Tutorial 1: Neutrino detection

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β-decay: Fermi Theory (1)

We can derive the electron spectrum of the process $n \rightarrow p + e^- + \bar{\nu}_e$ from first principles in Quantum mechanics:

- Assume free particle states:

$$|i\rangle = |n\rangle \qquad \Rightarrow \qquad \psi_i(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_n \cdot \vec{x}_1}$$

$$|f\rangle = |p, e^-, \bar{\nu}_e\rangle \qquad \Rightarrow \qquad \psi_f(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_p \cdot \vec{x}_2}\; e^{i\,\vec{p}_e \cdot \vec{x}_3}\; e^{i\,\vec{p}_\nu \cdot \vec{x}_4}$$

- We can assume the interaction to be point-like:

$$V_W(x_1, x_3, x_3, x_4) = G_F\; \delta^{(3)}(\vec{x}_1 - \vec{x}_2)\; \delta^{(3)}(\vec{x}_2 - \vec{x}_3)\; \delta^{(3)}(\vec{x}_3 - \vec{x}_4)$$

New interaction constant

β-decay: Fermi Theory (2)

The matrix element for this process is given by

$$\langle i\,|\,V_W\,|\,f\rangle = \int d^3x_1 \ldots d^3x_4\ \psi_i^*\ V_W(x_1, x_2, x_3, x_4)\ \psi_f$$

Inserting the expression and integrating over x_1, x_2, x_3 yields

$$\langle i\,|\,V_W\,|\,f\rangle = G_F \int d^3x^4\ e^{i\,(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n)\cdot \vec{x}_4} = (2\pi)^3\ G_F\ \delta^{(3)}(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n)$$

With the matrix element we can compute the decay rate of the process using **Fermi's golden rule**:

$$d\Gamma_{if} = 2\pi\ |\,V_{if}|^2\ \rho(E_i)$$

With the density of states

$$\rho(E_i) = \delta(Q - E_e - E_\nu)\ \frac{d^3p_p}{(2\pi)^3}\frac{d^3p_e}{(2\pi)^3}\frac{d^3p_\nu}{(2\pi)^3} \qquad \textbf{with} \quad Q = m_n - m_p$$

β-decay: Fermi Theory - Exercise

Exercise 1. Using **Fermi's golden rule** find an expression for the electron spectrum in the beta decay, *i.e.* $d\Gamma_{if}/dE_e$

Hint 1: Remember that $\int dx\ \delta(x-a)\ f(x) = f(a)$

Hint 2: Integrate first over the proton momentum p_p and use the relation

$$\int d^3p_p\ \delta^{(3)}(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n)\ \delta^{(3)}(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n) = \delta^{(3)}(0) = \frac{V}{(2\pi)^3},$$

where V is a normalisation volume that can be set to 1.

Hint 3: Integrate over p_ν and substitute p_e using the relations

$$d^3p_\nu = 4\pi\ E_\nu^2\ dE_\nu$$

$$d^3p_e = 4\pi\ p_e^2\ dp_e = 4\pi\ \sqrt{E_e^2 - m_e^2}\ E_e\ dE_e$$

β-decay: Fermi Theory - Exercise

Exercise 1. Solution: Starting with **Fermi's golden rule** and using Hint 2 we obtain the expression:

$$d\Gamma_{if} = \frac{G_F^2}{(2\pi)^5}\ \delta(Q - E_e - E_\nu)\ d^3p_e\ d^3p_\nu$$

Next, we integrate over the neutrino momentum p_ν using the first part of Hint 3 since it cannot be measured:

$$d\Gamma_{if} = \frac{G_F^2}{(2\pi)^5}\ \delta(Q - E_e - E_\nu)\ d^3p_e\ 4\pi\ E_\nu^2\ dE_\nu = \frac{G_F^2}{8\pi^4}\ (Q - E_e)^2\ d^3p_e$$

