Robustness of the dispersive representation of K_{l3} form factors and analysis of KTeV data

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- Papers: 2 papers in preparation

In the memory of Jan Stern

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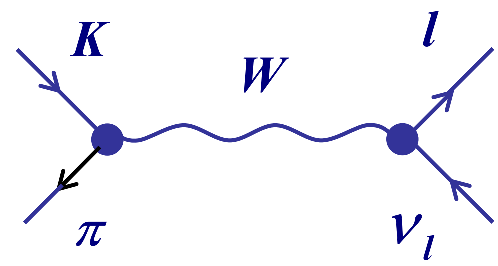
Outline

1. Introduction and motivation
2. Dispersive representation of the scalar and vector form factors
3. Results of the dispersive analysis with KTeV data
4. Conclusions and outlook

1. Introduction and motivation

1.1 Definition



- K_{l3} decays $K \to \pi l \nu_l$
- The hadronic matrix element :

$$\langle \pi(p_\pi) | \bar{s}\gamma_\mu u | K(p_K) \rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

→ $f_+(t)$, $f_-(t)$: form factors

→ $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

- Analysis in terms of $f_+(t)$ and $f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$

→ Normalization $\bar{f}_+(t) = \frac{f_+(t)}{f_+(0)}$ and $\bar{f}_0(t) = \frac{f_s(t)}{f_+(0)}$, $\bar{f}_+(0) = \bar{f}_0(0) = 1$

- $\bar{f}_+(t)$ accessible in K_{e3} and $K_{\mu 3}$ decays
- $\bar{f}_0(t)$ only accessible in $K_{\mu 3}$ (suppressed by m_l^2/M_K^2) + correlations

→ difficult to measure

1.2 Extraction of Vus

- Decay rate formula for K_{l3} $\qquad l = (e, \mu)$

$$\Gamma_{K^{+/0}l3} = \frac{Br_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta^{EM}_{K^{+/0}l}\right) \left| f_+^{K^{+/0}}(0) \mathcal{V}_{us} \right|^2 I^l_{K^{+/0}}$$

Kaon life time

½ for K^+, 1 for K^0

$$I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}} \lambda^{3/2} F\left(t, \overline{f}_+(t), \overline{f}_0(t)\right)$$

- Experimental inputs: → $Br_{K^{+/0}l3}, \tau_{K^{+/0}}$ Kl3 branching ratios, Kaon life time, with good treatment of radiative corrections

  → $I^l_{K^{+/0}}$ Phase space integrals, need form factor shapes extracted from Dalitz plot, from **NA48, KTeV, KLOE and ISTRA+.**

- Theoretical inputs: → S_{EW} Short distance EW corrections **[Marciano&Sirlin]**

  → $\Delta^{EM}_{K^{+/0}l}$ Long distance EM corrections (use of ChPT)

[Bijnens&Prades'97],[Knecht et al'00], [Cirigliano et al '02'04] [Andre'04], [Gatti'05],[Moussallam et al'06], [Cirigliano, Giannotti, Neufeld '08]

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1.2 Extraction of Vus

- Decay rate formula for K_{l3} $l = (e, \mu)$

$$\Gamma_{K^{+/0} l3} = \frac{Br_{K^{+/0} l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta^{EM}_{K^{+/0} l}\right) \left| f_+^{K^{+/0}}(0) \mathcal{V}_{us} \right|^2 I^l_{K^{+/0}}$$

Kaon life time

½ for K⁺, 1 for K⁰

$$I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}} \lambda^{3/2} F\left(t, \bar{f}_+(t), \bar{f}_0(t)\right)$$

- Extraction of $f_+(0) |\mathcal{V}_{us}|$ $\xrightarrow{f_+(0)}$ $|\mathcal{V}_{us}|$: test of the CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \overset{?}{=} 1$$

$0^+ \to 0^+$ β decays — K_{l3} decays — Negligible (B decays)

- Extraction of $\Delta_{SU(2)}$: $\frac{f_+^{K^+}(0)}{f_+^{K^0}(0)} = 1 + \Delta_{SU(2)}$, access to the ratio of light quark mass

⇒ $R = \frac{m_s - \hat{m}}{m_d - m_u}$ Important role in ChPT, Prediction : $\Delta_{SU(2)} = 2.36(22)\%$, $R = 40.8(3.2)$

[Leutwyler'96], [Cirigliano et al'01]

Theoretical knowledge for the scalar FF: CT relation

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = \overline{f_0}(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

[Gasser & Leutwyler]

$$\Delta_{CT}^{NLO} = (-3.5 \pm 8).10^{-3}$$

in agreement with **[Bijnens&Gorbani'07]**
[Kastner&Neufeld'08]

1.3 Test of the Standard Model via the CT theorem

$$C_{SM} = \bar{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |V_{us}|}{F_\pi |V_{ud}|} \frac{1}{f_+(0)|V_{us}|} |V_{ud}|}_{B_{exp}} + \Delta_{CT}$$

- C is predicted in the Standard Model using the measured Br:

[Talk by G.D'Ambrosio]

$Br(K_{l2}/\pi_{l2})$, $\Gamma(K_{e3})$ and $|V_{ud}|$. ($|V_{us}|$ not needed in this prediction.)

$\Rightarrow$ $B_{exp} = 1.2446 \pm 0.0041$

$C_{SM} = 1.2446 \pm 0.0041 + \Delta_{CT}$

$\ln C_{SM} = 0.2188(35) + \Delta_{CT} / B_{exp}$

- Relation which tests the Standard Model very accurately for K^0.
If physics beyond the SM: ~1% difference between C and B_{exp}.
Uncertainties from Δ_{CT} and B_{exp} on the permile level $\Rightarrow$ opportunity to see a possible effect.

- Possible test of the lattice results for F_K/F_π, $f_+(0)$.

1.6 How to measure the form factor shapes ?

- Data available from **KTeV**, **NA48** and **KLOE** for K^0 and from **ISTRA+**, **NA48** and **KLOE** for K^+.
- Necessity to parametrize the 2 form factors $\bar{f}_+(t)$ and $\bar{f}_0(t)$ to fit the measured distributions.
- Different parametrizations available, 2 classes of parametrizations :

→ 1rst class: parametrizations based on mathematical rigourous expansion, the slope and the curvature are free parameters :

- Taylor expansion

$$\bar{f}_{+,0}(t) = 1 + \lambda'_{+,0}\frac{t}{m_\pi^2} + \frac{1}{2}\lambda''_{+,0}\frac{t}{m_\pi^2} + \ldots$$

- Z-parametrization, conformal mapping from t to z variable with |z|<1 improves the convergence of the series.

$$f_{+,s}(t) = f_{+,s}(t_0)\frac{\phi(t_0,t_0,Q^2)}{\phi(t,t_0,Q^2)}\sum_{k=0}^{n} a_k\left(t_0,Q^2\right)z\left(t,t_0\right)^k$$

[Hill'06]

Theoretical error can be estimated : for a specific choice of ϕ, $\sum_{k=0}^{n} a_k^2$ bounded ⇨ use of some high-energy inputs (τ data …).

→ 2^{nd} class: parametrizations which by using physical inputs impose specific relations between the slope and the curvature

⇨ reduce the correlations, only one parameter fit.

– Pole parametrization, the dominance of a resonance is assumed

$$\bar{f}_{+,0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t}$$

$m_{V,S}$ is the parameter of the fit

– Dispersive parametrization: use of the low energy $K\pi$ scattering data and presence of resonances to contrain by dispersion relations the higher order terms of the expansion. Analysis from **[Jamin, Oller, Pich'04], [Bernard, Oertel, E.P, Stern'06],** for the scalar form factor and from **[Moussallam'07], [Jamin, Pich & Portoles'08], [Boito, Escribano, Jamin'08]** for the vector form factor using τ data.

