QSFP 2021

"Atom interferometry and gravitational wave detection"

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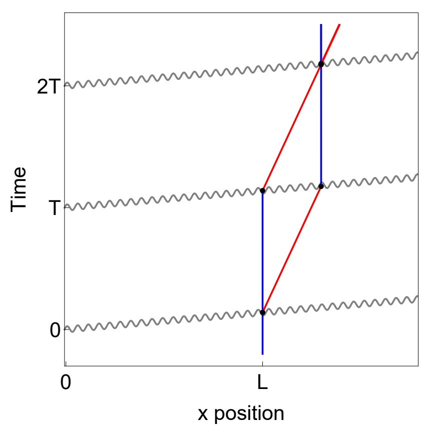
Tutorial 2: Gravitational wave sensitivity

Here we will calculate the atom interferometer phase shift due to a gravitational wave. In this problem we will make use of the relativistic treatment of atom interferometry. Consider the gravitation wave metric in the TT gauge:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 + h(t)(dx^2 - dy^2) \tag{1}$$

with $h(t) \equiv h\cos\left(\omega(t - \frac{z}{c}) + \phi_0\right)$ the dimensionless stain. In proposed detectors such as MAGIS, a pair of atom interferometers separated by a large baseline are used to make a differential measurement that suppresses laser technical noise. In this problem, we will begin by analyzing the phase shift of a single atom interferometer at position $x = L$ as shown in the figure below. A laser source is assumed to be located



at $x = 0$ on the left and emits pulse that propagate to the right at times 0, T, and $2T$ to implement the $\frac{\pi}{2} - \pi - \frac{\pi}{2}$ sequence. The paths of the light pulses are shown in gray. The resulting atom paths are shown for both the ground (blue) and excited (red) states. Assume the excited state has energy $\hbar\omega_A$ with respect to the ground state. The following trigonometric identities are useful for part 4:

$$2\sin(a+b) - \sin(a+2b) - \sin(a) = 4\sin^2\left(\frac{b}{2}\right)\sin(a+b)$$

$$\sin(A+B) - \sin(A) = 2\sin\left(\frac{B}{2}\right)\cos\left(\frac{B}{2} + A\right)$$

1. Find the null geodesic trajectory $x(t)$ for light propagating along the x-direction, to first order in the strain h, assuming initial conditions $x(t_0) = x_0$.

2. Invert the previous result to find the arrival time $t(x, x_0, t_0)$ of the light pulse at position x, assuming the pulse propagates along x from an initial position x_0, t_0. As before, use $h \ll 1$.

3. Calculate the atom propagation phases $\phi_{u,l} = \int_{u,l} L dt$ along the upper and lower interferometer arms, assuming three laser pulses interact with the atoms as shown in the figure and described in the following:

   (a) The first pulse begins at $x_0 = 0,\ t_0 = 0$, and when it reaches the atoms it puts the upper arm of the interferometer into the excited state while leaving the lower arm in the ground state

(b) The middle pulse begins at $x_0 = 0,\ t_0 = T$ and excites the lower arm while de-exciting the upper arm

(c) The final pulse begins at $x_0 = 0,\ t_0 = 2T$, de-exciting the lower arm while leaving the upper arm in the ground state.

Hint: The atom propagation phase accumulates as $\omega_A \Delta t_{u,l}$ along each arm, for time intervals $\Delta t_{u,l}$ spent in the excited state. Hint 2: when calculating the intersection points of the light and atom paths, you can neglect the motion of the atom due to the recoil velocity $v_r = \hbar k/m$ (i.e. assume that the atom position is fixed at $x = L$ for both the upper and the lower arm). What condition must hold for this approximation to be valid?

4. Show that, to first order in L, the interferometer phase is $\Delta\phi = 2hL\omega_A \sin^2(\omega T/2)\cos(\phi_0 + \omega T + \pi)$ (that is, assuming $L\omega/c \ll 1$).

5. Find the strain sensitivity $\overline{h}(\omega)$ (in strain per $\sqrt{\text{Hz}}$) assuming a phase noise amplitude spectral density of $\overline{\delta\phi}$ (in rad/$\sqrt{\text{Hz}}$). What atom flux (in atoms/second) is required to achieve a phase resolution of $\overline{\delta\phi} = 10^{-3}$ rad/$\sqrt{\text{Hz}}$, assuming the atom shot noise limit?