Rare Kaon Decays

Euroflavour 08
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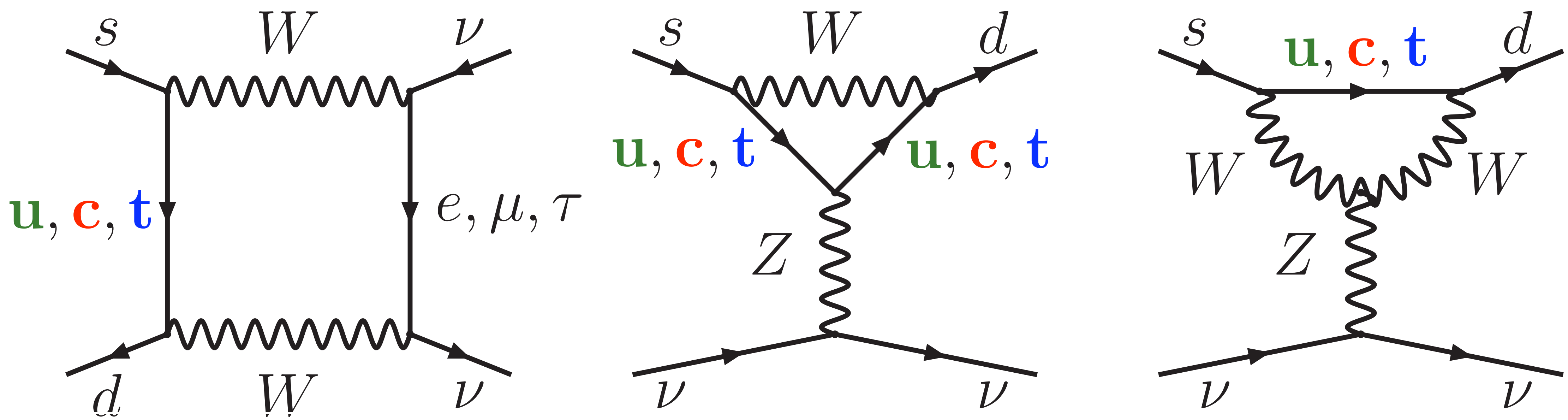
Introduction: $K \to \pi \nu \bar{\nu}$



- **Dominant Operator:** $Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda^2}{M_W^2}$

Use isospin symmetry and normalize to: $K^+ \to \pi^0 e^+ \nu$

$s \to d$ and New Physics (NP)

$b \to s:$ $|V_{tb}^* V_{ts}| \propto \lambda^2$

$b \to d:$ $|V_{tb}^* V_{td}| \propto \lambda^3$

$s \to d:$ $|V_{ts}^* V_{td}| \propto \lambda^5$

Rare K Decays: Additional Cabbibo suppression λ^5

$$\mathcal{L}_{\text{eff}} = \frac{C(b \to s)}{\Lambda_{NP}^2}(\bar{b}\Gamma s)(\bar{\nu}\Gamma\nu) + \frac{C(b \to d)}{\Lambda_{NP}^2}(\bar{b}\Gamma d)(\bar{\nu}\Gamma\nu) + \frac{C(s \to d)}{\Lambda_{NP}^2}(\bar{s}\Gamma d)(\bar{\nu}\Gamma\nu)$$

Low NP scale $\Lambda_{\text{NP}} \simeq 1$ TeV

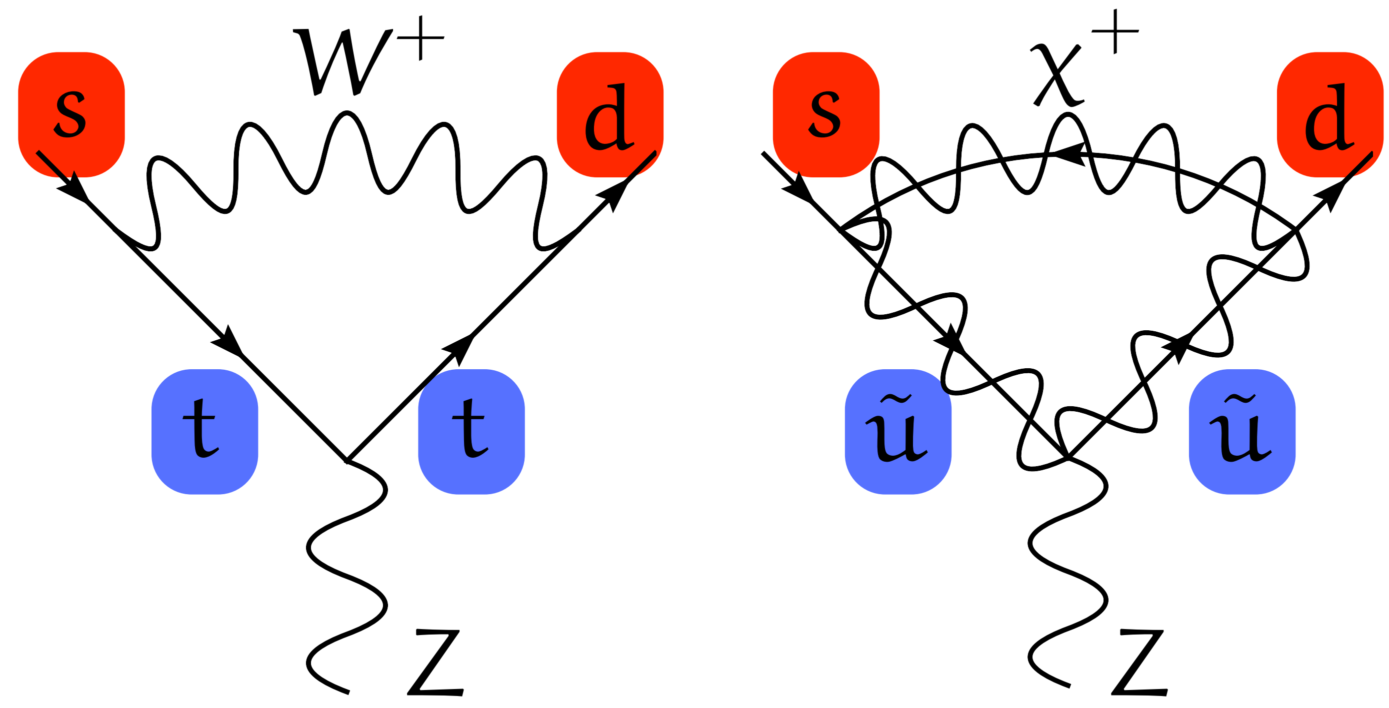
NP Flavour Sector $C(s \to d) < \lambda^5$

For Generic NP $C(s \to d) \simeq 1$

New Physics scale $\Lambda_{\text{NP}} > 75$ TeV

Rare K decays and New Physics:

- Test deviation of flavour alignment (Minimal Flavour Violation MFV)

