Atom interferometry and gravitational wave detection

Quantum Sensors for Fundamental Physics

QSFP School 2021

Jason Hogan
Stanford University
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Outline

Lecture 1

- General introduction/motivation
- Non-relativistic atom interferometer phase theory
- Example applications
- **Tutorial**: Accelerometer phase response

Lecture 2

- General relativistic phase shift theory
- GR effects and clock interferometry
- Gravitational wave detection and MAGIS
- Aharonov-Bohm phase shifts
- **Tutorial**: Gravitational wave phase response

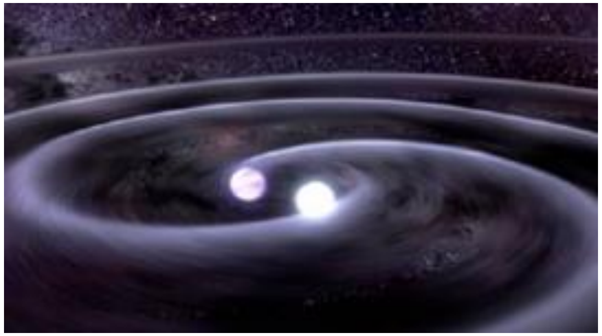
Lecture 3

- Advanced atom optics (large momentum transfer techniques)
- Systematic errors, backgrounds, and mitigations
- Supporting tools: matter wave lensing, optical lattices, phase shear readout

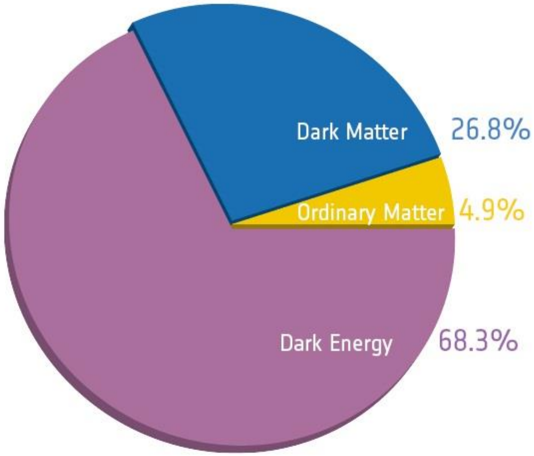
Lecture 1

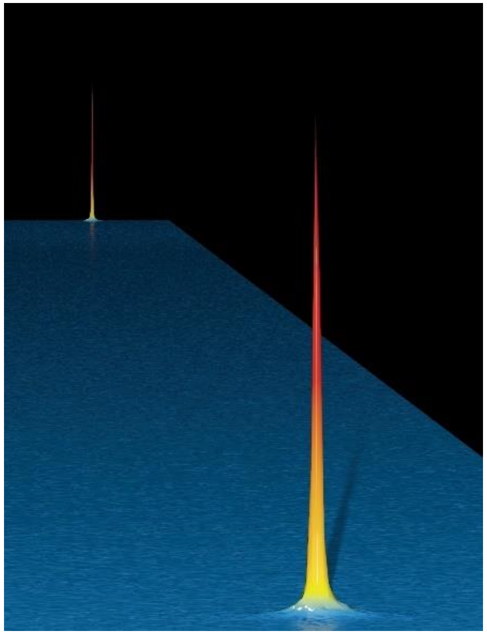
Science applications

- Gravitational wave detection
- Quantum mechanics at macroscopic scales
- QED tests (alpha measurements)
- Quantum entanglement for enhanced readout
- Equivalence principle tests, tests of GR
- Short distance gravity
- Search for dark matter
- Atom charge neutrality



Compact binary inspiral



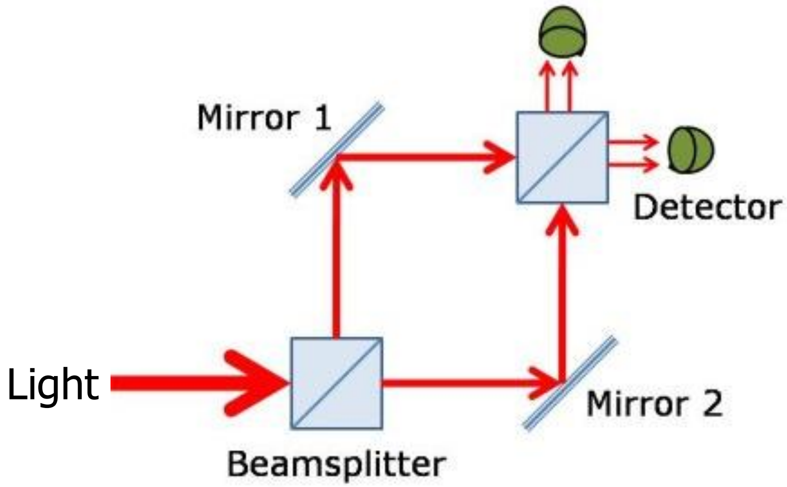


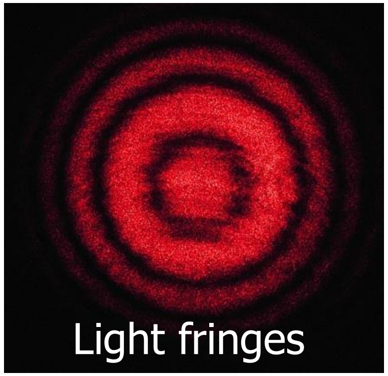
Rb wavepackets separated by 54 cm

Image: https://www2.physics.ox.ac.uk/research/dark-matter-dark-energy

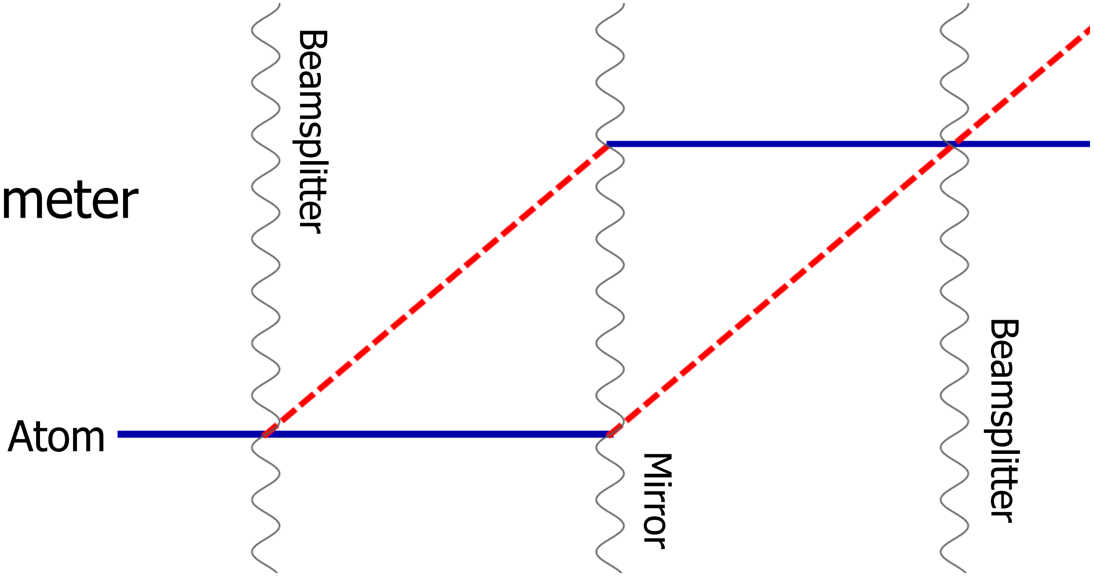
Atom interference

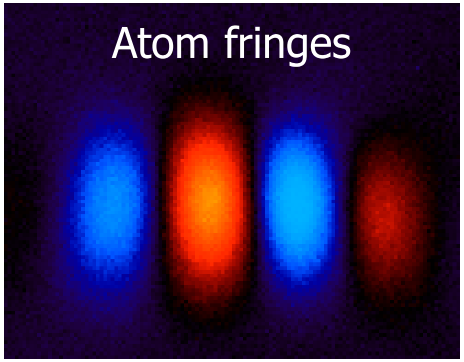
Light interferometer





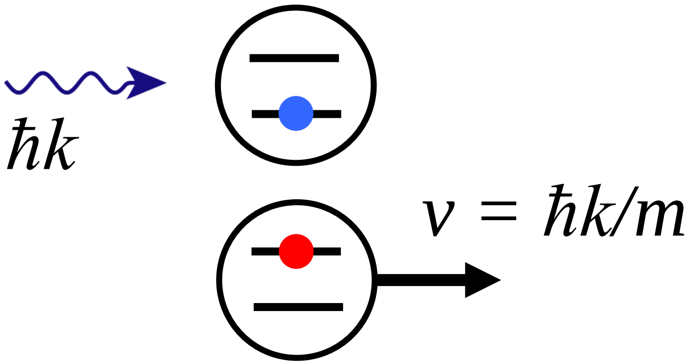
Atom interferometer



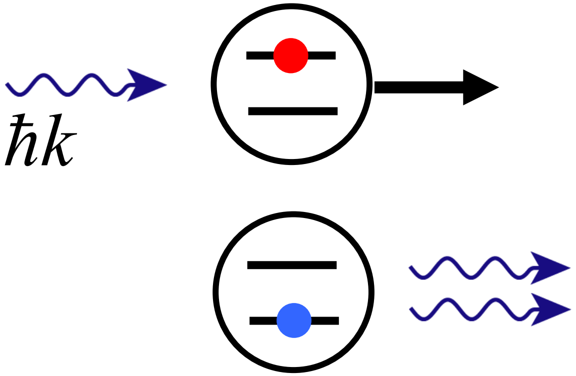


http://scienceblogs.com/principles/2013/10/22/quantum-erasure/
http://www.cobolt.se/interferometry.html

(1) Light absorption:

