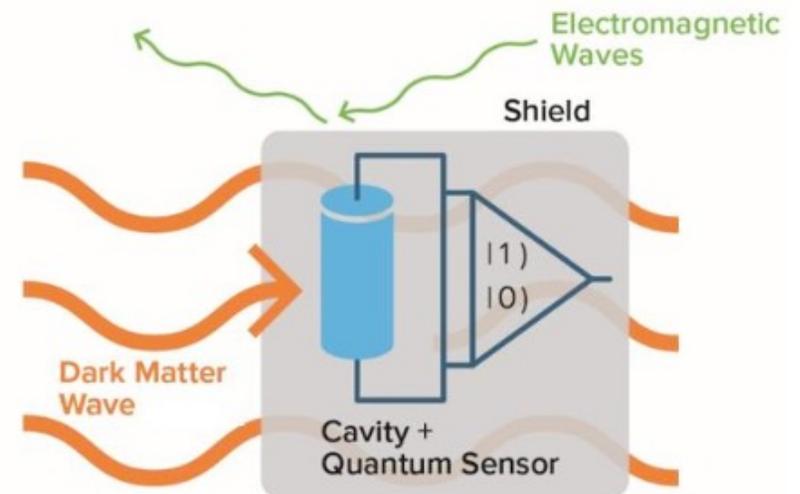


# Clever Quantum Tricks for Detecting Dark Matter Waves

Aaron S. Chou (Fermilab)  
QSFP School

Tutorials (repeated today and Thursday):  
Chelsea Bartram (U.Washington): axion experimental techniques  
Samantha Lewis (Fermilab): axions and microwave cavities  
Akash Dixit (U.Chicago): single photon sensing

Detect Wave  
Dark Matter  
in the Laboratory



# Plan for lectures

(30 minutes each)

