# Day1:Basics of atomic clocks

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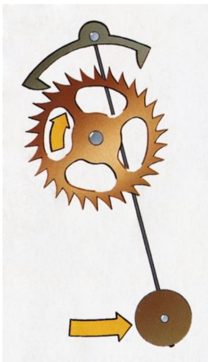
Hamilton ChronoMatic

# What will we cover?

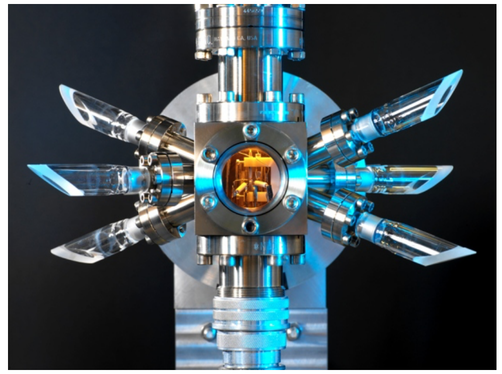
- Day 1: Atomic clock concepts
- Day 2: Variation of fundamental constants and dark matter searches
- Day 3: Gravitation wave detection and multi-messenger astronomy

Tutors: Charles Baynham, Tyler Daykin, Hoang Bao Tran Tan

# Atomic clocks as quantum sensors

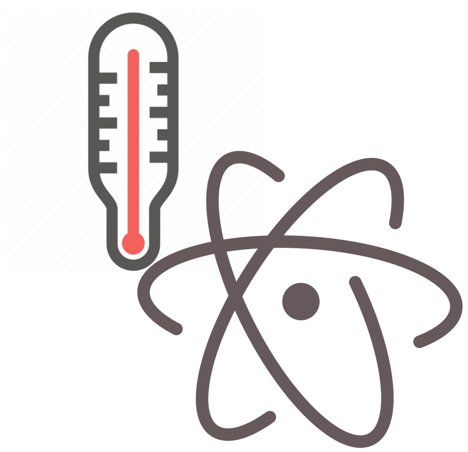


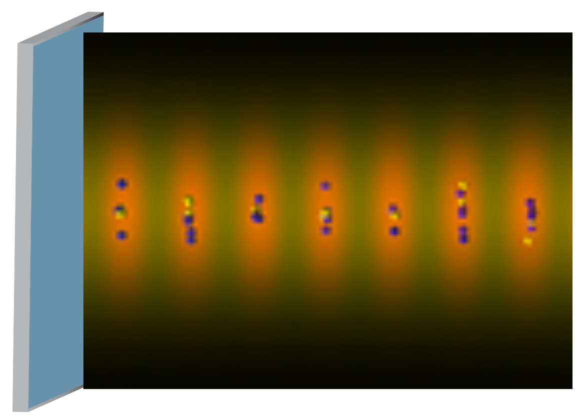
$\Rightarrow$

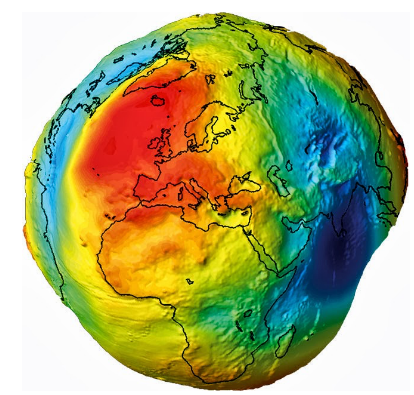


$$|0\rangle + |1\rangle \overset{?}{\Rightarrow} |0\cdots0\rangle + |1\cdots1\rangle$$

"Conventional"

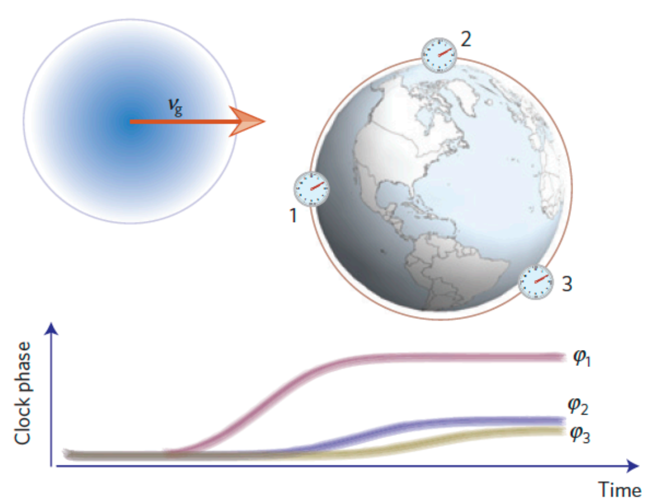


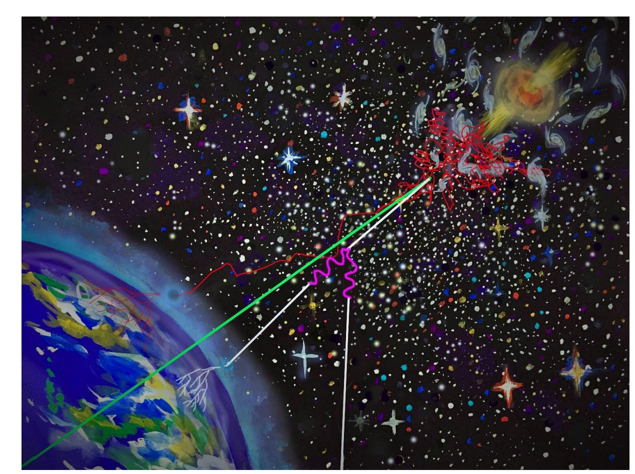




"Fundamental"

$$\dot{\alpha} \,\&\, \vec{\nabla}\alpha$$



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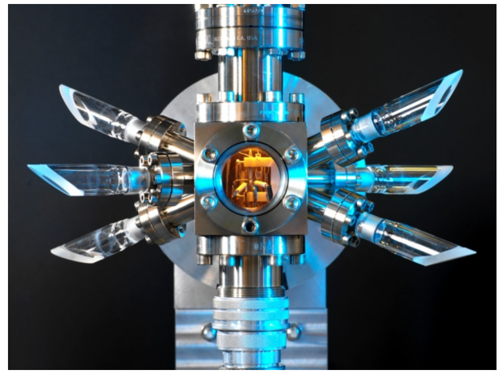

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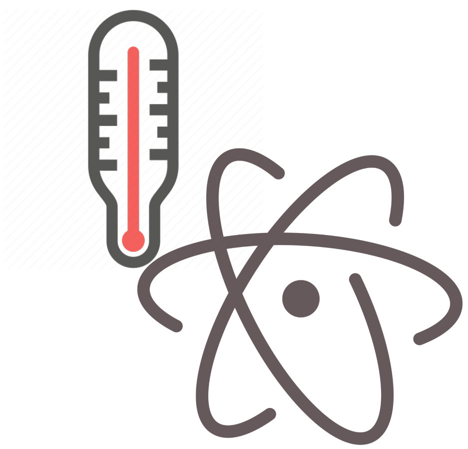

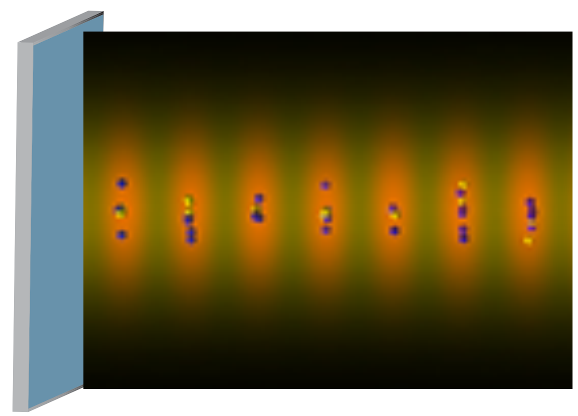

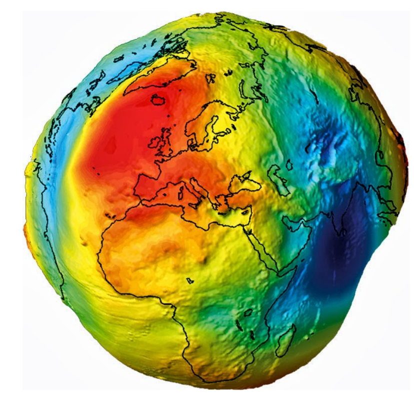

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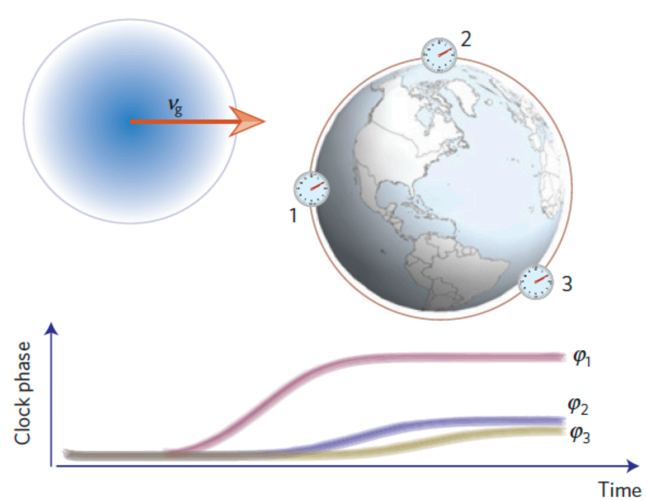


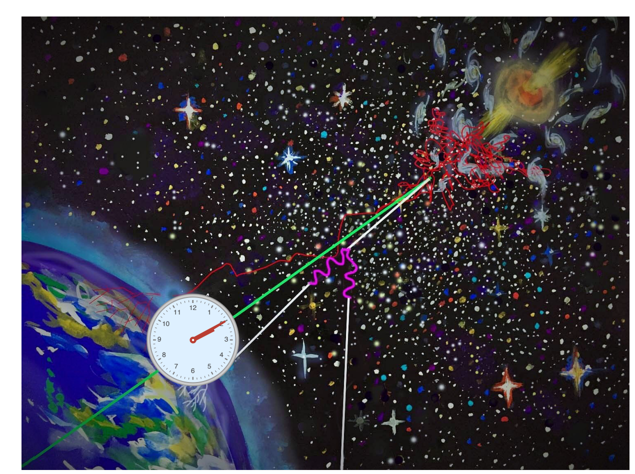

# Today

- ❑ Basis concepts of quantum time keeping
- ❑ Atomic clocks as quantum sensors
- ❑ Rabi and Ramsey detection schemes
- ❑ Stability vs accuracy
- ❑ Standard quantum and Heisenberg limits
- ❑ Toward massive entanglement

# What is time?

TIME (according to Merriam-Webster)

a :  the measured or measurable period during which an action, process, or condition exists or continues

b :  a nonspatial continuum that is measured in terms of events which succeed one another from past through present to future

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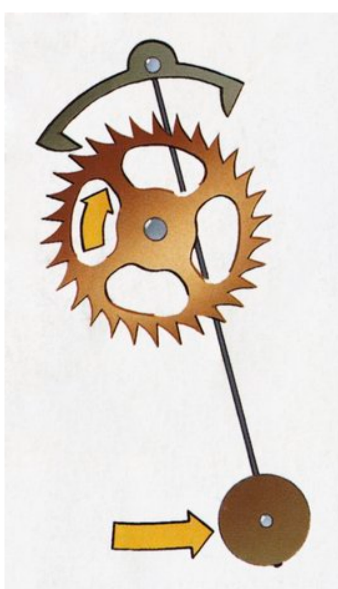
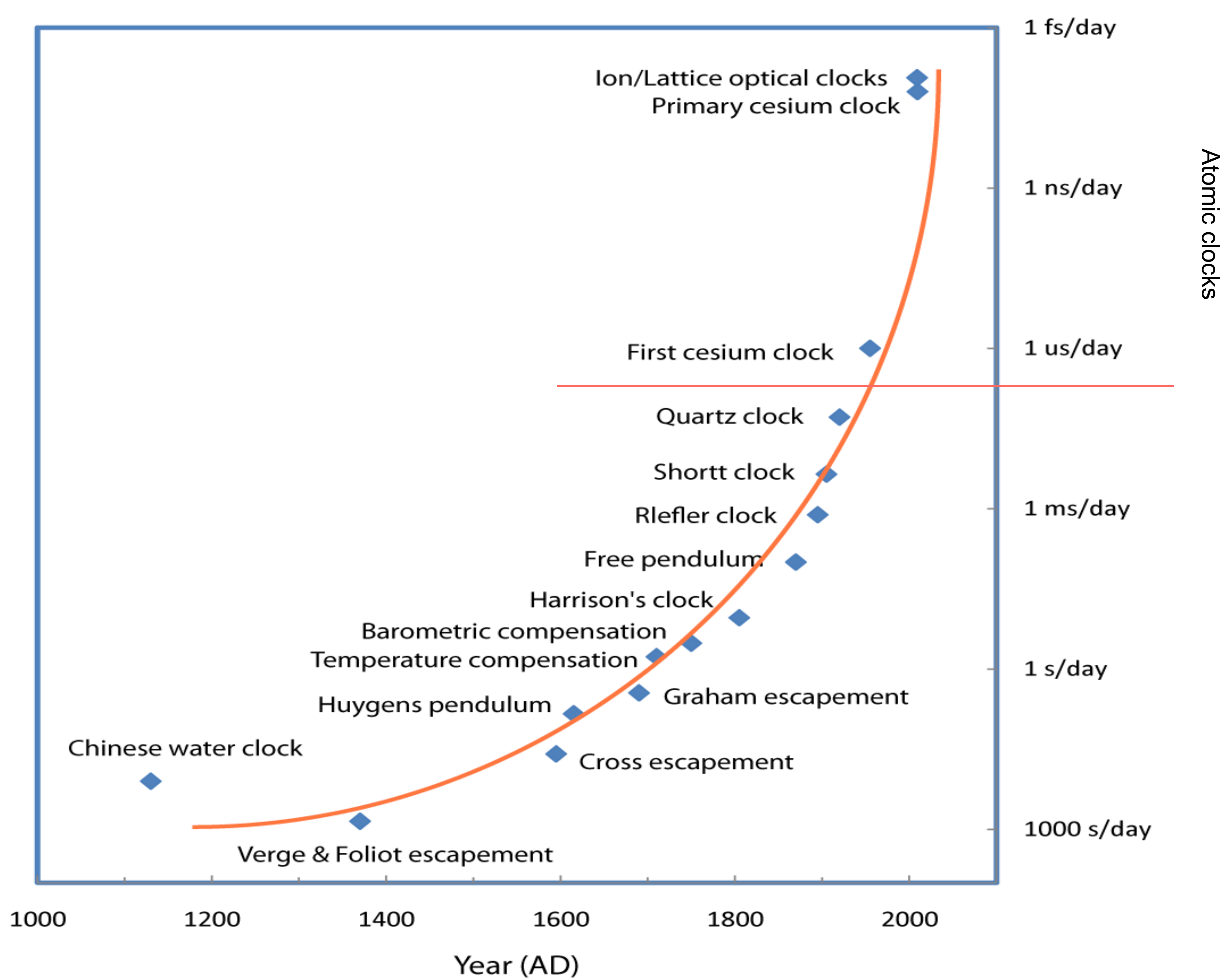