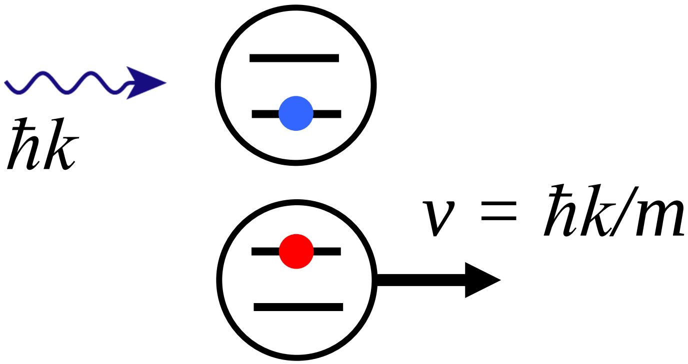


(2) Stimulated emission:

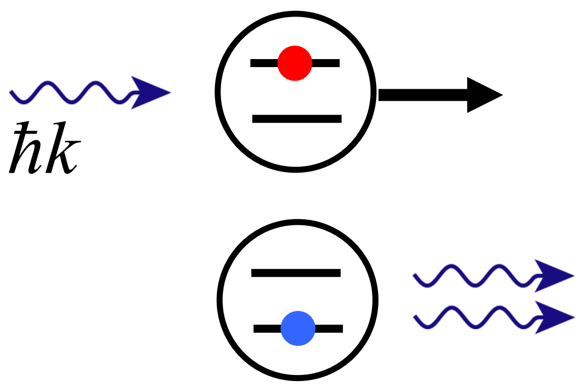


Atom optics using light

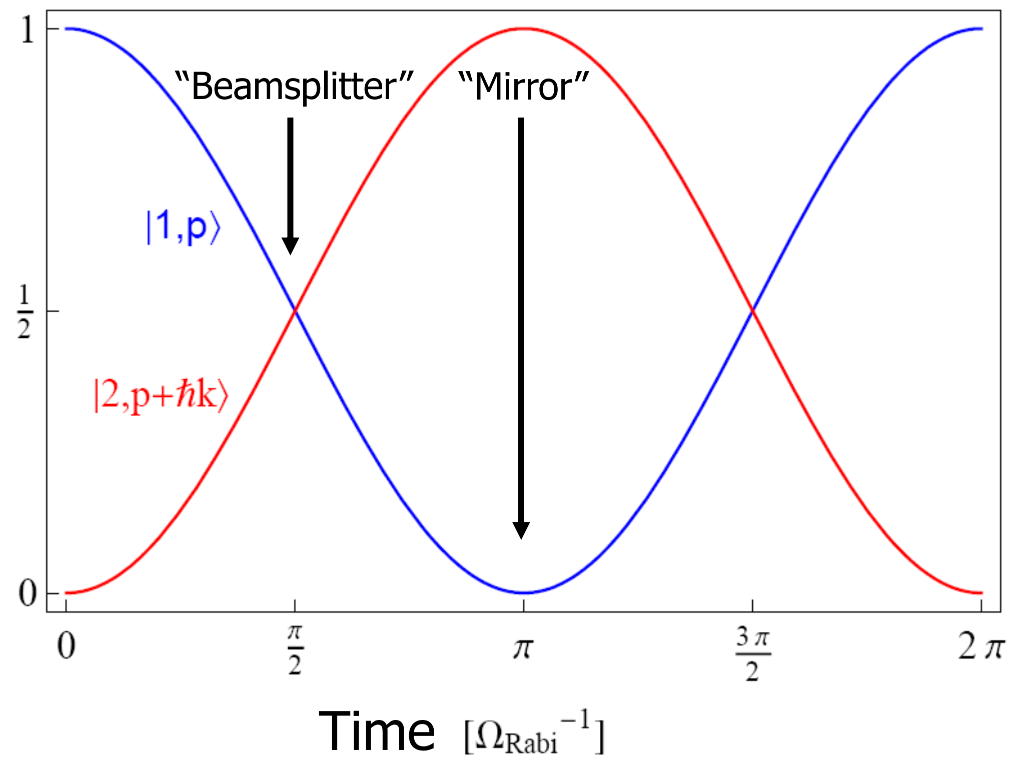
(1) Light absorption:



(2) Stimulated emission:



Rabi oscillations



Light Pulse Atom Interferometry

$$|\text{atom}\rangle = |p\rangle$$

π/2 pulse "beamsplitter"

$$\frac{1}{\sqrt{2}}(|p\rangle + e^{i\Delta\phi}|p + \hbar k\rangle)$$

π pulse "mirror"

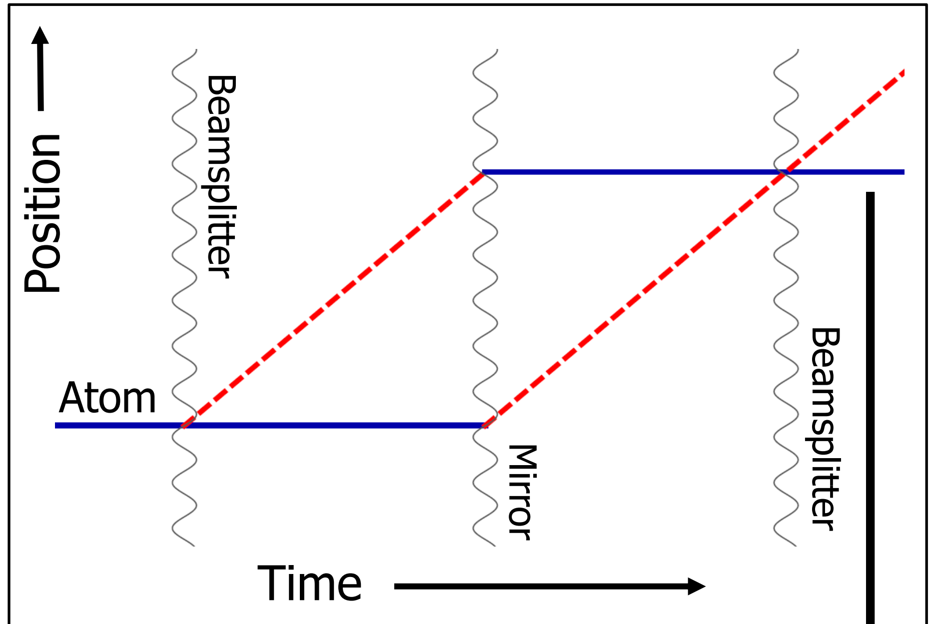
$$\frac{1}{\sqrt{2}}(|p + \hbar k\rangle + e^{i\Delta\phi}|p\rangle)$$

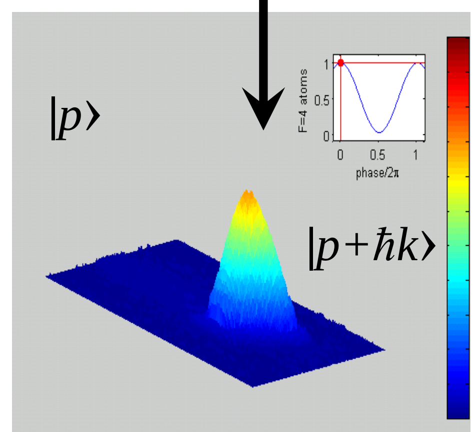
π/2 pulse "beamsplitter"

$$\frac{1}{2}((1 + e^{i\Delta\phi})|p\rangle + ((1 - e^{i\Delta\phi}))|p + \hbar k\rangle)$$

$$\text{Probability in State } |p\rangle = \cos^2\left(\frac{\Delta\phi}{2}\right)$$

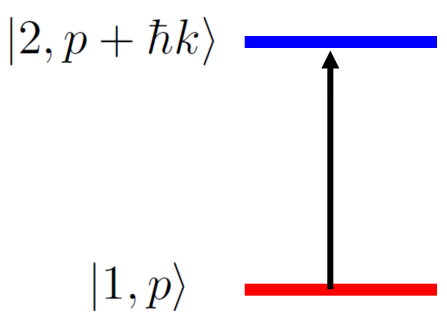
$$\text{Probability in State } |p + \hbar k\rangle = \sin^2\left(\frac{\Delta\phi}{2}\right)$$