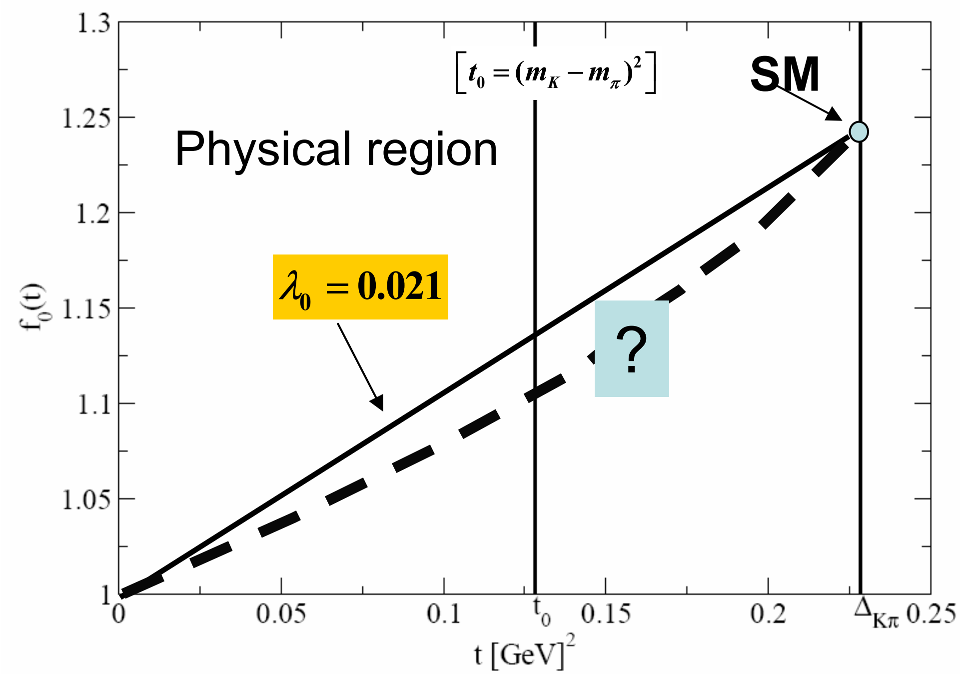
- Two requirements in the measurements of the form factor shapes from the K_{l3} data
  - Try to measure the form factor shapes from the data with the best accuracy and the slope and curvature of the scalar form factor are of special interest to allow for a matching with the 2 loop ChPT calculations. **[Bijnens&Talavera'03]**
  - Measurement of $\bar{f}_0(\Delta_{K\pi}) \equiv C$ to test the Standard Model via the CT correction

- Experimental constraints: if one uses a parametrization from 1rst class, for example a Taylor expansion,
  - Only two parameters measurable for the vector form factor, λ'_+ and λ''_+
  - Only one parameter accessible for the scalar form factor λ'_0
  - The correlations are strong,

λ'_0	1	-0.9996	-0.97	0.91
λ''_0		1	0.98	-0.92
λ'_+			1	-0.98
λ''_+				1

[Franzini, Kaon'07]

- Using a linear parametrization, it is impossible to extrapolate with a good precision up to the CT point !



- Necessity to use a second class parametrization which reduces the correlations, only one parameter is fitted.
  - For the vector form factor ⇨ pole parametrization with dominance of the K*(892) in good agreement with the data.
  - For the scalar form factor, not a such obvious dominance ⇨ necessity of a dispersive parametrization to improve the extraction of the ff parameters.

2. Dispersive Representation of the Kπ form factors

2.1 Dispersive parametrization for the scalar FF

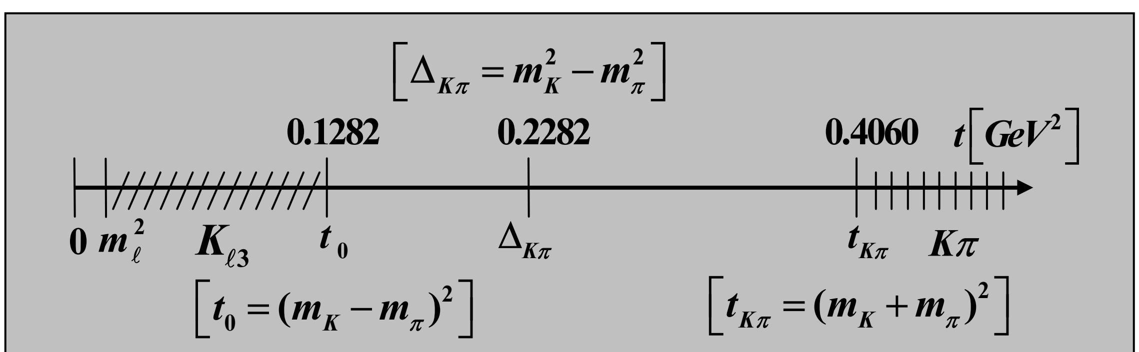
- Problem : How to construct a very precise representation of $\bar{f}_0(t)$ between 0 and $\Delta_{K\pi}$?
- Knowledge :
  - → $\bar{f}_0(0) = 1$
  - → $\bar{f}_0(\Delta_{K\pi}) = C$, Callan-Treiman point
  - → $K\pi$ scattering phase
  - → Asymptotic behaviour of the form factor : $\bar{f}_0(s) \overset{s\to-\infty}{=} \mathcal{O}(1/s)$
- A dispersion relation with two substractions at 0 and $\Delta_{K\pi}$ for $\ln(\bar{f}_0(t))$, assuming that $\bar{f}_0(t)$ has no zero

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

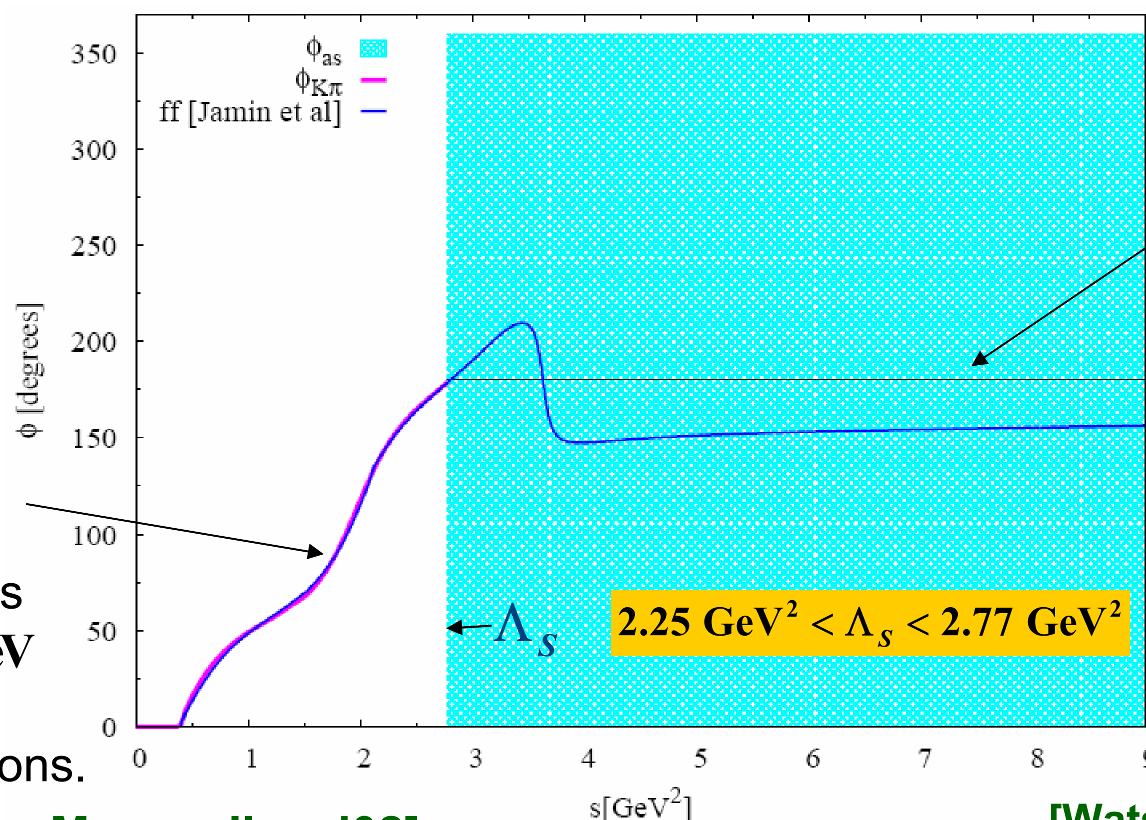
with

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{t_{K\pi}}^{\infty} \frac{ds}{s} \frac{\phi(s)}{(s - \Delta_{K\pi})(s - t)}$$

  - → $\phi(t)$ phase of the form factor : $\bar{f}_0(t) = \left|\bar{f}_0(t)\right| e^{i\phi(t)}$



- Phase used



Asymptotic value

$\phi(s) \underset{s\to -\infty}{\longrightarrow} \pi$

$\bar{f}_0(s) = \mathcal{O}(1/s)$

Kπ scattering data

Experimental inputs for $1\,\text{GeV} < E < 2.5\,\text{GeV}$ **[Aston et al]** + Roy Steiner equations.