Finally, substituting dp_e using the second part of Hint 3 we arrive at

$$d\Gamma_{if} = \frac{G_F^2}{2\pi^3}\ (Q - E_e)^2\ \sqrt{E_e^2 - m_e^2}\ E_e\ dE_e$$

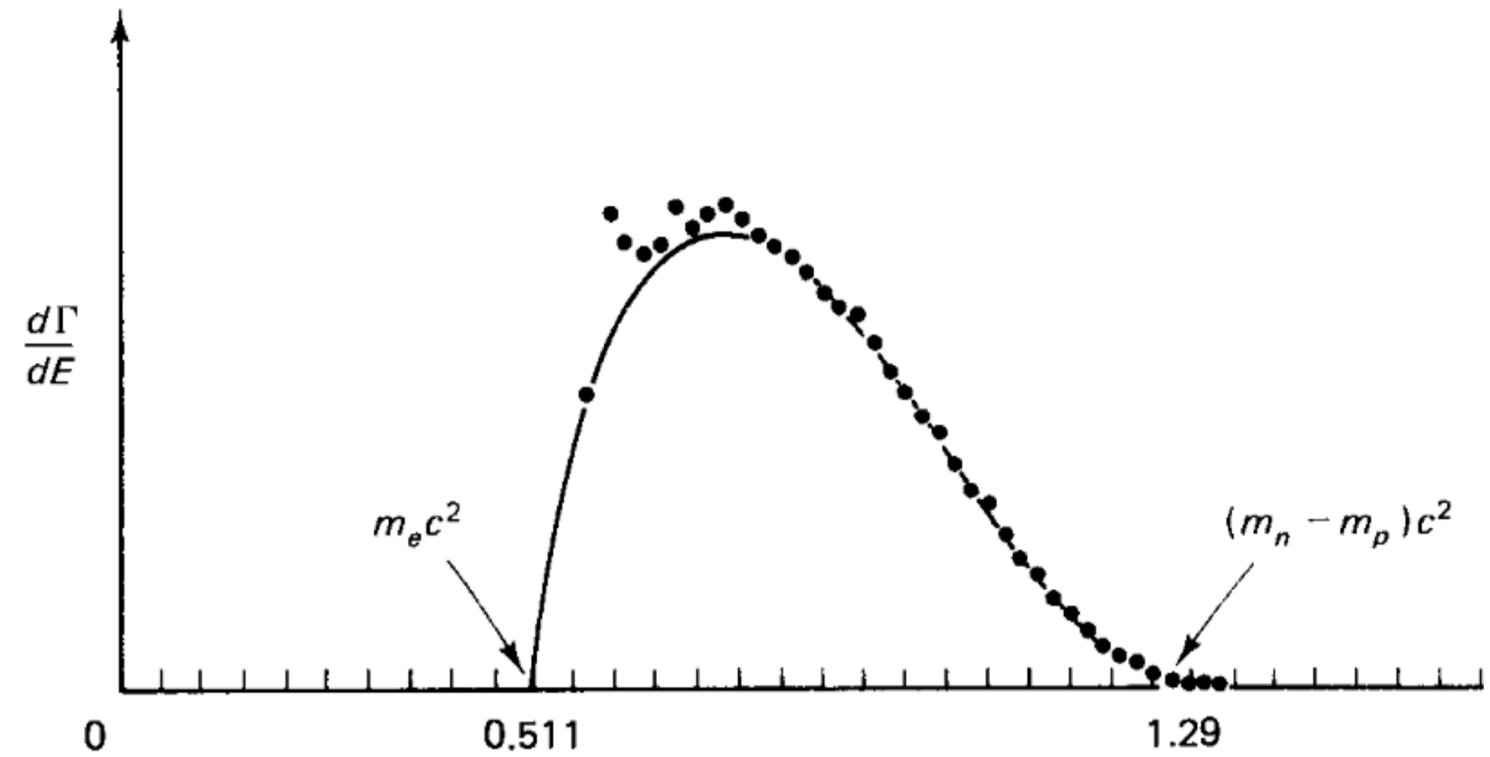
or

$$\frac{d\Gamma_{if}}{dE_e} = \frac{G_F^2}{2\pi^3}\ (Q - E_e)^2\ \sqrt{E_e^2 - m_e^2}\ E_e$$

