Non-perturbative test of HQET in two-flavour QCD

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Motivation

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2g_0^2}\mathrm{tr}\{F_{\mu\nu}F_{\mu\nu}\} + \sum_f \overline{\psi}_f D_\mu\gamma_\mu\psi_f + m\,\overline{\psi}_{\rm b}\psi_{\rm b}$$

$$\updownarrow$$

$$\mathcal{L}_{\rm HQET} = \overline{\psi}_{\rm h}\Big[\underbrace{D_0 + m}_{\text{static limit}} - \frac{\omega_{\rm kin}}{2m}\,\mathbf{D}^2 - \frac{\omega_{\rm spin}}{2m}\,\sigma\mathbf{B}\Big]\psi_{\rm h} + \ldots\,,$$

m : heavy quark mass

- systematic expansion in $1/m$, accurate for $m \gg \Lambda_{\rm QCD}$, renormalizable & has a continuum limit
- matching $\{m, \omega_{\rm spin}, \cdots\} \Leftrightarrow \{\text{QCD parameters}\}$ required to make HQET an effective theory of QCD
- consider HQET as expansion of QCD in $1/z \equiv 1/(LM)$ and verify that its large-z behaviour complies with HQET

$$\lim_{\mu\to\infty}\left\{\left[2b_0\bar{g}^2(\mu)\right]^{-d_0/(2b_0)}\overline{m}(\mu)\right\} = M$$

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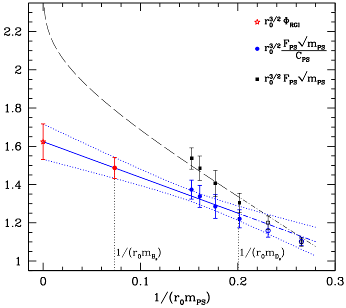
$$\mathcal{L}_{\rm HQET} = \overline{\psi}_{\rm h} \Big[\underbrace{D_0 + m}_{\text{static limit}} - \frac{\omega_{\rm kin}}{2m}\, \mathbf{D}^2 - \frac{\omega_{\rm spin}}{2m}\, \sigma \mathbf{B} \Big] \psi_{\rm h} + \ldots \,,$$

- tests may justify interpolations between the charm region (slightly above of it) and the static limit to the b-scale also in large-volume physics applications, e.g. to determine $F_{\rm B}$ [Alpha:JHEP02(2008)078]:



- comparison to tests of quenched QCD [Heitger etal:JHEP11(2004)048]

Requirements

Finetuning

- **line of constant physics**; within our strategy to do a NP matching between QCD and HQET, we are working at

$$\bar{g}^2(L_1) \approx 4.484 \qquad L_1 m_{\rm l} \approx 0 \qquad z \equiv L_1 M \approx \textbf{const}$$

  M : renormalization group invariant heavy quark mass

- *mapping between bare & renormalized parameters of the theory*
  ✓improvement coefficients and renormalization constants non-perturbatively [Della Morte etal:PoS(LATTICE 2007)246]

- **computations in finite (small) volume at $\mathbf{L_1/a \in \{20, 24, 32, 40\}}$**

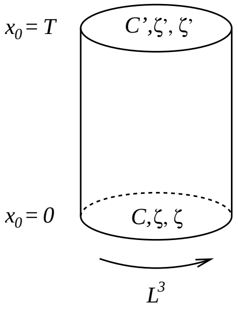
- we choose $\mathbf{z \equiv L_1 M \in \{4, 6, 7, 9, 11, 13, 15, 18, 21\}}$ to cover a wide range of masses $\leftrightarrow$ $M \sim (1.5, \ldots, 8.3)\mathrm{GeV}$
  (reference scale $L^* \approx 0.6\mathrm{fm}$ [Alpha:JHEP07(2008)037] $\rightsquigarrow$ $L_1 \approx 0.48\mathrm{fm}$)

Framework

The Schrödinger functional as finite renorm. scheme

- *periodic* b.c. in space and Dirichlet in time
- mass independent renormalization scheme
  $D_\mu\gamma_\mu$ has a gap $\rightsquigarrow$ in the massless limit ...
  - ... no infrared divergences
  - ... lattice simulations are possible
- fermion fields periodic in space up to a phase

$$\psi(x+\hat{k}L) = \mathrm{e}^{i\theta}\psi(x)$$
$$\overline{\psi}(x+\hat{k}L) = \mathrm{e}^{-i\theta}\overline{\psi}(x)\ , \qquad \theta \in \{0, 0.5, 1\}$$