[Buettiker, Descotes, Moussallam '02]

[Watson theorem]

- Elastic up to ~1.5 GeV ⟹ $t < \Lambda:\ \phi(t) = \phi_{K\pi}(t) = \delta^{s,\frac{1}{2}}_{\pi,K}(t) \pm \Delta\delta^{s,\frac{1}{2}}_{\pi,K}(t)$

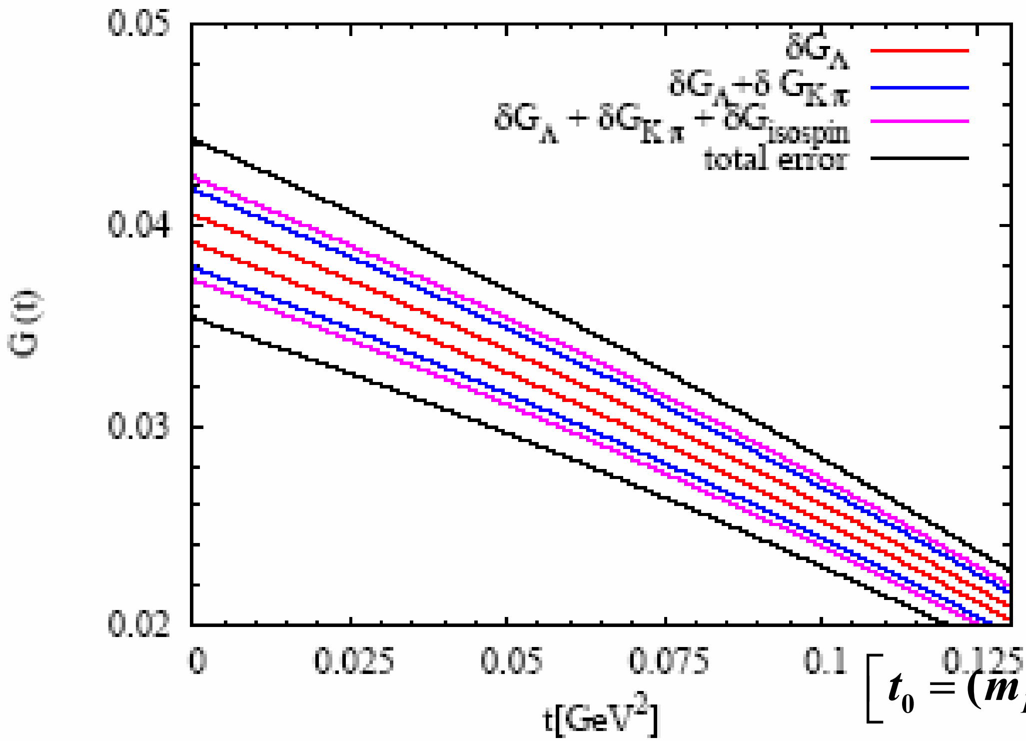
$t > \Lambda:\ \phi(t) = \phi_{as}(t) = \pi \pm \pi$

- 2 subtractions ⟹ Rapid convergence of G(t)

- Sum Rule $\ln C = G(-\infty) = \dfrac{\Delta_{K\pi}}{\pi}\int_{t_{K\pi}}^{\infty} \dfrac{ds}{s}\dfrac{\phi(s)}{(s-\Delta_{K\pi})}$ ⟹ not enough precise to determine lnC from it !

2.2 Study of the robustness

[Bernard, Oertel, E.P., Stern, work in progress]



$\left[t_0 = (m_K - m_\pi)^2 \right]$

G(t) with the uncertainties added in quadrature

$$G(0) = \mathbf{0.0398 \pm 0.0044}$$

G(t) does not exceed 20% of the expected value of lnC

lnC~0.20

- Result not sensitive to the exact position of Λ_S (G(0) remains stable) →δG_Λ
- Study of isospin breaking corrections →$\delta G_{isospin}$
- Apart from the parameter (lnC) to be determined by the fit, very precise parametrization of the form factor in the physical region.

2.2 Dispersive parametrization of the Kπ vector form factor

- We can also write a dispersion relation for the vector form factor, improving the pole parametrization. In this case the presence of K*(892) is assumed.
- In the same way as for the scalar form factor, 2 subtraction points at low energy : $\bar{f}_+(0)=1$, $\bar{f}'_+(0)=\Lambda_+/m_\pi^2$. Assuming $\bar{f}_+(t)$ has no zero

$$\bar{f}_+(t)=\exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right] \quad \text{with} \quad H(t)=\frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\varphi(s)}{(s-t)}$$

- Extrapolation of the scattering phase from the data for $0.825\ \mathrm{GeV} < E < 2.5\ \mathrm{GeV}$ **[Aston et al]** down to the threshold by the construction of the partial wave amplitude : Breit-Wigner (K*(892)) a la Gounaris-Sakourai (analyticity, unitarity and correct threshold behavior).

  Inputs: mass and width of K*(892).
- Study of the robustness: H(t) precisely known. The presence of a zero is excluded from τ data.

Study of the influence of a zero for the scalar ff

- A zero can be real or complex and in the physical or unphysical region.

- Studies of a presence of zeros for the form factor in the case of the pion form factor **[Raziller,Schmidt& Sabba-Stefanescu'76][Leutwyler'02].**

- **Case of a real zero at t=T_0**, $F(t)=\bar{f}_0(t)/T_0$ has no zero. One can write a dispersion relation for ln(F(t))

$$\bar{f}_0(t) = \left(1 - \frac{t}{T_0}\right) \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - \ln\left(1 - \frac{\Delta_{K\pi}}{T_0}\right) - G(t)\right)\right]$$

Jump of the phase by π

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{K\pi}}^{\infty}\frac{ds}{s}\frac{\phi(s) - \pi\ \theta(s - T_0)}{(s - \Delta_{K\pi})(s - t)}\right)\right]$$

