# QSFP 2021

## "Atom interferometry and gravitational wave detection"

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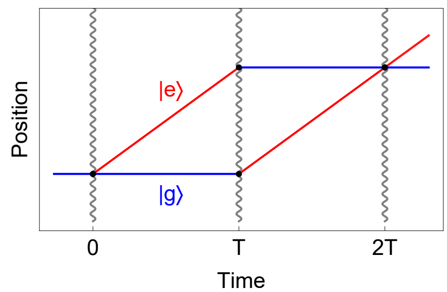
## Tutorial 1: Atom interferometer accelerometer phase shift

Here we will calculate the phase shift of a light-pulse atom interferometer subject to a uniform gravitational acceleration $g$. The trajectories of the atomic ground and excited states are shown in blue and red, respectively, while the light pulses are shown in gray at times 0, $T$, and $2T$. The interferometer uses a three-pulse




Mach-Zehnder sequence ($\frac{\pi}{2} - \pi - \frac{\pi}{2}$) with time $T$ between pulses and beamsplitter momentum $\hbar k$ at each atom-light interaction point (i.e., single-photon atom optics). Use the non-relativistic phase shift analysis and ignore any delays due to the finite speed of light.

1. First we solve for the phase shift using perturbation theory. Work out the propagation phase difference between interferometer arms, treating gravity as a perturbation (i.e., calculate the atom propagation phase along *unperturbed* trajectories shown in the figure).
   *Hint: The atom propagation phases $\phi_{u,l}^{prop}$ along the upper and lower arms of the interferometer $(u, l)$ are given by $\phi_{u,l}^{prop} = \frac{1}{\hbar}\int_0^{2T} L_{u,l}(t)\,dt$, where $L_{u,l}$ is the perturbing Lagrangian.*

2. Now calculate the phase shift again, this time without using perturbation theory. Unlike in part 1, this means that, for example, the trajectories in the lab frame are parabolas. For simplicity, analyze the problem in the freely falling frame.
   *Hint: The atom propagation phase in the free-falling frame is zero (discuss why) - consider the acceleration of atoms towards the laser and the associated laser phase imprinted on the interferometer arms.*

3. Find an expression for the acceleration sensitivity (in $g/\sqrt{\mathrm{Hz}}$) assuming the phase noise is limited by atom shot noise. Assume an average detected atom flux of $n$ atoms per second.

4. Now assume the interferometer uses large momentum transfer (LMT) atom optics, with momentum $N\hbar k$ at each interaction. Show that the above phase shift due to gravity is $N$ times larger.
   *Hint: Consider your solutions to parts 1 and 2 - how does LMT change this picture?*

5. Calculate the phase shift (to lowest order) due to a gravity gradient. Assume the strength of gravity varies linearly as $g(z) = g + T_{zz}z$, where $T_{zz}$ is the gravity gradient.
   *Hint: Consider the gravitational potential energy in the presence of a uniform gravity gradient, and use a perturbative approach.*