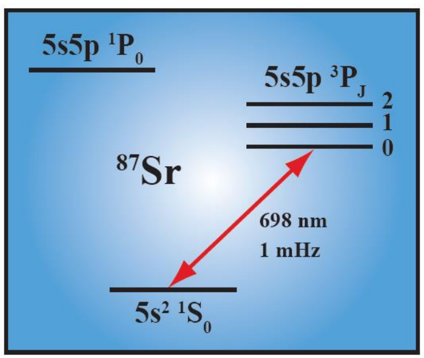




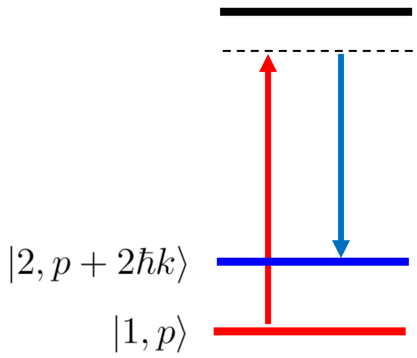
Common atom optics processes

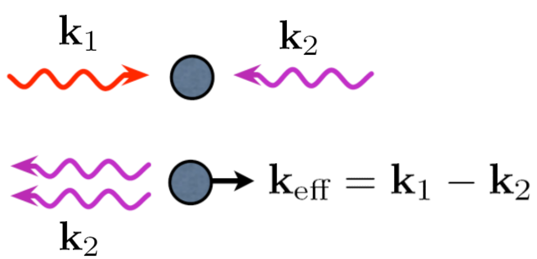
Single photon



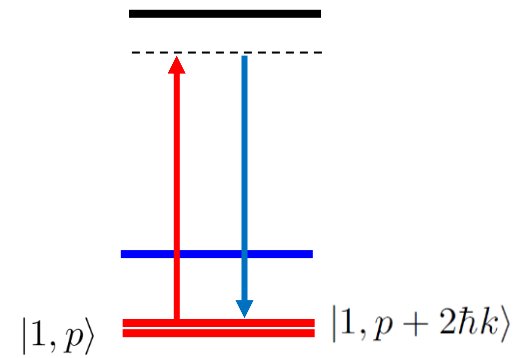


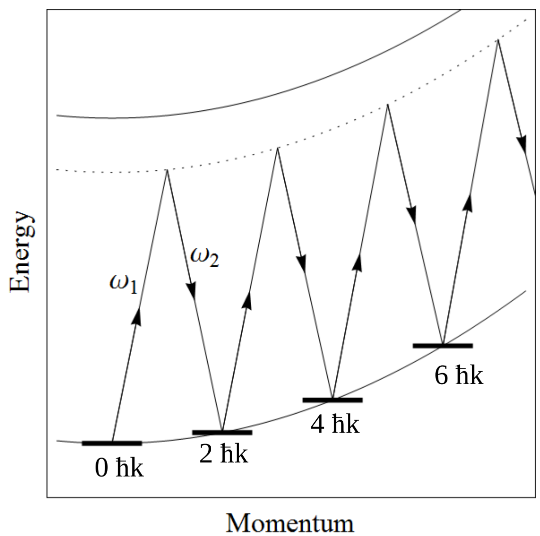
Raman





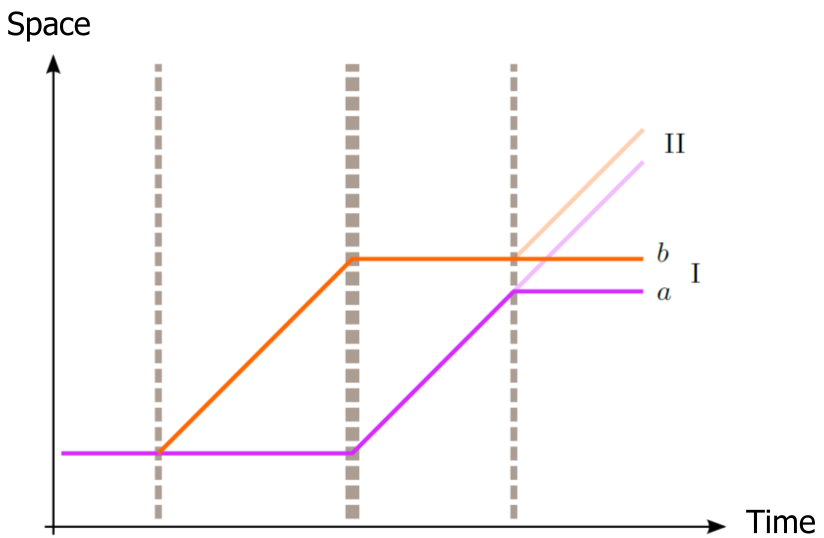
Bragg

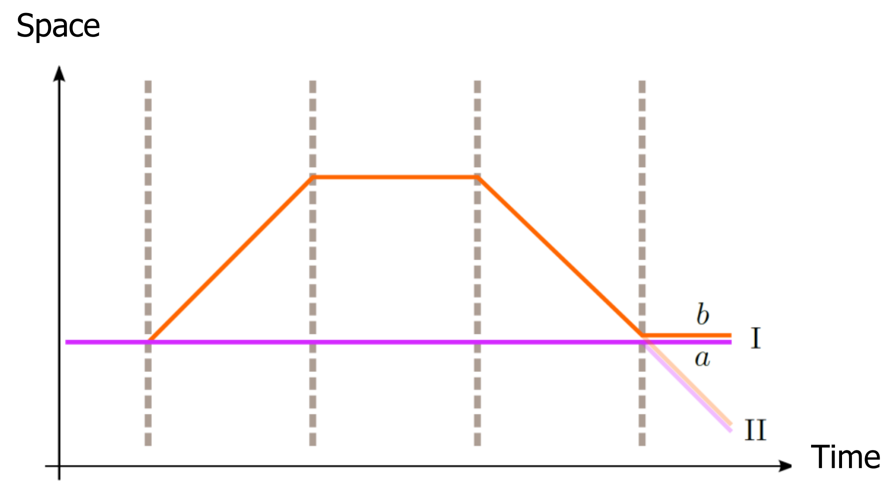




Spontaneous emission in alkali atoms require 2-photon atom optics

Example interferometer geometries





<u>Mach-Zehnder interferometer</u>

Phase shift measures acceleration

Example: Equivalence principle tests, inertial sensing

Also: Gyroscopes (space-space instead of space-time)

<u>Ramsey-Borde interferometer</u>

Phase shift measures kinetic energy difference (due to absorbed photons)

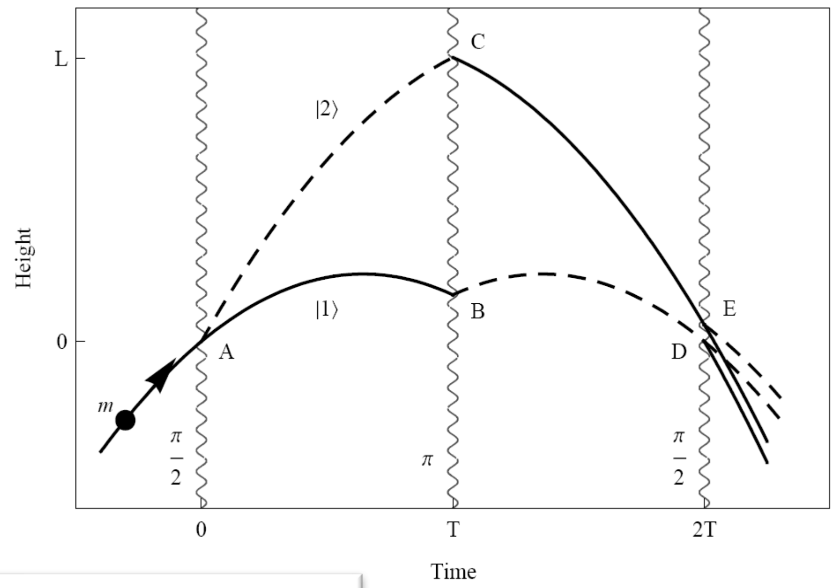
Example: fine structure constant measurements

Figures from: A. Roura, Phys. Rev. X 10, 021014 (2020)

Atom interferometer phase shift analysis

Phase shift can be decomposed as:

- **Laser** phase at each node
- **Propagation** phase along each path
- **Separation** phase at end of interferometer



$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{propagation}} + \Delta\phi_{\text{separation}} + \Delta\phi_{\text{laser}}$$

This approach mostly follows "Light-pulse atom interferometry" (2008), as well as Bongs/Kasevich (2006) and others.

Other approaches:

- C. Borde, ABCD formalism, e.g., Metrologia 39, 435-463, (2002)
- Storey, Cohen-Tannoudji. "The Feynman path integral approach to atomic interferometry. A tutorial" (1994)
- Representation-free approach: Kleinert (2015)
- Wigner function approach: Dubetsky (2006)

Non-relativistic phase shift calculation

$$\hat{H}_{\rm tot} = \hat{H}_{\rm a} + \hat{H}_{\rm ext} + \hat{V}_{\rm int}(\hat{\mathbf{x}})$$

Internal External Interaction

Internal: $i\partial_t |A_i\rangle = \hat{H}_{\rm a} |A_i\rangle = E_i |A_i\rangle \qquad |A_i\rangle = |i\rangle\, e^{-iE_i(t-t_0)}$

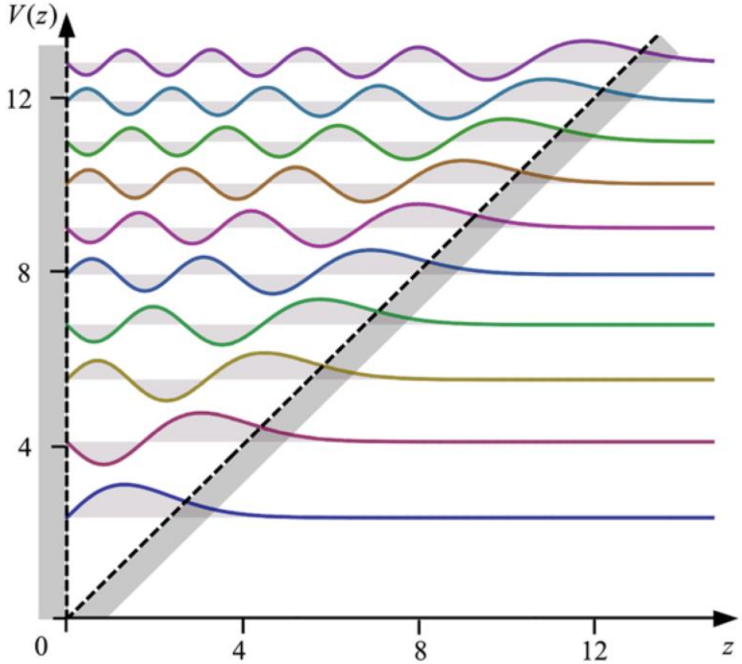
- Atomic energy levels
- No need to calculate, can look up, etc.

External: $i\partial_t |\psi\rangle = H(\hat{\mathbf{x}}, \hat{\mathbf{p}}) |\psi\rangle$

$\hat{H}_{\rm ext}$

- Includes kinetic energy $p^2/2m$
- mgz gravity
- Gravity gradients (quadratic and higher)
- Rotations (mix position and momentum)
- Magnetic field gradients
- …

External Hamiltonian: Eigenfunction analysis?