and $\phi(s) \xrightarrow{s \to +\infty} 2\pi$

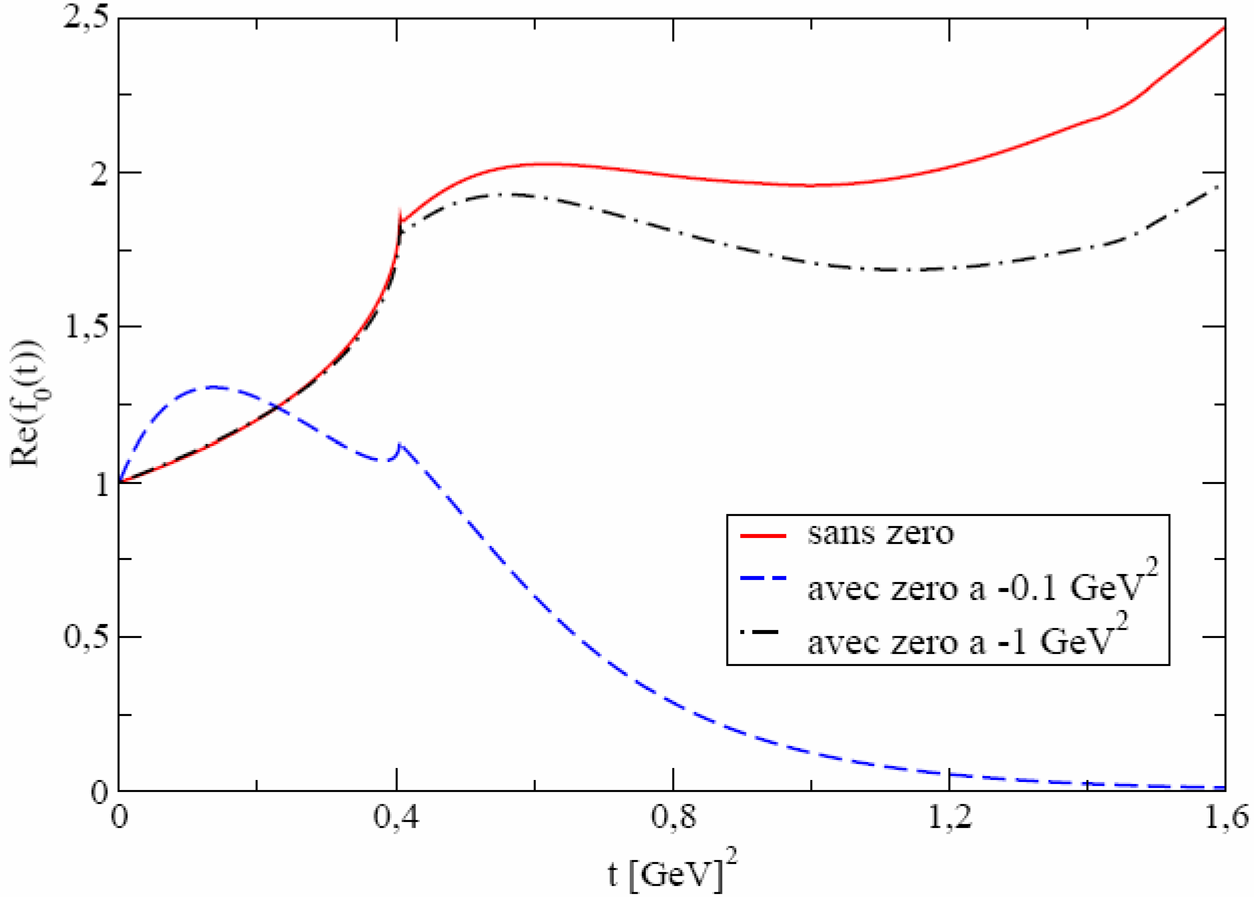
- In the physical or time like region: no zeros in the elastic part since it would be observed in the $K\pi$ phase shift data ⇨ $T_0 \geq \Lambda_S$~$2.77 GeV^2$.

  ⇨ No influence on the form factor.

- In the unphysical or space-like region: $t < 0$, a zero is less probable because the ff ⇔ Fourier transform of the charge density **[Leutwyler'02]** similar to the one of the electron in the ground state of the hydrogen atom. Charge density proportional to the squared of the wave function

  ⇨ $f_0(t) > 0$ for $t<0$.

- In the unphysical region:

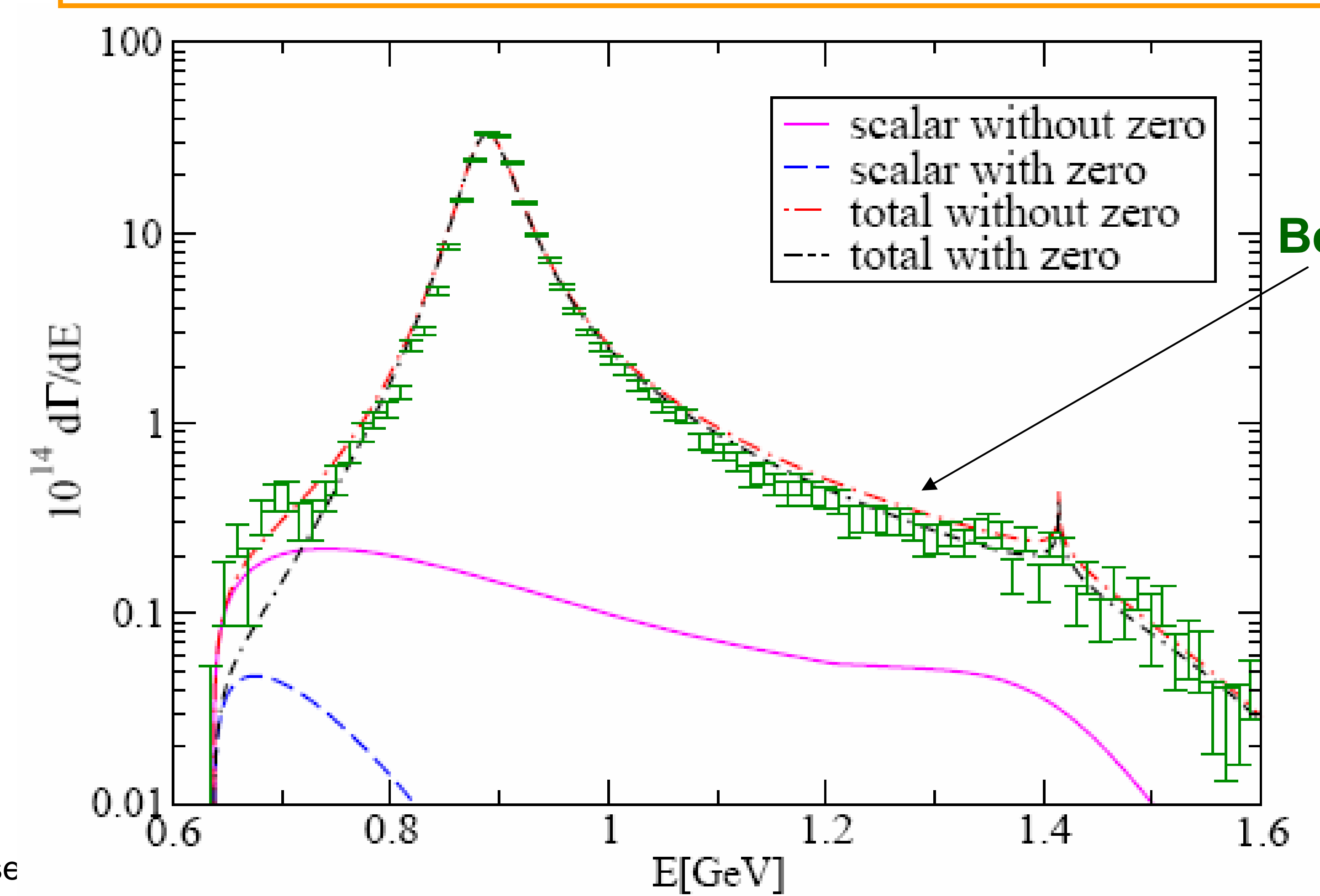


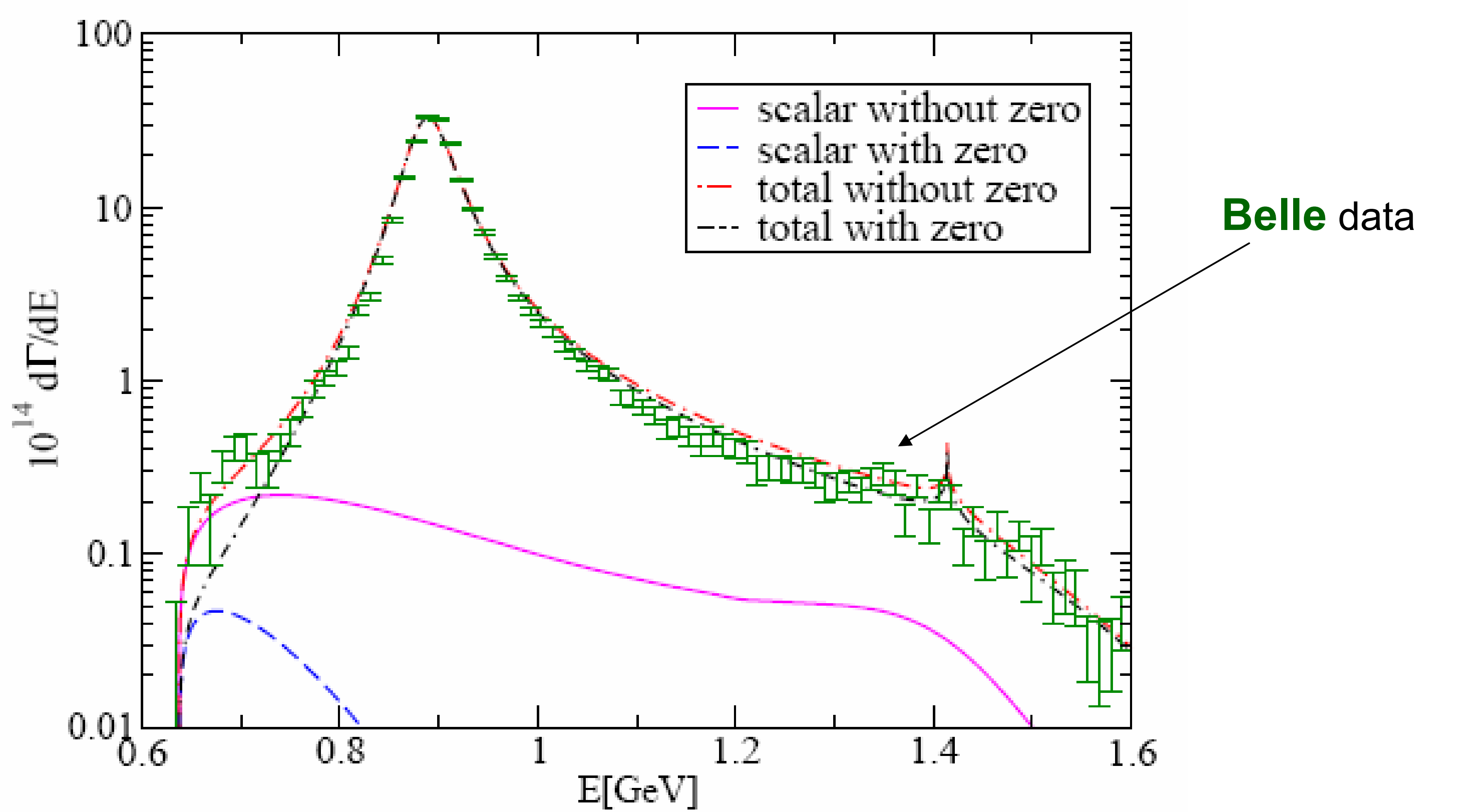
Shape of the ff completely different for small t. Large slopes for small $|T_0|$ as for instance T_0 = - 0.1 GeV². This seems excluded by $K_{\mu 3}$ data

- Use of τ data

Kinematic factor

$$\frac{d\Gamma_{K\pi}(t)}{d\sqrt{t}} = \frac{|V_{us}|^2 G_F^2 M_\tau^3}{128\pi^3} q_{K\pi}(t) \left(1 - \frac{t}{M_\tau^2}\right)^2 \times$$
$$\left[\left(1 + \frac{2t}{M_\tau^2}\right) \frac{4q_{K\pi}^2(t)}{t} |\bar{f}_+(t)|^2 + \frac{3(M_K^2 - M_\pi^2)^2}{t^2} |\bar{f}_0(t)|^2\right]$$





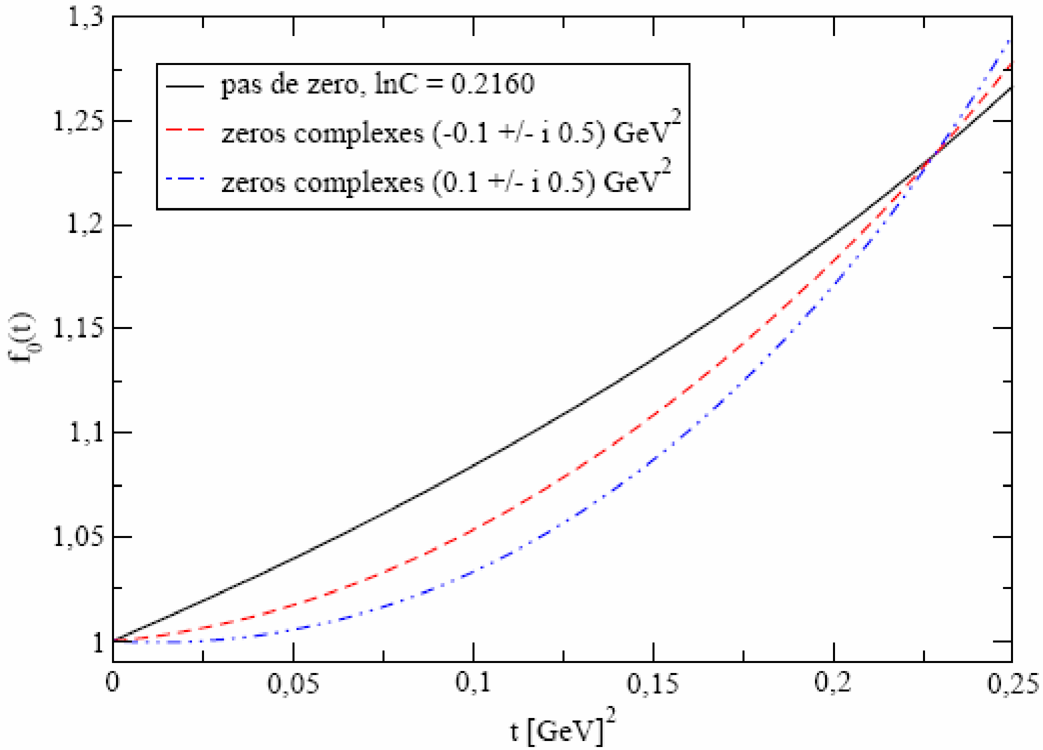
- Plot for T_0= - 0.1 GeV^2 with the Belle data and only the experimental uncertainties, no uncertainties from the ff included. Similar data from Babar but not publicly accessible.
- Dominance of the K*(892) between ~0.8 GeV and ~1.2GeV, a vector ff without zeros fits the data very well. Influence of a zero for the scalar ff close to the threshold ⇨ seems excluded by the τ data. More precise results are awaited.

- **Case of a complex zero :** ff real before the cut ⇨ if a complex zero Z_0 ⇨ 2 zeros, itself and its complex conjugate.

$$\bar{f}_0(t) = (1-t/Z_0)(1-t/\overline{Z}_0)\exp\left[\frac{t}{\Delta_{K\pi}}(\ln C - \ln(1-\Delta_{K\pi}/Z_0) - \ln(1-\Delta_{K\pi}/\overline{Z}_0) - G(t))\right]$$

and $\phi(s) \xrightarrow{s\to+\infty} 3\pi$

- In this case the zeros can be very close to the physical region ⇨ it dramatically affects the scalar ff



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Can one exclude complex zeros ?

- 3 sum rules to satisfy simultanously due to $\overline{f}_0(t) \overset{t\to -\infty}{=} \mathcal{O}(1/t)$

$$G(-\infty) = \ln C - \ln(1 - \Delta_{K\pi}/Z_0) - \ln(1 - \Delta_{K\pi}/\bar{Z}_0)$$

$$\int_{t_{\pi K}}^{\infty} ds \frac{\mathrm{Im}\bar{f}_0(s)}{(1 - s/Z_0)\left(1 - s/\overline{Z}_0\right)} = 0$$

$$\int_{t_{\pi K}}^{\infty} ds\ s\ \frac{\mathrm{Im}\bar{f}_0(s)}{(1 - s/Z_0)(1 - s/\bar{Z}_0)} = 0\ .$$

- Possibility to find a phase that satisfies these 3 sum rules

- But equivalent to the CT theorem at $M_\pi^2 - M_K^2$

$$C_{\pi K} = \overline{f_0}(\Delta_{\pi K}) = \frac{F_\pi}{F_K f_+^{K^0}(0)} + \widetilde{\Delta}_{CT}$$