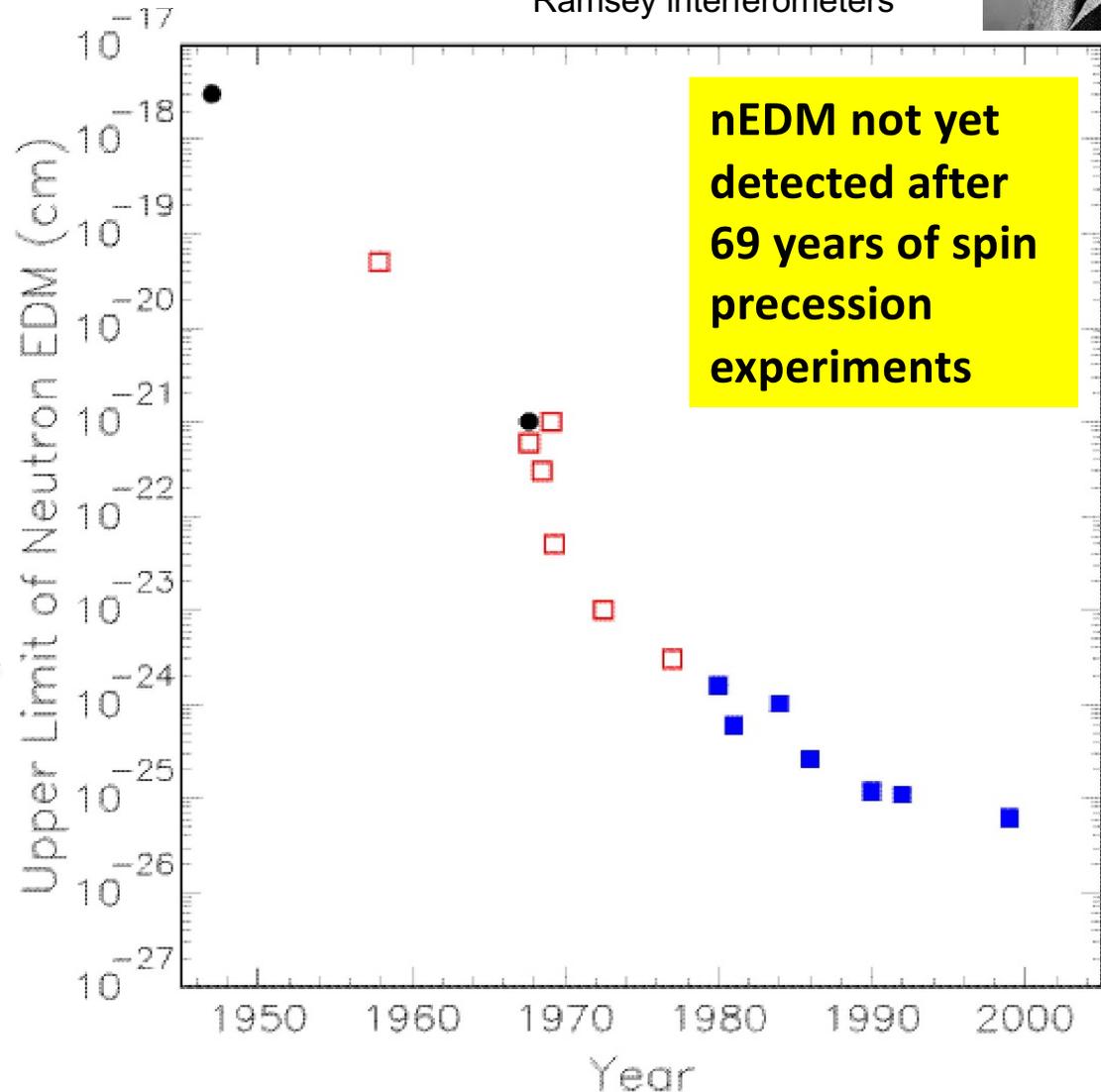
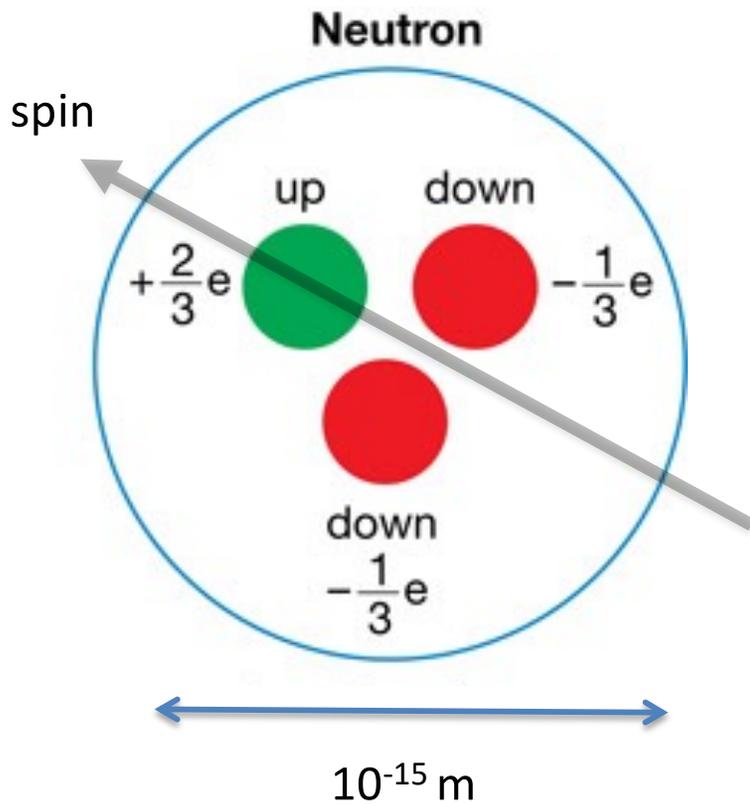
- Dark matter waves, the strong-CP problem and axions
- Axion haloscopes and the Standard Quantum Limit on noise
- Quantum optics and quantum non-demolition measurements
- Stimulated emission using non-classical state preparation
- Single photon detection and Micro-calorimetry

# QCD axion motivated by the Strong-CP Problem: Why is the neutron electric dipole moment so small?

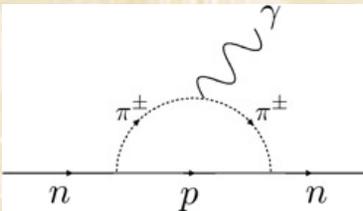


Norman Ramsey  
Nobel Prize 1989.  
Neutrino oscillation expts are  
"Ramsey interferometers"

Naive estimate gives  
 $nEDM \approx 10^{-16} \text{ e-cm}$



# The CP Problem of Strong Interactions



Characterizes degenerate QCD ground state ( $\Theta$  vacuum)

Phase of Quark Mass Matrix

Standard QCD Lagrangian contains a CP violating term

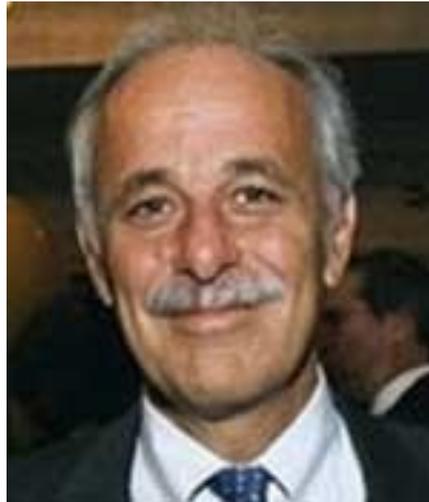
$$L_{CP} = -\frac{\alpha_s}{8\pi} \underbrace{(\Theta - \arg \det M_q)}_{0 \leq \Theta \leq 2\pi} \text{Tr } \tilde{G}_{\mu\nu} G^{\mu\nu}$$