Example

$$V(z) = \begin{cases} mgz, & z \geq 0, \\ \infty, & z \leq 0, \end{cases}$$

$$u_E(z) = \mathcal{N}_E \cdot \mathrm{Ai}\left(\left(\frac{2}{m\hbar^2\tilde{g}^2}\right)^{1/3}[m\tilde{g}z - E]\right)$$

Airy functions

$$E_n = \left(\frac{m\hbar^2\tilde{g}^2}{2}\right)^{\frac{1}{3}} a_{n+1}$$

- Freely falling wavepacket is a superposition
- Different energy eignevalues results in time dependence of wavepacket…

Some issues

- “Tunneling into the classically forbidden region”
- “New” dependence on the inertial and gravitational mass? (but note the hard wall at z=0)
- How to handle higher order terms in the potential?

$$E_n = \left(\frac{1}{2}\hbar^2 g^2\right)^{\frac{1}{3}} m_g^{\frac{2}{3}} m_i^{-\frac{1}{3}} a_{n+1}$$

Possible approach, but not necessarily the most useful

Propagation phase

Time evolution of atom's state between laser pulses:

Galilean transformation operator: $\hat{G}_c \equiv \hat{G}(\mathbf{x}_c, \mathbf{p}_c, L_c) = e^{i\int L_c dt} e^{-i\hat{\mathbf{p}}\cdot\mathbf{x}_c} e^{i\mathbf{p}_c\cdot\hat{\mathbf{x}}}$

Phase Translation Boost

Initial state

$$\langle \mathbf{x}|\psi, A_i\rangle = \langle \mathbf{x}|\,\hat{G}_c\,|\phi_{CM}\rangle\,|A_i\rangle = e^{i\int_{t_I}^{t_F} L_c dt} e^{i\mathbf{p}_c\cdot(\mathbf{x}-\mathbf{x}_c)} \phi_{CM}(\mathbf{x}-\mathbf{x}_c)\,|i\rangle\, e^{-iE_i(t_F-t_I)}$$

External Internal

Action

Plane wave (carrier)

Wavepacket (envelope)

Internal

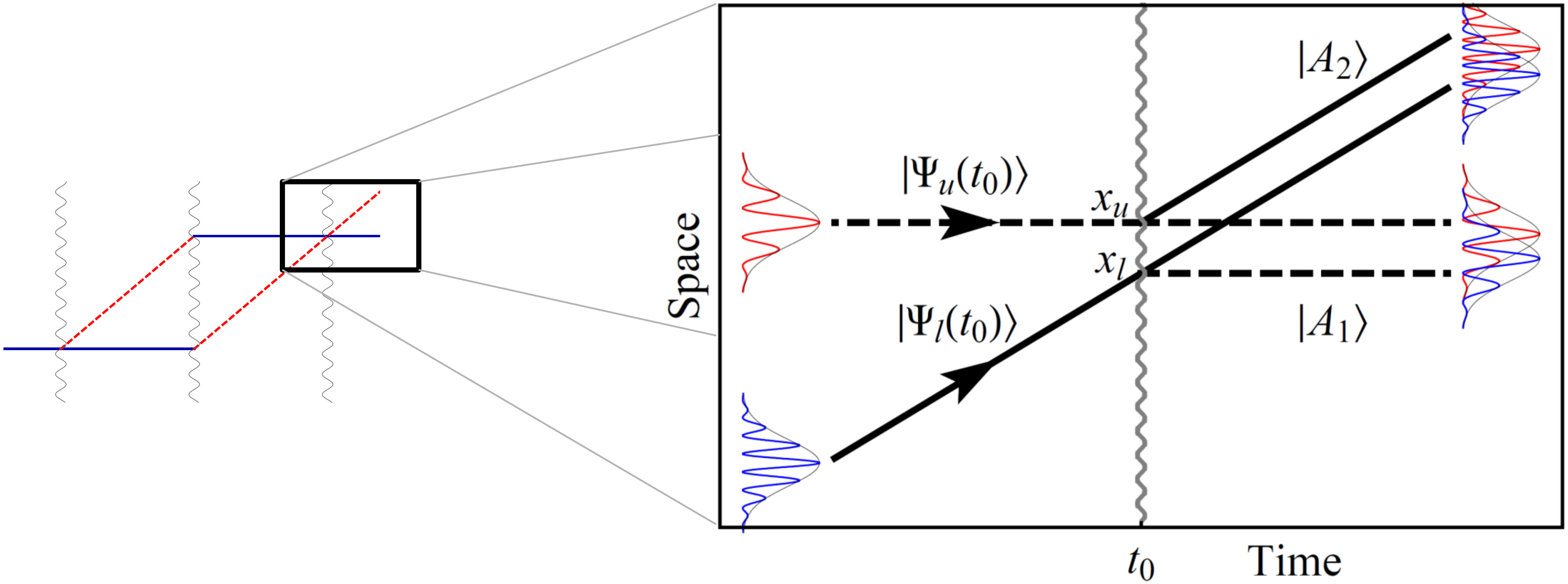
Propagation phase

- The phase of the center of the wavepacket is the classical action
- The carrier and wavepacket envelope move together along the classical path

$$\Delta\phi_{\text{propagation}} = \sum_{\text{upper}} \left(\int_{t_I}^{t_F} (L_c - E_i)dt \right) - \sum_{\text{lower}} \left(\int_{t_I}^{t_F} (L_c - E_i)dt \right)$$

Separation phase

Wavepackets do not always perfectly overlap at the final beamsplitter, due to tidal forces across wavepacket separation



$$\Delta\phi_{\text{separation}} = \bar{\mathbf{p}} \cdot \mathbf{\Delta x}$$

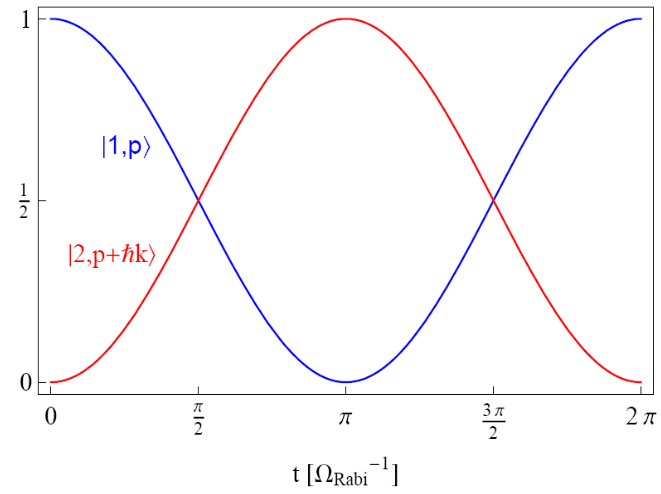
$$\mathbf{\Delta x} \equiv \mathbf{x}_l - \mathbf{x}_u$$

Conceptually similar to propagation phase; completes the loop.

Laser phase

$$|\Psi\rangle = \int d\mathbf{p} \sum_i c_i(\mathbf{p})\, |\psi_{\mathbf{p}}\rangle\, |A_i\rangle$$

Rabi oscillations



Atom-light interactions follow from Schrodinger equation (interaction picture):

$$\dot{c}_1(\mathbf{p}) = \frac{1}{2i}\Omega\, c_2(\mathbf{p}+\mathbf{k})e^{-i\phi_L}e^{-i\int_{t_0}^{t}\Delta(\mathbf{p})dt}$$

$$\dot{c}_2(\mathbf{p}+\mathbf{k}) = \frac{1}{2i}\Omega^* c_1(\mathbf{p})e^{i\phi_L}e^{i\int_{t_0}^{t}\Delta(\mathbf{p})dt}$$

Transition rules:

$$|\mathbf{p}\rangle \rightarrow |\mathbf{p}+\mathbf{k}\rangle\, e^{i\phi_L}$$

$$|\mathbf{p}+\mathbf{k}\rangle \rightarrow |\mathbf{p}\rangle\, e^{-i\phi_L}$$

$$\phi_L \equiv \mathbf{k}\cdot\mathbf{x}_c(t_0) - \omega t_0 + \phi$$

Atom position

- Laser phase is imprinted on the wavefunction at each pulse
- The position of the atom (at time of pulse) is encoded in the atom's wavefunction
- "Measures" the atom position with a wavelength-scale "ruler" → *corresponding momentum kick* (uncertainty principle)

Summary: Non-relativistic phase shift calculation

The atom interferometer phase shift can be decomposed as

$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{propagation}} + \Delta\phi_{\text{separation}} + \Delta\phi_{\text{laser}}$$

$$\Delta\phi_{\text{propagation}} = \sum_{\text{upper}} \left(\int_{t_I}^{t_F} (L_c - E_i) dt \right) - \sum_{\text{lower}} \left(\int_{t_I}^{t_F} (L_c - E_i) dt \right)$$

$$\Delta\phi_{\text{laser}} = \left(\sum_j \pm\phi_L(t_j, \mathbf{x}_u(t_j)) \right)_{\text{upper}} - \left(\sum_j \pm\phi_L(t_j, \mathbf{x}_l(t_j)) \right)_{\text{lower}}$$

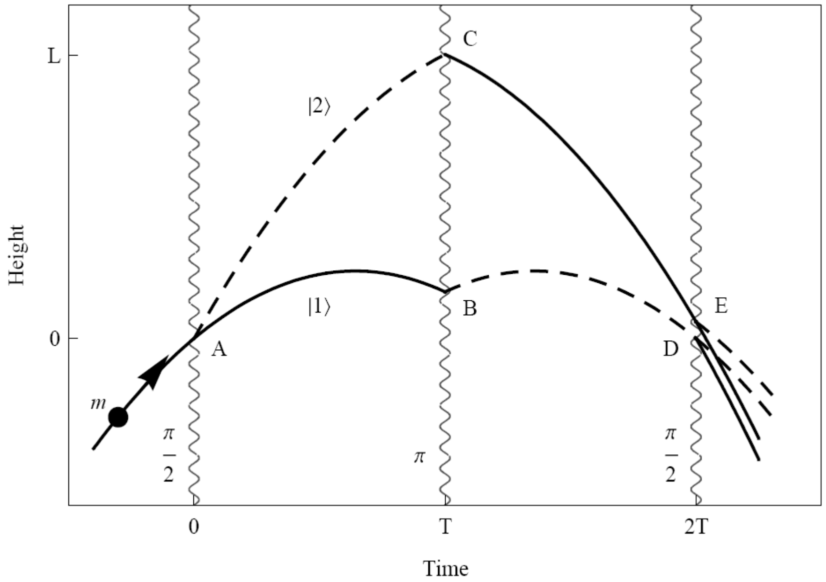
$$\Delta\phi_{\text{separation}} = \bar{\mathbf{p}} \cdot \mathbf{\Delta x}$$

Semi-classical phase shift analysis example

Three contributions:

- Laser phase at each node
- Propagation phase along each path
- Separation phase at end of interferometer

$$\Delta\phi_{\text{total}} = \Delta\phi_{\text{prop}} + \Delta\phi_{\text{laser}} + \Delta\phi_{\text{sep}}$$



Include all relevant forces in the classical Lagrangian:

$$L = \frac{1}{2}m(\dot{\mathbf{r}} + \mathbf{\Omega} \times (\mathbf{r} + \mathbf{R}_e))^2 - m\phi(\mathbf{r} + \mathbf{R}_e) - \frac{1}{2}\alpha \mathbf{B}(\mathbf{r})^2$$