$|e\rangle$

$|g\rangle$

$$\nu_{\text{clock}} = \frac{E_e - E_g}{h}$$


1 fs/day
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Atomic clocks
Ion/Lattice optical clocks
Primary cesium clock
First cesium clock
Quartz clock
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Verge & Foliot escapement
1000
1200
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1600
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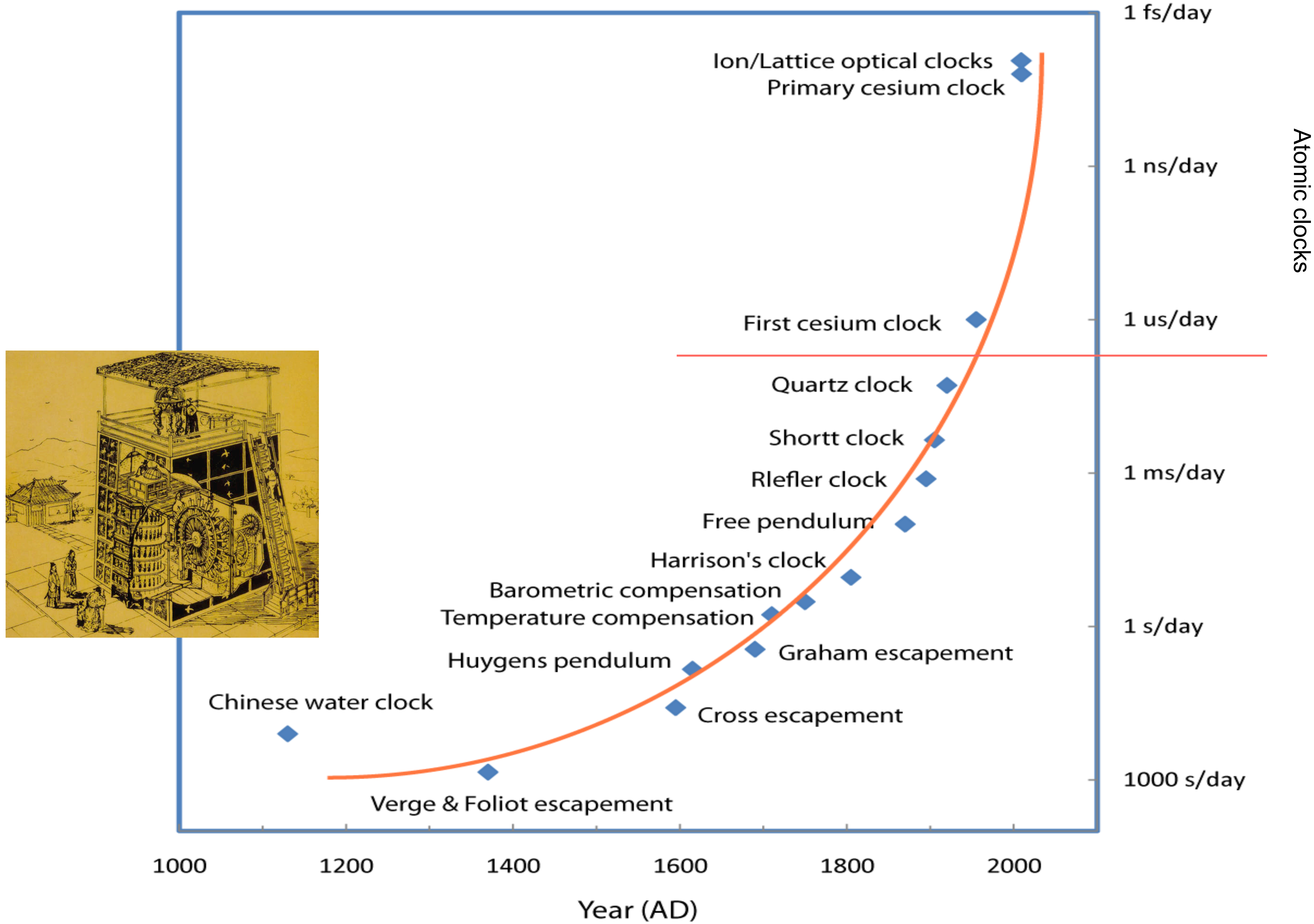
Year (AD)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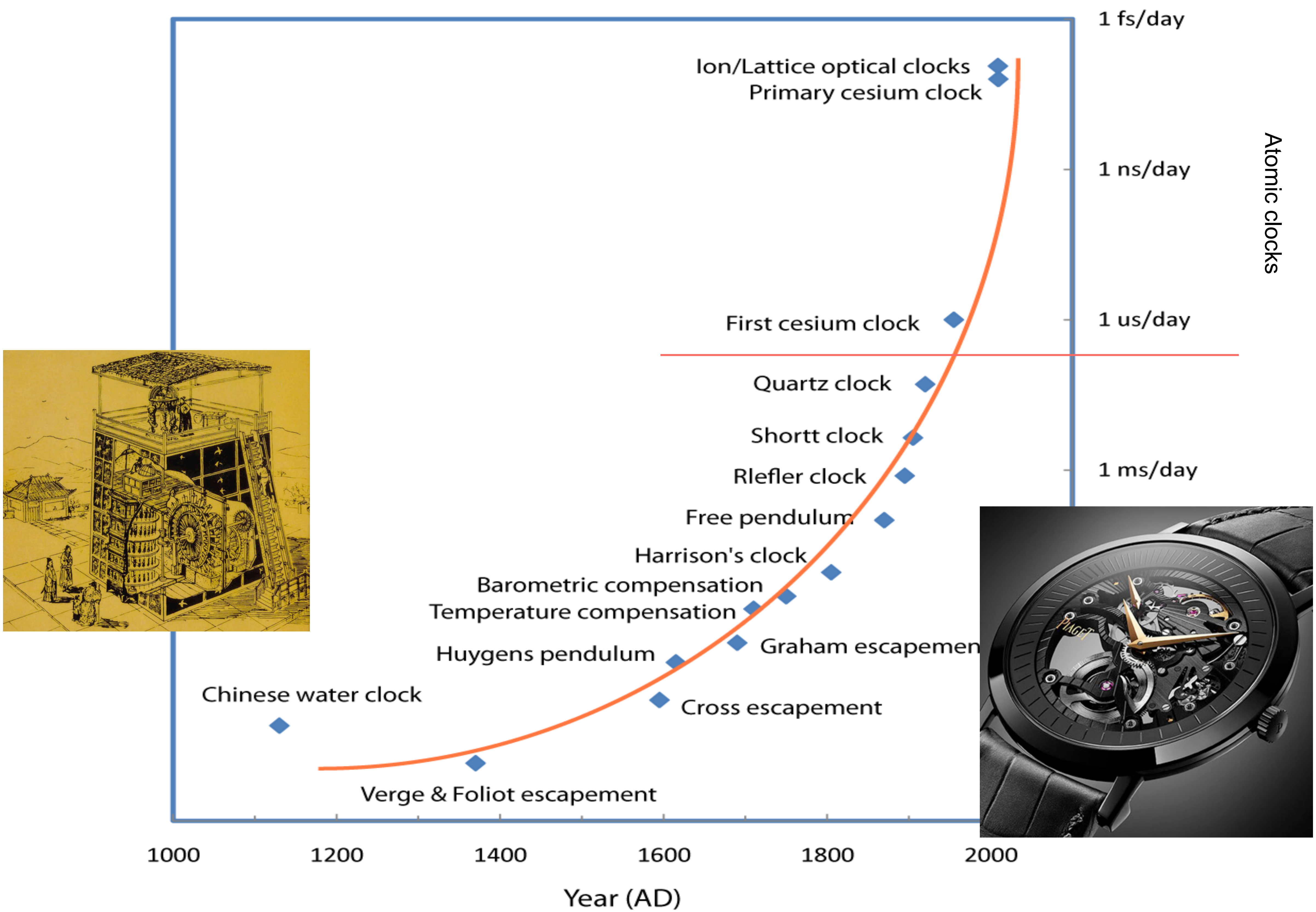

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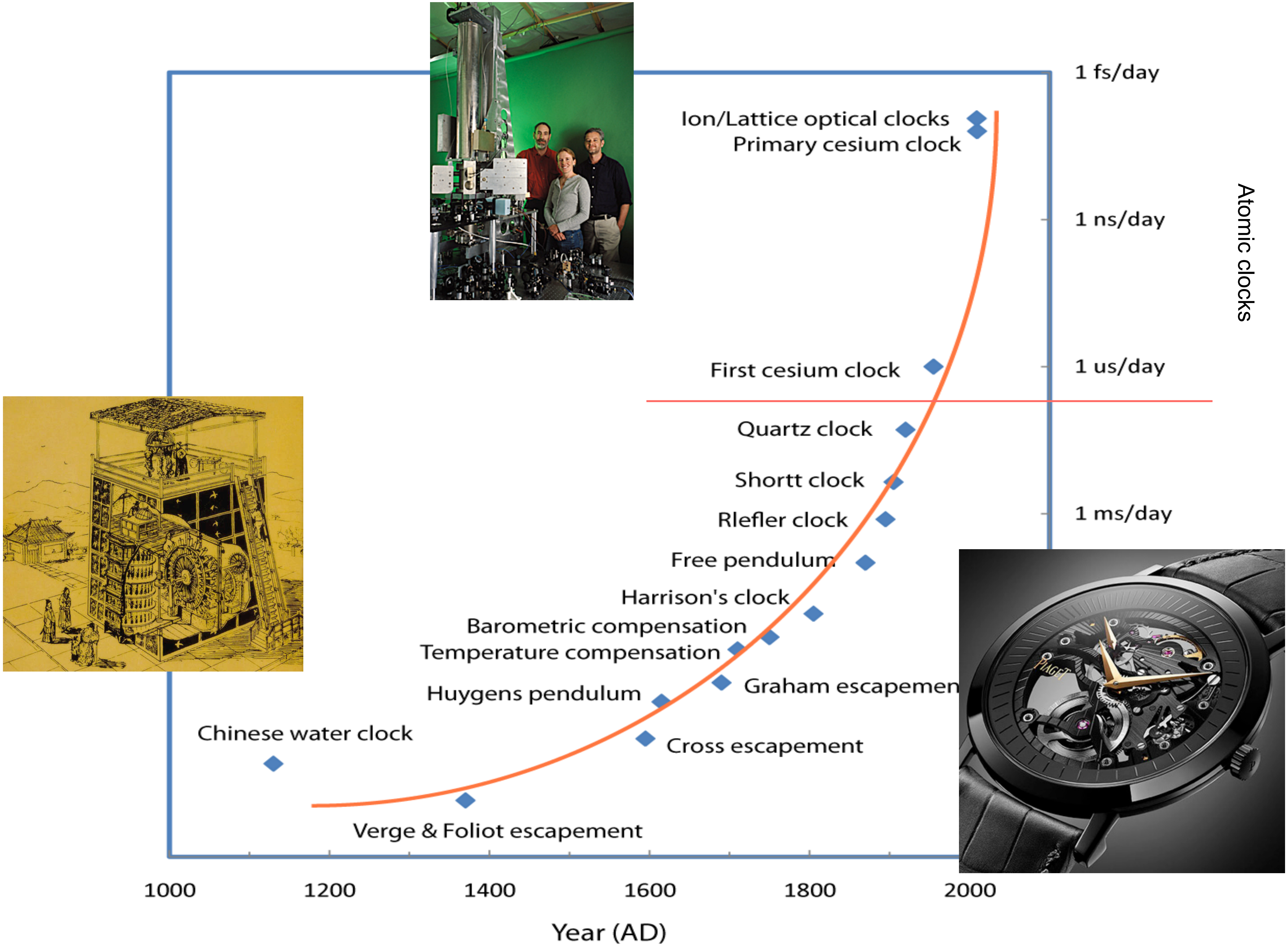
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# Atomic clocks

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- Modern nuclear/atomic clocks aim at 19 significant figures of precision
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$$\Delta t = \left(\frac{\Delta\omega}{\omega_{\text{clock}}}\right) \times t = 1\,\text{s}$$

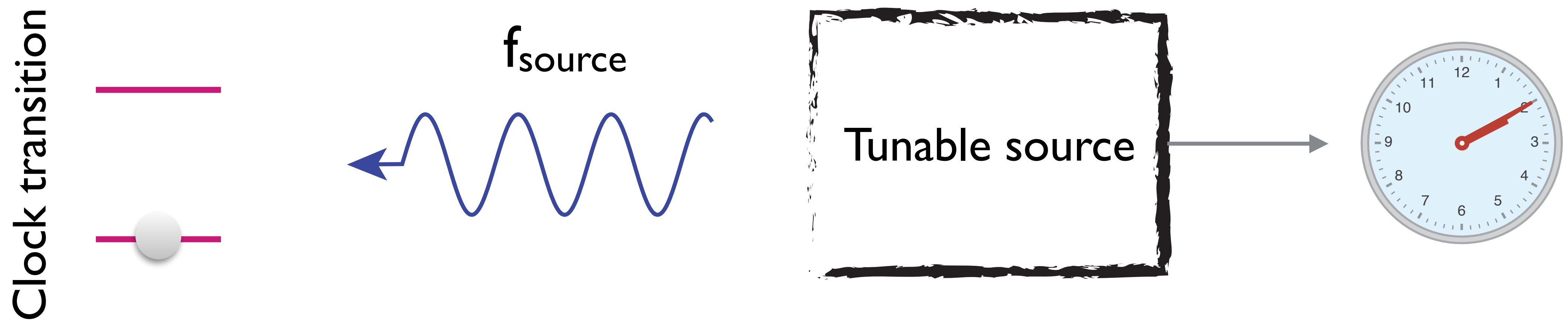
fractional inaccuracy
$10^{-18}$

Age of the universe $10^{18}$ s

# How do atomic clocks work?

# Overview

Time = (number of oscillations) x ($1/f_{clock}$)



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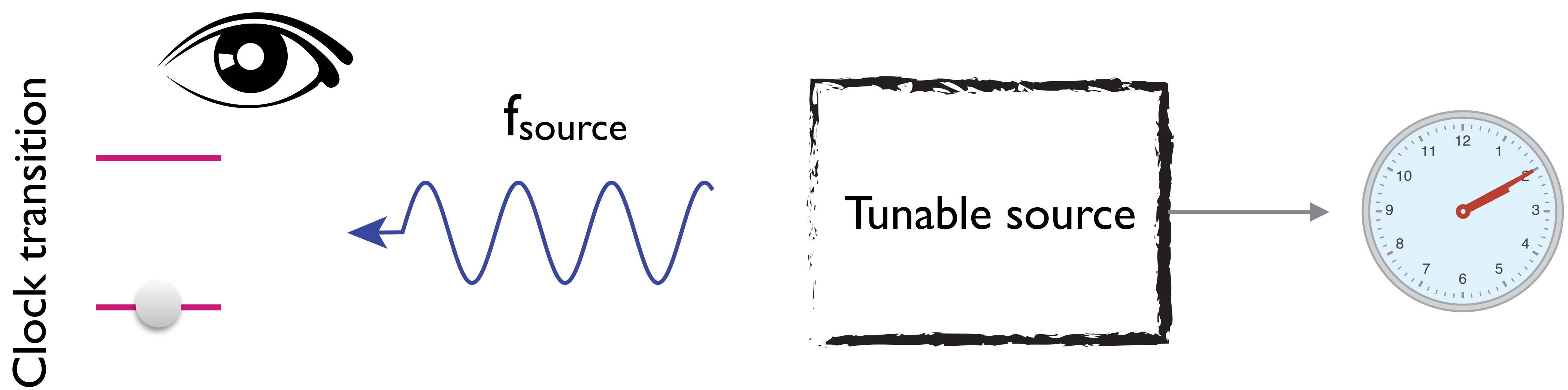

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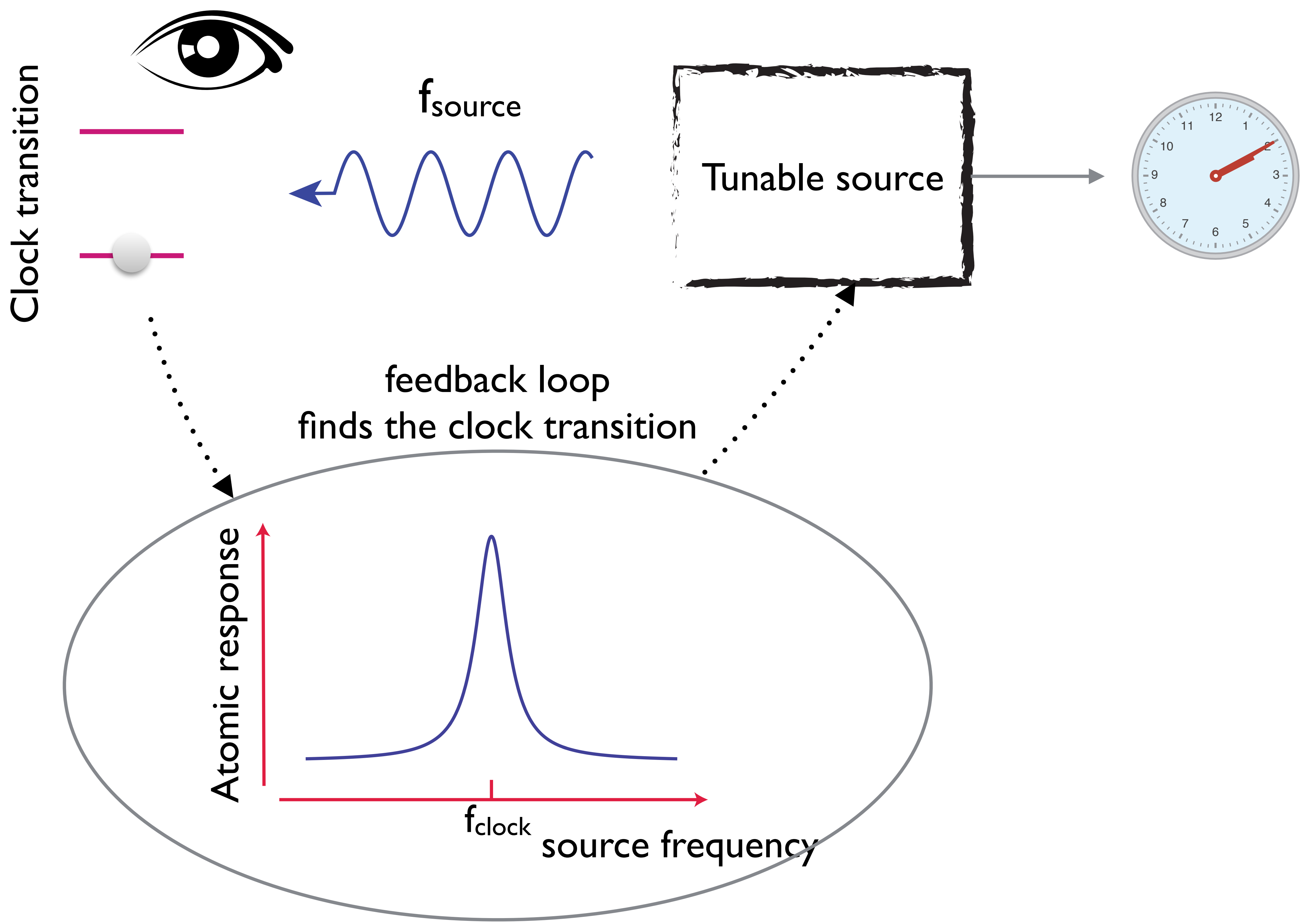
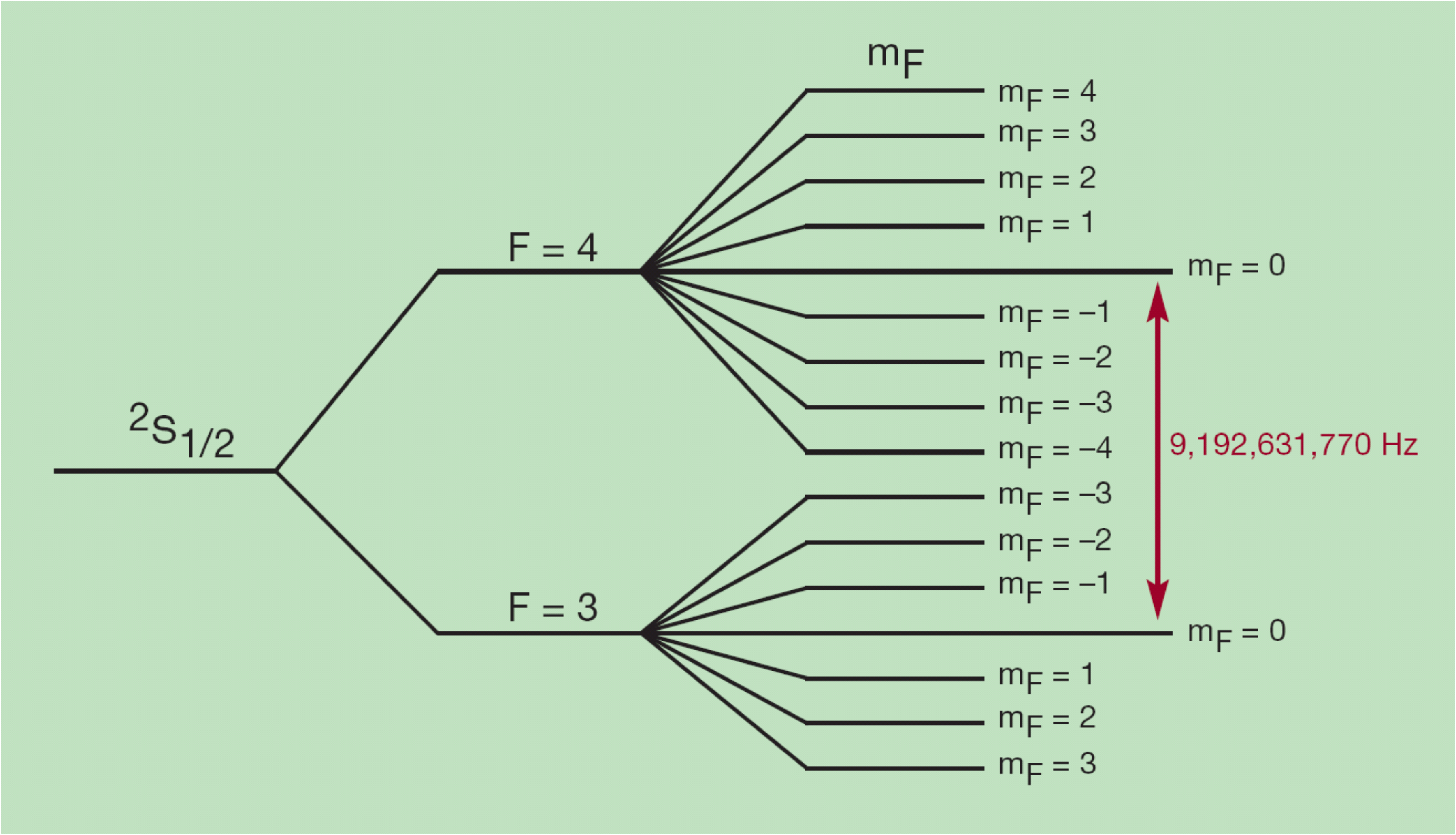