Preliminary

$$\Delta_{\pi K} = m_\pi^2 - m_K^2$$

- Assuming the SM EW couplings

$$C_{\pi K} = \frac{\hat{F}_\pi}{\hat{F}_K}\frac{1}{\hat{f}_+(0)} + \tilde{\Delta}_{CT}$$

$$\tilde{\Delta}_{CT} \sim \frac{M_K^2}{M_\pi^2}\Delta_{CT}$$ estimated in ChPT, SU(3) correction

$$\tilde{\Delta}_{CT}^{NLO} = (3 \pm 4).10^{-2}$$

[Gasser&Leutwyler] + estimate of uncertainties

$$C_{\pi K} = 0.9066 \pm 0.0763$$

Preliminary

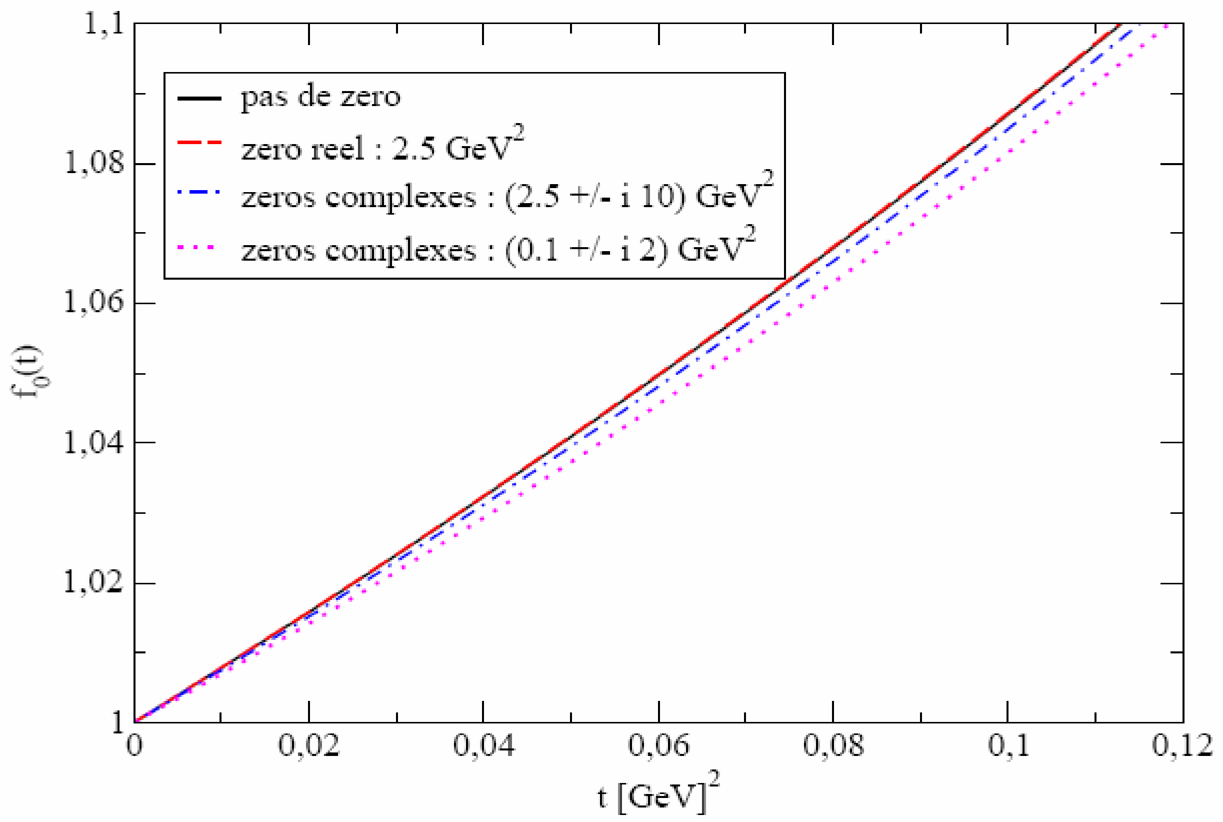
⇨ exclude zeros whose real and imaginary parts are too close to the physical region.

$Z_0 = \left(0.1 \pm \mathrm{i}\ 0.5\right)\mathrm{GeV}^2$ and $Z_0 = \left(-0.1 \pm \mathrm{i}\ 0.5\right)\mathrm{GeV}^2$ are excluded.

$\left(\overline{f_0}(\Delta_{\pi K}) = 1.2261\right)$

But for instance a zero at $Z_0 = \left(0.1 \pm \mathrm{i}\ 2.0\right)\mathrm{GeV}^2$ is not excluded $\left(\overline{f_0}(\Delta_{\pi K}) = 0.8923\right)$

- Such a zero will change the shape of the scalar ff., smaller slope.



- Impossible to exclude from a fit to K_{l3} decay data.

3. Results of the dispersive analysis with KTeV data

3.1 Presentation

	K_{e3} only	$K_{\mu3}$ only	K_{e3} and $K_{\mu3}$ Combined
$\Lambda_+ \times 10^3$	25.17 ± 0.58	24.57 ± 1.10	25.09 ± 0.55
$\ln C$	-	0.1947 ± 0.0140	0.1915 ± 0.0122
$\rho(\Lambda_+, \ln C)$	-	-0.557	-0.269
χ^2/dof	66.6/65	193/236	0.48/2
$\lambda'_+ \times 10^3$	25.17 ± 0.58	24.57 ± 1.10	25.09 ± 0.55
$\lambda''_+ \times 10^3$	1.22 ± 0.03	1.19 ± 0.05	1.21 ± 0.03
$\lambda'_0 \times 10^3$	-	13.22 ± 1.20	12.95 ± 1.04
$\lambda''_0 \times 10^3$	-	0.59 ± 0.03	0.58 ± 0.03
I_K^e	0.15450 ± 0.00028	0.15416 ± 0.00060	0.15446 ± 0.00025
I_K^μ	-	0.10207 ± 0.00032	0.10219 ± 0.00025
I_K^μ / I_K^e	-	0.6621 ± 0.0018	0.6616 ± 0.0015

- Only one parameter fitted ⇒ better precision in the results, reduce the correlations.
- Improvement in the precision of the calculation of the phase space integrals

  Previously, $I_K^e = 0.10165 \pm 0.00080$ and $I_K^\mu = 0.15350 \pm 0.00105$

  ⇒ better extraction of V_{us}, $\Delta_{SU(2)}$ **[KTeV'04]**
- Shape measured with a good precision ⇒ test of the EW couplings of the SM+ matching with the 2 loop CHPT calculation possible.

3.2 Influence of the vector form factor parametrization

	Parameterization Vector FF/Scalar FF			
Results	disp.(I)/disp.	pole(I)/disp.	quad (II)/disp.	z-param.(II)/disp.
Fit param. v_i	$\Lambda_+ = 24.57(83)$	$M_V = 890.00$ $(13.00)\mathrm{MeV}$	$\lambda'_+ = 17.5(3.4)$ $\lambda''_+ = 4.3(1.4)$	$a_1 = 1.057(63)$ $a_2 = 3.9(3.2)$
λ'_+	24.57(83)	24.59(72)	17.5(3.4)	20.00(2.60)
λ''_+	1.19(4)	1.21(7)	4.3(1.4)	2.5(6)
Fit param. $\ln C$	0.1947(91)	0.1944(93)	0.169(16)	0.170(16)
λ_0	13.22(78)	13.20(79)	11.03(1.37)	11.11(1.37)
λ'_0	0.59(2)	0.59(2)	0.54(3)	0.54(3)
$\rho(v_i, \ln C)$	-0.557	0.588	0.707	0.477
	-	-	-0.819	-0.766
χ^2/dof	193/236	193/236	189/235	189/235

- Only statistical uncertainties, 2 different results :

λ_0 x $10^3 \sim 13.2 \pm 0.8$ and λ_0 x $10^3 \sim 11.1 \pm 1.4$ central values differ by 2σ.

With more data, the 2 results should converge.