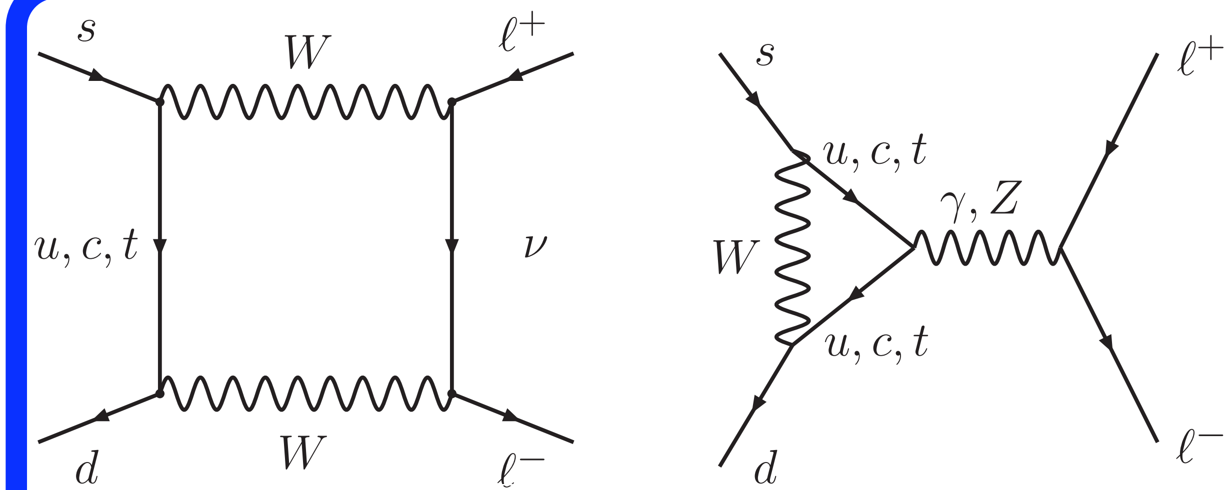


- Precise theory prediction

- Sensitive to small deviations from MFV

$$K_L \to \pi^0 \mu^+ \mu^- \qquad K_L \to \pi^0 \bar{\nu} \nu$$
$$K_L \to \pi^0 e^+ e^- \qquad K^+ \to \pi^+ \bar{\nu} \nu$$

$K_L \to \pi^0 l^+ l^-$: Three Contributions

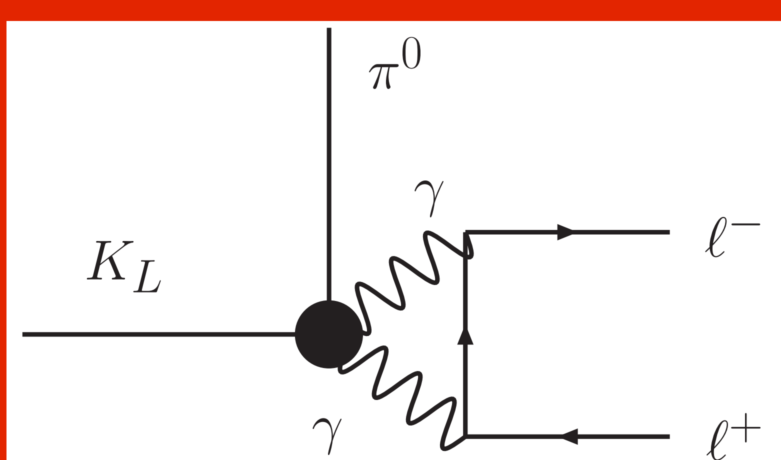


$$Q_{7V} = (\bar{s}_L \gamma_\mu d_L)(\bar{l}\gamma^\mu l) \to 1^{--}$$

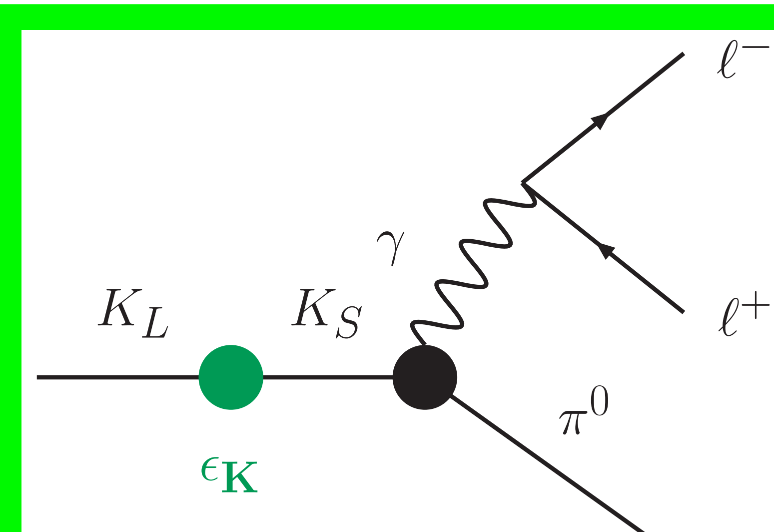
$$Q_{7A} = (\bar{s}_L \gamma_\mu d_L)(\bar{l}\gamma^\mu \gamma_5 l) \to 1^{++},\ 0^{-+}$$

Direct CP Violating

Wilson Coefficients: $y_{7V},\ y_{7A}$ at NLO [Buchalla et al. ´96]

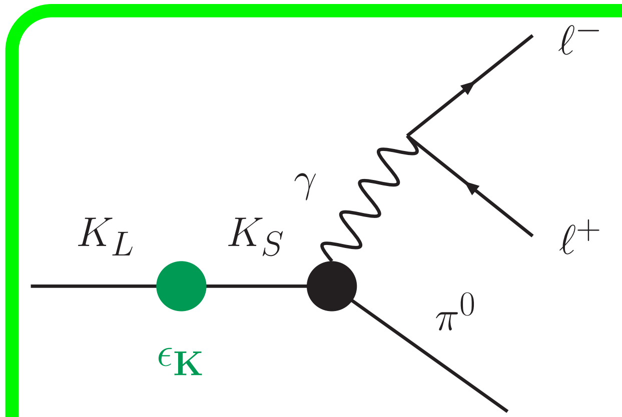


CP Conserving



Indirect CP Violating

$K_L \to \pi^0 l^+ l^-$: Three Contributions

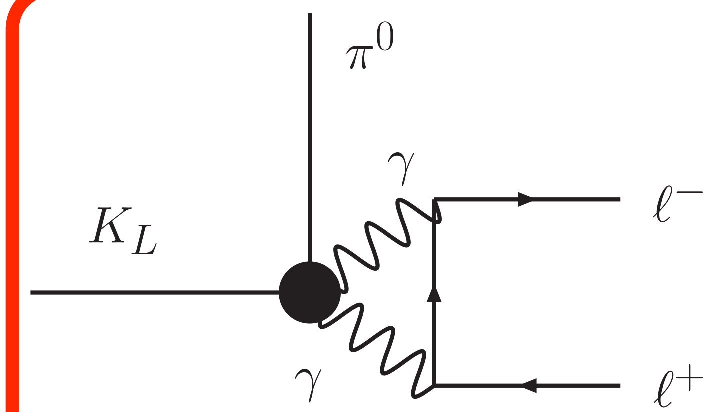


Counterterm $|a_S| = 1.2 \pm 0.2$ from $K_S \to \pi^0 l^+ l^-$

[D´Ambrosio et. al. ´98, Mescia et. al. ´06]

For 1^{--} interference with Q_{7V}

[Buchalla et. al. ´03, Friot et al. ´04]



Estimate from $K_L \to \pi^0\gamma\gamma$

[Isidori et. al. ´04]