# Illustrative example:
# Cs fountain clock

# SI definition of the second

**The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom. This definition refers to a cesium atom at rest at a temperature of 0 K.**



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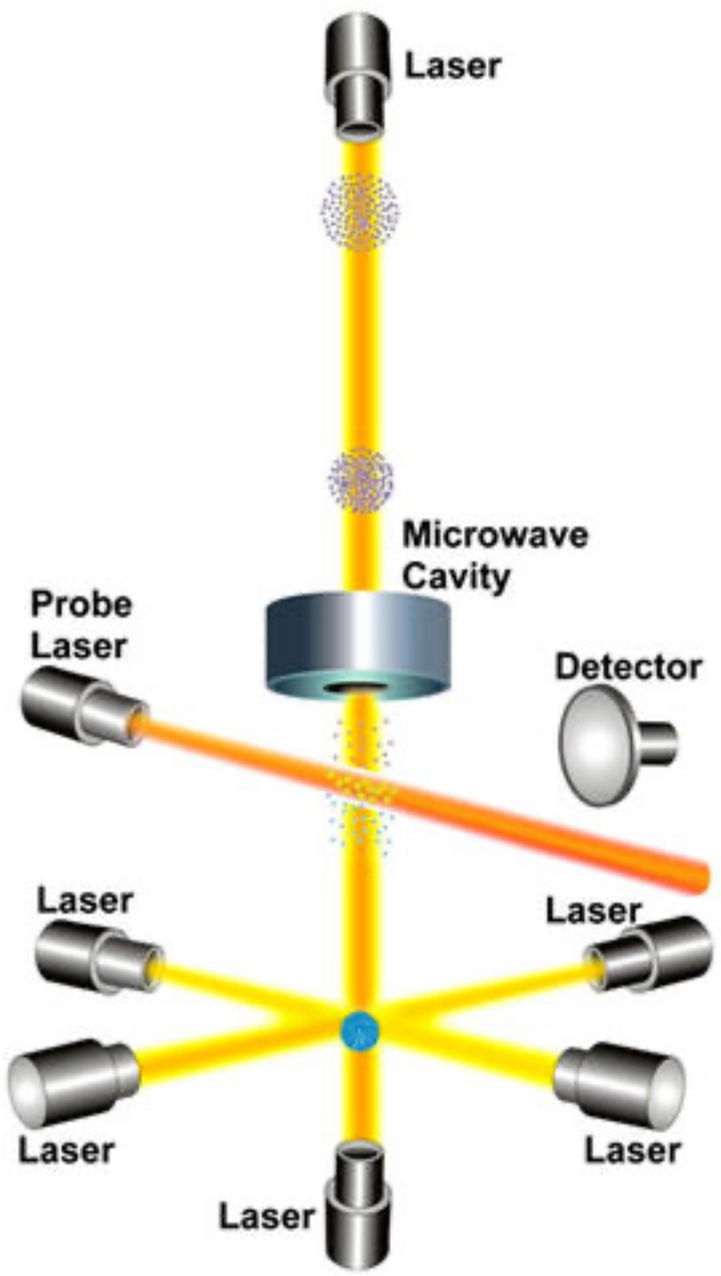


*https://www.nist.gov/pml/time-and-frequency-division/time-realization/primary-standard-nist-f1*

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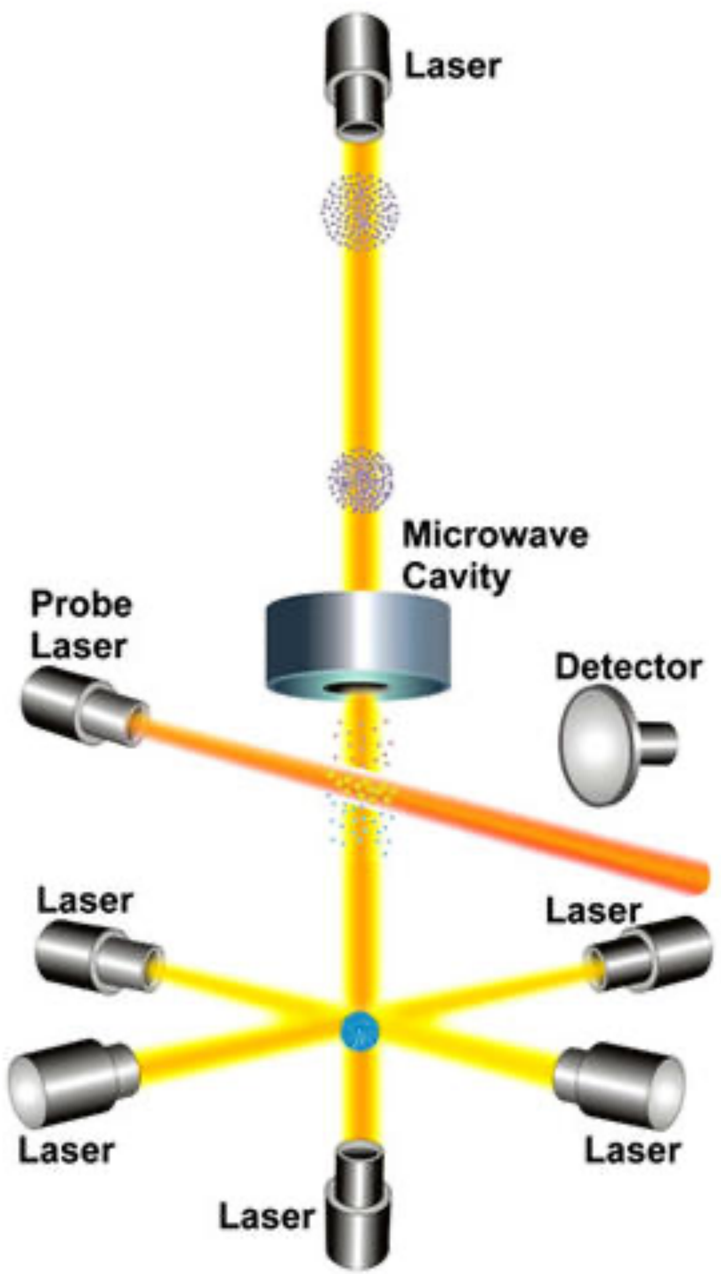


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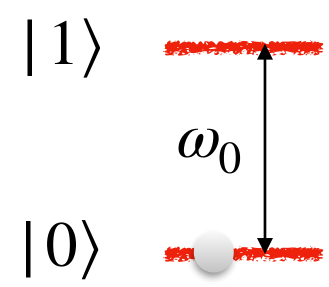
# Rabi interrogation scheme

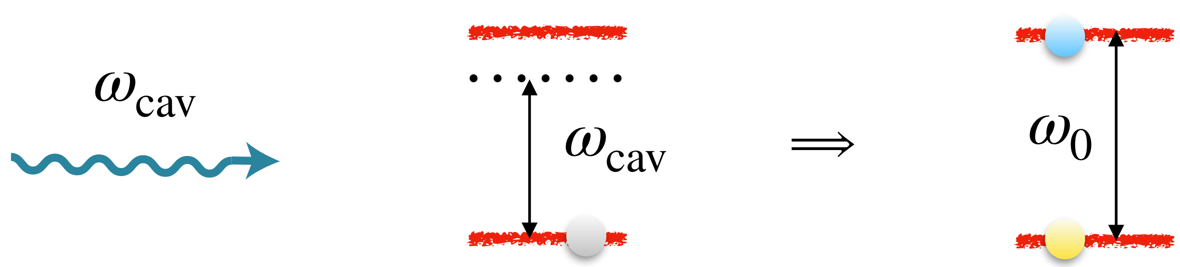



We need to tune cavity frequency $\omega_{\text{cav}}$ to the clock frequency

map frequency detuning $\Delta$ into population of $|1\rangle$ state

$$\Delta = \omega_{\text{cav}} - \omega_0$$

Cavity




+ projective measurement on $|1\rangle$

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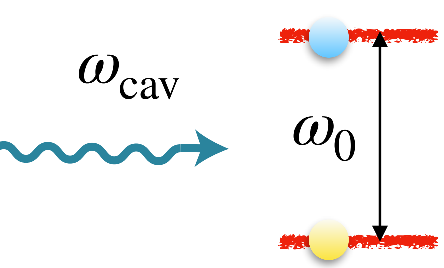


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$$\Delta = \omega_{\text{cav}} - \omega_0$$

Rabi frequency

$$\Omega = \langle 1 | H_{\gamma e}^{\text{int}} | 0 \rangle / \hbar$$

$$\Omega' = \sqrt{\Omega^2 + \Delta^2}$$

$$P_{0\to 1}(\tau_p | \Omega, \Delta) = \left(\frac{\Omega}{\Omega'}\right)^2 \sin^2\left(\Omega' \tau_p / 2\right)$$

pulse duration

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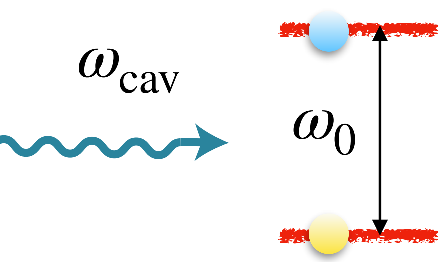


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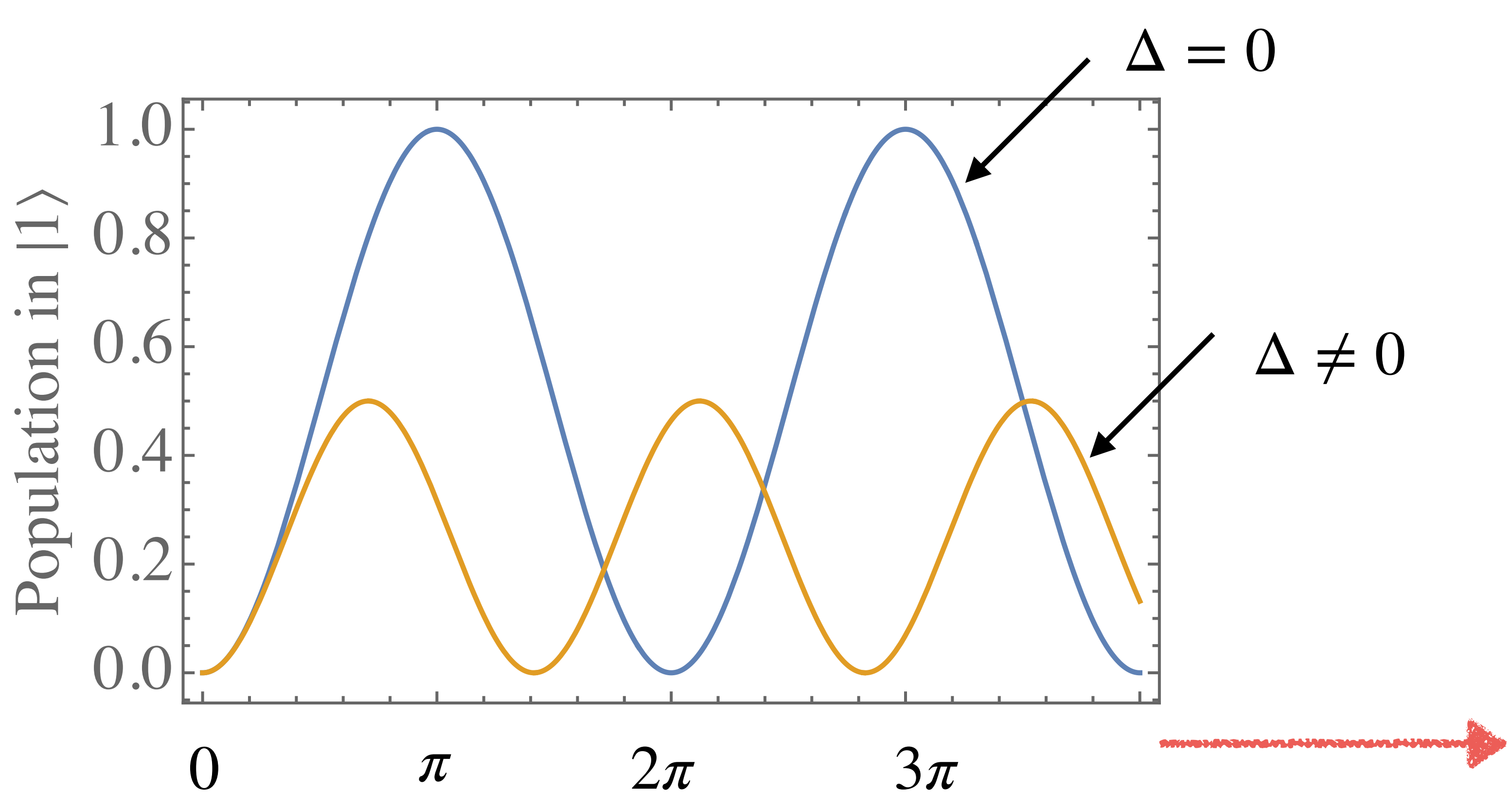
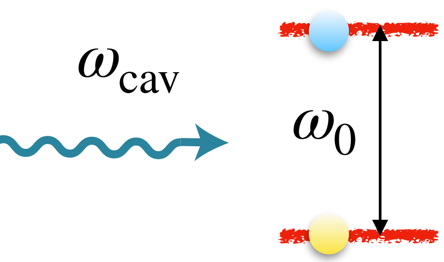



pulse duration × Rabi frequency

# Rabi interrogation scheme

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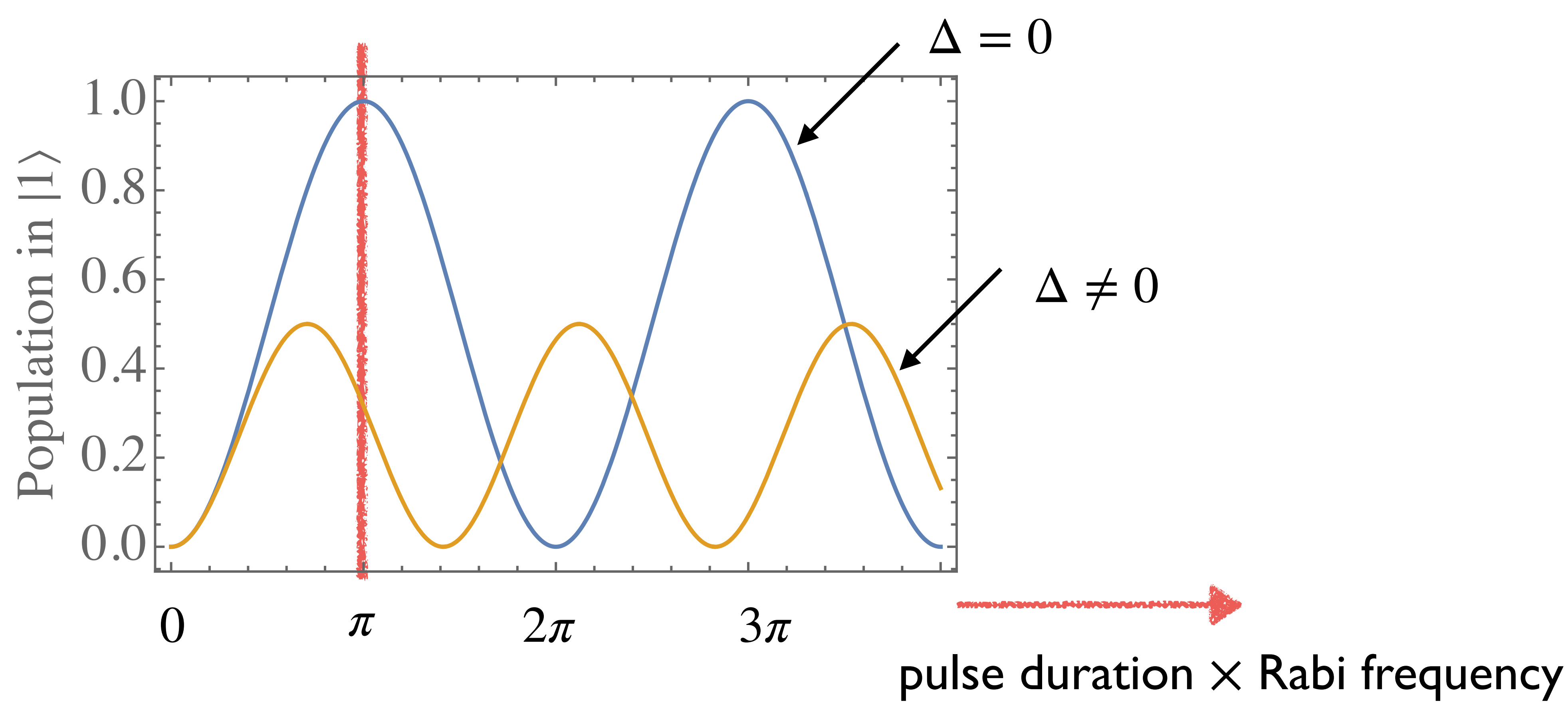

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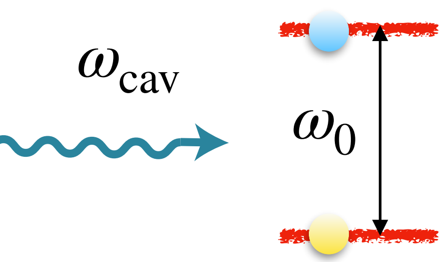