Rotation of Earth — Gravity gradients, etc. — Magnetic field shifts

$$\phi(\mathbf{r} + \mathbf{R}_e) = -\left(\mathbf{g} \cdot \mathbf{r} + \frac{1}{2!}(T_{ij})r_i r_j + \frac{1}{3!}(Q_{ijk})r_i r_j r_k + \frac{1}{4!}(S_{ijkl})r_i r_j r_k r_l\right)$$

Example phase shift result

Phase shifts (3-pulse accelerometer)

$$\Delta\phi_{\text{propagation}} = \frac{1}{\hbar}((S_{AC}+S_{CE})-(S_{AB}+S_{BD}))$$
$$\Delta\phi_{\text{laser}} = \phi_L(\mathbf{r}_A,0)-\phi_L(\mathbf{r}_C,T)-\phi_L(\mathbf{r}_B,T)+\phi_L(\mathbf{r}_D,2T)$$
$$\Delta\phi_{\text{separation}} = \frac{1}{2\hbar}(\mathbf{p}_D+\mathbf{p}_E)\cdot(\mathbf{r}_D-\mathbf{r}_E)$$

Solve using power series trajectories

$$r_i(t) = \sum_{n=0}^{N} a_{in}(t-t_0)^n$$

Includes gravity gradients, rotation (Coriolis forces), magnetic forces

Phase shift	Size (rad)	Fractional size
$-k_{\text{eff}}gT^2$	-2.85×10^{8}	1.00
$k_{\text{eff}}R_e\Omega_y^2T^2$	6.18×10^{5}	2.17×10^{-3}
$-k_{\text{eff}}T_{zz}v_zT^3$	1.58×10^{3}	5.54×10^{-6}
$\frac{7}{12}k_{\text{eff}}gT_{zz}T^4$	-9.21×10^{2}	3.23×10^{-6}
$-3k_{\text{eff}}v_z\Omega_y^2T^3$	-5.14	1.80×10^{-8}
$2k_{\text{eff}}v_x\Omega_yT^2$	3.35	1.18×10^{-8}
$\frac{7}{4}k_{\text{eff}}g\Omega_y^2T^4$	3.00	1.05×10^{-8}
$-\frac{7}{12}k_{\text{eff}}R_eT_{zz}\Omega_y^2T^4$	2.00	7.01×10^{-9}
$-\frac{\hbar k_{\text{eff}}^2}{2m}T_{zz}T^3$	7.05×10^{-1}	2.48×10^{-9}
$\frac{3}{4}k_{\text{eff}}gQ_{zzz}v_zT^5$	9.84×10^{-3}	3.46×10^{-11}
$-\frac{7}{12}k_{\text{eff}}Q_{zzz}v_z^2T^4$	-7.66×10^{-3}	2.69×10^{-11}
$-\frac{7}{4}k_{\text{eff}}R_e\Omega_y^4T^4$	-6.50×10^{-3}	2.28×10^{-11}
$-\frac{7}{4}k_{\text{eff}}R_e\Omega_y^2\Omega_z^2T^4$	-3.81×10^{-3}	1.34×10^{-11}
$-\frac{31}{120}k_{\text{eff}}g^2Q_{zzz}T^6$	-3.39×10^{-3}	1.19×10^{-11}
$-\frac{3\hbar k_{\text{eff}}^2}{2m}\Omega_y^2T^3$	-2.30×10^{-3}	8.06×10^{-12}
$\frac{1}{4}k_{\text{eff}}T_{zz}^2v_zT^5$	2.19×10^{-3}	7.68×10^{-12}
$-\frac{31}{360}k_{\text{eff}}gT_{zz}^2T^6$	-7.53×10^{-4}	2.65×10^{-12}
$3k_{\text{eff}}v_y\Omega_y\Omega_zT^3$	2.98×10^{-4}	1.05×10^{-12}
$-k_{\text{eff}}\Omega_y\Omega_zy_0T^2$	-7.41×10^{-5}	2.60×10^{-13}
$-\frac{3}{4}k_{\text{eff}}R_eQ_{zzz}v_z\Omega_y^2T^5$	-2.14×10^{-5}	7.50×10^{-14}
$\frac{31}{60}k_{\text{eff}}gR_eQ_{zzz}\Omega_y^2T^6$	1.47×10^{-5}	5.17×10^{-14}
$\frac{3}{2}k_{\text{eff}}T_{zz}v_z\Omega_y^2T^5$	-1.42×10^{-5}	5.00×10^{-14}
$-\frac{7}{6}k_{\text{eff}}T_{zz}v_x\Omega_yT^4$	1.08×10^{-5}	3.81×10^{-14}
$-2k_{\text{eff}}T_{xx}\Omega_yx_0T^3$	-6.92×10^{-6}	2.43×10^{-14}
$-\frac{7\hbar k_{\text{eff}}^2}{12m}Q_{zzz}v_zT^4$	-6.84×10^{-6}	2.40×10^{-14}
$-\frac{7}{6}k_{\text{eff}}T_{xx}v_x\Omega_yT^4$	-5.42×10^{-6}	1.90×10^{-14}
$-\frac{31}{60}k_{\text{eff}}gT_{zz}\Omega_y^2T^6$	4.90×10^{-6}	1.72×10^{-14}
$k_{\text{eff}}T_{xx}v_z\Omega_y^2T^5$	4.75×10^{-6}	1.67×10^{-14}
$\frac{3\hbar k_{\text{eff}}^2}{8m}gQ_{zzz}T^5$	4.40×10^{-6}	1.55×10^{-14}
$\frac{31}{360}k_{\text{eff}}R_eT_{zz}^2\Omega_y^2T^6$	1.63×10^{-6}	5.74×10^{-15}
$-\frac{31}{90}k_{\text{eff}}gT_{xx}\Omega_y^2T^6$	-1.63×10^{-6}	5.74×10^{-15}
$\frac{\hbar k_{\text{eff}}^2}{8m}T_{zz}^2T^5$	9.78×10^{-7}	3.43×10^{-15}
$-\frac{\hbar k_{\text{eff}}\alpha B_0(\partial_zB)T^2}{m}$	-7.67×10^{-8}	2.69×10^{-16}
$\frac{31}{60}k_{\text{eff}}gS_{zzzz}v_z^2T^6$	-7.52×10^{-8}	2.64×10^{-16}
$-\frac{1}{4}k_{\text{eff}}S_{zzzz}v_z^3T^5$	3.64×10^{-8}	1.28×10^{-16}
$\frac{31}{72}k_{\text{eff}}T_{zz}Q_{zzz}v_z^2T^6$	-3.13×10^{-8}	1.10×10^{-16}

Perturbative approach

The Feynman path integral approach to atomic interferometry. A tutorial

Pippa Storey and Claude Cohen-Tannoudji

(Received 22 September 1994, accepted 26 September 1994)

$$L = L_0 + \epsilon L_1$$ ← Any perturbing Lagrangian: magnetic fields, gravity, …

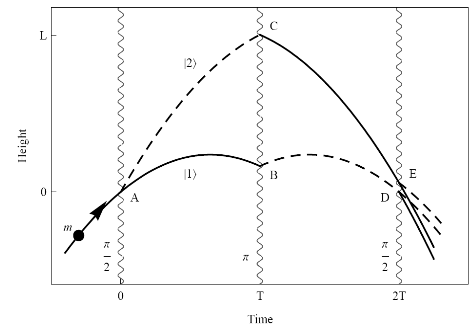
Can show (to first order in perturbation)

$$\delta\phi = \frac{\epsilon}{\hbar}\int_{\Gamma^{(0)}_{\mathrm{cl}}} L_1 \, \mathrm{d}t. \qquad \text{(to leading order)}$$

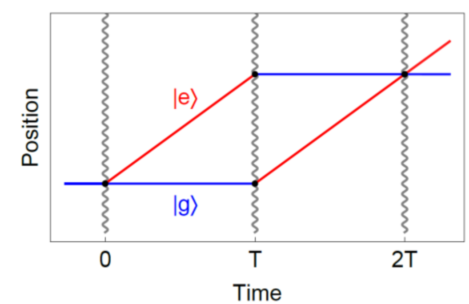
↑ *unperturbed* path

- Ignore affect of the perturbation on the atom trajectories
- Simple way to estimate leading order phase response
- Does not capture higher order effects

Perturbed paths

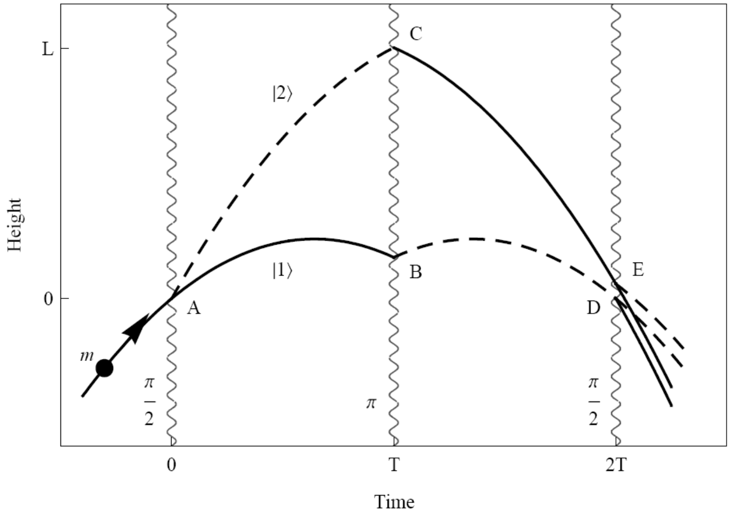


Unperturbed paths



Mach-Zehnder as discrete derivative sensor

Simple picture: Atom interferometer records the positions of the atom with respect to a wavelength-scale “laser ruler”



$$\phi_L \equiv \mathbf{k}\cdot\mathbf{x}_c(t_0) - \omega t_0 + \phi$$

- For Mach-Zehnder, propagation + separation phase tend to cancel

  …Except for high-order potentials (e.g., Aharonov Bohm effects)

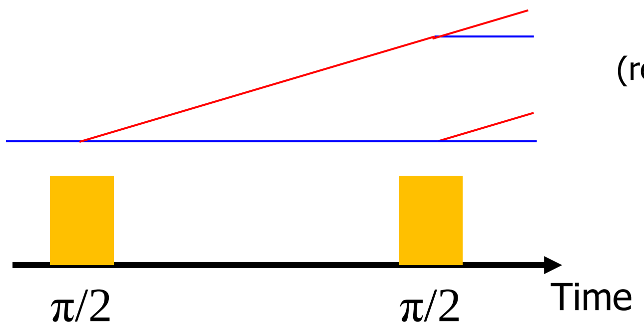
  See Overstreet et al., Am. J. Phys. 89, 324 (2021)

- Laser phase records the position of the atom at each pulse
- Total phase encodes differences (motion) between pulses
- “Discrete derivative sensor”: Records any spatial (or temporal) variation of atom (or background fields).