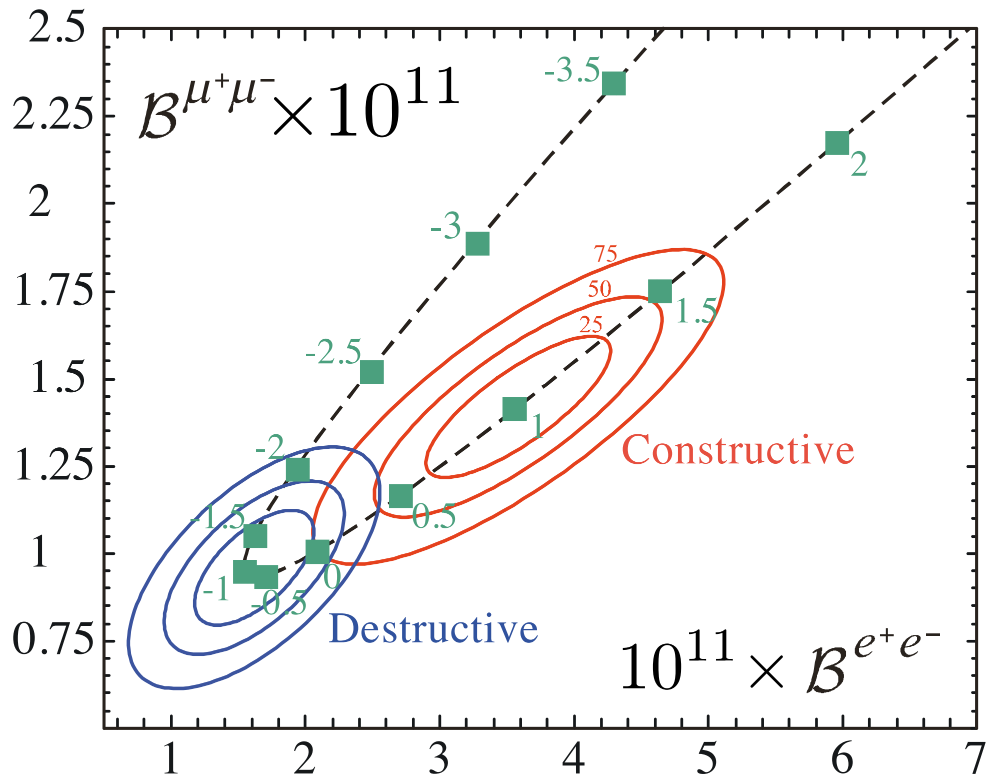
$$\mathcal{B}r(K_L \to \pi^0 \ell^+ \ell^-) = \left(C^\ell_{dir} \pm C^\ell_{int}|a_S| + C^\ell_{mix}|a_S|^2 + C^\ell_{\gamma\gamma}\right) \times 10^{-12}$$

ℓ	C^ℓ_{dir}	C^ℓ_{int}	C^ℓ_{mix}	$C^\ell_{\gamma\gamma}$
e	$(4.62 \pm 0.24)(y_V^2 + y_A^2)$	$(11.3 \pm 0.3)y_V$	14.5 ± 0.5	≈ 0
μ	$(1.09 \pm 0.05)(y_V^2 + 2.32 y_A^2)$	$(2.63 \pm 0.06)y_V$	3.36 ± 0.20	5.2 ± 1.6

$K_L \to \pi^0 l^+ l^-$: Improvements

- Measure both $\mathcal{B}r_{e^+e^-}$ and $\mathcal{B}r_{\mu^+\mu^-}$: [Mescia et. al. ´06]
  Disentangle short distance contribution $(y_{7V},\ y_{7A})$

- Dominant theory error in a_s:
  Forward backward asymmetry. [Mescia et. al. ´06]
  Better measurement of $K_S \to \pi^0 l^+ l^-$. [Smith ´07]

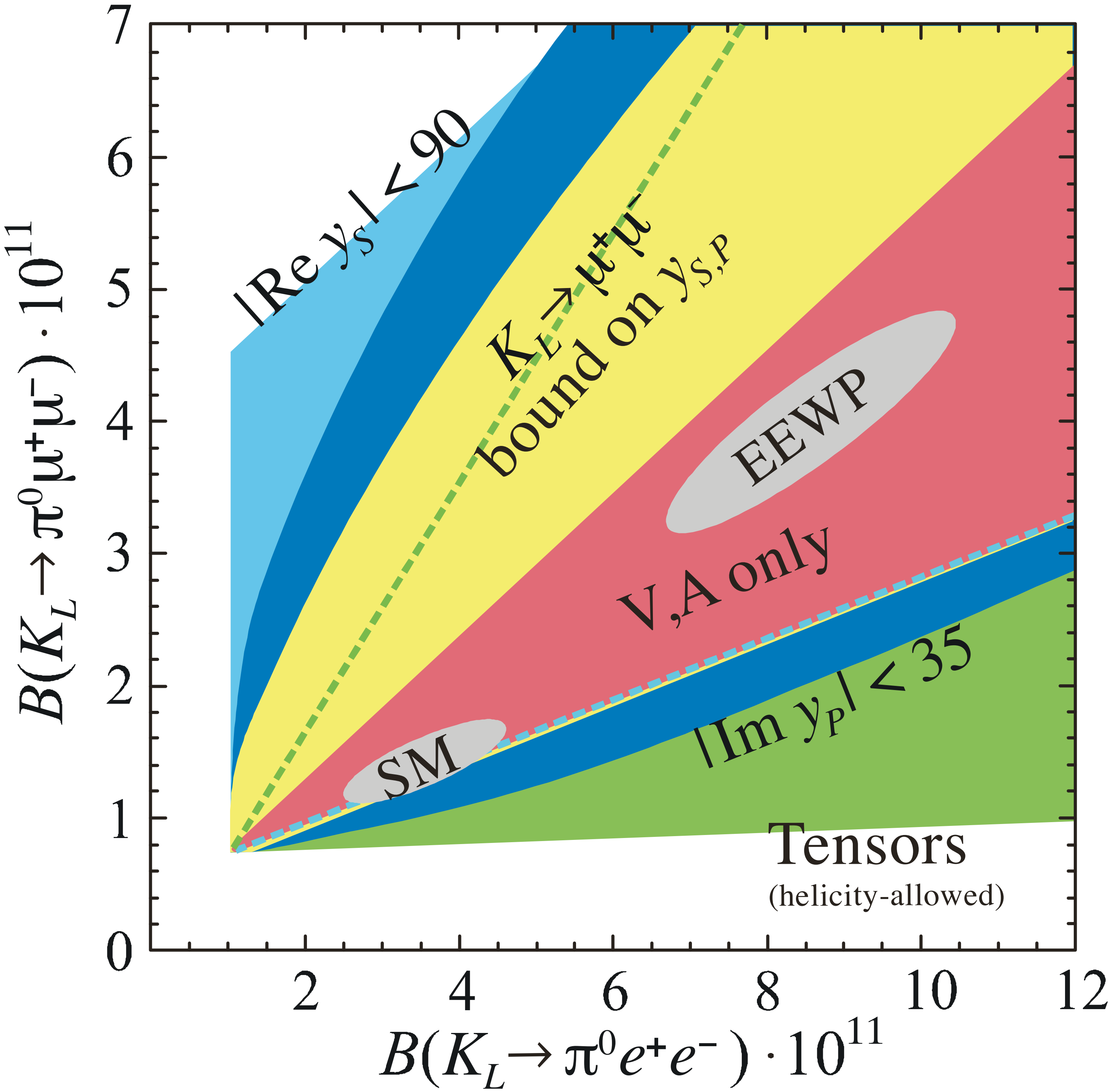


[Mescia et. al. ´06]