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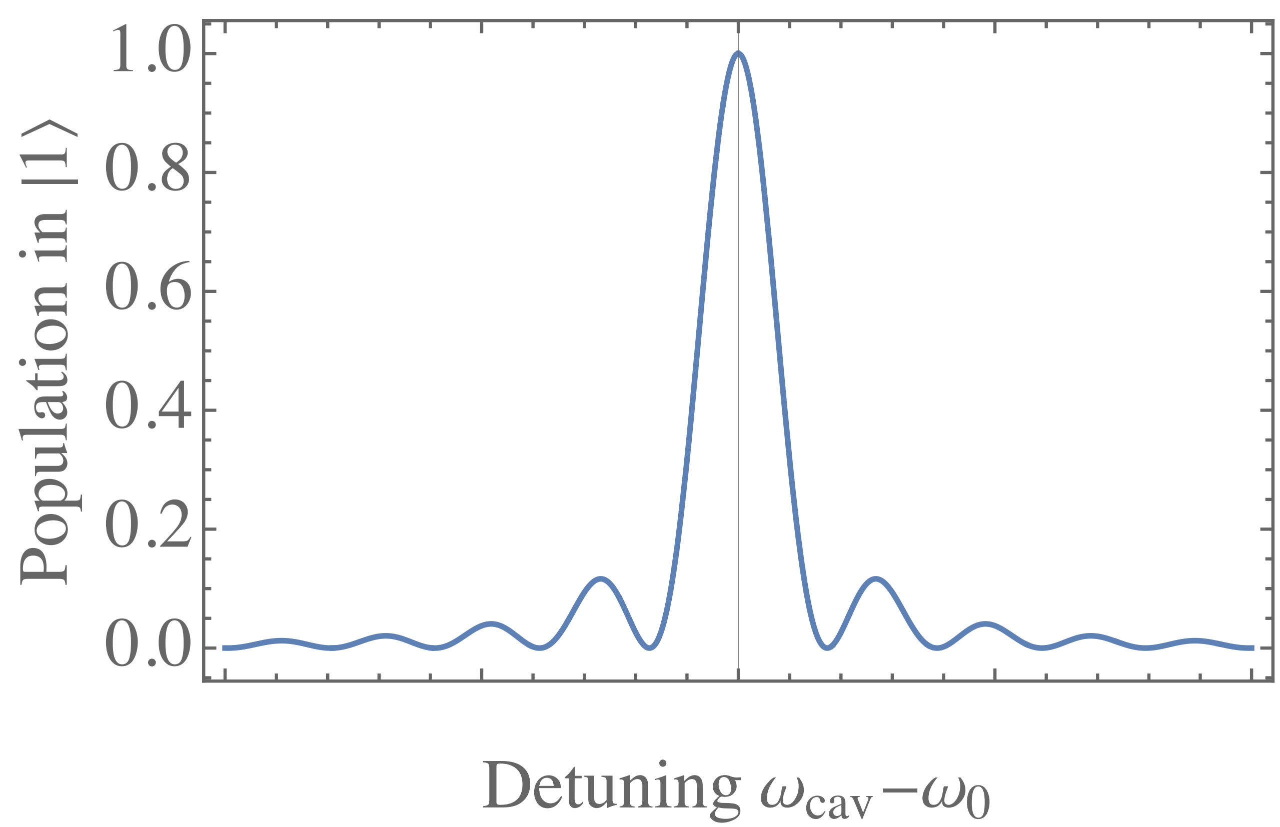

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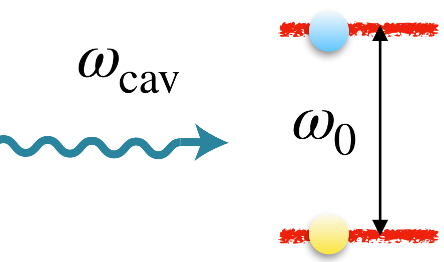


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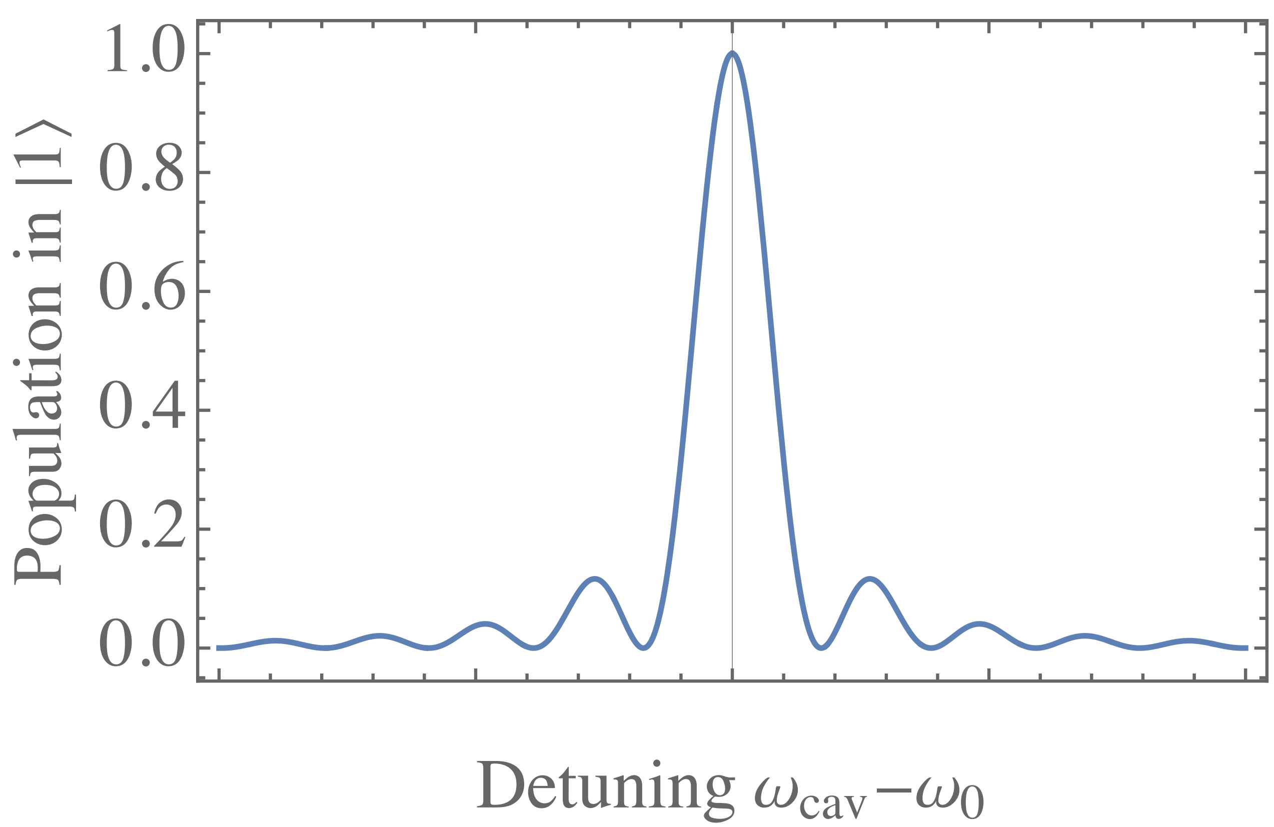

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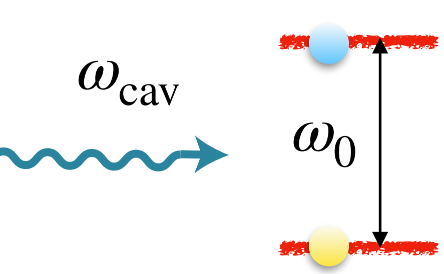


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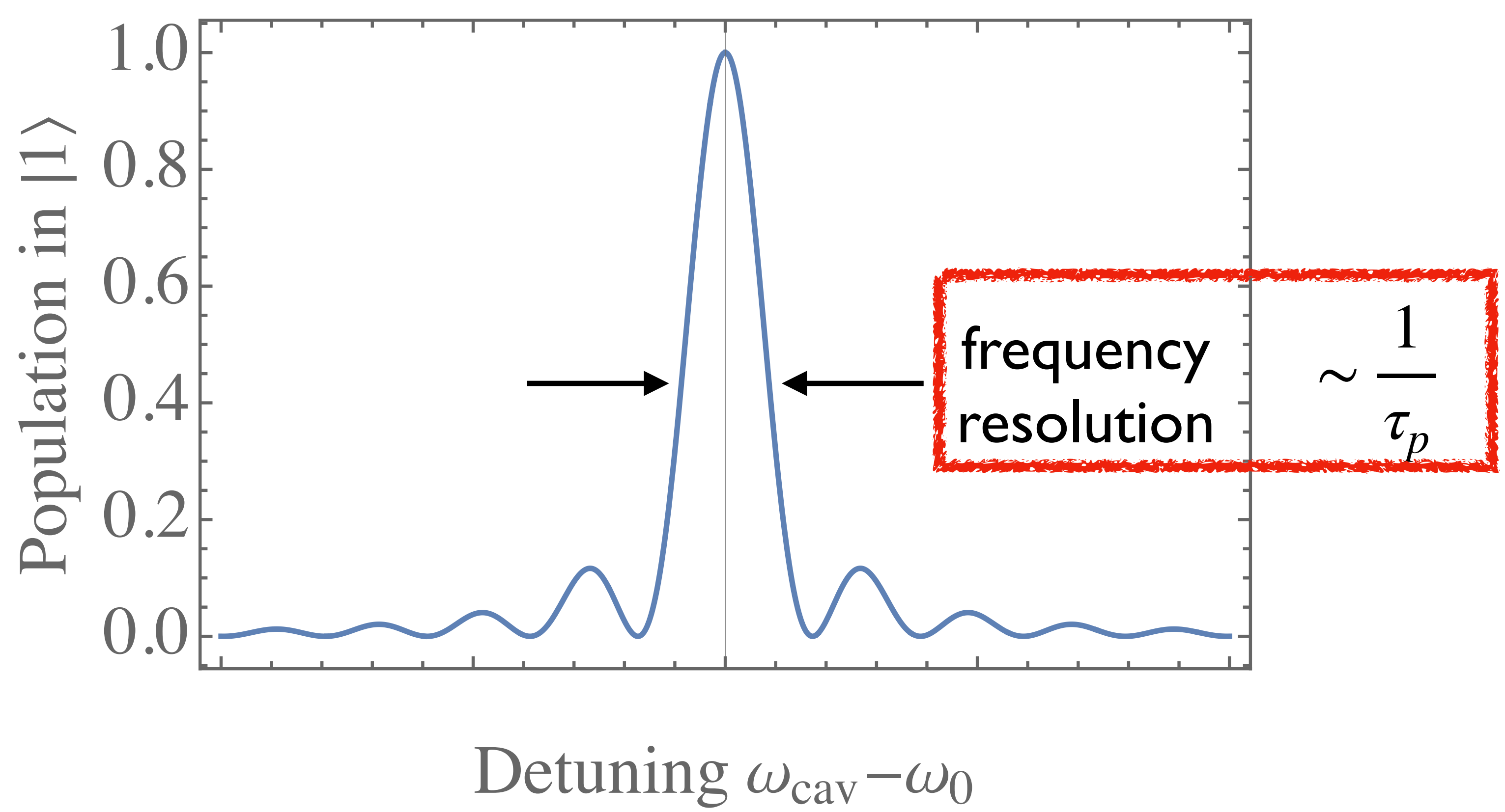
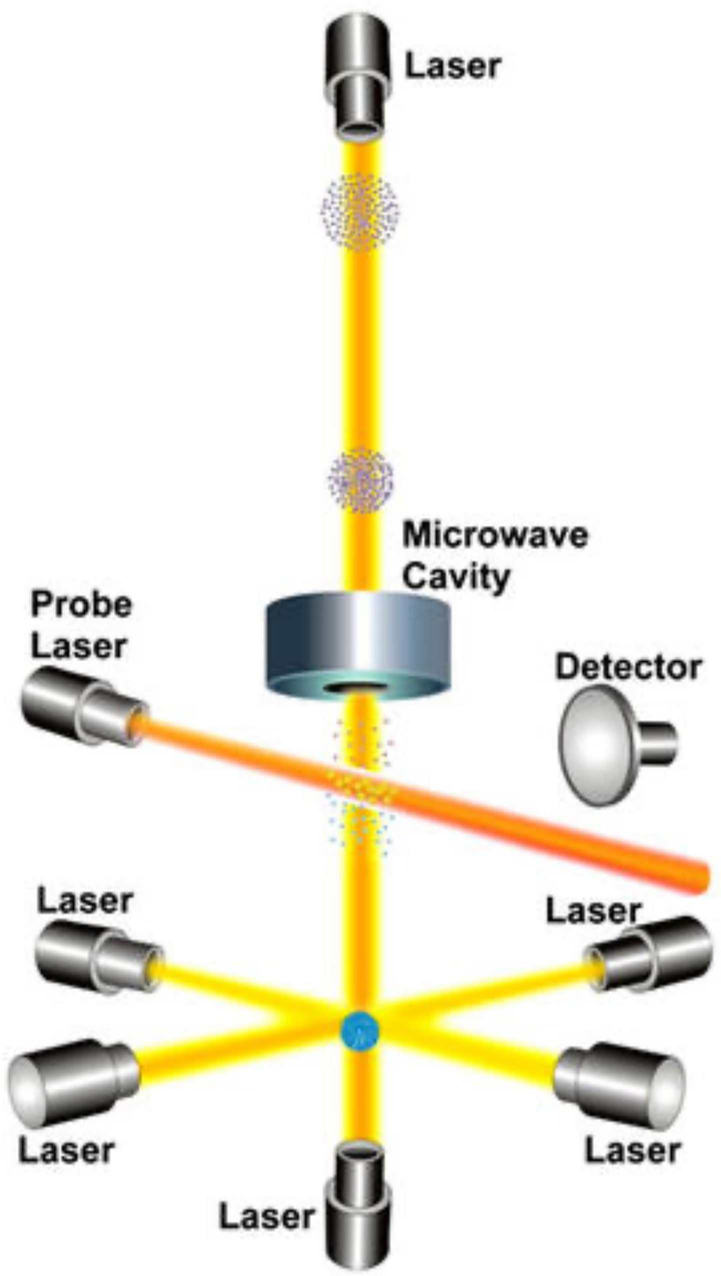


# Example: Cs fountain clock

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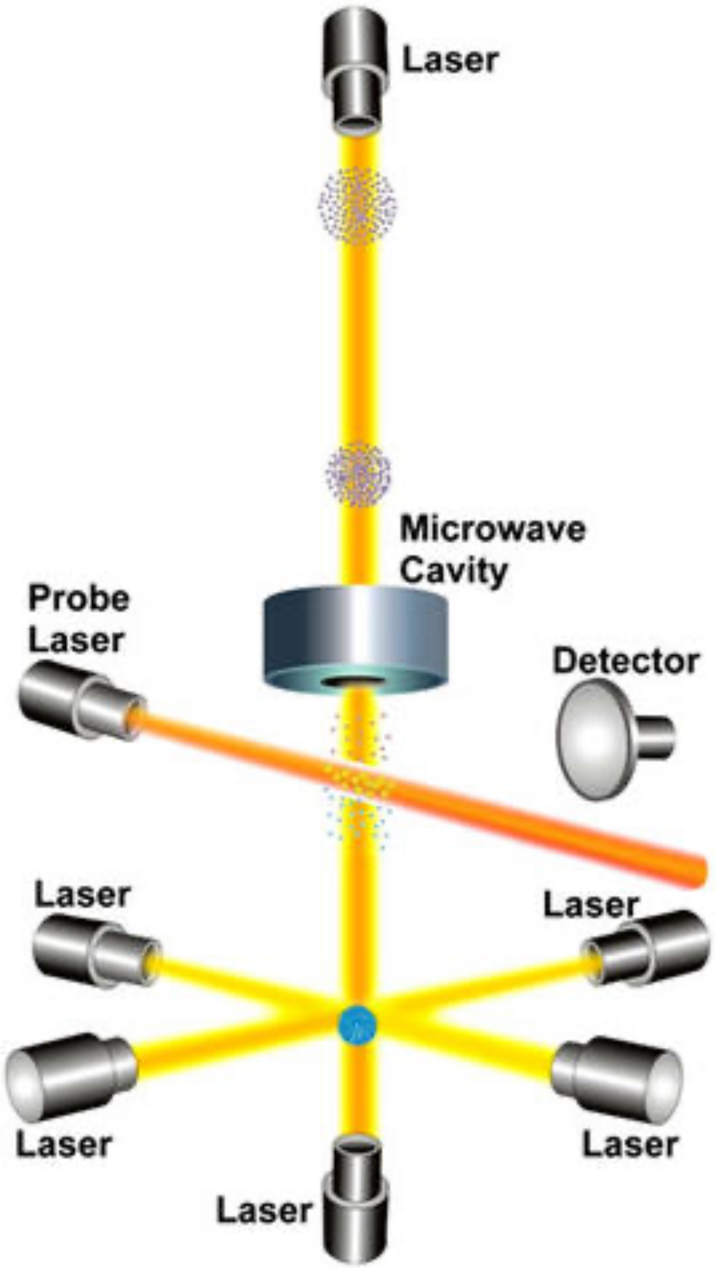


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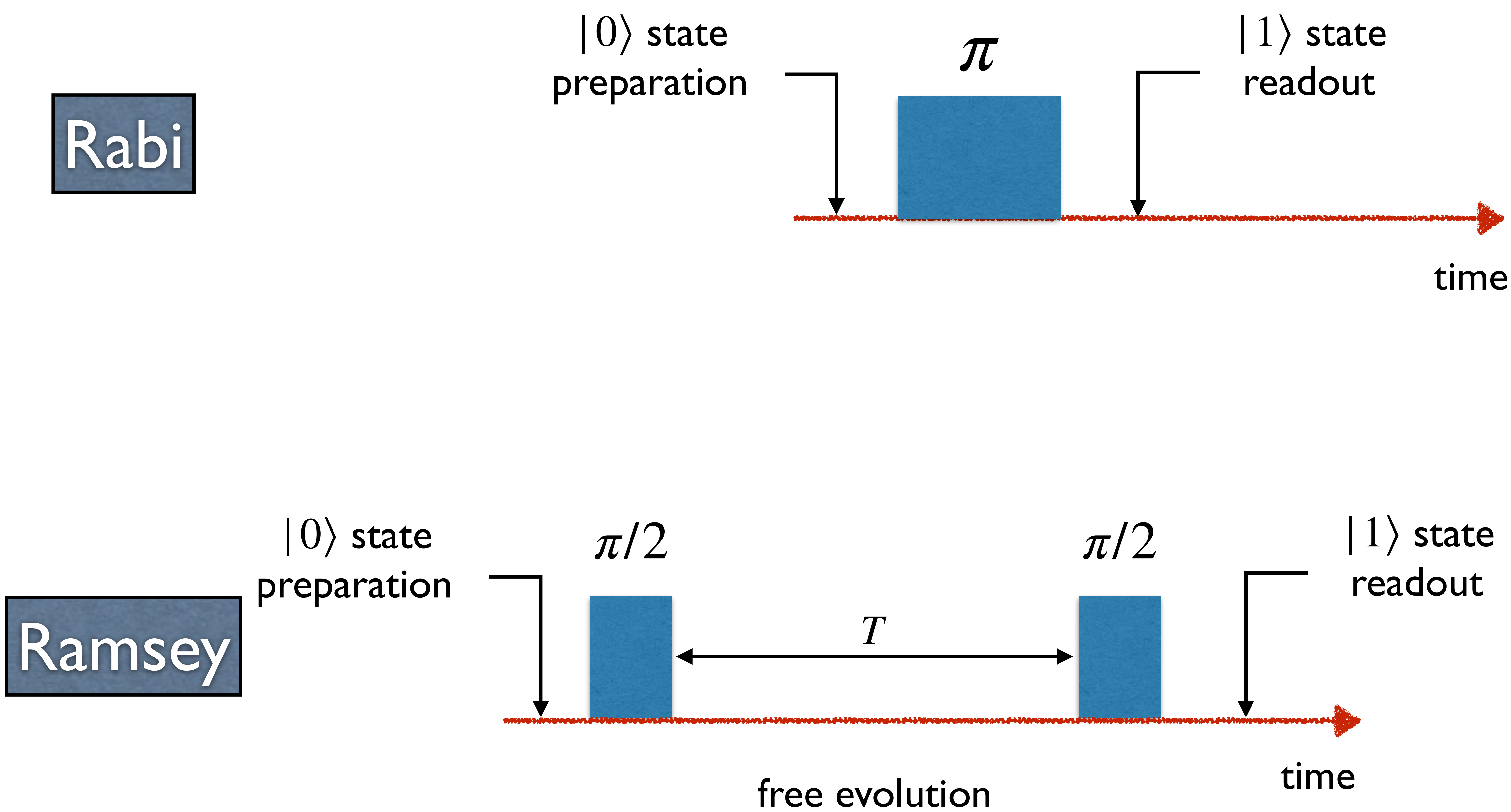
# Improving resolution: Ramsey method