3.3 Test of the SM EW couplings

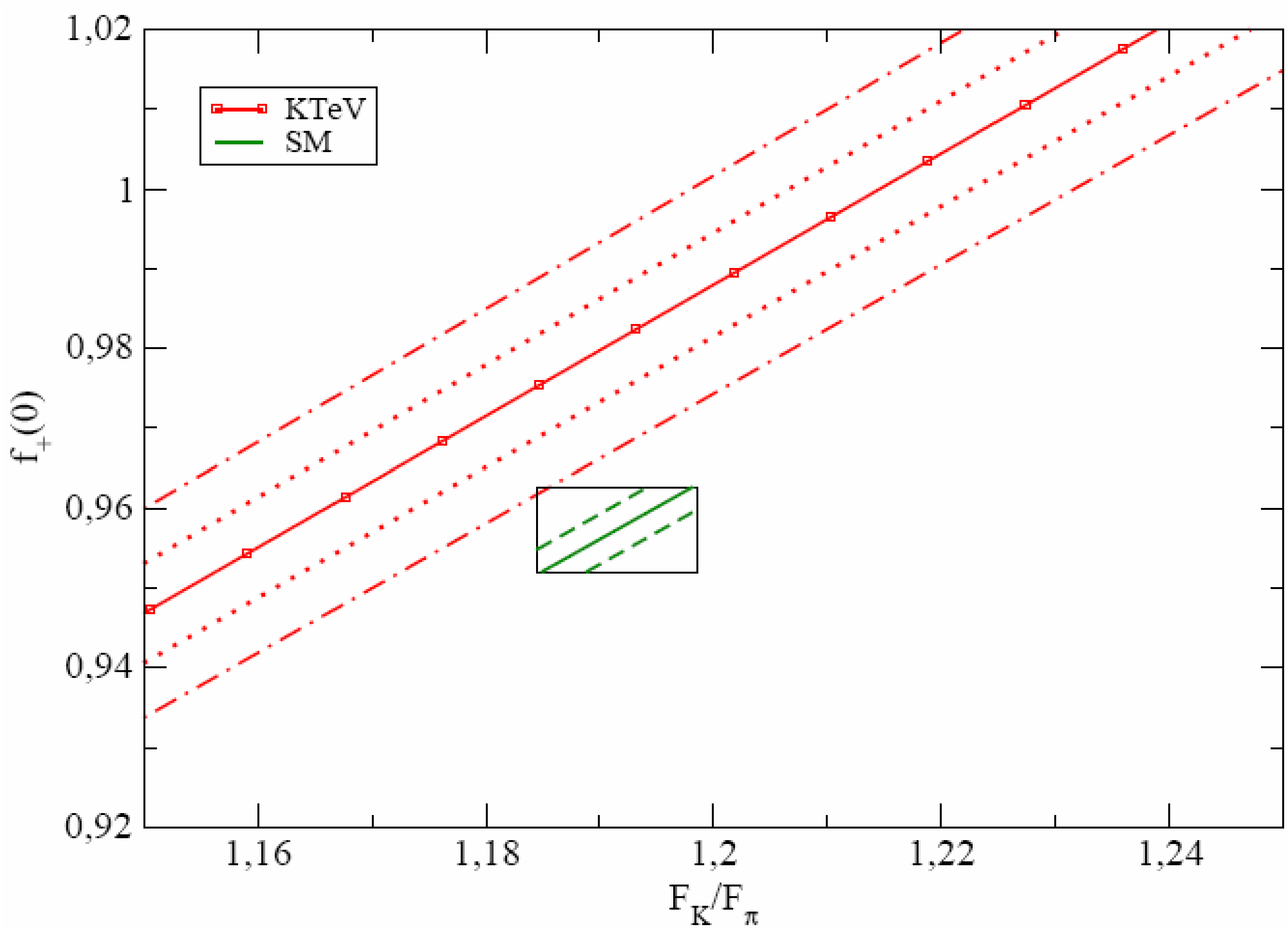
Experiment Ke3+Kμ3	ln C
KTeV+BOPS Prel.	0.192(12)
KLOE'08	0.204(25)
NA48'07 ($K_{\mu3}$ only)	0.144(14)

- To be compared with

$$\ln C_{SM} = 0.2160(35)(64)$$

KTeV in agreement with KLOE and at ~1.5σ from the SM.
NA48 4.5σ away from the SM !

- A deviation from the SM prediction can be explained :
  - Test of RHCs appearing at NLO of an EW low energy effective theory as a signature of exchange of new particules (W_R,…) at high energy. [Bernard, Oertel, E.P., Stern'06]
  - Presence scalar couplings (charged Higgs) : [Hou]
    MFV + large tanβ : hard to explain a 4.5σ effect (~several% level) [Isidori, Paradisi'06]
  - Existence of a complex zero and its complex conjugate for the form factor [Bernard, Oertel, E.P., Stern, work in progress]



Black rectangle: SM constraint on F_K/F_π and $f_+(0)$

$$\Rightarrow \quad \frac{\hat{F}_K}{\hat{F}_\pi} = 1.192(7)$$

$$\hat{f}_+(0) = 0.9574(52)$$

The green band :

$$B_{exp} = \left.\frac{F_K}{F_\pi f_+^{K^0}(0)}\right|_{exp}$$

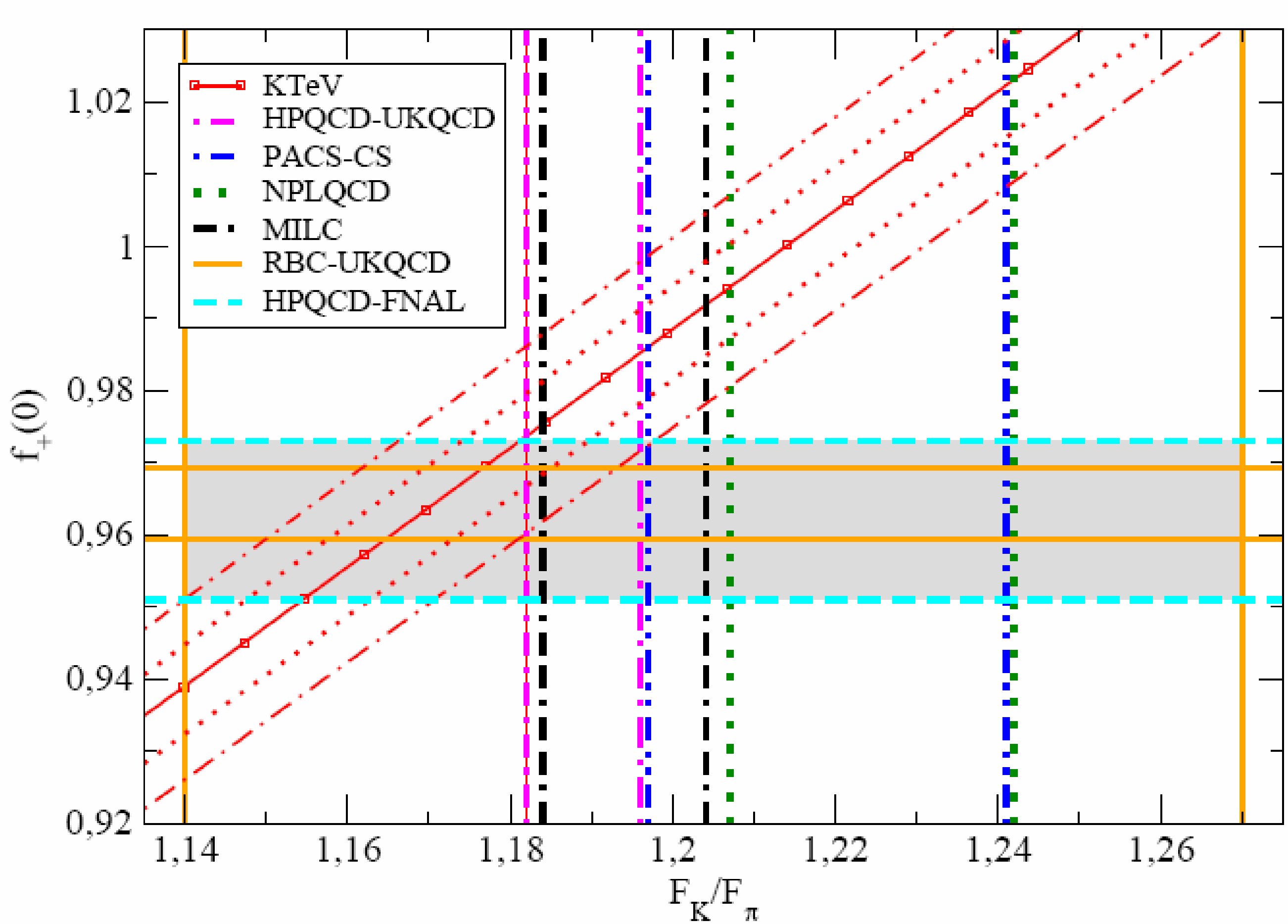
- Measurement of KTeV $\Rightarrow$ prediction for $F_K/F_\pi/f_+(0)$ via the CT theorem

$$\frac{F_K}{F_\pi f_+^{K^0}(0)} = C - \Delta_{CT}$$

we take $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

$\Rightarrow$ The red band : KTeV results are at ~1.5σ from the SM

3.4 Combination with N_f=2+1 lattice results



- All the combined lattice results ⇨ grey band.
- The KTeV result selects values for F_K/F_π on the lower side of lattice results and values for $f_+(0)$ on the upper side.

$$1.182 \leq \frac{F_K}{F_\pi} \leq 1.197$$ and $$0.959 \leq f_+(0) \leq 0.969$$

4. Conclusions and Outlook

- Dispersive parametrization very useful to analyse $K^L_{\mu 3}$ decays: parametrization physically motivated which allows with one parameter to determine the shape of the form factor, quite robust
  - Allows for a more precise determination of V_{us}.
  - Allows for a test of the SM electroweak couplings via the CT theorem.
  - Allows for a matching with the 2 loop ChPT calculations.