[KTEV ´04]	[KTEV ´00]
$Br_{e^+e^-}$	$Br_{\mu^+\mu^-}$
$< 28 \times 10^{-11}$	$< 38 \times 10^{-11}$

$K_L \rightarrow \pi^0 l^+ l^-$: New Physics

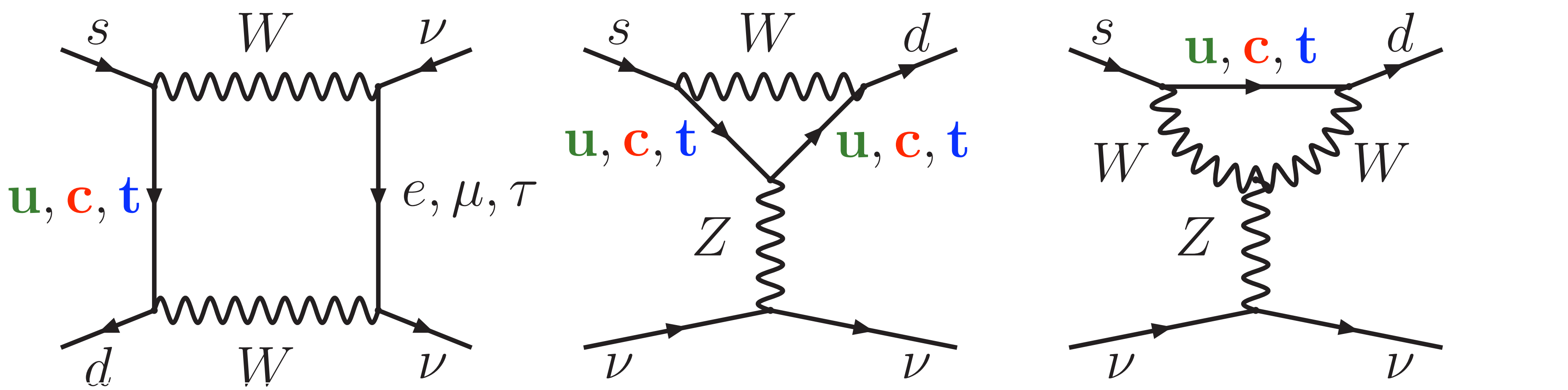
- EW penguin operators correlated with $K \rightarrow \pi \bar{\nu} \nu$, while QCD ones fixed by data



- $Q_s = (\bar{s}d)(\bar{l}l)$ and $Q_p = (\bar{s}d)(\bar{l}\gamma_5 l)$ generated in the MSSM with large tan beta.
- Effect only $K_L \rightarrow \pi^0 \mu^+ \mu^-$ correlated with $K_L \rightarrow \mu^+ \mu^-$
- Sensitve to tensor operators

[Mescia et. al. ´06]

$K_L \to \pi^0 \bar{\nu}\nu$ Effective Hamiltonian



CP violating: DCPV : ICPV : CPC = 1 : 10^{-2} : < 10^{-4}

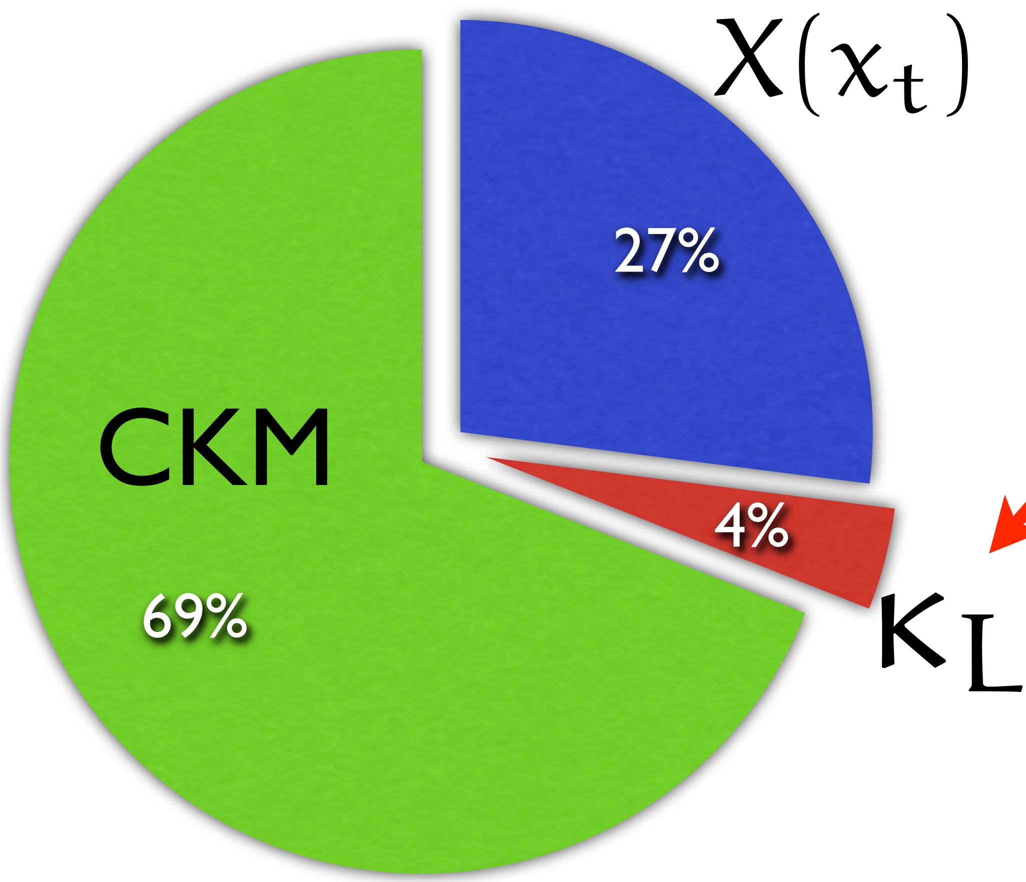
[Buchalla, Isidori ´96]

Only top quark contributes: $H_{\rm eff} = \frac{G_F}{\sqrt{2}} \frac{8\, V^*_{ts} V_{td}}{2\pi \sin^2\Theta_W} X(x_t) Q_\nu$

Use isospin symmetry and normalize to: $K^+ \to \pi^0 e^+ \nu$

$$\mathcal{B}r(K_L \to \pi^0 \bar{\nu}\nu) = \kappa_L \left(\frac{\mathrm{Im}(V^*_{ts} V_{td})}{\lambda^5} X(x_t) \right)^2$$

$K_L \to \pi^0 \bar{\nu} \nu$ Theoretical Status



Matrix element extracted from K_{l3} decays. $N^{\frac{3}{2}}LO$ χPT
[Mescia, Smith ´07; Bijnens, Ghorbani ´07]

No further long distance uncertainty

$X(x_t)$: NLO QCD calculation: ±1% error
[Misiak, Urban ´99; Buchalla, Buras ´99]

$X(x_t)$: Electroweak (EW) corrections: ±2% error
[Buchalla, Buras ´99]