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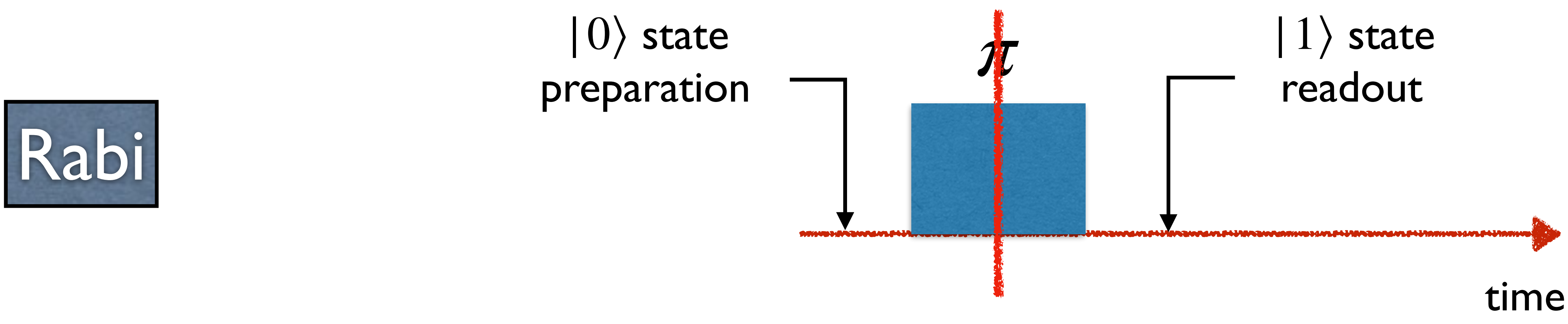


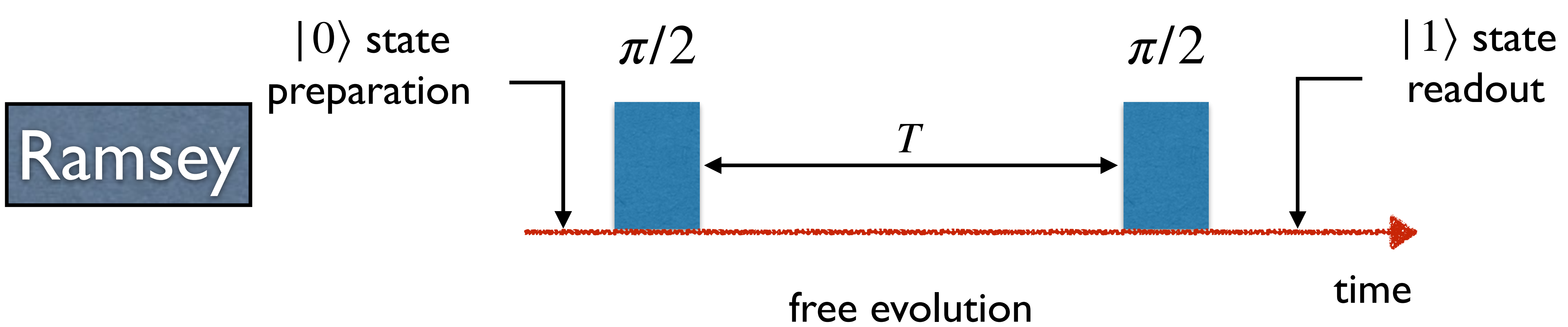

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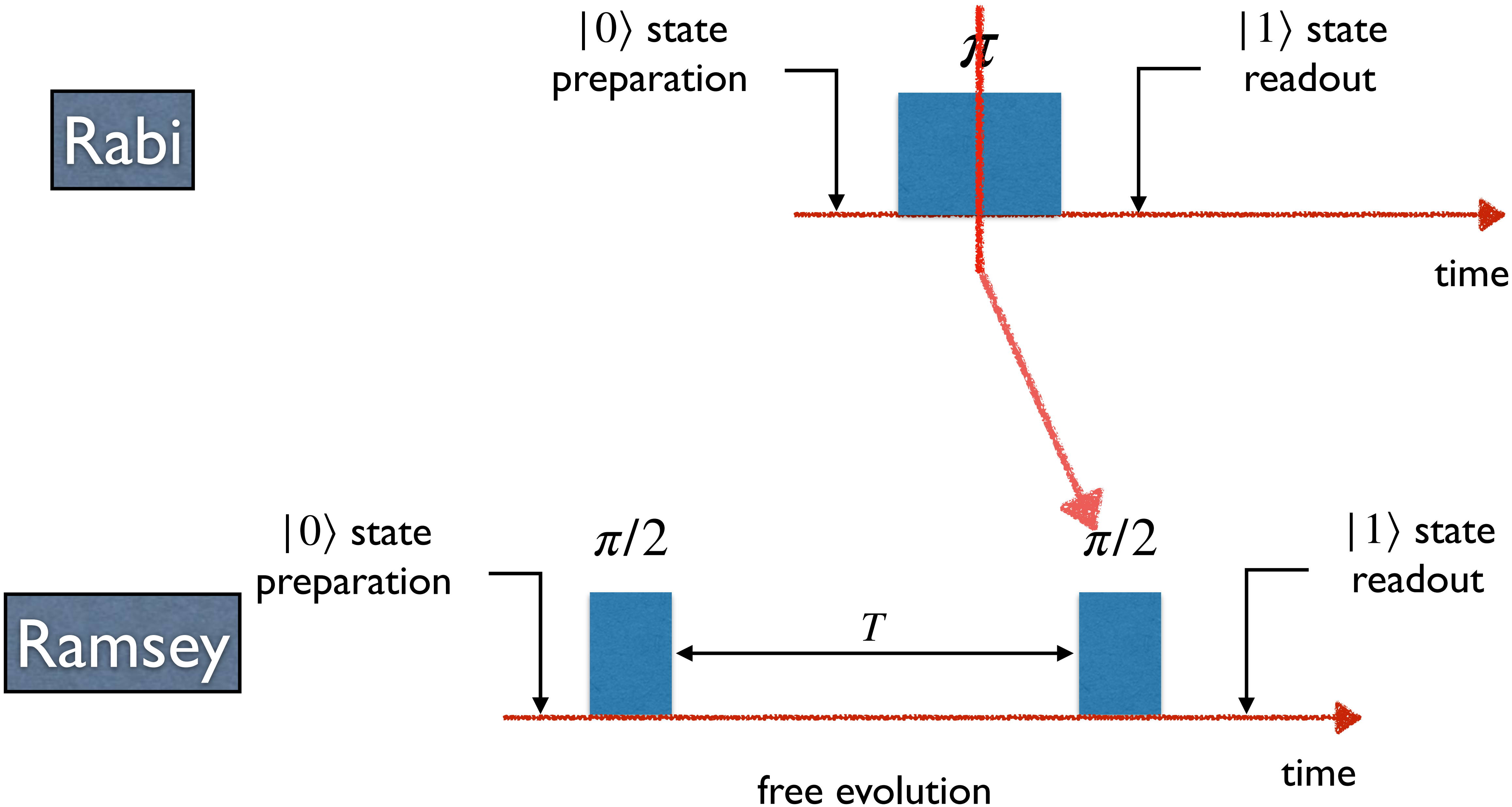

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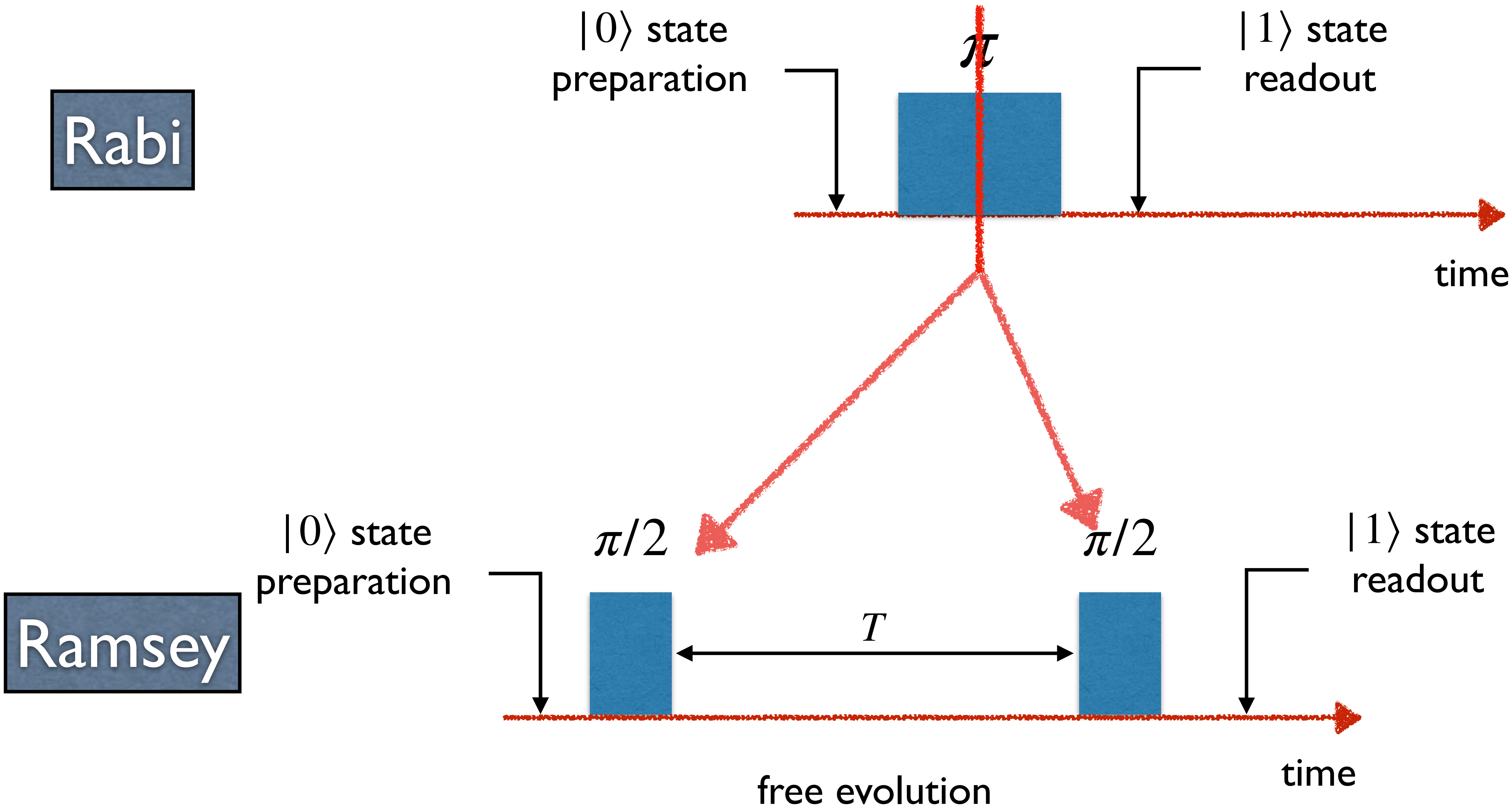

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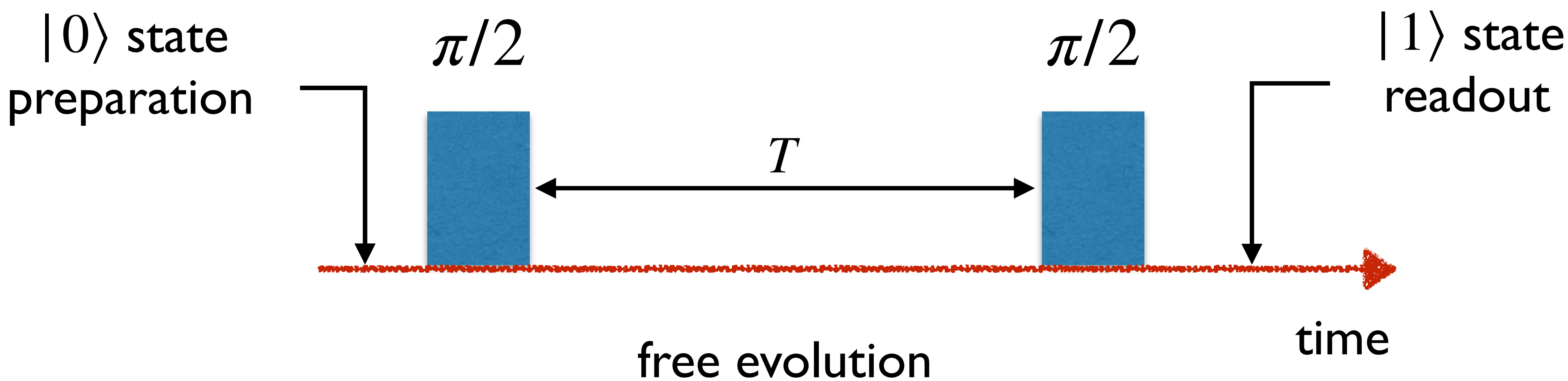


superposition $$\frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right)$$

differential phase accumulation $$\frac{1}{\sqrt{2}}\left(e^{-iT\Delta/2}|0\rangle + ie^{+iT\Delta/2}|1\rangle\right)$$

result: $$P_{|1\rangle} \propto \cos^2\left(\frac{T\Delta}{2}\right)$$

Internal state interferometry

# Improving resolution: Ramsey method

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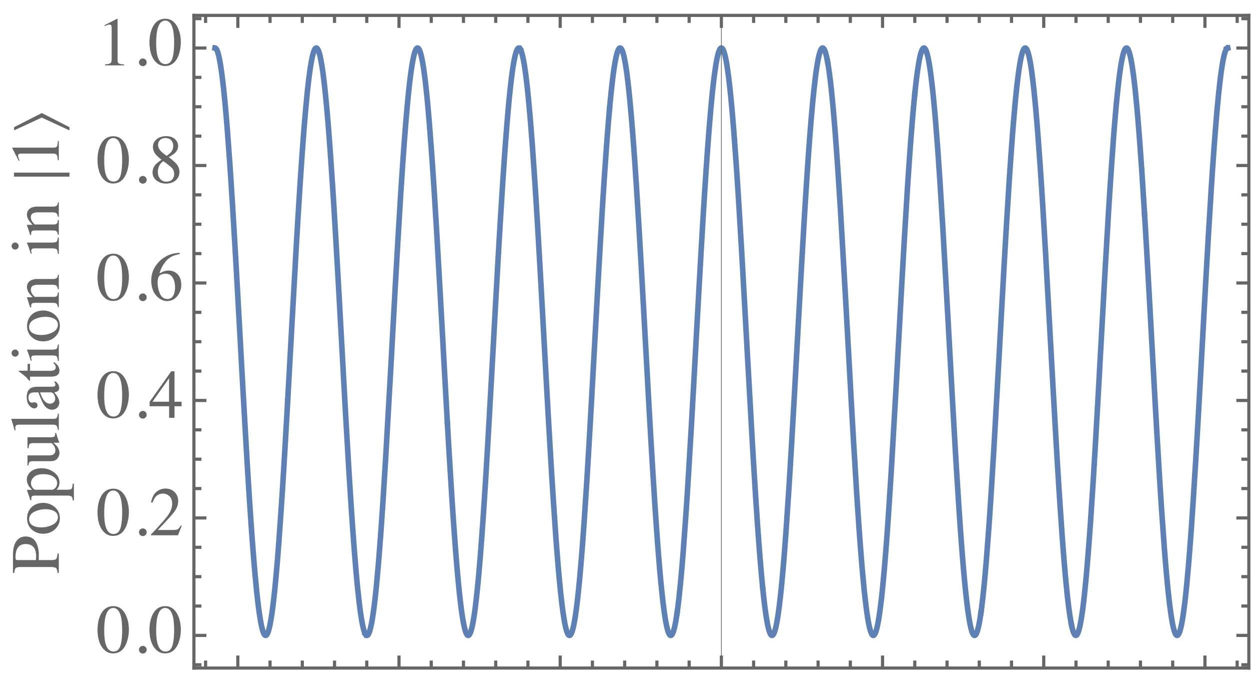