β-decay: Fermi Theory (3)

The expression we obtained from our quantum mechanics calculation exactly describes the measured β-decay electron spectrum

$$\frac{d\Gamma_{if}}{dE_e} = \frac{G_F^2}{2\pi^3} (Q - E_e)^2 \sqrt{E_e^2 - m_e^2}\ E_e$$



This allows to extract the **Fermi constant** G_F from the spectrum and determine it to

$$G_F \approx 1.15 \times 10^{-5}\ \text{GeV}^{-2}$$

Inverse β-decay

With the methods developed before we can also find an expression for the **inverse β-decay**, or neutrino-proton scattering $\bar{\nu}_e + p \to n + e^+$.

We proceed exactly analogous, but with the initial and final states as

$$|i\rangle = |\bar{\nu}_e, p\rangle \quad \Rightarrow \quad \psi_i(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_\nu \cdot \vec{x}_1}\, e^{i\,\vec{p}_p \cdot \vec{x}_2}$$

$$|f\rangle = |n, e^+\rangle \quad \Rightarrow \quad \psi_f(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_n \cdot \vec{x}_3}\, e^{i\,\vec{p}_e \cdot \vec{x}_4}$$

Assuming the proton to be at rest $\vec{p}_p = 0$, we can derive the cross section from

$$d\sigma = 2\pi\, |V_{if}|^2\, \delta(E_\nu + m_p - E_n - E_e)\, \frac{d^3p_e}{(2\pi)^3}\, \frac{d^3p_n}{(2\pi)^3}$$

Inserting the expression for V_{ij} from before and integrating over p_n yields the simple expression for the total cross section (neglecting m_e and taking $m_n \approx m_p$)

$$\sigma_{\text{tot}} \approx \frac{G_F^2}{\pi}\, \frac{E_\nu^2\, m_p}{2E_\nu + m_p}$$

Inverse β-decay - Exercise

Exercise 2. Using the expression for the neutrino-proton scattering cross section

$$\sigma_{\rm tot} \approx \frac{G_F^2}{\pi} \frac{E_\nu^2 m_p}{2E_\nu + m_p}$$

give a rough estimate for the expected cross section of reactor neutrinos with an energy of $E_\nu \approx 1$ MeV. Express your results in units of barn and cm^2.

Compare your result to Thomson scattering of a photon on a non-relativistic electron, $\sigma_{\rm Thomson} \approx 10^{-24}$ cm^2.

Hint: 1 barn $\equiv 10^{-24}$ cm^2 $= 2.57 \times 10^3$ GeV^{-2}

Inverse β-decay - Exercise

Exercise 2. Solution Since $E_\nu \ll m_p$ we can further approximate (we don't have to)

$$\sigma_{\text{tot}} \approx \frac{G_F^2}{\pi} E_\nu^2$$

Inserting the numbers then yields

$$\sigma_{\text{tot}} \approx 4 \times 10^{-17} \text{ GeV}^{-2} \approx 10^{-5} \text{ fb} \approx 10^{-44} \text{ cm}^2$$

The neutrino-proton scattering cross section is 20 (!) orders of magnitudes smaller than Thomson scattering. We need huge detectors and large fluxes to detect neutrinos!

Neutrino detection

Exercise 3. Consider a research nuclear reactor with a power of $P_{\mathrm{reactor}} = 20\ \mathrm{MW}$ which mainly has ${}^{235}_{92}U$. If an average of 6 neutrinos are emitted from each fission, what would be the neutrino flux at a location 150 m away from the core?