Reduce error with 2 loop electroweak calculation

$K^+ \to \pi^+ \bar{\nu}\nu$ and $K_L \to \pi^0 \bar{\nu}\nu$

Different from $K_L \to \pi^0 \bar{\nu}\nu$

- CP conserving: Charm & top contribute

$$\mathcal{B}r\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) = \kappa_+(1+\Delta_{EM}) \times \left|\frac{V_{ts}^* V_{td} X_t(m_t^2) + \lambda^4 \mathrm{Re} V_{cs}^* V_{cd}\left(P_c(m_c^2) + \delta P_{c,u}\right)}{\lambda^5}\right|^2 .$$

$\frac{m_c^2}{M_W^2}$ suppression lifted by $\log(\frac{m_c}{M_W})\frac{1}{\lambda^4}$

Like in $K_L \to \pi^0 \bar{\nu}\nu$

- Only Q_ν: Quadratic GIM & Isospin symmetry
- Top quark contribution like in $K_L \to \pi^0 \bar{\nu}\nu$

$K^+ \to \pi^+ \bar{\nu}\nu$ Long distance

- Matrix element extracted from K_{l3} decays
  [Mescia, Smith ´07]

- $K^+ \to \pi^+ \nu\bar{\nu}$ is $K^+ \to \pi^+ \nu\bar{\nu}(\gamma)$
  QED radiative corrections included:

$$\Delta_{EM}(E_\gamma < 20\text{MeV}) = -0.003$$

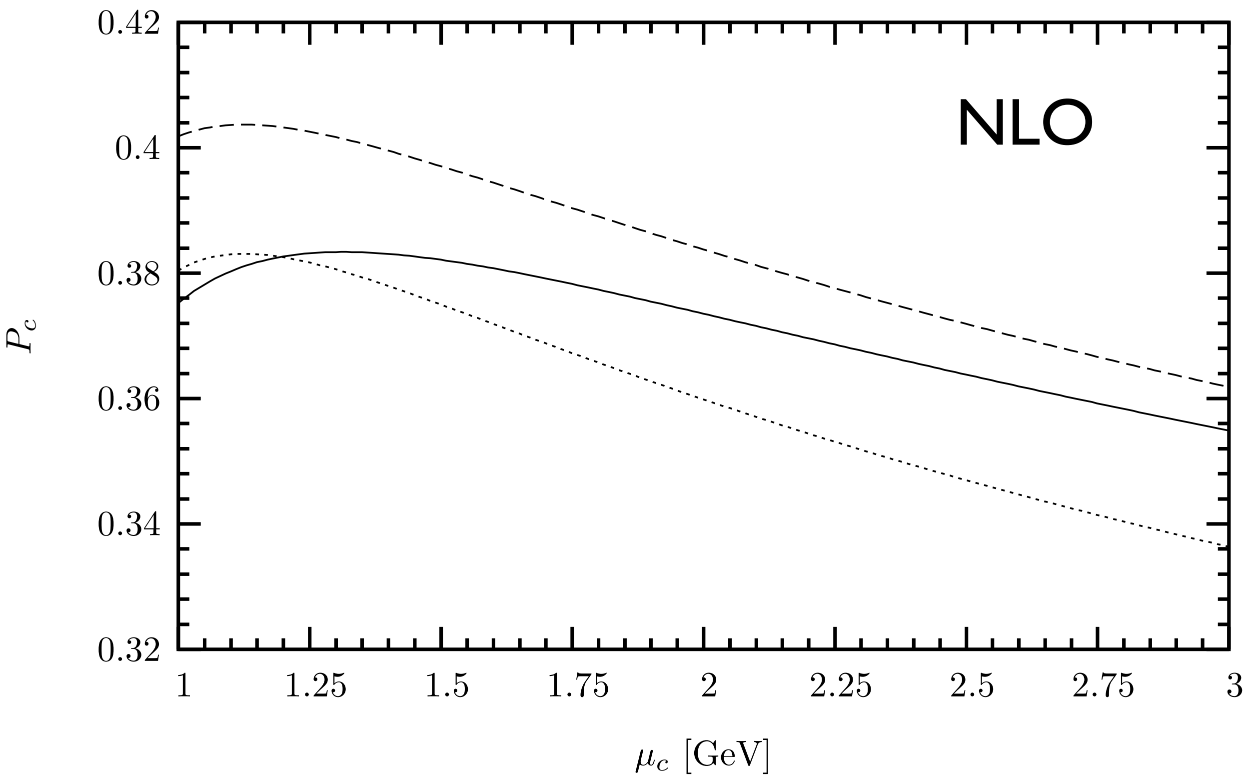
- Uncertainty in $\kappa_+(1 - \Delta_{EM})$ reduced by $\frac{1}{7}$

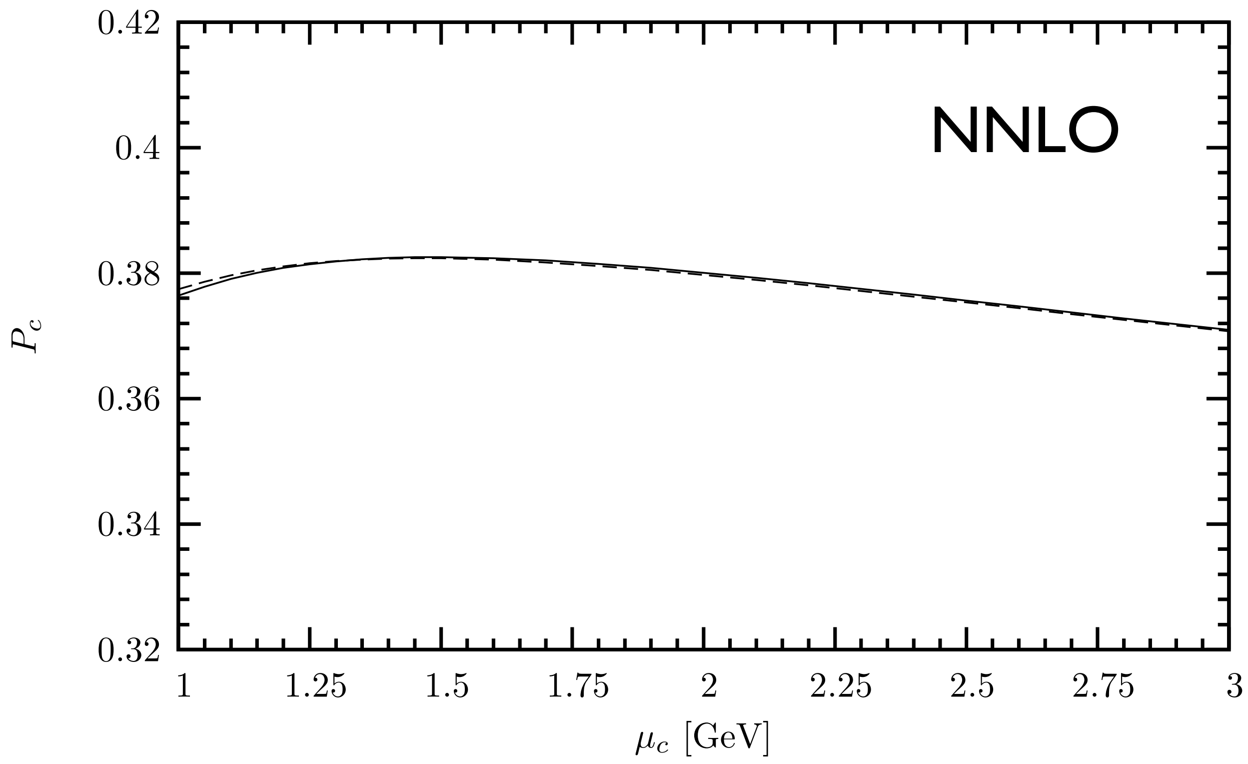
- Below charm scale: Dimension 8 operators
  [Falk et. al. ´01]