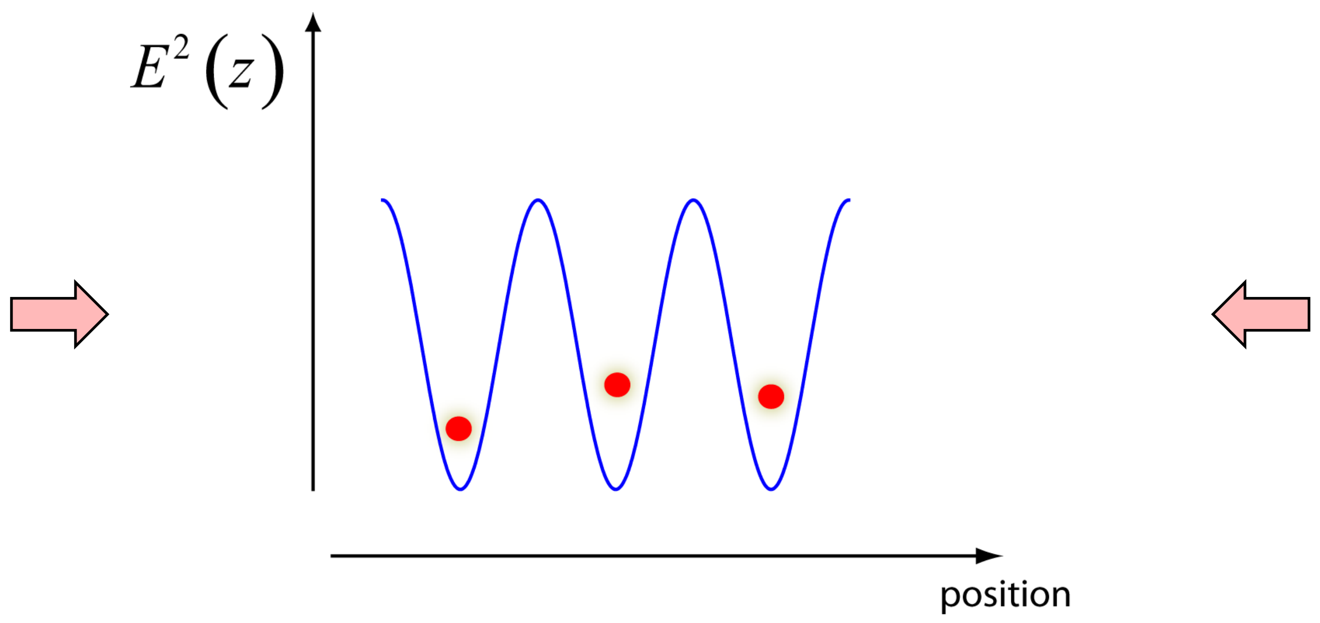

Ramsey fringes

Dramatic increase in frequency resolution

$$\Delta \sim 1/\tau_p \rightarrow 1/T$$

# Optical lattice clocks

Optical lattice : counter-propagating laser beams = standing wave




$$V(z) = -\frac{1}{2}\alpha(\omega_L)E^2(z)$$

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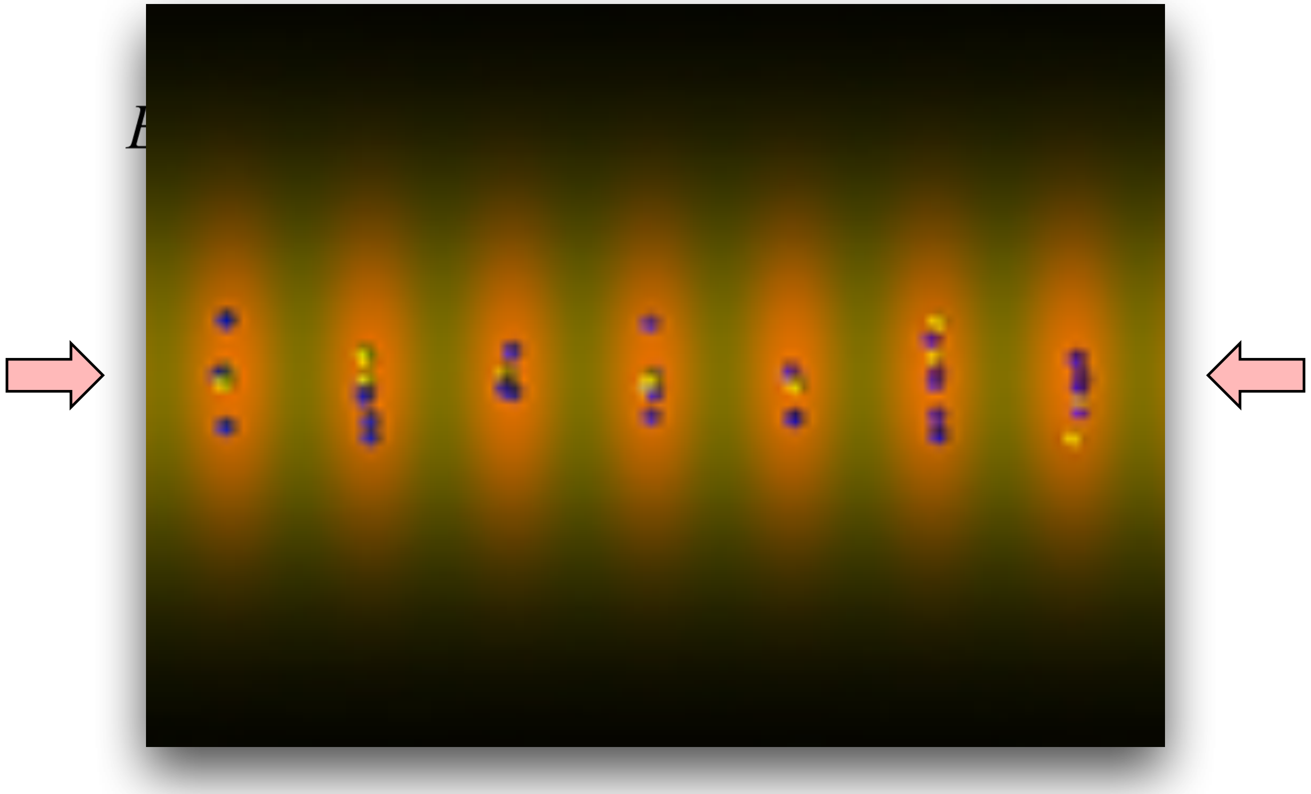

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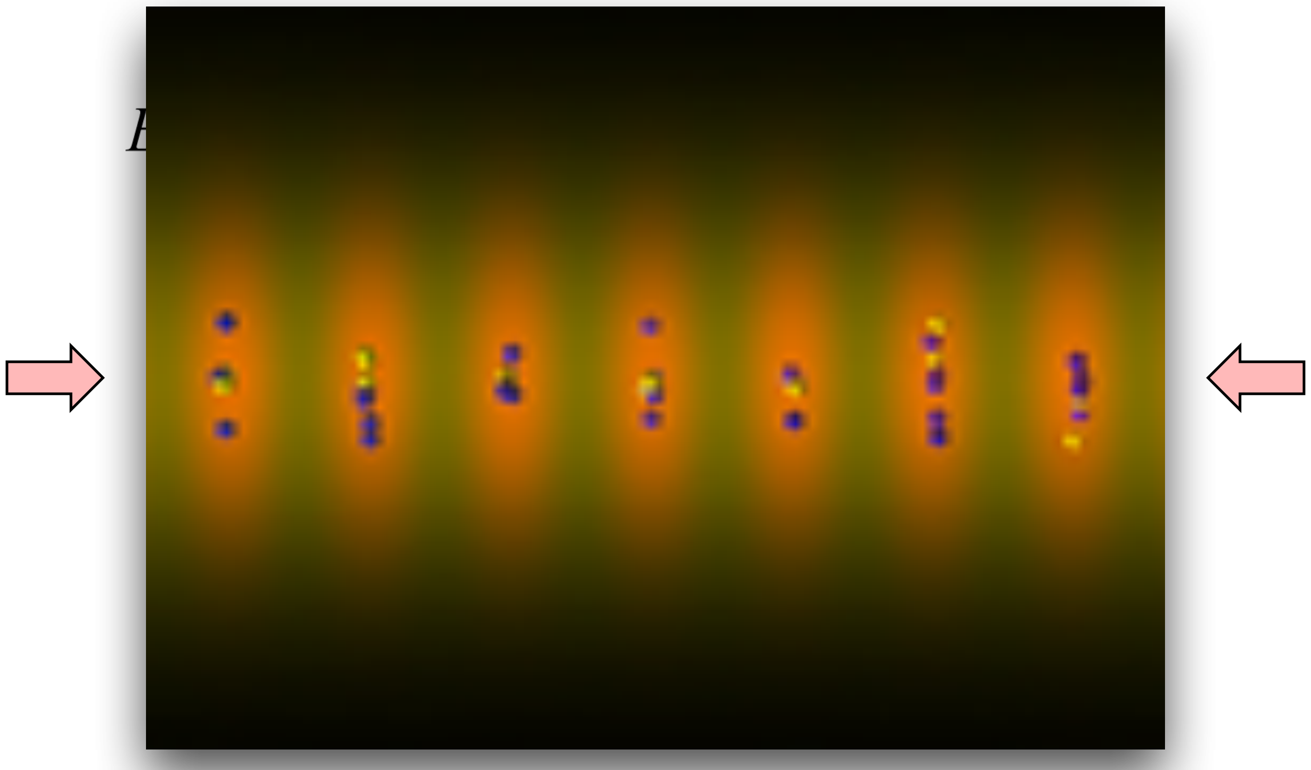

$$V(z) = -\frac{1}{2}\alpha(\omega_L)E^2(z)$$

Problem: Lasers perturb clock levels

$$\delta\nu_{\text{clock}} = \frac{1}{2h}\left[\alpha_g(\omega_L) - \alpha_e(\omega_L)\right]E^2(z)$$

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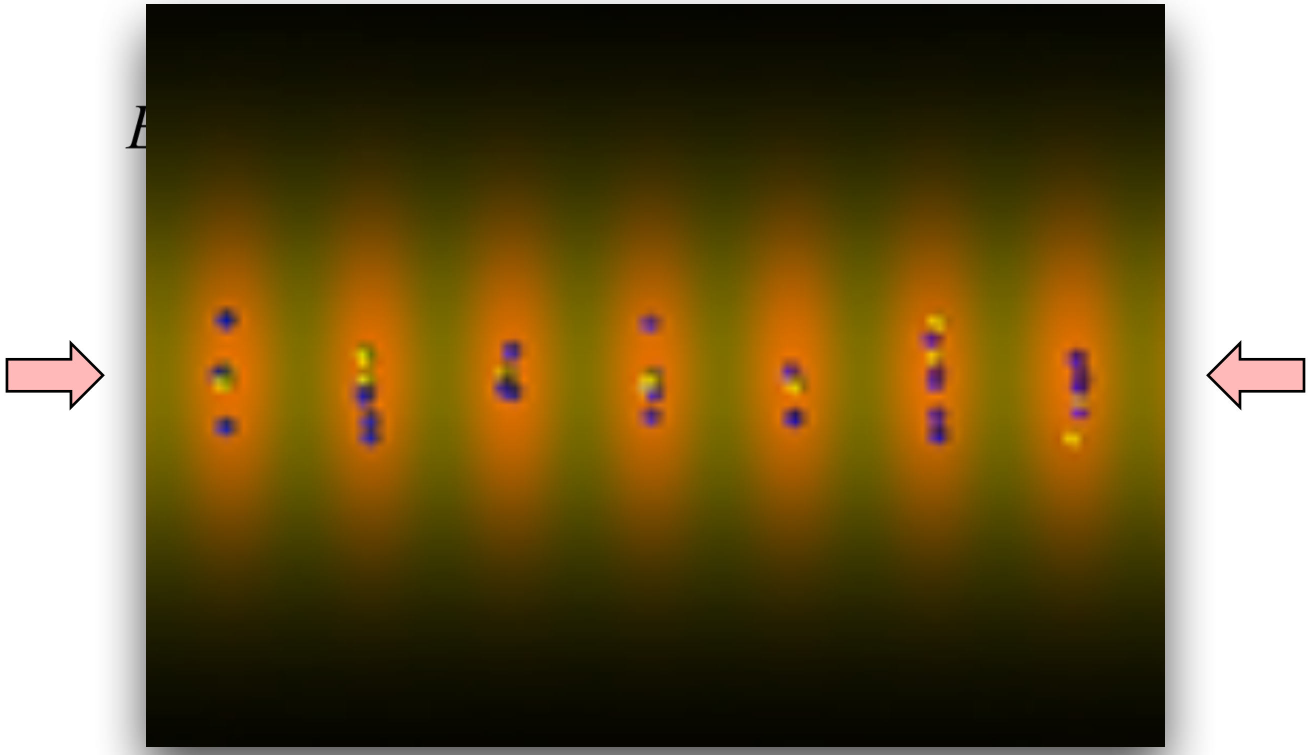


$$V(z) = -\frac{1}{2}\alpha(\omega_L)E^2(z)$$

Problem: Lasers perturb clock levels

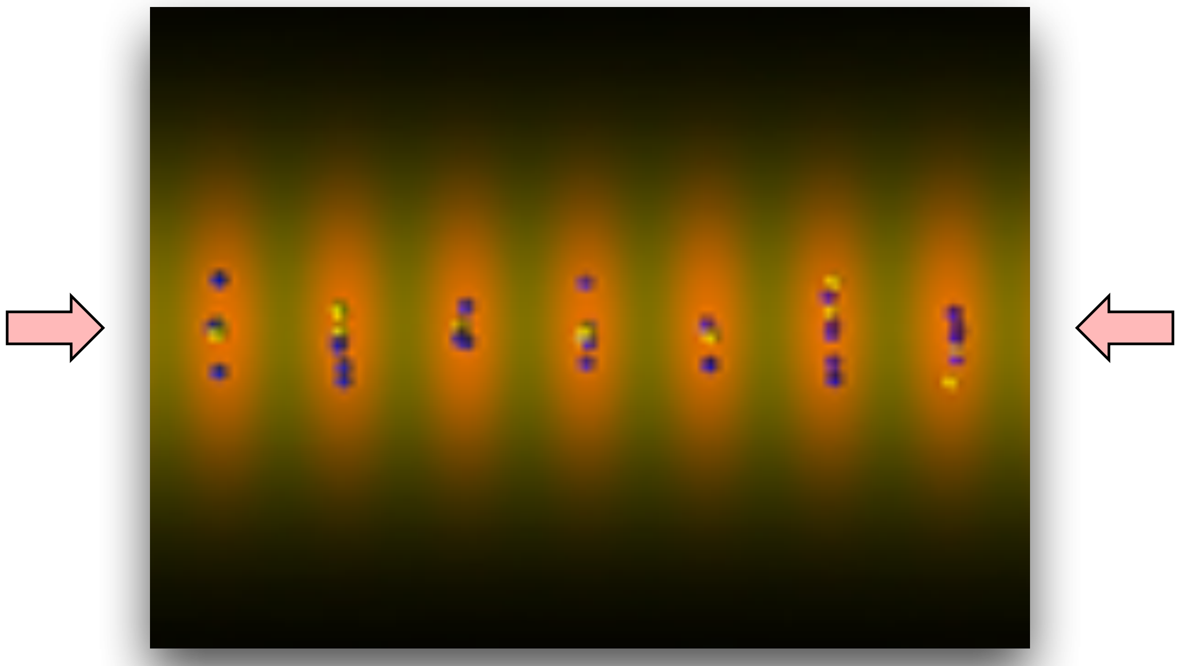
$$\delta\nu_{\text{clock}} = \frac{1}{2h}\left[\alpha_g(\omega_L) - \alpha_e(\omega_L)\right]E^2(z)$$

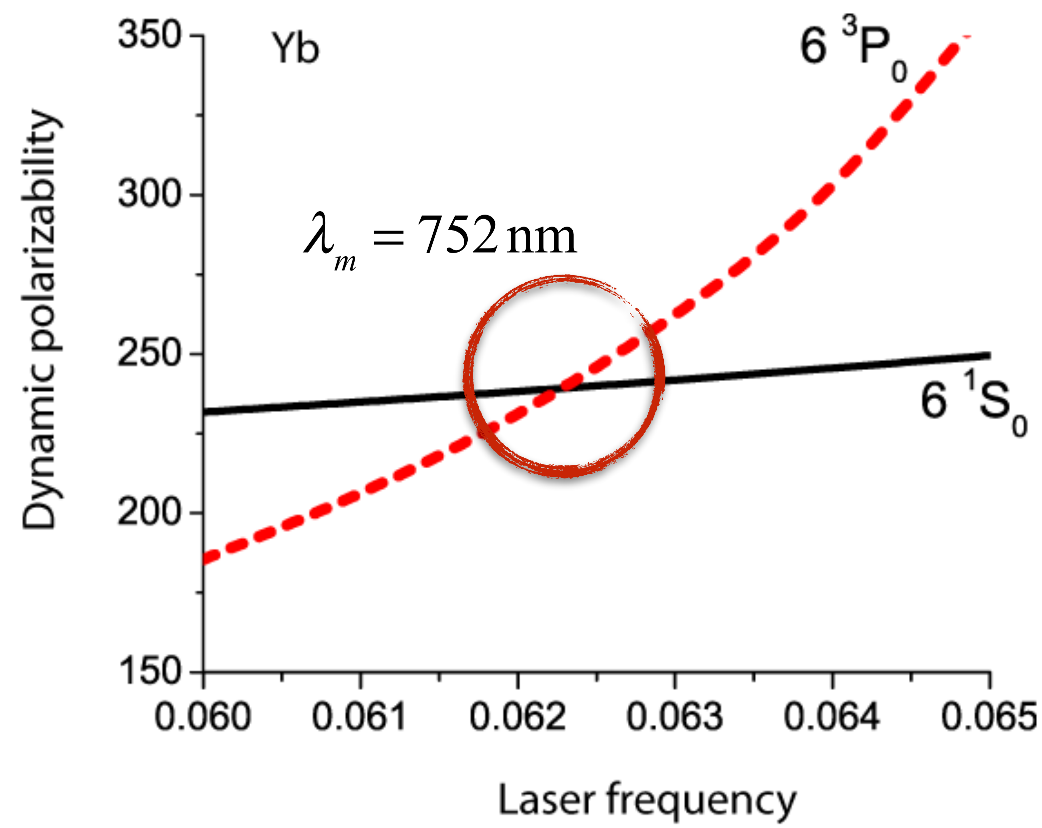
Tune laser frequency :: magic frequency (Katori)

$$\alpha_g(\omega_{\text{magic}}) \equiv \alpha_e(\omega_{\text{magic}}) \qquad \Longrightarrow \qquad \delta\nu_{\text{clock}} = 0$$

# Optical lattice clocks

Optical lattice : counter-propagating laser beams = standing wave






Porsev, Derevianko & Fortson PRA 69, 021403(R) (2004)

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Review: *Derevianko & Katori, Rev. Mod. Phys. 83, 331 (2011)*

PHYSICAL REVIEW A **69**, 021403(R) (2004)

# Possibility of an optical clock using the $6\,{}^1S_0 \rightarrow 6\,{}^3P_0^o$ transition in ${}^{171,173}$Yb atoms held in an optical lattice

Sergey G. Porsev
*Department of Physics, University of Nevada, Reno, Nevada 89557, USA*
*and Petersburg Nuclear Physics Institute, Gatchina, Leningrad District, 188300, Russia*

Andrei Derevianko
*Department of Physics, University of Nevada, Reno, Nevada 89557, USA*

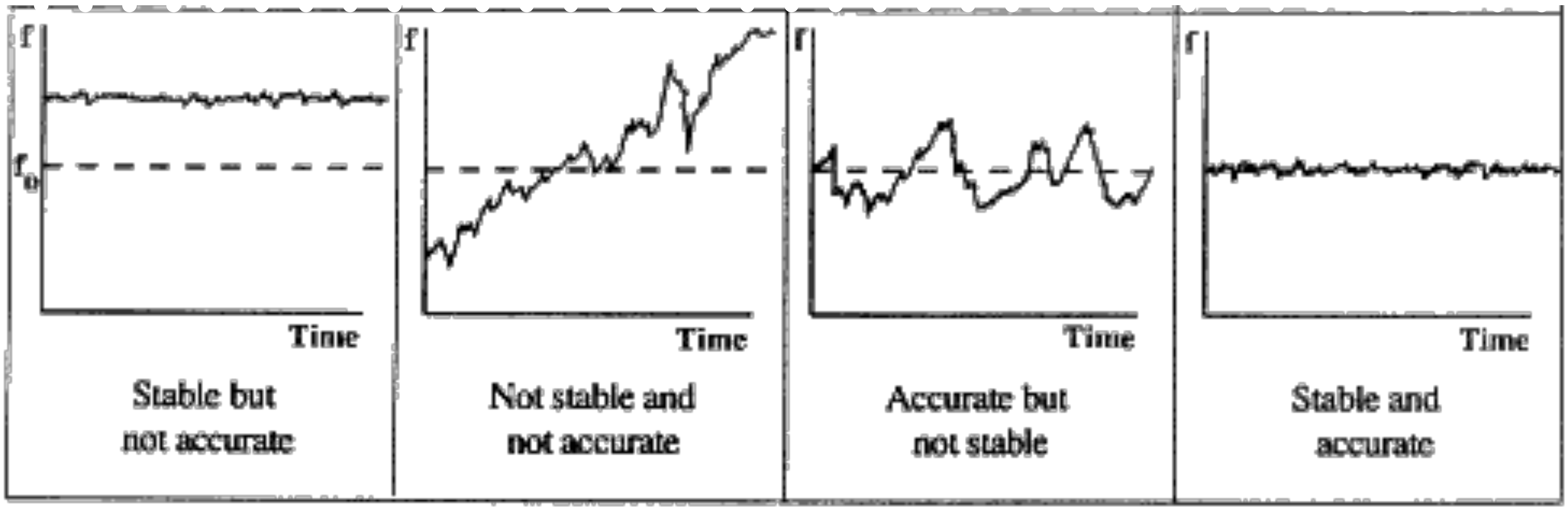
E. N. Fortson
*Department of Physics, University of Washington, Seattle, Washington 98195, USA*
(Received 16 October 2003; published 24 February 2004)

We report calculations assessing the ultimate precision of an atomic clock based on the 578 nm $6\,{}^1S_0 \rightarrow 6\,{}^3P_0$ transition in Yb atoms confined in an optical lattice trap. We find that this transition has a natural linewidth less than 10 mHz in the odd Yb isotopes, caused by hyperfine coupling. The shift in this transition due to the trapping light acting through the lowest order ac polarizability is found to become zero at the *magic* trap wavelength of about 752 nm. The effects of Rayleigh scattering, multipole polarizabilities, vector polarizability, and hyperfine induced electronic magnetic moments can all be held below 1 mHz (about one part in $10^{18}$). In the case of the hyperpolarizability, however, larger shifts due to nearly resonant terms cannot be ruled out without an accurate measurement of the magic wavelength.