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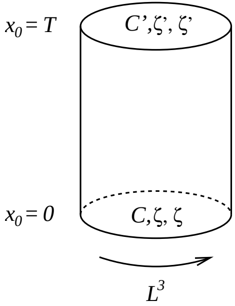

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- multiplicative renormalization scheme where the kinematical parameters L, $T/L, \theta$ fixes the renormalization prescription
- $N_\mathrm{f} = 2$ degenerate massless sea quarks ($m_\mathrm{l} \equiv m_\mathrm{light} = 0, \theta = 0.5$)
- correlation functions are build from heavy-light valence quarks; light quark mass is set to the sea-quark mass

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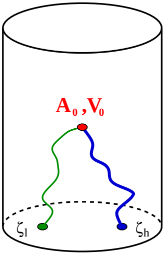
Finite volume observables

SF correlation functions ...

Boundary-to-bulk:

$$f_{\rm A}(x_0,\theta) = -\frac{a^6}{2L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z}} \left\langle \overline{\psi}_{\rm l}(x)\gamma_0\gamma_5\psi_{\rm h}(x)\,\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z})\right\rangle$$

$$k_{\rm V}(x_0,\theta) = -\frac{a^6}{6L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z},k} \left\langle \overline{\psi}_{\rm l}(x)\gamma_k\psi_{\rm h}(x)\,\zeta_{\rm h}(\mathbf{y})\gamma_k\zeta_{\rm l}(\mathbf{z})\right\rangle$$



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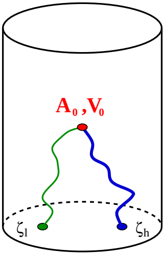

Boundary-to-boundary:

$$f_1(\theta) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \left\langle \overline{\zeta}_{\rm l}{}'(\mathbf{u})\gamma_5\zeta_{\rm h}{}'(\mathbf{v})\,\zeta_{\rm h}(\mathbf{y})\gamma_5\zeta_{\rm l}(\mathbf{z})\right\rangle$$

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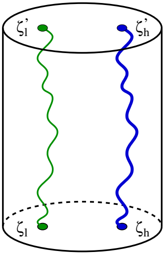

and additionally $f_{\rm P}$, $k_{\rm T}$ to improve $f_{\rm A}$, $k_{\rm V}$ respectively

Finite volume observables

... and derived quantities . . .

- provided that A_μ, V_μ denote *renormalized* currents,

$$Y_{\rm PS}(L,M) \equiv +\frac{f_{\rm A}(T/2)}{\sqrt{f_1}}\,, \qquad Y_{\rm V}(L,M) \equiv -\frac{k_{\rm V}(T/2)}{\sqrt{k_1}}\,,$$

$$R_{\rm A/V}(L,M) \equiv -\frac{f_{\rm A}(T/2)}{k_{\rm V}(T/2)}\,, \qquad R_{\rm A/P}(L,M) \equiv -\frac{f_{\rm A}(T/2)}{f_{\rm P}(T/2)}\,,$$

$$R_{\rm spin}(L,M) \equiv \frac{1}{4}\ln\frac{f_1}{k_1}\,,$$

are finite quantities; our test observables

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- *interpretation of SF CFs in terms of matrix elements possible; e.g.*

$$Y_{\rm PS}(L,M) \equiv \frac{\langle\Omega(L)|\mathcal{A}_0|B(L)\rangle}{\||\Omega(L)\rangle\| \cdot \||B(L)\rangle\|}\,, \quad \begin{cases} |B(L)\rangle = e^{-T\mathcal{H}/2}|\phi_{\rm B}(L)\rangle \\ |\Omega(L)\rangle = e^{-T\mathcal{H}/2}|\phi_0(L)\rangle \end{cases}$$

Finite volume observables

... and derived quantities

- for the same purpose **effective energies** are defined by

$$\Gamma_{\rm PS}(L,M) \equiv -\frac{\rm d}{{\rm d}x_0} \ln\left[\, f_{\rm A}(x_0)\,\right]\Big|_{x_0=T/2} = -\frac{f'_{\rm A}(T/2)}{f_{\rm A}(T/2)}\,,$$

$$\Gamma_{\rm V}(L,M) \equiv -\frac{\rm d}{{\rm d}x_0} \ln\left[\, k_{\rm V}(x_0)\,\right]\Big|_{x_0=T/2} = -\frac{k'_{\rm V}(T/2)}{k_{\rm V}(T/2)}\,,$$

$$\Gamma_{\rm av}(L,M) \equiv \tfrac{1}{4}\left[\Gamma_{\rm PS}(L,M) + 3\Gamma_{\rm V}(L,M)\right]$$