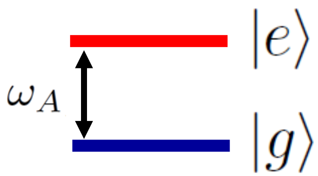
Two pulse atomic clock sequence

Atomic clocks are closely related to atom interferometers

Consider a *microwave* atomic clock (Ramsey sequence)



(recoil exaggerated)



Laser phase: $\phi_L \equiv \mathbf{k}\cdot\mathbf{x}_c(t_0) - \omega t_0 + \phi$

Can ignore separation phase (recoil is negligible for a microwave transition)

$$\Delta\phi = \Delta\phi_{\text{prop}} + \Delta\phi_{\text{laser}} = \omega_A T + \phi_1 - \phi_2$$

$$= (\omega - \omega_A)T + kx_1 - kx_2$$

$$= (\omega - \omega_A)T + kvT$$

(atom velocity v)

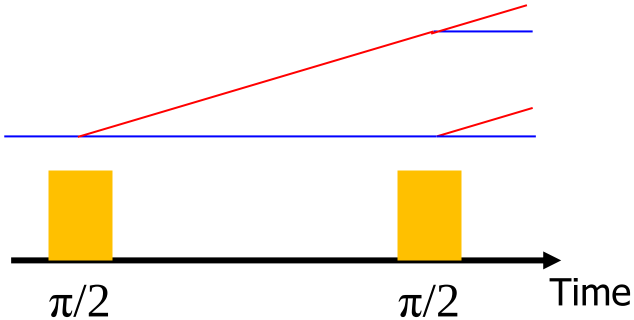
Sensitive to atom velocity (Doppler shift)

For atom interferometers with optical transitions, recoil must be managed

→ Requires more pulses

Atom interferometer as discrete derivative sensor

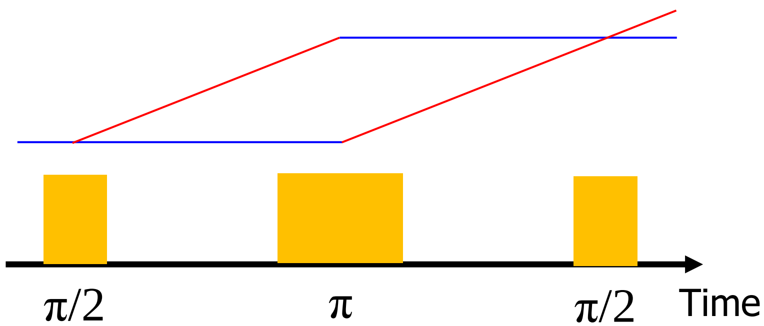
Ramsey sequence (clock)



$$\Delta\phi = \phi_1 - \phi_2 = (\omega - \omega_A)T + kx_1 - kx_2$$
$$= (\omega - \omega_A)T + \underline{kvT}$$

- Measures velocity

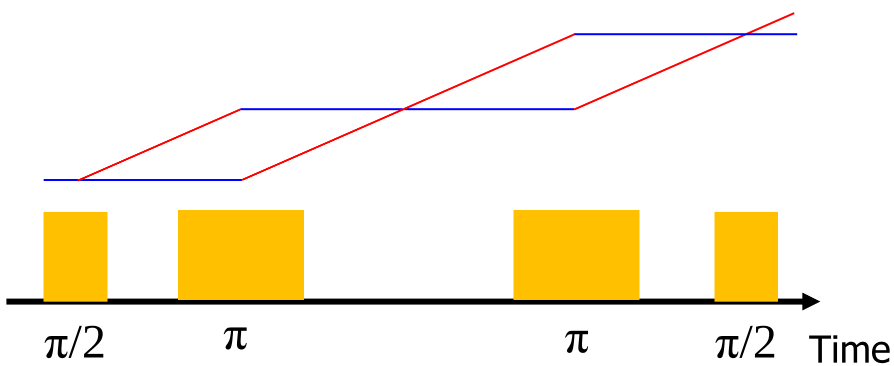
Mach-Zehnder



$$\Delta\phi = (\phi_1 - \phi_2) - (\phi_2 - \phi_3)$$
$$= kv_1T - kv_2T = \underline{kaT^2}$$

- "Difference" of two Ramsey sequences
- Measures acceleration

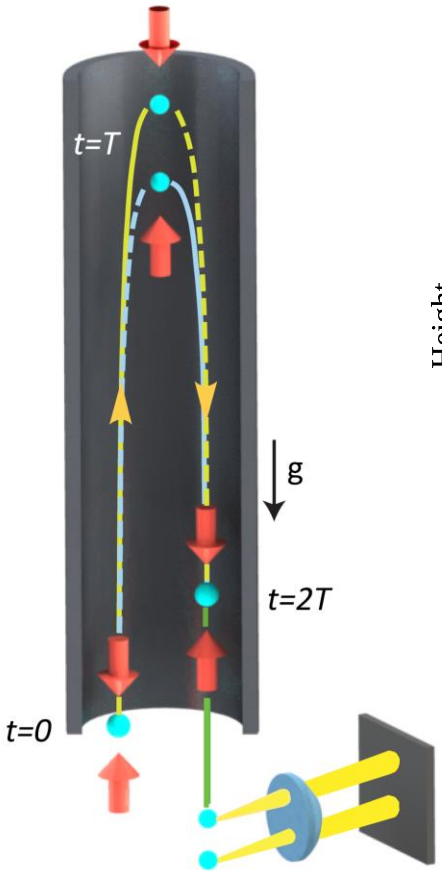
"Double diamond"

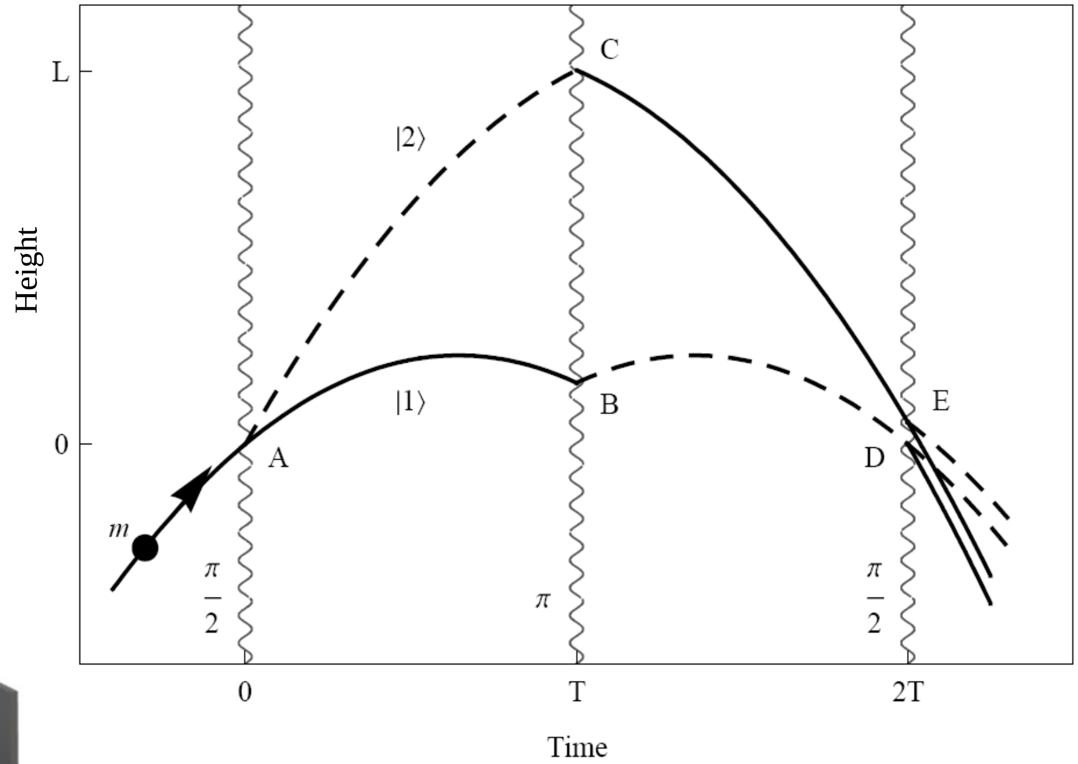


$$\Delta\phi = ka_1T^2 - ka_2T^2 = k\,\delta a\,T^3$$

- Difference of two MZ loops
- Measures acceleration gradient (in space and/or time)

Accelerometer sensitivity





$$\Delta\phi = k_{\text{eff}}\, g T^2$$

Proportional to spacetime area enclosed.

$$\frac{\delta g}{g} \sim \frac{\delta \phi}{k_{\text{eff}}\, g T^2}$$

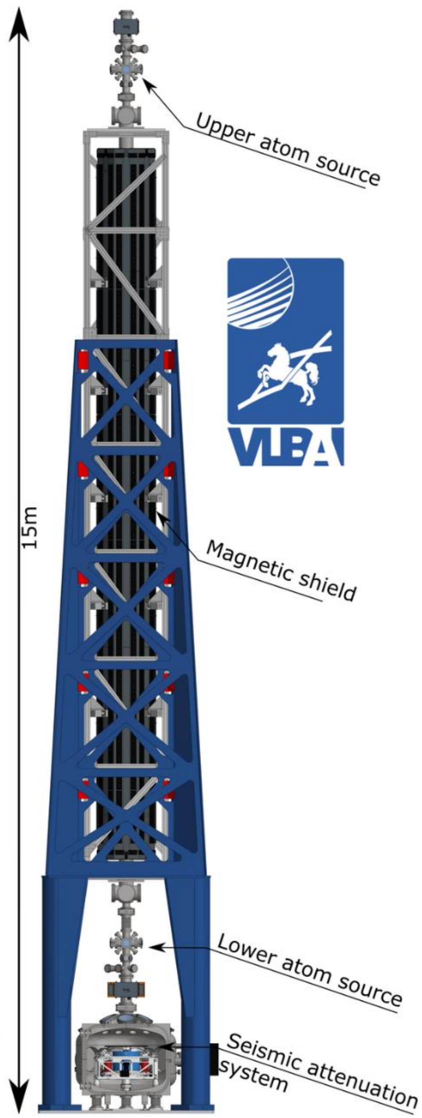
Sensitivity

$$\delta\phi \sim \frac{1}{\sqrt{N}}$$

Shot noise

T :	Long duration
k_{eff} :	Large wavepacket separation
$\delta\phi$:	High flux, spin squeezing