Proposed: 2004 — Experimentally realized: 2006 (NIST-Boulder)

Yb lattice clocks around the globe:
USA (NIST, MIT), Germany, Italy, S. Korea, China, Japan,…

# Accuracy and stability






# Systematics and statistics

# Accuracy (systematics)

Quantum oscillator must be well protected from
the environmental perturbations (no systematic shifts)

# Example of evaluating accuracy

**Single-Ion Nuclear Clock for Metrology at the 19th Decimal Place**

C. J. Campbell, A. G. Radnaev, A. Kuzmich, V. A. Dzuba, V. V. Flambaum, and A. Derevianko

**Phys. Rev. Lett. 108, 120802 (2012)**

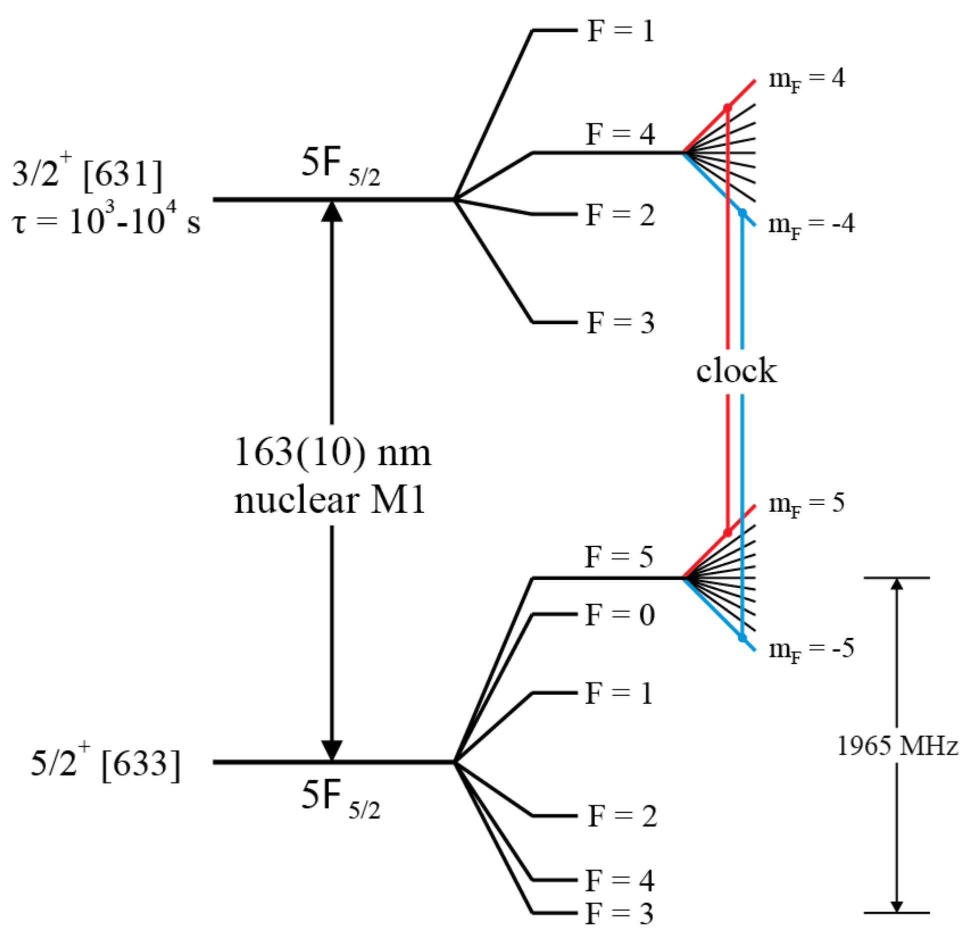



TABLE I. Estimated systematic error budget for a $^{229}\mathrm{Th}^{3+}$ clock using realized single-ion clock technologies. Shifts and uncertainties are in fractional frequency units $(\Delta\nu/\nu_{clk})$ where $\nu_{clk} = 1.8\,\mathrm{PHz}$. See text for discussion.

| Effect | $\lvert\text{Shift}\rvert$ $(10^{-20})$ | Uncertainty $(10^{-20})$ |
|---|---|---|
| Excess micromotion | 10 | 10 |
| Gravitational | 0 | 10 |
| Cooling laser Stark | 0 | 5 |
| Electric quadrupole | 3 | 3 |
| Secular motion | 5 | 1 |
| Linear Doppler | 0 | 1 |
| Linear Zeeman | 0 | 1 |
| Background collisions | 0 | 1 |
| Blackbody radiation | 0.013 | 0.013 |
| Clock laser Stark | 0 | $\ll 0.01$ |
| Trapping field Stark | 0 | $\ll 0.01$ |
| Quadratic Zeeman | 0 | 0 |
| Total | 18 | 15 |

# Atomic clock taxonomy

Clock frequency

## Microwave

Hyperfine transitions

Cs fountains[$10^{-16}$]
Rb beam/cell
Hg+ ion clock
Hydrogen maser
….

## Optical

Al+,Yb+,… ion [$10^{-18}$]
Sr,Yb,.. optical lattice [$10^{-18}$]
Th nuclear [$10^{-19}$]
Highly-charged ions [$10^{-19}$]
….

## XUV

Highly-charged ions

[projected fractional accuracy]

# Why higher clock frequency is better?

$|e\rangle$

$|g\rangle$

shifts remain approximately the same

$$\Delta t = \left( \frac{\Delta\omega}{\omega_{\text{clock}}} \right) \times t$$

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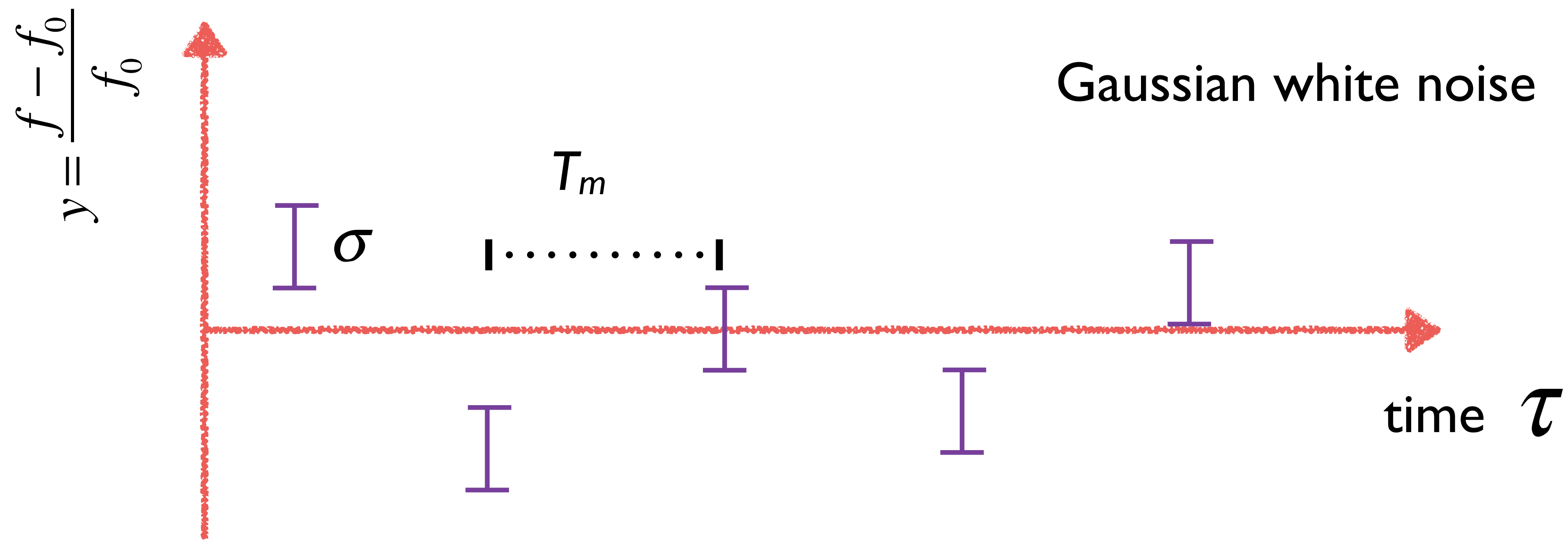
# Why nuclear/HCI clocks would have a better accuracy?

Couplings to external field ~ size of the quantum oscillator

# Stability

# Statistical uncertainties depend on how long you measure

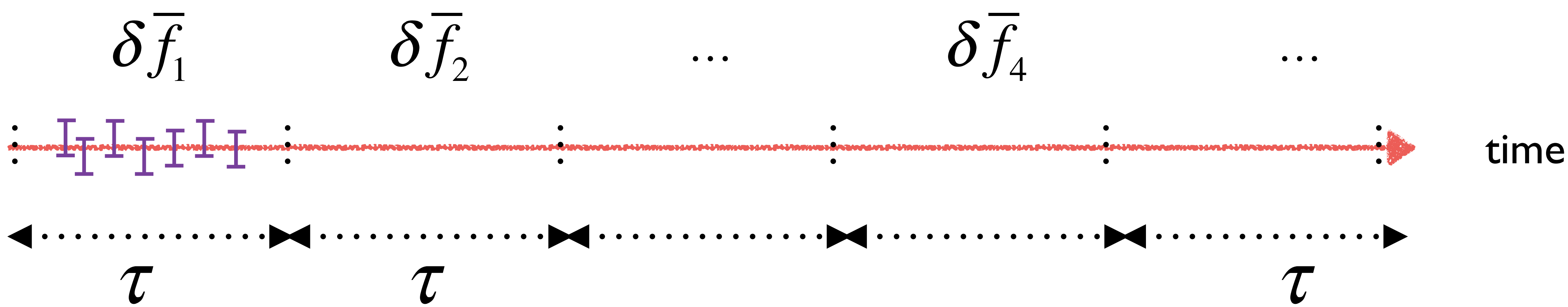



$$\sigma(\tau) = \frac{\sigma}{\sqrt{N_m}} = \frac{\sigma}{\sqrt{\tau / T_m}} \quad \Longrightarrow \quad \sigma(\tau) \propto \frac{1}{\sqrt{\tau}}$$

"Integrating out white frequency noise"

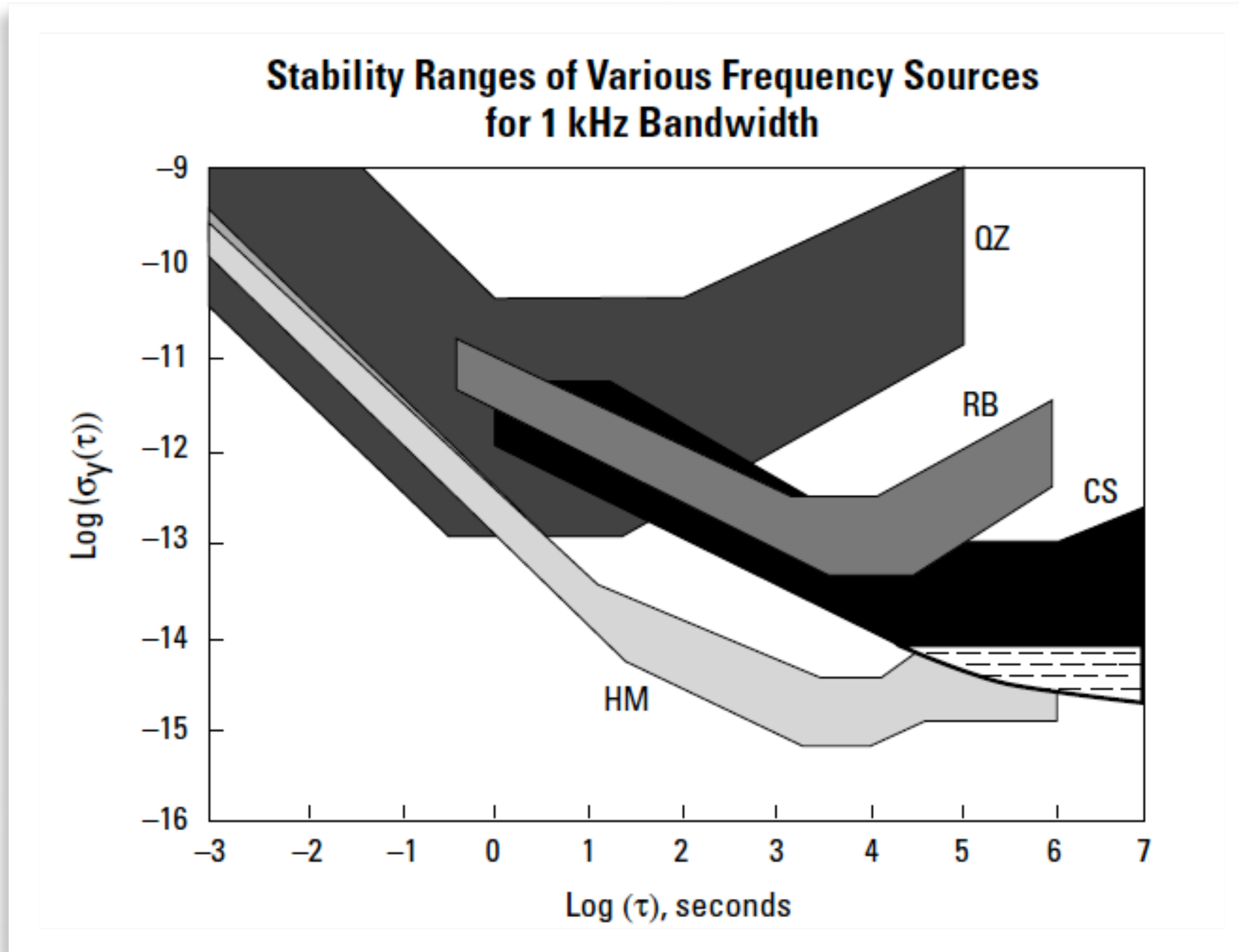
# Allan variance as a characteristic of stability

## Dealing with random walks and drifts




$$\sigma_y^2(\tau) \equiv \frac{1}{2}\left\langle \left( \frac{\delta\bar{f}_{n+1} - \delta\bar{f}_n}{f_0} \right)^2 \right\rangle_{\text{average over } n}$$

For the white noise still $\sigma_y(\tau) \propto \frac{1}{\sqrt{\tau}}$ flicker noise $\sigma_y(\tau) \propto \text{const}$




Usual scaling of long-term instability $\sigma_y(T) \propto 1/\sqrt{T}$

Typical values for microwave clocks $\sigma_y(1\ \mathrm{s}) > 10^{-13}$

Projected stability (optical lattice clocks) $\sigma_y(1\ \mathrm{s}) \sim 10^{-18}$

# Summary of basic concepts

- Time = (number of oscillations) x (known period)
- Atomic clocks work by locking sources of EM radiation to atomic transitions.
  Oscillations are counted at the source.
- Quantum oscillator must be well protected from the environmental perturbations (no systematic shifts)
- Clocks are characterized by accuracy (systematics) and stability (statistics, Allan variance)

D. Allan, N. Ashby, and C. Hodge, Hewlett Packard Appl. Note 1289 (1997).

# Entangling the lattice clock: Towards the Heisenberg-limited timekeeping

# Quantum metrology

Metrology: estimation of parameters of physical systems

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Classical metrology:

$$\hat{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad\qquad \sigma_{\hat{x}} = \frac{\sigma_1}{\sqrt{N}} \propto \frac{1}{\sqrt{N}}$$

# Quantum metrology

Metrology: estimation of parameters of physical systems

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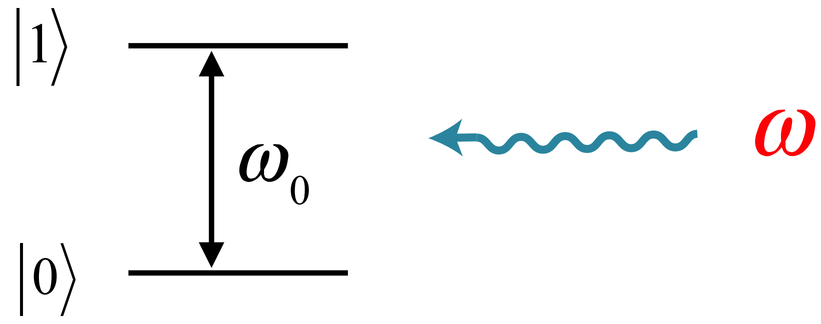
Quantum metrology (*N* ***entangled*** systems):

$$\sigma_{\hat{x}} = \frac{\sigma_1}{N} \propto \frac{1}{N}$$

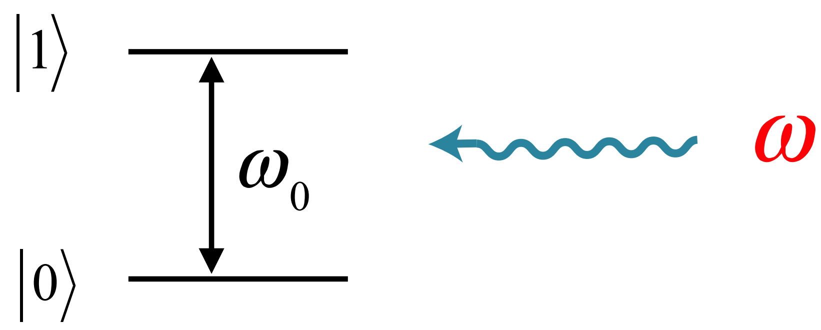
$N^{1/2}$ gain in the accuracy/sensitivity

# Quantum projection noise




What is the laser frequency?