- meaning of the observables from their large-volume behaviour (up to normalizations)

$$L \to \infty \;:\quad Y_{\rm PS}, Y_{\rm V} \to F_{\rm PS}, F_{\rm V} \qquad : \text{heavy-light decay constant,}$$

$$R_{\rm spin} \to m_{{\rm B}_0^*} - m_{{\rm B}_0} \qquad : \text{mass splitting}$$

Effective theory predictions

at the classical level:

- current matrix elements expected to posses a power series expansion in $1/z \equiv 1/(LM)$
- leading term in expansion of CFs by replacing $\psi_{\rm b} \to \psi_{\rm h}$ & dropping O(1/m) terms $\rightsquigarrow$ static limit

$$f_{\rm A} \to f_{\rm A}^{\rm stat} \qquad \frac{f_{\rm A}^{\rm stat}(T/2)}{\sqrt{f_1^{\rm stat}}} \equiv X(L) = \lim_{z\to\infty} Y_{\rm PS}(L,M)$$

$$= \lim_{z\to\infty} Y_{\rm V}(L,M)$$

due to *heavy quark spin-symmetry* ($A_0^{\rm stat} \Leftrightarrow V_0^{\rm stat}$)

Effective theory predictions

correspondence of HQET and QCD in quantum theory:

- scale dependent ren. of HQET implies logarithmic modifications

$$\text{axial current renorm.} \qquad X_R(L,\mu) = Z_{\mathrm{A}}^{\mathrm{stat}}(\mu) X_{\mathrm{bare}}(L)$$

  depends logarithmically on the chosen renorm. scale μ

- no scheme dependence when going over to renormalization group invariants (RGI)

$$\lim_{\mu\to\infty} \left\{ \left[2b_0\bar{g}^2(\mu)\right]^{-\gamma_0/(2b_0)} X_R(L,\mu) \right\} = X_{\mathrm{RGI}} = Z_{\mathrm{RGI}} X_{\mathrm{bare}}(L)$$

$$\text{where} \qquad b_0 = \frac{11-2N_{\mathrm{f}}/3}{(4\pi)^2}\,, \qquad \gamma_0 = -\frac{1}{(4\pi)^2}\,,$$

  are first order coeff.s of β and of the anomalous dimension of the axial current, respectively

- large-mass behaviour of the QCD observables:
  (RGIs of the eff. theory)×(logarithmically mass dependent functions C)

Conversion to the matching scheme

translation to another renormalization scheme

Definition of the matching scheme: for arbitrary renormalized matrix elements Φ_{R} in QCD & the effective theory it should hold

$$\Phi_{\mathrm{R}}^{\mathrm{QCD}} = \Phi_{\mathrm{R}}^{\mathrm{HQET}}(\mu)\Big|_{\mu=m} + O(1/m)$$

- in perturbative QCD, m typically can either be the pole mass m_Q or the $\overline{\mathrm{MS}}$ mass $\overline{m}_*$

Matching coefficients $\mathbf{C_X(\Lambda_{\overline{MS}}/M)}$

more convenient choice of the argument of the conversion functions $\widehat{C}_X$:

- change argument of $\widehat{C}_X$ to the ratio of RGIs, $M/\Lambda_{\overline{\rm MS}}$
  $\Rightarrow$ functions $C_X(M/\Lambda_{\overline{\rm MS}})$
- $M =$ RGI quark mass, advantage: fixed in lattice calculations without perturbative uncertainties

one then expects the (heavy) quark mass dependence to obey

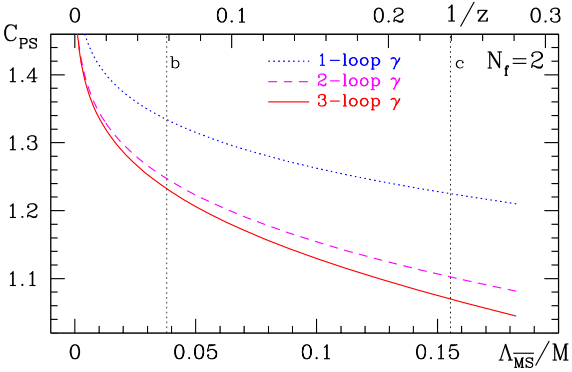
$$Y_X(L,M) \overset{M\to\infty}{\sim} C_{\rm X}\left(M/\Lambda_{\overline{\rm MS}}\right) X_{\rm RGI}(L)\left(1+{\rm O}(1/z)\right), \qquad \begin{array}{c} X = {\rm PS}, {\rm V}, \\ z = ML, \end{array}$$