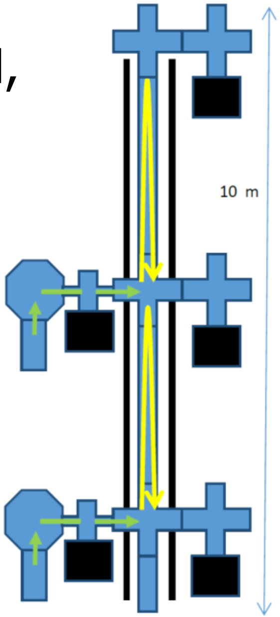
10-meter scale atom drop towers



Hannover, Germany

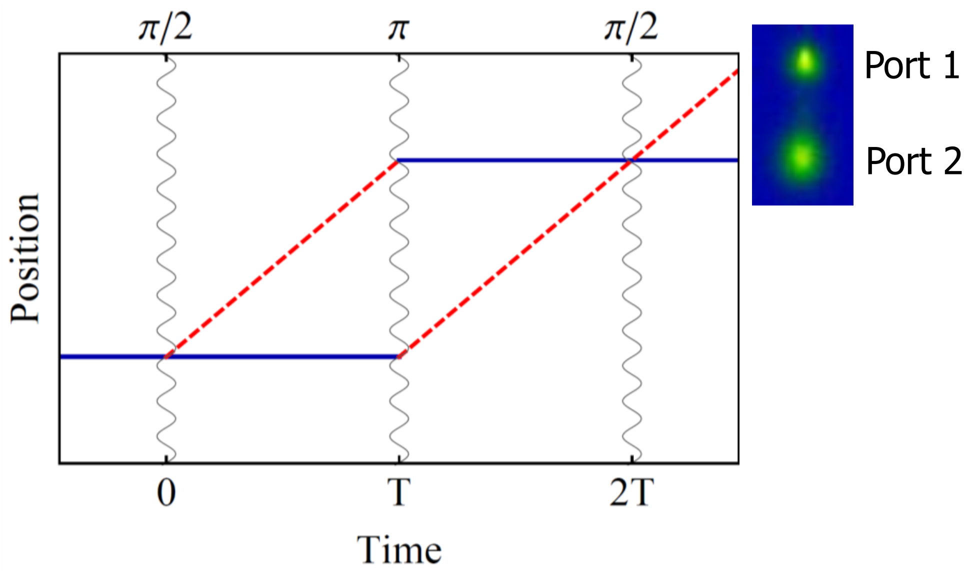
Wuhan, China

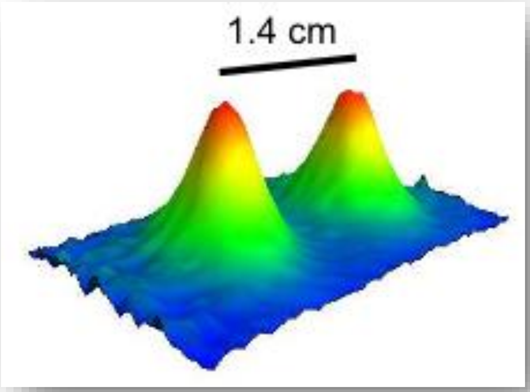
AION, UK



Stanford University

Interference at long interrogation time



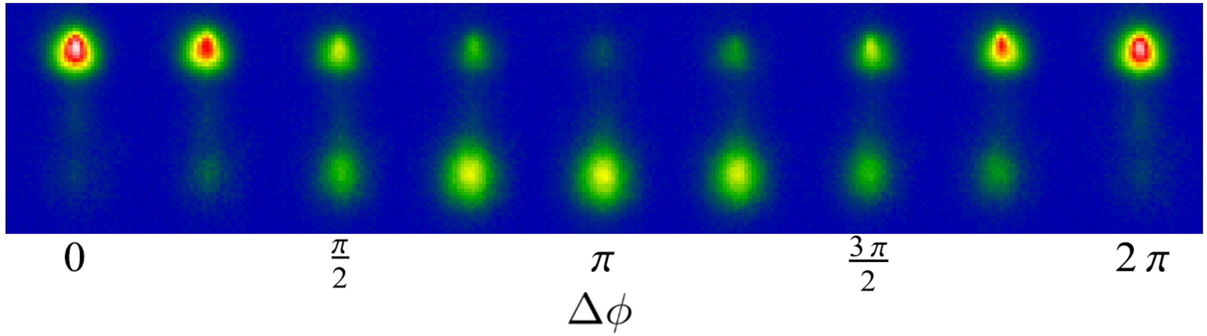


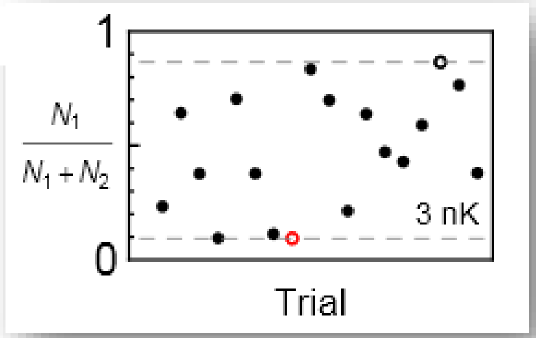
Wavepacket separation at apex (this data 50 nK)

2T = 2.3 seconds

1.4 cm wavepacket separation

Interference (3 nK cloud)





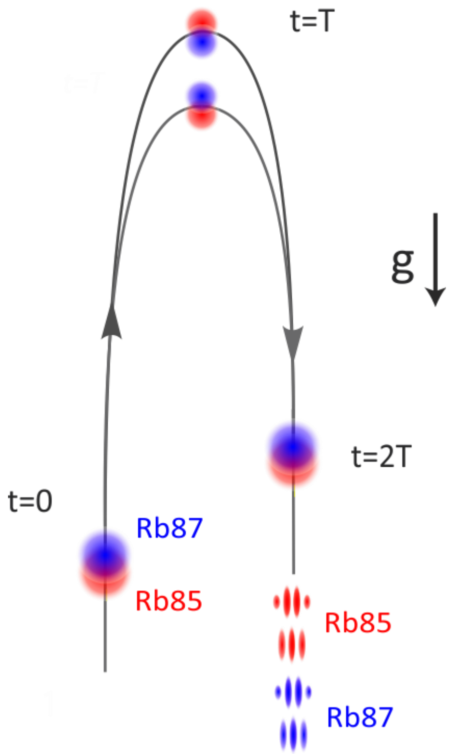
Dickerson, et al., PRL **111**, 083001 (2013).

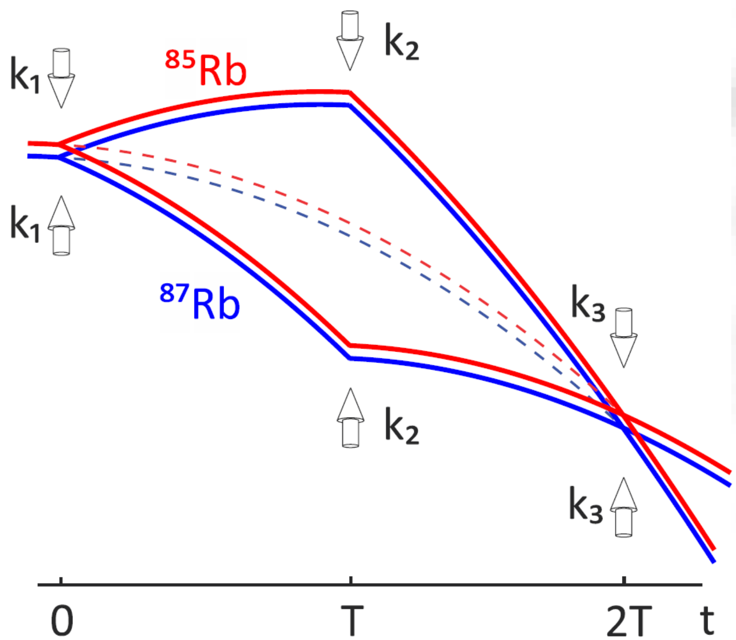
Example Applications

- Tests of the equivalence principle
- Search for new forces
- Measurements of the fine structure constant α
- Measurements of the gravitational constant G
- Gravitational wave detection
- Dark matter detection
- Testing atom charge neutrality
- Tests of quantum mechanics

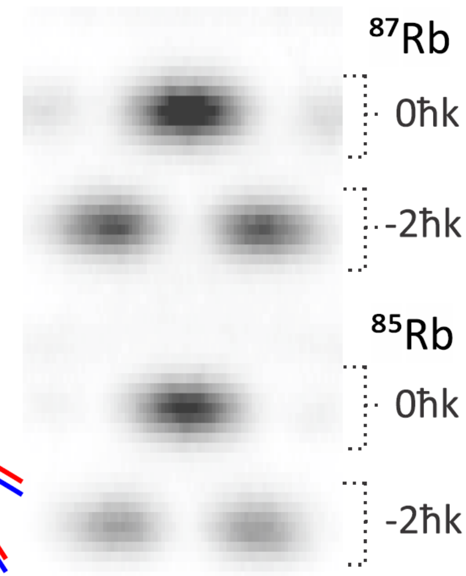
10-meter fountain equivalence principle test

Simultaneous Dual Interferometer





Dual interferometer fringes



Recent results:

$$\eta = [1.6 \pm 1.8\ (\mathrm{stat}) \pm 3.4\ (\mathrm{sys})] \times 10^{-12}$$

+ Laser system upgrade to improve sensitivity further!