# Quantum projection noise




What is the laser frequency?

$$|0\rangle \xrightarrow[\text{over time T}]{\text{Coherent interrogation}} c_1(\omega)|1\rangle + c_0(\omega)|0\rangle \xrightarrow[\text{measurement}]{\text{Projective}} \begin{cases} |1\rangle \\ |0\rangle \end{cases}$$

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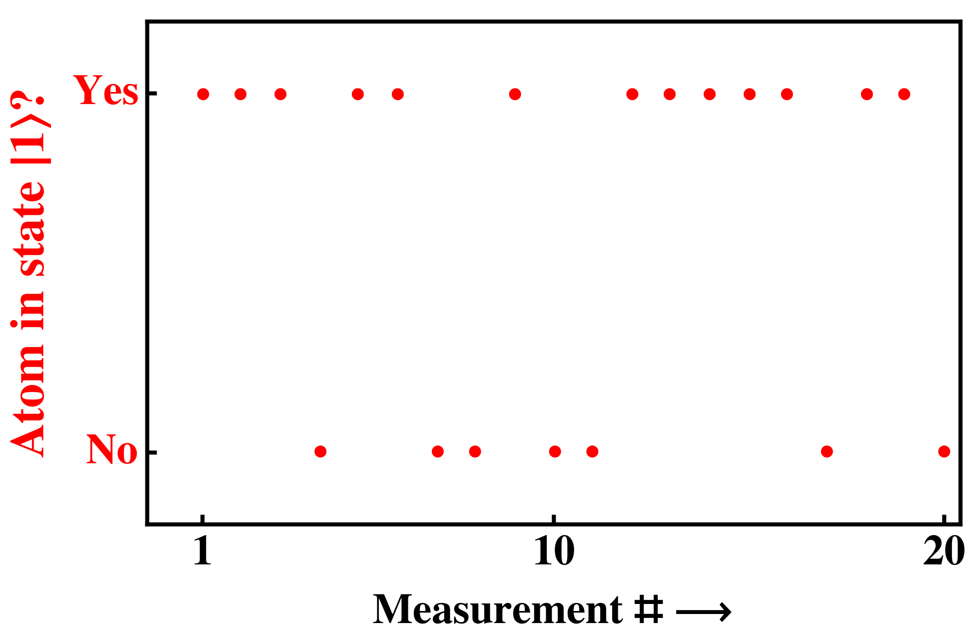

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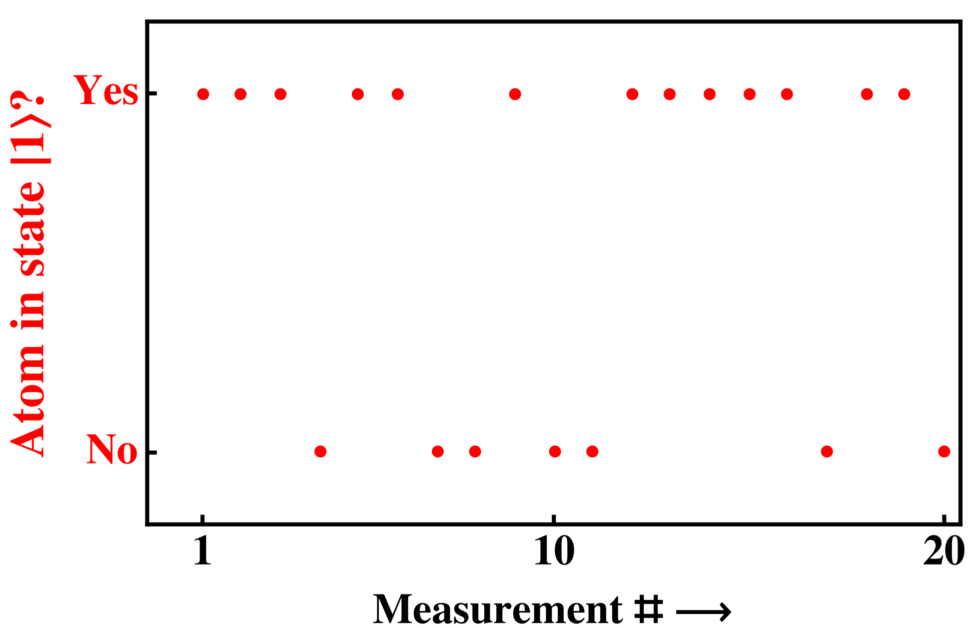


$$|c_1(\omega)|^2 = \frac{N_{\text{Yes}}}{N} \pm \frac{\text{const}}{\sqrt{N}}$$

$$\boxed{\sigma_\omega^{\text{QPN}} \sim \frac{1}{T}\frac{1}{\sqrt{N}}}$$

Standard Quantum Limit

# The power of entanglement

$E_1 = \hbar\omega_0$ —— $|1\rangle$

$E_0 = 0$ —— $|0\rangle$

$$|\Psi(t=0)\rangle \propto |0\rangle + |1\rangle$$

$$|\Psi(t=T)\rangle \propto |0\rangle + \exp(-i\omega_0 T)|1\rangle$$

$E_N = N\hbar\omega_0$ —— $|1\cdots1\rangle$

$\vdots$

$E_0 = 0$ —— $|0\cdots0\rangle$

$$|\Psi_{\mathrm{GHZ}}(t=0)\rangle \propto |0\cdots0\rangle + |1\cdots1\rangle$$

$$|\Psi_{\mathrm{GHZ}}(t=T)\rangle \propto |0\cdots0\rangle + \exp(-iN\omega_0 T)\ |1\cdots1\rangle$$

Phase accumulates *N* times faster

N entangled atoms

# The power of entanglement

***N* independent atoms**

$E_1 = \hbar\omega_0$ —— $|1\rangle$

$E_0 = 0$ —— $|0\rangle$

$$|0\rangle + |1\rangle \xrightarrow{T} |0\rangle + \exp(-i\omega_0 T)|1\rangle$$

$$\boxed{\sigma_\omega^{\mathrm{SQL}} \sim \frac{1}{T}\frac{1}{\sqrt{N}}}$$

Standard Quantum Limit

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Standard Quantum Limit

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$$|0\cdots0\rangle + |1\cdots1\rangle \xrightarrow{T} |0\rangle + \exp(-iN\omega_0 T)|1\rangle$$

Phase accumulates *N* times faster

$$\boxed{\sigma_{N\omega} \sim \frac{1}{T} \quad \Rightarrow \quad \sigma_\omega^{\mathrm{HL}} \sim \frac{1}{NT}}$$

Heisenberg Limit

# Entanglement and how to attain it

$$|0\rangle_a|0\rangle_b + |1\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |1\rangle_a|1\rangle_b$$

$$|0\rangle_a|0\rangle_b + \qquad\qquad |1\rangle_a|1\rangle_b$$

# Entanglement and how to attain it

$$|0\rangle_a|0\rangle_b + |1\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |1\rangle_a|1\rangle_b$$
$$= \left(|0\rangle + |1\rangle\right)_a \left(|0\rangle + |1\rangle\right)_b$$

$$|0\rangle_a|0\rangle_b + \qquad\qquad\qquad |1\rangle_a|1\rangle_b$$

# Entanglement and how to attain it

Factorizable

$$|0\rangle_a|0\rangle_b + |1\rangle_a|0\rangle_b + |0\rangle_a|1\rangle_b + |1\rangle_a|1\rangle_b$$

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Non-factorizable

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$$|0\rangle_a|0\rangle_b + \qquad\qquad\qquad |1\rangle_a|1\rangle_b$$

Factorizable states can be easily produced by single qubit rotations

Entangled states -non-trivial generation -
needs conditional quantum logic - multi-particle interactions

# Entanglement and how to attain it

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Entangled states -non-trivial generation -
needs conditional quantum logic - multi-particle interactions

Neutral atom examples:

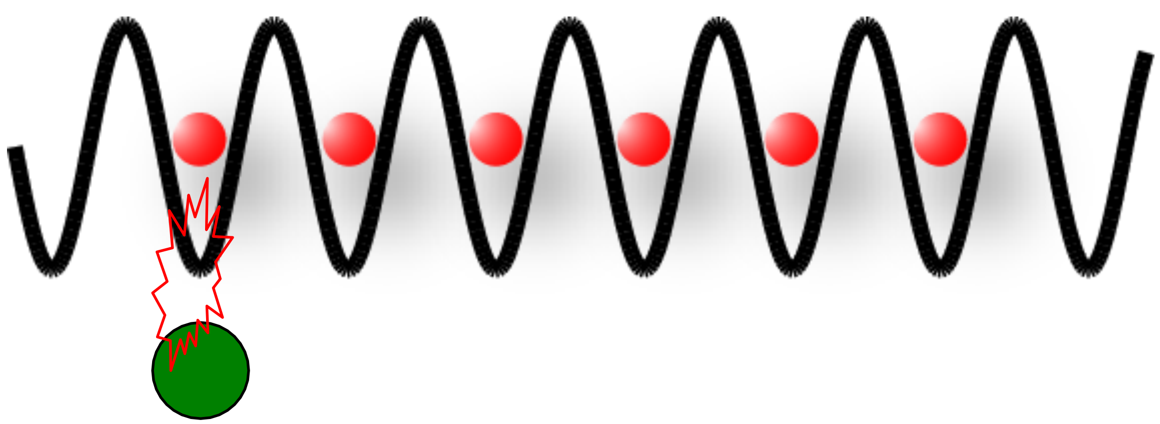
- Controlled collisions
- Rydberg atom interactions
- Magnetic interactions

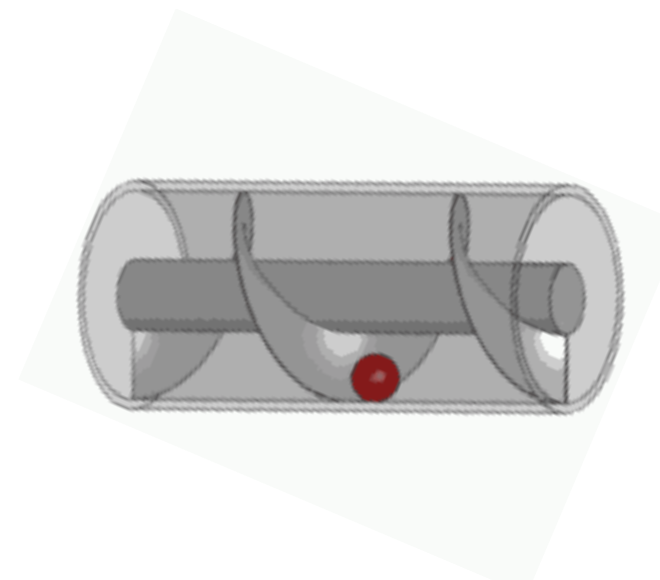
PHYSICAL REVIEW A **81**, 030302(R) (2010)

# Entangling the lattice clock: Towards Heisenberg-limited timekeeping

Jonathan D. Weinstein, Kyle Beloy, and Andrei Derevianko
*Department of Physics, University of Nevada, Reno, Nevada 89557, USA*
(Received 4 December 2009; published 29 March 2010)

A scheme is presented for entangling the atoms of an optical lattice to reduce the quantum projection noise of a clock measurement. The divalent clock atoms are held in a lattice at a “magic” wavelength that does not perturb the clock frequency—to maintain clock accuracy—while an open-shell $J = 1/2$ “head” atom is coherently transported between lattice sites via the lattice polarization. This polarization-dependent “Archimedes’ screw” transport at magic wavelength takes advantage of the vanishing vector polarizability of the scalar, $J = 0$, clock states of bosonic isotopes of divalent atoms. The on-site interactions between the clock atoms and the head atom are used to engineer entanglement and for clock readout.





- Challenge: adding extra pieces to the clock should not affect systematics
- Clock register + “head” atom (Aluminum)
- Archimedes’ screw transport through the optical lattice
- Collisional phase gate between the “head” and clock atoms

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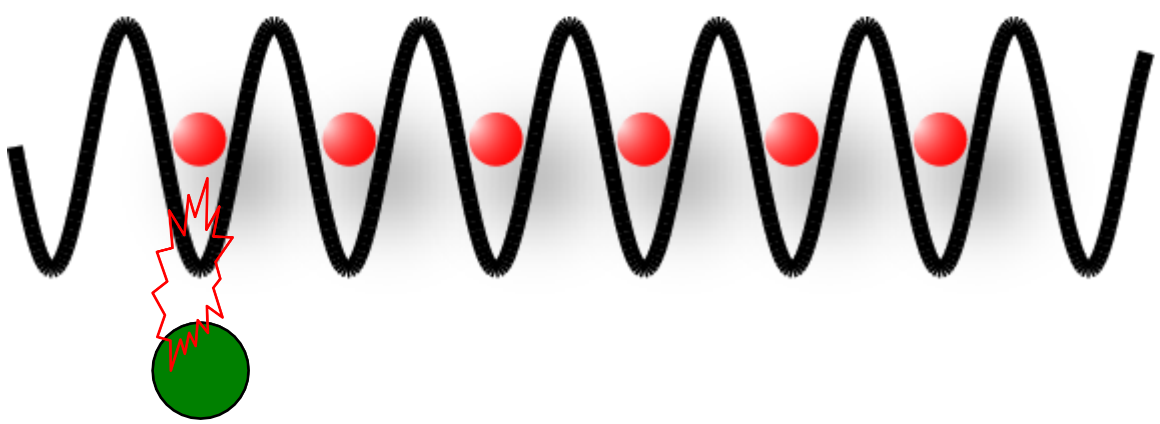

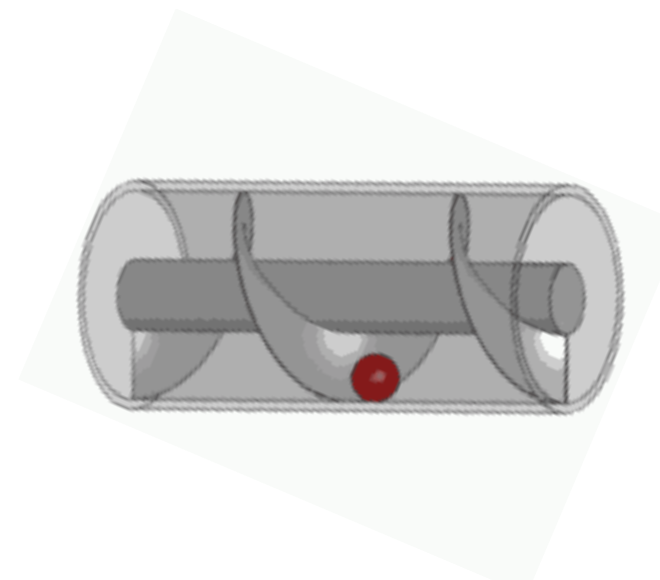

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PRL 117, 060506 (2016) PHYSICAL REVIEW LETTERS week ending 5 AUGUST 2016

# Quantum Network of Atom Clocks: A Possible Implementation with Neutral Atoms

P. Kómár,[1] T. Topcu,[1,2,3] E. M. Kessler,[1,3] A. Derevianko,[1,2,3] V. Vuletić,[4] J. Ye,[5] and M. D. Lukin[1]
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[2]*Department of Physics, University of Nevada, Reno, Nevada 89557, USA*
[3]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA*
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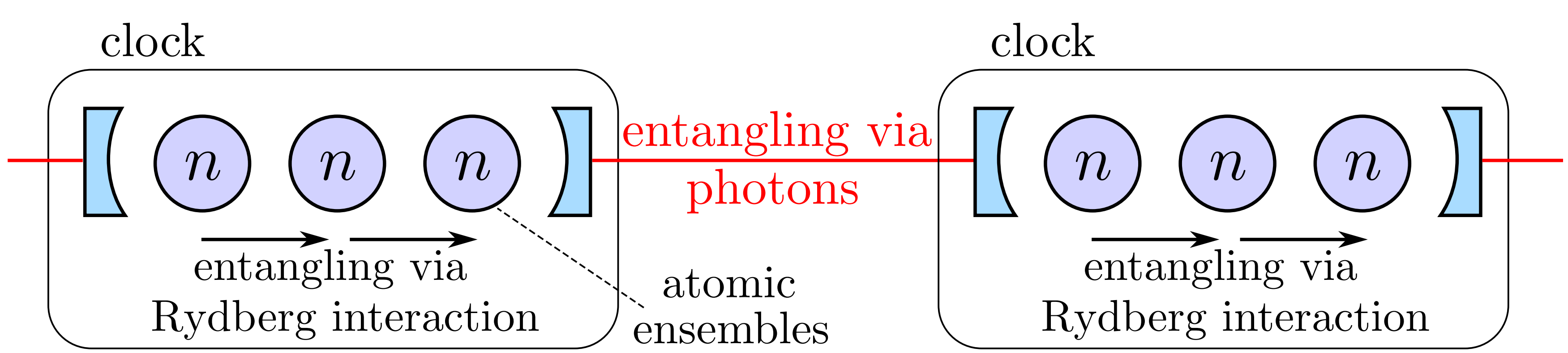
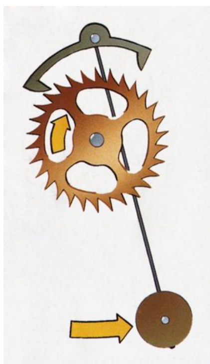
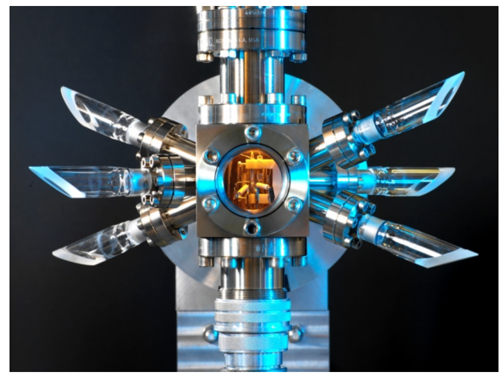
[5]*JILA, NIST, Department of Physics, University of Colorado, Boulder, Colorado 80309, USA*
(Received 22 March 2016; published 5 August 2016)

We propose a protocol for creating a fully entangled Greenberger-Horne-Zeilinger-type state of neutral atoms in spatially separated optical atomic clocks. In our scheme, local operations make use of the strong dipole-dipole interaction between Rydberg excitations, which give rise to fast and reliable quantum operations involving all atoms in the ensemble. The necessary entanglement between distant ensembles is mediated by single-photon quantum channels and collectively enhanced light-matter couplings. These techniques can be used to create the recently proposed quantum clock network based on neutral atom optical clocks. We specifically analyze a possible realization of this scheme using neutral Yb ensembles.




Entangling protocol is partially based on Saffman & Mølmer PRL 102, 240502 (2009)

# Summary: Day 1

 $\Rightarrow$ 

$$|0\rangle + |1\rangle \overset{?}{\Rightarrow} |0\cdots 0\rangle + |1\cdots 1\rangle$$

- ❑ Arguably the most precise quantum sensors ever built
- ❑ A wealth of techniques form a toolbox of quantum information science
- ❑ Natural long-coherence qubits, state preparation, coherent state manipulation
- ❑ Ion clocks - first demonstration of high-fidelity entangling gates
- ❑ Quantum oscillator (qubit) is well protected and characterized $\Longrightarrow$

  novel applications in fundamental physics