- Together with light quarks: $\delta P_{c,u} = 0.04 \pm 0.02$
  [Isidori et. al. ´05]

- Could be Improved by Lattice [Isidori et. al. ´05]

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (QCD)





- Resum $\log \frac{m_c}{M_W}$ in P_c

P_c at NLO: ±10% (theory)
[Buras et. al. ´05]

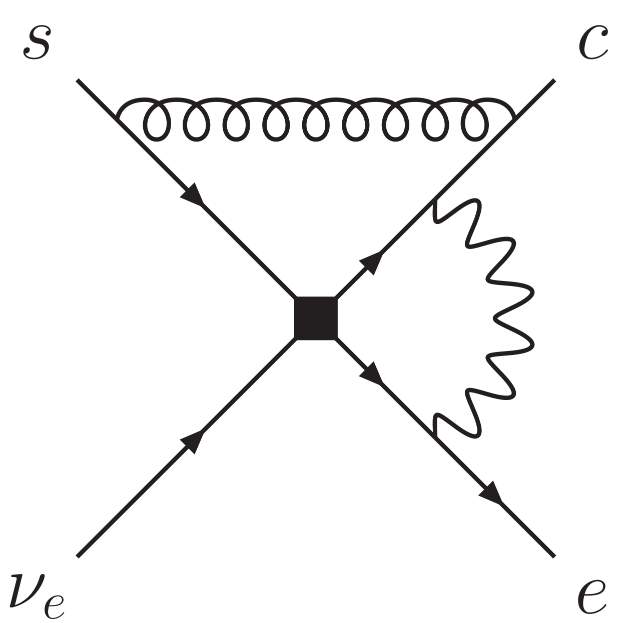
P_c at NNLO: ±2.5% (theory)
[Buras et. al. ´05]

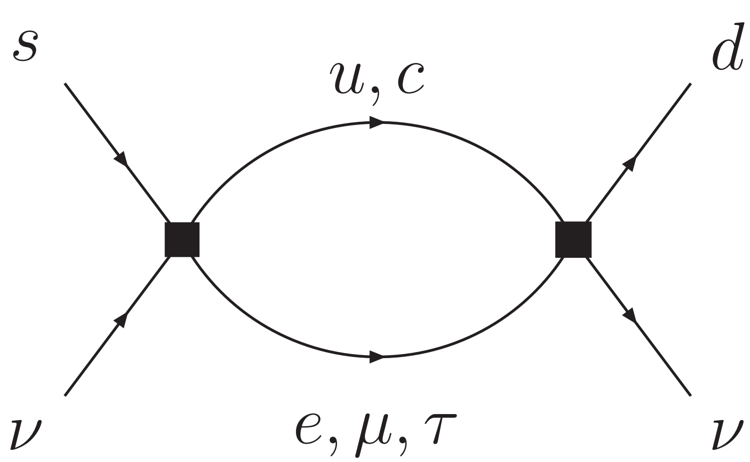
- Dominant parametric uncertainty in P_c:

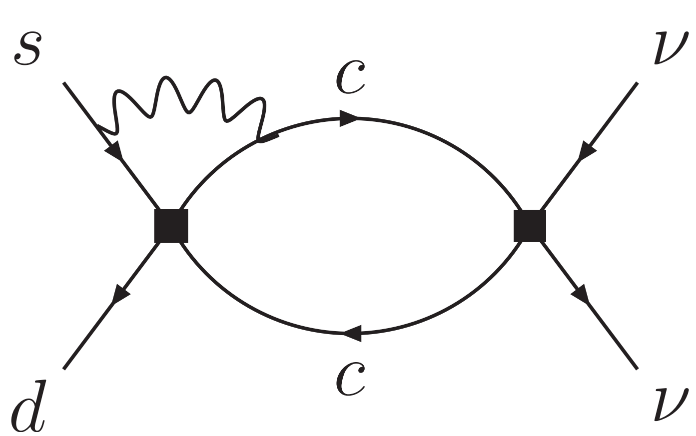
$m_c = (1.3 \pm 0.05)\text{GeV}$ **75%**

$\alpha_s = 0.1187 \pm 0.002$ **25%**

$K^+ \to \pi^+ \bar{\nu} \nu$ charm contr. (EW)

