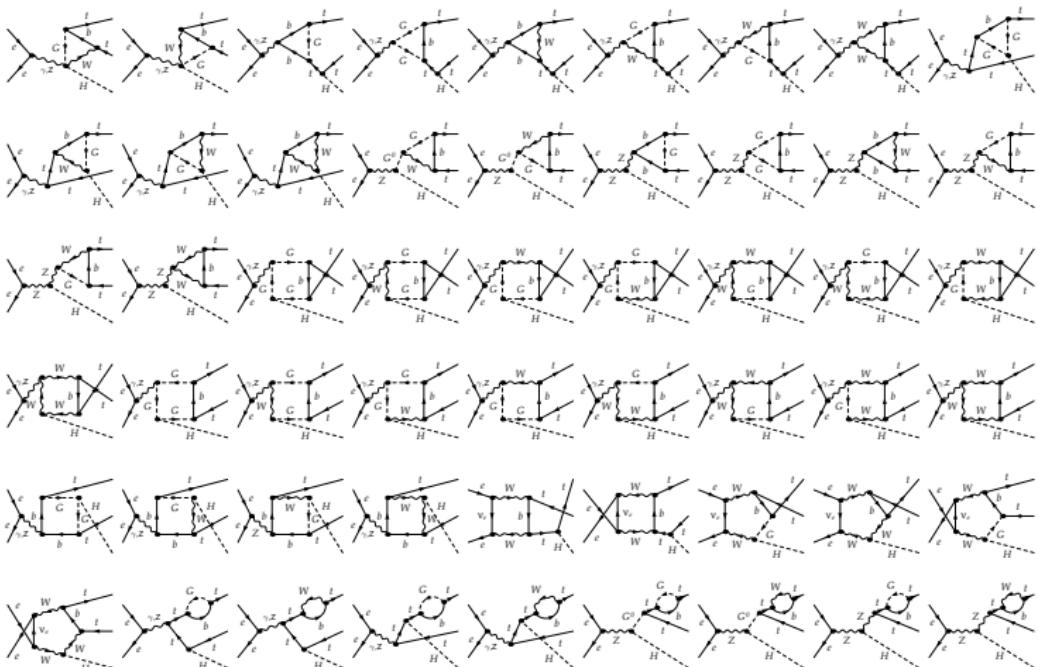
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## The relevant diagrams for $t\bar{t}H$



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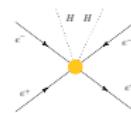
Conclusion

## ● Sensitivity to $\text{Re}[G]$

Optical Theorem:  $\sigma_{\text{tot}} \sim \text{Im}[c_w G]$

$$G_{\text{Coulomb}}^{\text{LL}}(0, 0, E) = \frac{m_t^2}{4\pi} \left\{ i\nu - c_F \alpha_s \left[ \frac{1}{4\epsilon} + \ln\left(\frac{-im_t\nu}{\mu}\right) + \psi\left(1 - \frac{i c_F \alpha_s}{2\nu}\right) \right] \right\}$$

⇒ RGE running of new operator



## ● Phase Space Matching:

double-resonant region: NR expansion is valid

⇒ Cut on  $Q^2$  of the tops

# Electroweak Effects

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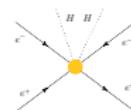
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- Sensitivity to  $\text{Re}[G]$

Optical Theorem:  $\sigma_{\text{tot}} \sim \text{Im}[c_w G]$

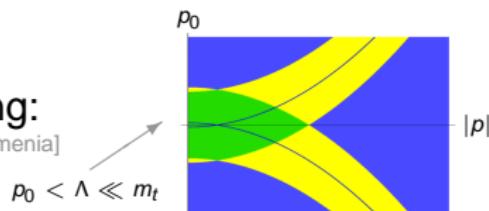
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⇒ RGE running of new operator



- Phase Space Matching:

[Hoang, Reisser, Ruiz-Femenia]



double-resonant region: NR expansion is valid

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# Electroweak Effects

## Results

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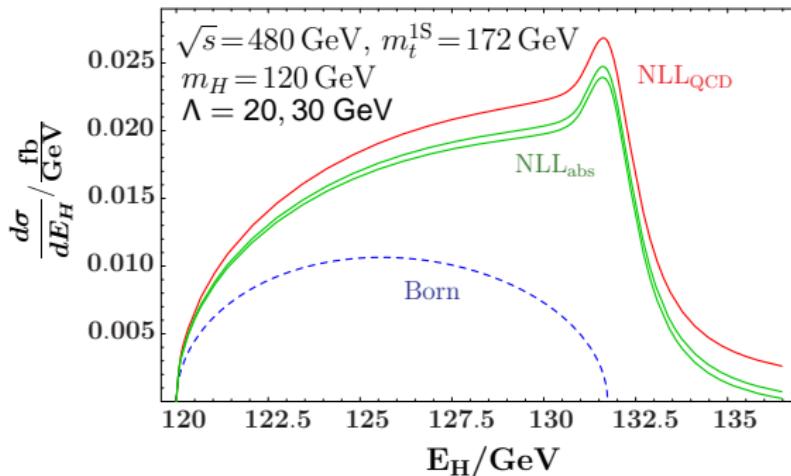
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Numerical effect: 10-15%

CS goes to zero above threshold

# Conclusion

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- Top Yukawa coupling for test of EWSB
- ILC at 500 GeV: Phase space is non-relativistic  
⇒ vNRQCD
- Completed:
  - Strong and electroweak matching conditions at  $\mathcal{O}(NLL)$
  - Effects of  $e^+ e^-$  polarization
  - Formula for the total cross section
  - Electroweak NLL decay effects
  - Phase space matching
- Increase of total cross section of up to 400%
  - $\delta Y_t / Y_t \approx 10 - 15\%$  might be possible