Hint: The reaction is ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{55}Cs + {}^{90}_{37}Rb + 2{}^{1}_{0}n,$
and the atomic masses for each element are:

$$m({}^{235}U) = 235.044u,$$
$$m({}^{144}Cs) = 143.932u,$$
$$m({}^{90}Rb) = 89.915u,$$
$$m({}^{1}_{0}n) = 1.009u.$$

$$1u = 931.5\ \mathrm{MeV}$$

Hint 2: $1\ \mathrm{MW} \approx 6.25 \cdot 10^{18}\ \mathrm{MeV/s}$

Neutrino detection

Exercise 3. Solution

The excess mass is given by the difference between the fission products and the fuel: $\Delta m = 235.044u + 1.009u - 143.932u - 89.915u - 2.018u = 0.188u$, and the energy released in each fission is then

$$\Delta E = \Delta mc^2 = 0.188 \cdot 931.5 \text{ MeV} = 175.1 \text{ MeV}$$

The neutrino flux will be given by

$$\phi_\nu = \frac{6 \cdot N_{\text{fission}}}{4\pi L^2 \Delta t} = \frac{6 \cdot P_{\text{reactor}}}{4\pi L^2 \Delta E} = 1.5 \cdot 10^9 \text{ cm}^{-2} \text{ s}^{-1}$$

Neutrino detection

Exercise 4. If we wanted to detect these reactor neutrinos through inverse beta decay (IBD), how much mass of the liquid scintillator $C_{18}H_{30}$ would we need to have an event rate of about 10 events/day?

Hint1: Remember that the IBD cross section on free protons is $\sigma_{\text{IBD}} \sim 10^{-44} cm^2$, and the event rate would be given by

$$N_{\text{IBD}}/\Delta t = \phi_\nu \sigma_{\text{IBD}} N_{\text{targets}}.$$

Hint2: Assume that the number of free protons per molecule is given by the number of H atoms

The molar mass for $C_{18}H_{30}$ is $M_{C_{18}H_{30}} = 246.4 \text{ g/mol}$

Avogadro's number is $N_A = 6.022 \times 10^{23}/\text{mol}$

Neutrino detection

Exercise 4. Solution

The number of free protons would be given by the number of hydrogen atoms in the $C_{18}H_{30}$ molecule times the number of molecules. Thus

$$N_{\text{targets}} = 30 \cdot \frac{m_{\text{detector}}}{M_{C_{18}H_{30}}} \cdot N_A$$

Using the previously calculated flux and cross section, we find

$$m_{\text{detector}} = \frac{N_{\text{IBD}}}{\phi_\nu \sigma_{\text{IBD}} \Delta t} \cdot \frac{M_{C_{18}H_{30}}}{30 N_A} \approx 105 \text{ tons}$$

Neutrino detection

Exercise 5. Apart from IBD, there are other processes through which neutrinos can interact with matter. A particularly interesting one is coherent neutrino nucleus scattering (CEvNS), in which low energy neutrinos interact with the whole atomic nucleus instead of interacting with individual nucleons.

How large would a detector made of CsI(Na) need to be to observe an event rate of about 10 events/day?

Compare this to the previous detector relying on IBD.

Hint: The CEvNS cross section on Cs for neutrinos with $E_\nu \sim 2 \text{ MeV}$ is $\sigma_{\text{CE}\nu\text{NS}} \sim 10^{-40} \text{ cm}^2$.

The molar mass of CsI is $M_{\text{CsI}} = 259.8 \text{ g/mol}$

Neutrino detection

Exercise 5. Solution

Now we only want to know the number of Cs atoms in our detector. Thus

$$N_{\text{targets}} = \frac{m_{\text{detector}}}{M_{\text{CsI}}} \cdot N_A$$

Using the previously calculated flux and cross section, we find

$$m_{\text{detector}} = \frac{N_{\text{CE}\nu\text{NS}}}{\phi_\nu \sigma_{\text{CE}\nu\text{NS}} \Delta t} \cdot \frac{M_{\text{CsI}}}{N_A} \approx 333 \text{ kg}$$