$$R_{\rm spin}(L,M) \overset{M\to\infty}{\sim} C_{\rm spin}\left(M/\Lambda_{\overline{\rm MS}}\right) \frac{X_{\rm RGI}^{\rm spin}(L)}{z}\left(1+{\rm O}(1/z)\right),$$

$$L\Gamma_{\rm av}(L,M) \overset{M\to\infty}{\sim} C_{\rm mass}\left(M/\Lambda_{\overline{\rm MS}}\right) \times z \;+{\rm O}(1),$$

Matching coefficients $C_X(\Lambda_{\overline{MS}}/M)$

C_X : integrate perturbative RG equations (in the effective theory) in the matching scheme, using 4-loop $\beta(g)$, $\tau(g)$



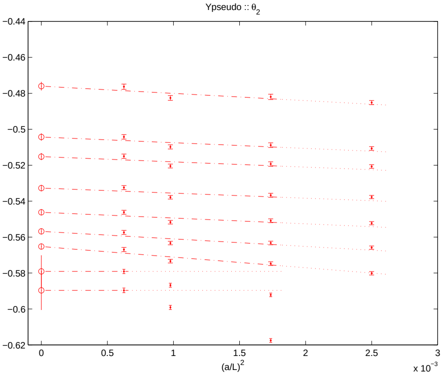
- 3-loop $\gamma_2^{\overline{MS}}$ anomalous dimension (AD) from [Chetyrkin&Grozin,2003]

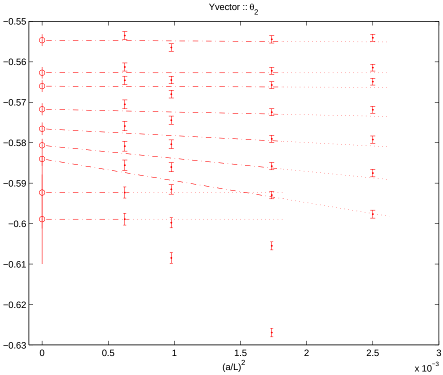
Results

Continuum extrapolations with asymptotics

"Decay constants"

Y_{PS} Y_{V}





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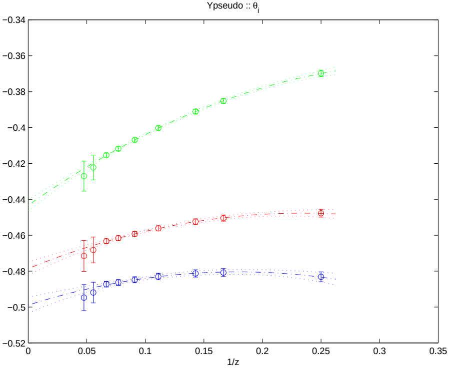
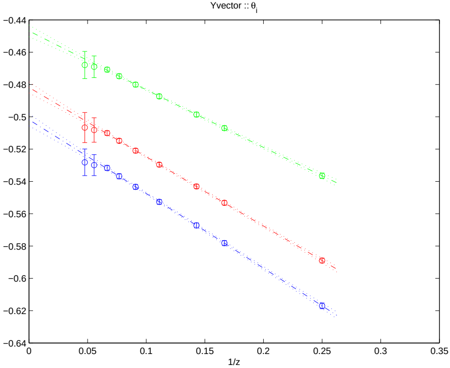
Results

Continuum extrapolations with asymptotics

"Decay constants"

$$Y_{\mathrm{PS}}\,/\,C_{\mathrm{PS}} \propto X_{\mathrm{RGI}}\big(1+\mathrm{O}(1/z)\big) \qquad Y_{\mathrm{V}}\,/\,C_{\mathrm{V}} \propto X_{\mathrm{RGI}}\big(1+\mathrm{O}(1/z)\big)$$