Kasevich group, Stanford

Asenbaum et al., Phys. Rev. Lett. 125, 191101 (2020)

New force tests

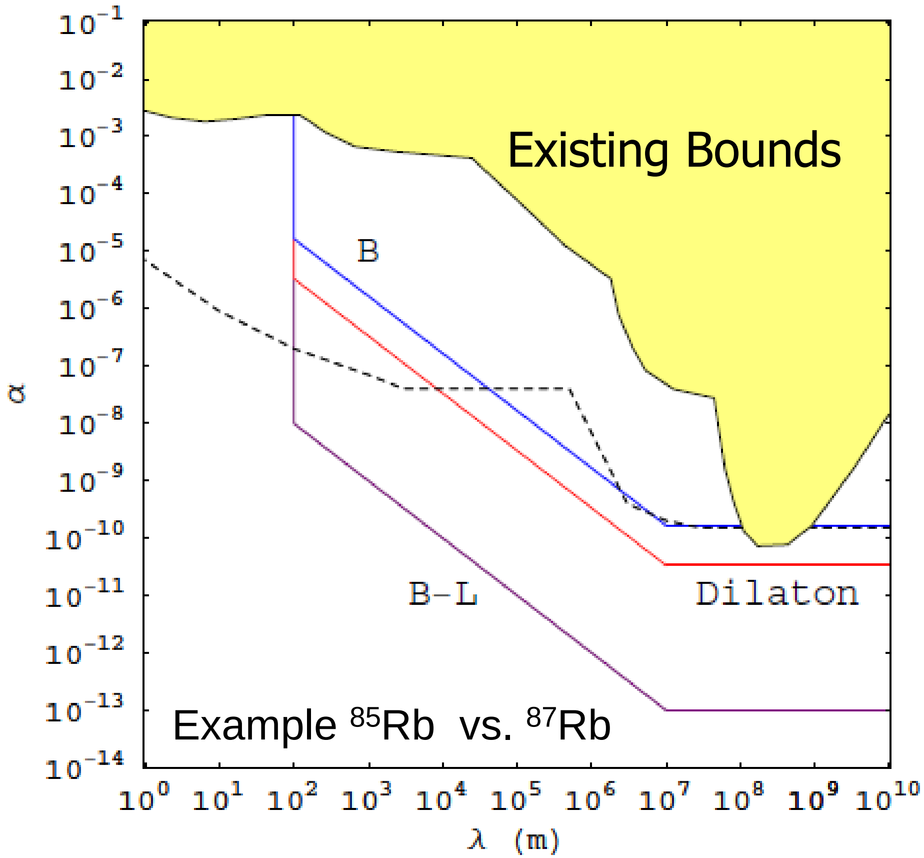
Violations of EP due to "fifth forces"

Yukawa type:

$$V(r) = -\frac{GM_1M_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

EP tests are sensitive to "charge" differences of new forces

Typically, new forces violate EP



Fine Structure Constant Measurement

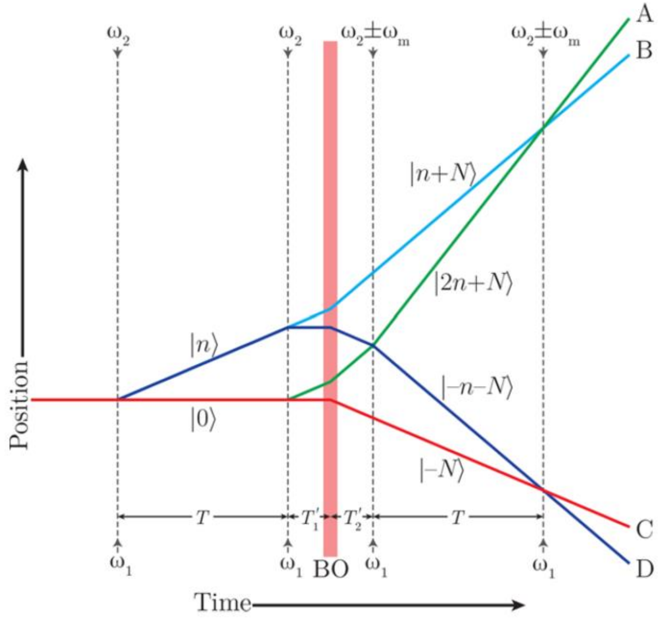
Measure the fine structure constant α to test QED

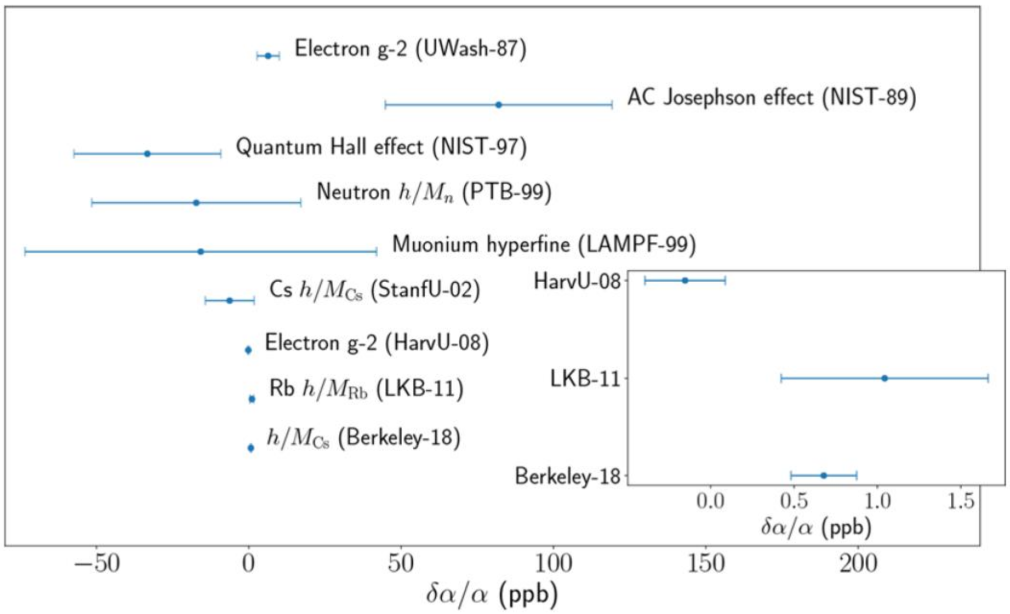
$$\frac{1}{\alpha} = 137.035999046(27)$$

- Ramsey-Borde sequence phase sensitive to the recoil frequency: $16n(n+N)\omega_r T$

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_{\mathrm{At}}}{m_e}\frac{h}{m_{\mathrm{At}}}$$

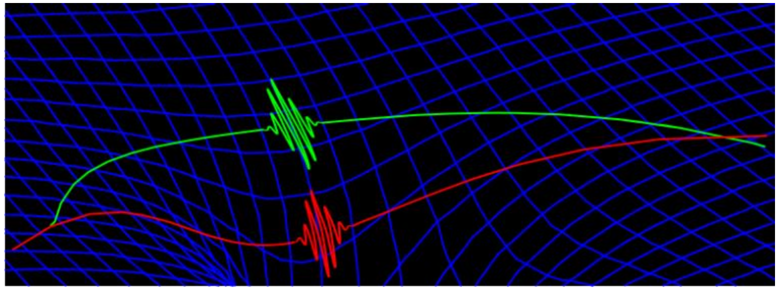
- Use recoil measurement to determine h/m: $\omega_r = \frac{\hbar k^2}{2m_{\mathrm{At}}}$





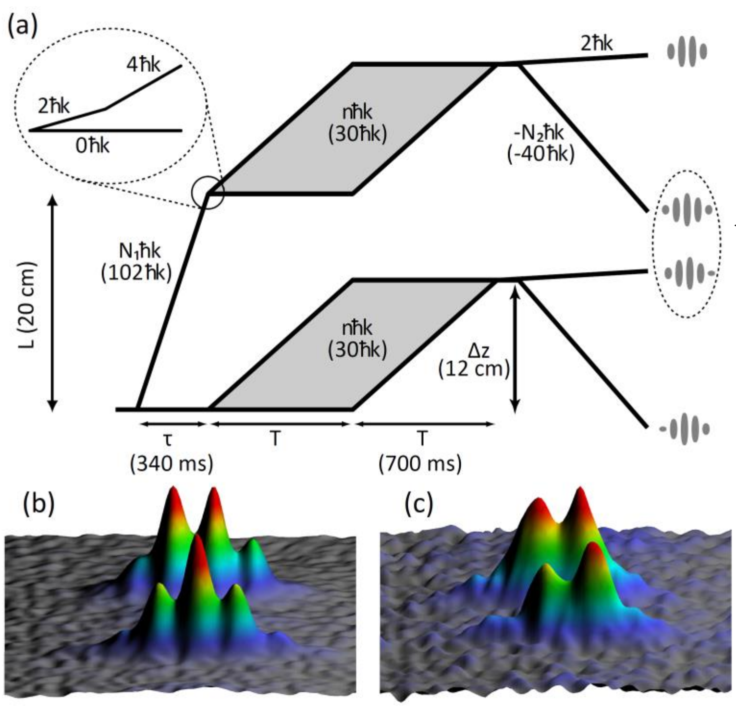
"Measurement of the fine-structure constant as a test of the Standard Model," R. H. Parker, et al., Science 360, 191-195 (2018)

Phase shift from spacetime curvature



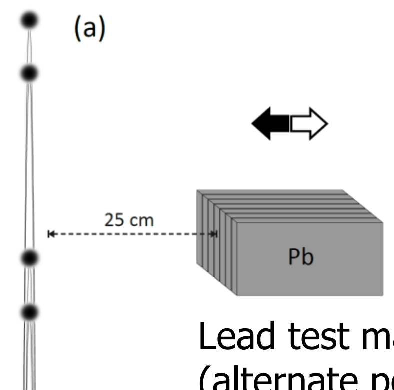
In GR, 'true' gravity is **spacetime curvature** (a uniform acceleration can be transformed away)

Gravity Gradiometer



Measure *phase difference*

Gradiometer interference fringes



Lead test mass (alternate position)

Observed of the effect of spacetime curvature across a single particle's wavefunction

Asenbaum et al., PRL **118**, 183602 (2017)