NL^3 — a scheme for merging NLO+PS

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Outline

Introduction

Standard CKKW(-L) merging
- Tree-level ME's
- Mixing merging/ordering scales
- Reconstruction and reweighting

Adding one-loop ME's
- One-loop ME's
- Expanding the CKKW-L weights

Implementation details
- α_s considerations
- Generating the expanded Sudakov
- The full algorithm
- Mixing merging/ordering scales

Outlook

Introduction

- Starting point is CKKW-L
- Most of this is easily applicable also to plain CKKW
- We want to add events generated according to loop ME's
- The corresponding terms must be subtracted from the standard CKKW-L events.

Standard CKKW(-L) merging

Start out with events generated according to tree-level ME's

$$d\sigma_{+n}^{\mathrm{tree}} = C_n(\Omega_n)\alpha_{\mathrm{s}}^n(\mu)d\Omega_n$$

where $\Omega_n = (q_1, \ldots, \mathrm{q}_m; p_1, \ldots, p_n)$ is the phase spcae for an m-particle Born process with n extra jets.

The divergencies are regularized by a jet-like phase space cut, $k_{\perp MS}$.

Here we will assume that the parton shower is ordered in ρ, which is the same variable as $k_{\perp MS}$.

In this way we don't have to worry about vetoed/trunkated showers. We can simply add a shower below $k_{\perp MS}$ (except for the highest jet multiplicity).

Mixed ordering/mergin scales will be discussed later.

First we do a mapping to the patron shower phase space

$$\Omega_n \mapsto \Omega_n^{\mathrm{PS}} = (q_1, \ldots, \mathrm{q}_m; \rho_1, x_1 \ldots, \rho_n, x_n)$$

I.e. a parton shower history is constructed, with emissions (ρ_i, x_i).

Then we reweight

$$d\sigma_{+n}^{\mathrm{CKKW-L}} = C_n(\Omega_n)\alpha_{\mathrm{s}}^n(\mu) \prod_{i=1}^{n} \frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1}) d\Omega_n^{\mathrm{ps}}$$

with $\rho_{n+1} = k_{\perp MS}$.

$\alpha_s^{\rm PS}(\rho_i)$ is the coupling the shower would have used in the corresponding emission.

$\Delta_{S_i}(\rho_i, \rho_{i+1})$ is the no-emission probability in the shower from the reconstructed state S_i between the scales ρ_i and ρ_{i+1}. This is by definition the Sudakov form factor used in the shower.

Adding one-loop ME's

Now we want to look at n-jet events generatet to one-loop order

$$d\sigma_{+n}^{\mathrm{loop}} = C_n(\Omega_n)\alpha_{\mathrm{s}}^n(\mu)\left[1 + C_{n,1}(\Omega_n)\alpha_{\mathrm{s}}(\mu)\right] d\Omega_n$$

Where $C_{n,1}$ is the virtual and real corrections integrated up to the merging scale $k_{\perp MS}$.

We can't use CS-dipole subtraction directly. Subtraction terms need to be cutoff at $k_{\perp MS}$ (cf. yesterdays discussion).

$$\sigma_{+n}^{\mathrm{NLO}} = \sigma_{+n}^{\mathrm{loop}} + \sigma_{+n+1}^{\mathrm{tree}}$$

- $\sigma_{+n}^{\mathrm{CKKW-L}}$ gives exclusive n-jet states approximately correct (as far as the PS is correct) to all orders in α_{s}.
- $\sigma_{+n}^{\mathrm{loop}}$ gives exclusive n-jet states exactly correct to the first two orders in α_{s}.

In both cases we can add a shower below $k_{\perp MS}$.

We want to use $\sigma_{+n}^{\mathrm{loop}}$ and add $\sigma_{+n}^{\mathrm{CKKW-L}}$ with the first two orders in α_{s} subtracted.

$$\prod_{i=1}^{n}\frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu)}\prod_{i=0}^{n}\Delta_{S_i}(\rho_i,\rho_{i+1}) = 1+\alpha_{\mathrm{s}}(\mu)B^{\mathrm{PS}}+\mathcal{O}\left(\alpha_{\mathrm{s}}^2(\mu)\right)$$

So we reweight the tree-level events by

$$\left[\prod_{i=1}^{n}\frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu)}\prod_{i=0}^{n}\Delta_{S_i}(\rho_i,\rho_{i+1}) - 1 - \alpha_{\mathrm{s}}(\mu)B^{\mathrm{PS}}\right]$$

- $\sigma_{+n}^{\mathrm{loop}} + \sigma_{+n}^{\mathrm{PScorr}}$ gives exclusive n-jet states exactly correct to the first two orders in α_{s} and approximately correct to all other orders in α_{s}.

So far we have not added anything to new as compared to Nagy–Soper.

Implementation details

- Note that $\alpha_s(\mu) \neq \alpha_s^{PS}(\mu)$.
- ${\alpha_s}^{PS}$ is typically a one- or two-loop α_s with Λ_{QCD} a free parameter fitted reproduce event shapes at LEP.
- $\alpha_s(\mu)$ is here just a fixed number corresponding to the "world-average" $\alpha_s(M_Z)$ running with $\Lambda_{\bar{MS}}$

Typically $\alpha_s^{PS}(M_Z) > \alpha_s(M_Z)$ because the parton shower underestimates the emission probabilities, and needs to *boost* the probabilities to fit the data.

Rather than tuning Λ_{QCD}, we could say we are using $\Lambda_{\bar{MS}}$ and instead tune a scale factor, $\alpha_s^{\mathrm{PS}}(\rho) = \alpha_s(b\rho)$.