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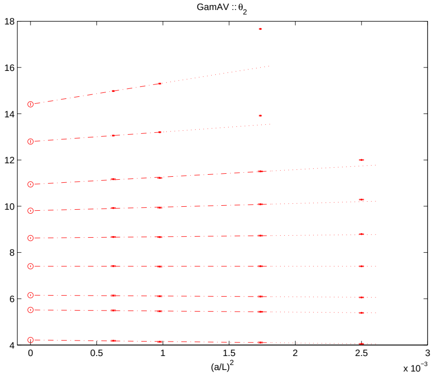
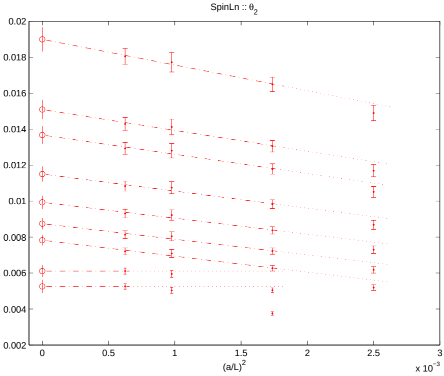
Continuum extrapolations with asymptotics

"Spin averaged mass & Spin splitting"

$\Gamma_{\rm av}$ $R_{\rm spin}$





Results

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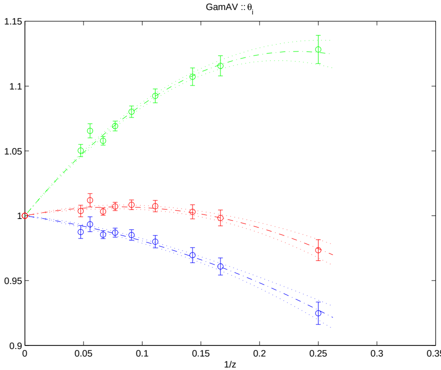
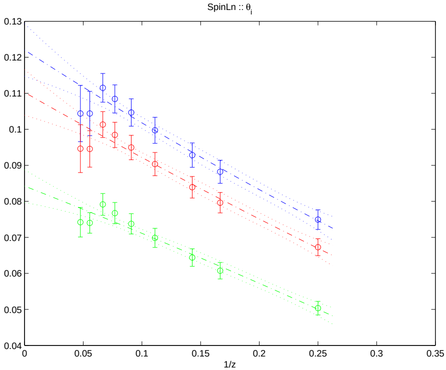
"Spin averaged mass & Spin splitting"

$$(L\Gamma_{\mathrm{av}})/(zC_{\mathrm{mass}}) \propto 1+\mathrm{O}(1/z) \qquad zR_{\mathrm{spin}}/C_{\mathrm{spin}} \propto X^{\mathrm{spin}}_{\mathrm{RGI}}\left(1+\mathrm{O}(1/z)\right)$$





Conclusions & perspectives

conclusions that can be drawn (maybe):

- nearly linear (1/z)-behaviour down to $1/z = 0.1 \leftrightarrow M \sim 4\mathrm{GeV}$ for all observables investigated so far
- small $(1/z)^2$ corrections in spin splitting over the whole range of z covered
- correlated fits for a reliable error estimate done
  (all z's at constant L computed on the same gauge background)
- overall behaviour similar to quenched $\rightsquigarrow$ NP matching of QCD and HQET should also be as well behaved as in the quenched case

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what still need to be done:

- apply more reasonable fits to CL at largest z
- remove tree-level cutoff effect before extrapolating to CL
- connect data of *heavy-light decay constant* to the one computed in HQET [DellaMorte etal: JHEP 0702:079,2007]
- compute $X_{\mathrm{RGI}}^{\mathrm{spin}}$ in HQET

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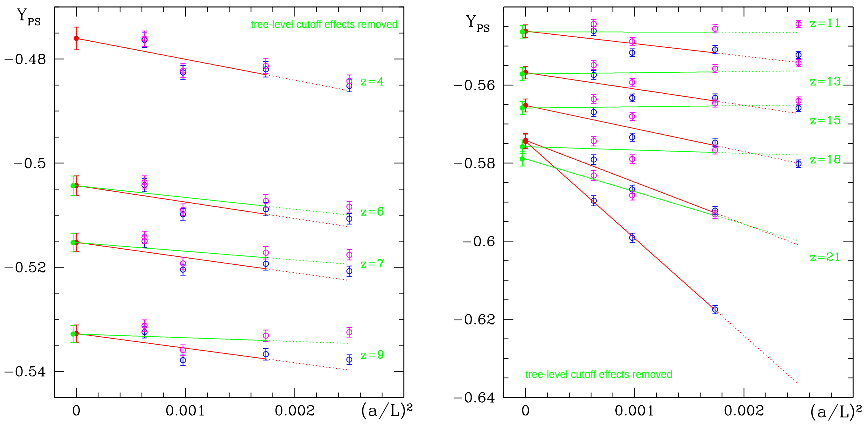
Tree-level cutoff effect

preliminary



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PCAC mass in the SF

at $L/a = 40$