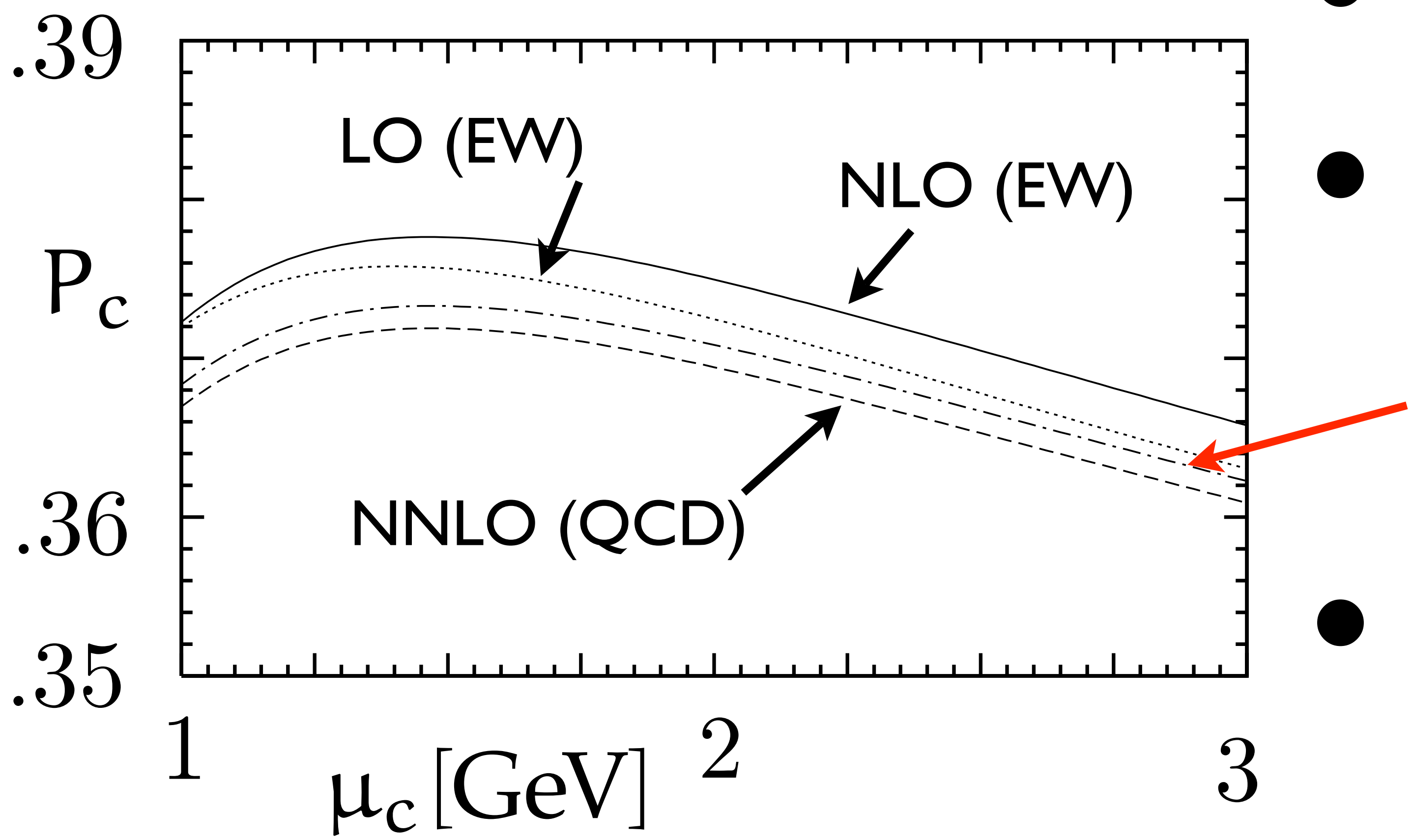






- Large QED logs? Does Q_ν run?
- Semileptonic operator has QED running and mixes into Q_ν.
- No $\mathcal{O}(\alpha/\alpha_s)$ but $\mathcal{O}(\alpha)$ corrections: NLO QEDxQCD calculation
- Bilocal mixing is $\mathcal{O}(G_F^2)$
- What is the parameter $x_c = \frac{m_c^2}{M_W^2}$
- EW corrections define M_W

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (EW)



[Brod, Gorbahn ´08]

- Use $\overline{MS}$ scheme
- Normalize to G_F
- use

$$x_c = \sqrt{2}\frac{\sin^2\theta_W}{\pi\alpha} G_F m_c^2(\mu_c)$$

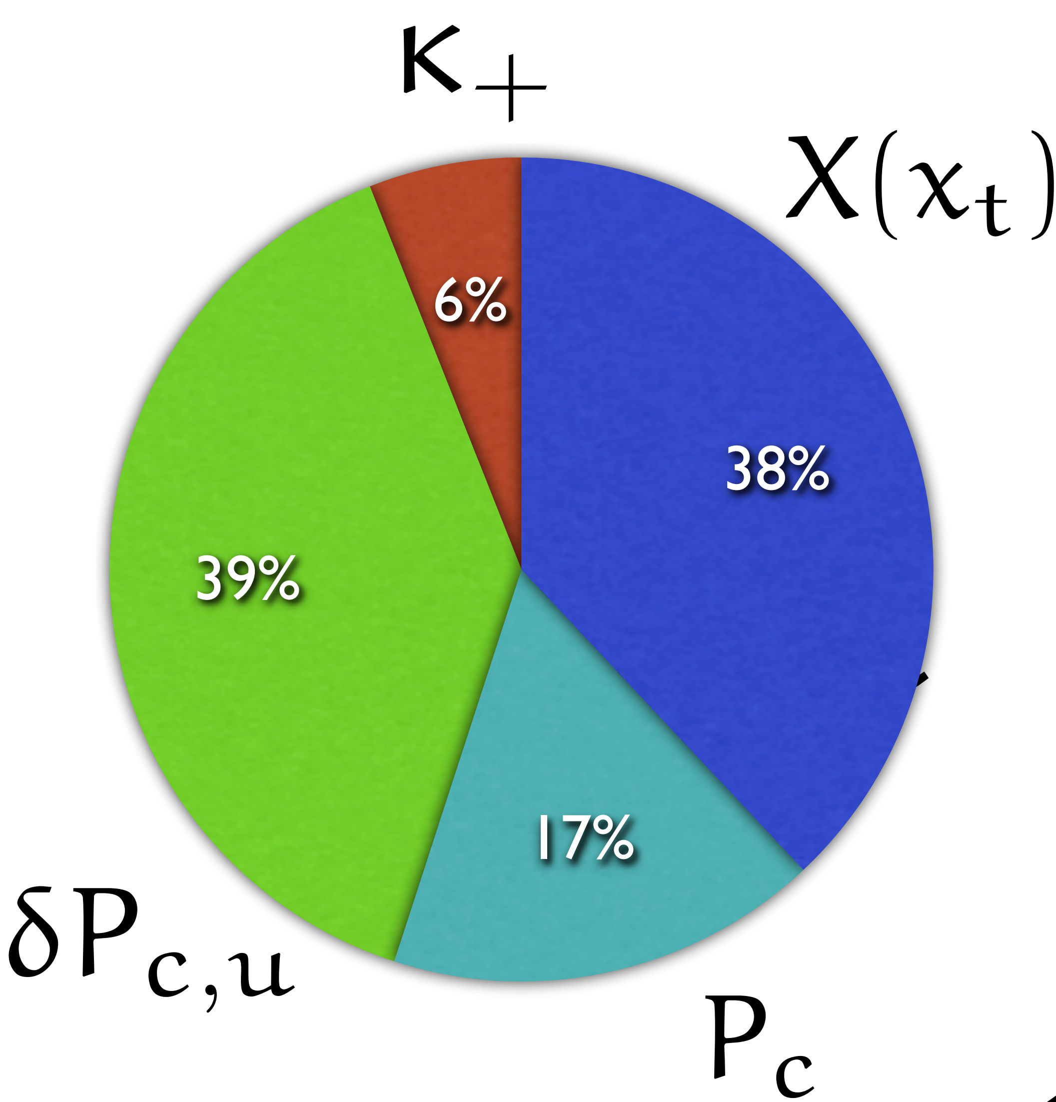
- instead of

$$x_c = \frac{m_c(\mu)^2}{M_W^2}$$

- P_c enhanced by up to 2% for all EW

$K^+ \to \pi^+ \bar{\nu} \nu$ Error budget

Theory error budget



for $m_c(m_c) = (1286 \pm 13)\text{MeV}$

[Kühn et. al. ´07]

$$\mathcal{B}r_{K^+} = (0.85 \pm 0.07) \times 10^{-10}$$

Theory error 30%

for $m_c(m_c) = (1224 \pm 57)\text{MeV}$

[Hoang et. al. ´05]

$$\mathcal{B}r_{K^+} = (0.80 \pm 0.08) \times 10^{-10}$$

Experiment [E787, E949 ´08]