- Study of influence of zeros, real zeros close to the physical region are excluded but impossibility to exclude complex zeros that affect the scalar form factor shape (small slope).
- Experimental results of dispersive analysis with KTeV data agree with KLOE analysis and marginally with the SM but not with NA48 result which is at 4.5σ from the SM ⇨ results for K^+

- Strong influence of the vector ff parametrization on the result.

- Combination of the dispersive analysis of the KTeV data with the lattice results ⇨ interesting constraints on the values of F_K/F_π and $f_+(0)$. New and more precise results from lattice are awaited.

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- Outlook:
  - Systematic study of IB corrections, dispersive analysis of the form factors at 2 loops.
  - Update of the matching with the 2 loop ChPT calculations.

Additional slides

1.3 Theoretical knowledge for the scalar FF: CT relation

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = \overline{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

- Corrections of order m_u, m_d $\quad \Delta_{CT} \sim \mathcal{O}\left(m_{u,d} / 4\pi F_\pi\right)$

  → No chiral logarithms, in the isospin limit m_d=m_u :

  $\Delta_{CT}^{NLO} \sim -3.5.10^{-3}$ **[Gasser & Leutwyler]**

  → K^0 decay : no small denominators due to π^0 - η mixing $\left(\mathcal{O}\left((m_d - m_u)/m_s\right)\right)$.

  → K^+ decay case : enhancement by π^0 - η mixing in the final state

  ⇨ $\Delta_{CT}^{K^+} \sim few\ 10^{-2}$ (K^0 ideal decay)

- Estimations of the higher order terms: corrections in $\mathcal{O}\left(m_{u,d}.m_s\right)$

  ⇨ $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$ in agreement with **[Bijnens&Gorbani'07]** **[Kastner&Neufeld'08]**

1.5 Matching with the ChPT 2 loop calculations

- Measurement of slope and curvature of the scalar form factor from experiments ⇨ determination of
  - F_K/F_π
  - $f_+(0)$
  - C_{12} and C_{34}, 2 $\mathcal{O}(p^6)$ LECs which enters K_{l3} decays
  - Callan-Treiman correction Δ_{CT}

- Why ?
  - LECs play an important role in ChPT calculations, enter different processes. Ex: C_{12} into $\eta \to 3\pi$ **[Bijnens&Ghorbani'07].**
  - Possibility to test the lattice calculation and resonance model estimates
  - Test of the Standard Model, knowledge of $f_+(0)$ ⇨ extraction of V_{us}

2.2 Dispersive parametrization of the Kπ vector form factor

- We can also write a dispersion relation for the vector form factor, improving the pole parametrization. In this case the presence of K*(892) is assumed.
- In the same way as for the scalar form factor, a dispersion relation with two subtractions for ln($\bar{f}_+(t)$) : 2 subtraction points at low energy :

  → $\bar{f}_+(0) = 1$

  → $\bar{f}'_+(0) = \Lambda_+ / m_\pi^2$

  Assuming $\bar{f}_+(t)$ has no zero

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$

with

$$H(t) = \frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\varphi(s)}{(s-t)}$$

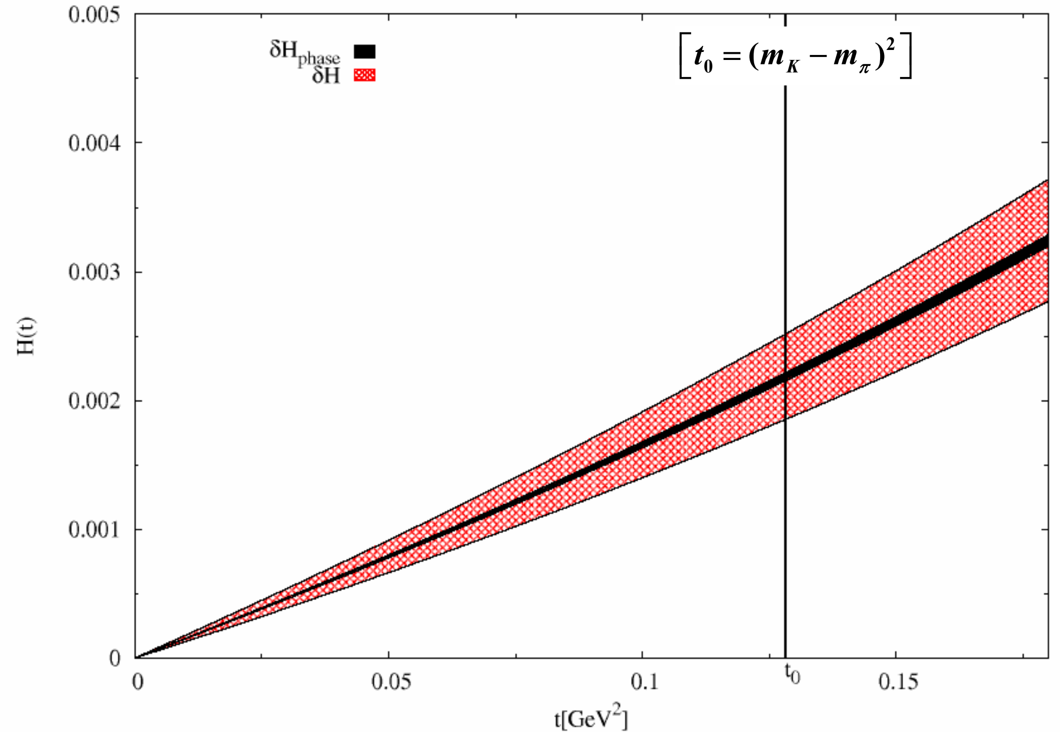
  → φ(t) phase of form factor : $\bar{f}_+(t) = \left|\bar{f}_+(t)\right| e^{i\varphi(t)}$

- φ(s) a priori unknown but

  → $\bar{f}_+(s) \overset{s\to -\infty}{=} \mathcal{O}(1/s)$ ⟹ For large s, φ(s) → π. Rapid convergence of H(t)

  → At « low energy » $\varphi(s) = \delta^{1/2}_{K\pi}(s)$, P wave I=1/2 Kπ scattering phase [**Watson theorem**].

- $K\pi$ scattering phase
  - Experimental input for $0.825\ \mathrm{GeV} < E < 2.5\ \mathrm{GeV}$ **[Aston et al].**
  - Extrapolation of the phase down to threshold complicated ⇨ lack of relevant experimental inputs.
  - Construction of the partial wave amplitude : Breit-Wigner (K*(892)) a la Gounaris-Sakourai (Analyticity, Unitarity and Correct threshold behavior) Inputs: mass and width of K*(892).



- Study of the robustness: H(t) precisely known. The presence of a zero is excluded from τ data.

2.2 Study of the robustness

- Influence of Λ_S end of the physical region **$2.25\ \text{GeV}^2 < \Lambda_S < 2.77\ \text{GeV}^2$**
  - G(0) quasi insensitive to the exact position of Λ_S
  - G(-∞) quite sensitive to Λ_S