Hence, we can write

$$\frac{\alpha_s^{\mathrm{PS}}(\rho)}{\alpha_s(\mu)} = 1 - \frac{\log\frac{b\rho}{\mu}}{\alpha_0}\alpha_s(\mu) + \mathcal{O}\left(\alpha_s^2(\mu)\right)$$

$$\Delta_{S_i}(\rho_i,\rho_{i+1}) = \exp\left(-\int_{\rho_{i+1}}^{\rho_i} d\rho\alpha_s(\rho)\Gamma_{S_i}(\rho)\right)$$

We know how to generate this from the shower:

- Start the shower from the state S_i, with ρ_i as the maximum scale.
- Generate one emission giving a scale ρ.
- The probability that $\rho < \rho_{i+1}$ is exactly $\Delta_{S_i}(\rho_i,\rho_{i+1})$.

- If $\rho > \rho_{i+1}$, restart from S_i and generate again one emission starting from ρ as maximum scale.
- Continue until we find a $\rho < \rho_{i+1}$.
- Count the number of emissions n_{acc} before going below ρ_{i+1}.

$$\langle n_{acc} \rangle = -\log \Delta_{S_i}(\rho_i, \rho_{i+1}) = \int_{\rho_{i+1}}^{\rho_i} d\rho \alpha_s(\rho) \Gamma_{S_i}(\rho)$$

We want to calculate the first two term

$$\begin{aligned}\Delta_{S_i}(\rho_i, \rho_{i+1}) &= 1 - \int_{\rho_{i+1}}^{\rho_i} d\rho \alpha_s(\rho) \Gamma_{S_i}(\rho) + \mathcal{O}\left(\alpha_s^2(\mu)\right) \\ &= 1 - \alpha_s(\mu) \int_{\rho_{i+1}}^{\rho_i} d\rho \Gamma_{S_i}(\rho) + \mathcal{O}\left(\alpha_s^2(\mu)\right)\end{aligned}$$

Using the method above we get $-\log\Delta$, but we generate it using a fixed $\alpha_s(\mu)$ in the shower (if available as option).

Now we have everything we need:

- Generate events with $0 \leq n \leq N$ extra jets according to the tree-level ME cut off at $k_{\perp MS}$.
- Generate events with $0 \leq n < N$ extra jets according to the one-loop ME cut off at $k_{\perp MS}$.
- Reconstruct $\Omega_n \mapsto \Omega_n^{\mathrm{PS}}$.
- For one-loop events, add PS below $k_{\perp MS}$.
- For three-level events, reweight with $n = N$, reweight with

$$\prod_{i=1}^{n} \frac{\alpha_{\mathrm{s}}^{\mathrm{PS}}(\rho_i)}{\alpha_{\mathrm{s}}(\mu)} \prod_{i=0}^{n-1} \Delta_{S_i}(\rho_i, \rho_{i+1})$$

  and continue below ρ_n.

- For three-level events with $n < N$, reweight with

$$\prod_{i=1}^{n} \frac{\alpha_s^{\mathrm{PS}}(\rho_i)}{\alpha_s(\mu)} \prod_{i=0}^{n} \Delta_{S_i}(\rho_i, \rho_{i+1})$$
$$-1$$
$$+\alpha_s(\mu) \sum_{i=1}^{n} \frac{\log \frac{b\rho_i}{\mu}}{\alpha_0}$$
$$+\alpha_s(\mu) \sum_{i=0}^{n} \int_{\rho_{i+1}}^{\rho_i} d\rho \Gamma_{S_i}(\rho)$$

and add PS below $k_{\perp MS}$

All weights are positive as long as

- $k_{\perp MS}$ is large enough for the loop ME to be positive
- $\mu < b\rho_i$

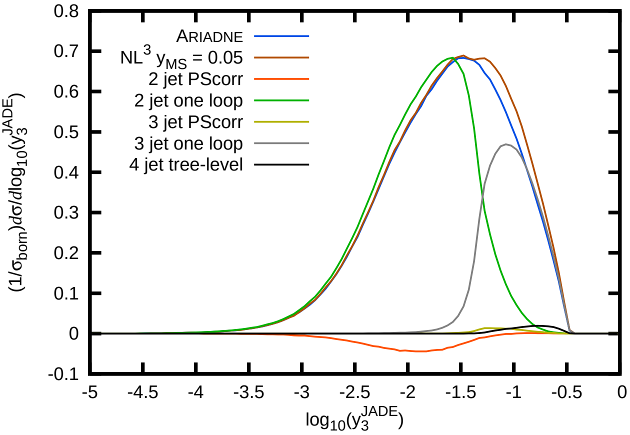
The net result is events generated so that all n-jet observables (above the merging scale and $n < N$) will be correct to NLO with a PS-simulated resummation. And N-jet observables will correct to LO+PSresum.

If the scale for the merging is not the same as in the shower evolution, things becom a bit more complicated.

Also in the one-loop events we then need to multiply with Sudakov form factors (no-emission probability below the cut but above the reconstructed shower scales).

n-jet observables will be correct to NLO+PSresum only if the *n* hardest (according to the PS) are above the merging scale.

Why combine several different jet multiplicities? If we are looking at three-jet observables, why generate two-jet events? Isn't it enough to generate 3-jet loop + 4-jet tree?



Outlook

- CKKW-L-like NLO+PS merging works.
- So far only for e^+e^-
- Should be trivial to apply to standard CKKW as well (the Sudakovs can be expanded analytically)
- Doing it for *pp* collisions, eg. Drell-Yan should be possible, but not necessarily trivial.

In CKKW-L, the main difference is the ratios of PDF's which enters into the Sudakov form factors and in the weight directly.

We may need to worry about renormalization scheme dependencies as well.

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