$$\mathcal{B}r_{K^+} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

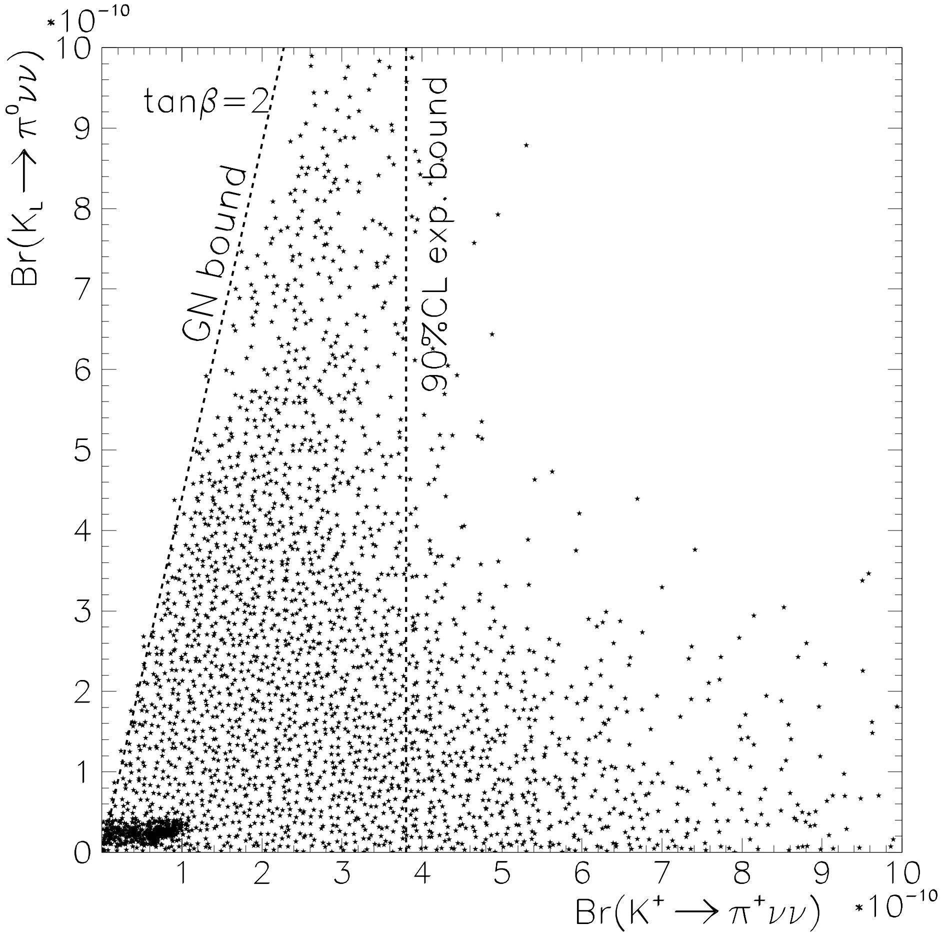
NA62 10% error

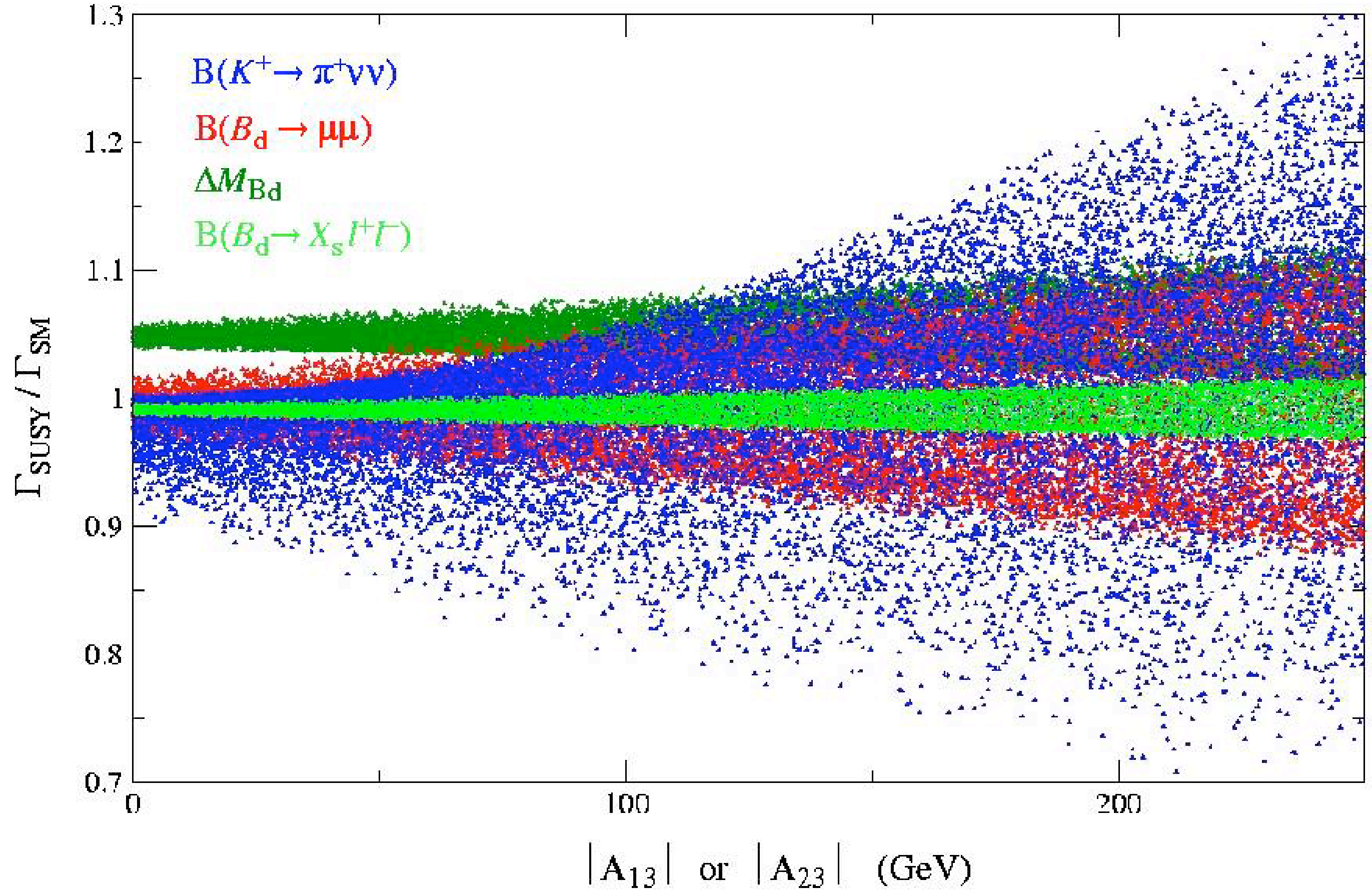
[Talk by Ceccucci]

$K \to \pi \bar{\nu} \nu$ and non MFV

$K \to \pi \bar{\nu} \nu$: strong sensitivity on A_u

[Isidori et. al. ´05]





MSSM 66 parameter scan: Grossmann-Nir bound can be saturated [Buras´04]

Large effects possible for $K_L \to \pi^0 \bar{\nu} \nu$

Conclusions

$K \to \pi \bar{\nu} \nu$ and $K_L \to \pi^0 l^+ l^-$ provide a unique test of the SM and its extensions

$K \to \pi \bar{\nu} \nu$ the cleannest + future improvements

$K_L \to \pi^0 l^+ l^-$ different sensitivity to New Physics
Theory prediction could be improved by exp.

	Theory	Experiment
$K_L \to \pi^0 e^+ e^-$	$(3.54\,^{+0.98}_{-0.85}) \times 10^{-11}$	$< 28 \times 10^{-11}$ KTEV
$K_L \to \pi^0 \mu^+ \mu^-$	$(1.41\,^{+0.28}_{-0.26}) \times 10^{-11}$	$< 38 \times 10^{-11}$ KTEV
$K_L \to \pi^0 \nu \bar{\nu}$	$(2.76 \pm 0.40) \times 10^{-11}$	$< 6.7 \times 10^{-8}$ E391a
$K^+ \to \pi^+ \nu \bar{\nu}$	$(8.51 \pm 0.70) \times 10^{-11}$	$(1.73\,^{+1.15}_{-1.05}) \times 10^{-10}$ E787 E949