Induces a neutron electric dipole moment (EDM) much in excess of experimental limits

$$d_n \approx \bar{\Theta} 10^{-16} \text{ e cm} \approx \frac{\bar{\Theta}}{10^2} \mu_n < 3 \times 10^{-26} \text{ e cm}$$

$$\bar{\Theta} \lesssim 10^{-10} \quad \text{Why so small?}$$

# The 1977 Peccei-Quinn solution to the strong-CP problem

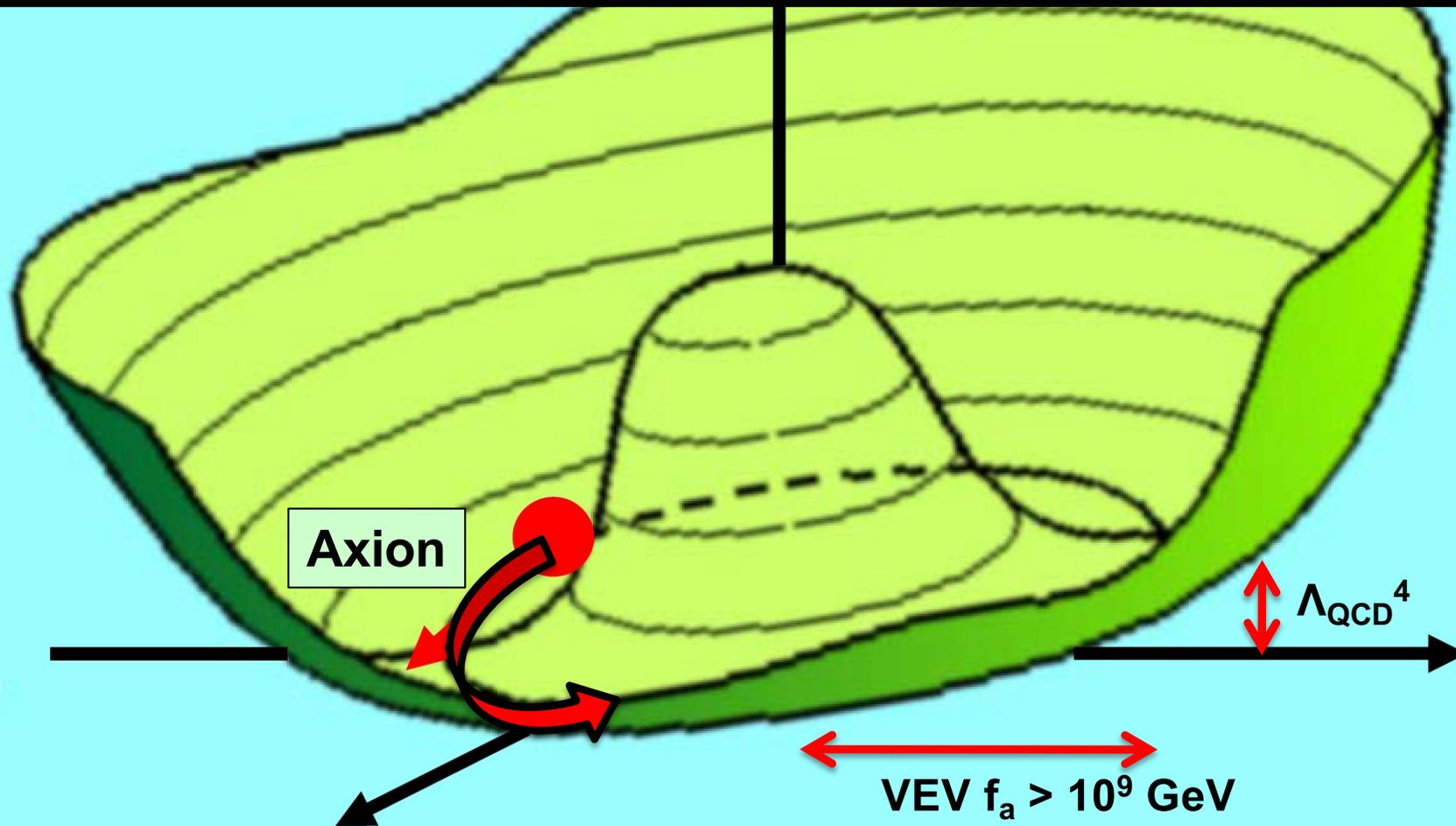


Dirac Medal  
(2000)

- Promote theta to become a new **dynamical** scalar field which has a two-gluon coupling. (dynamical = can vary in space and time)
- Think like an electrical engineer: Use this field in a cosmological feedback loop to dynamically zero out any pre-existing CP-violating phase angles.

# Natural potential energy function

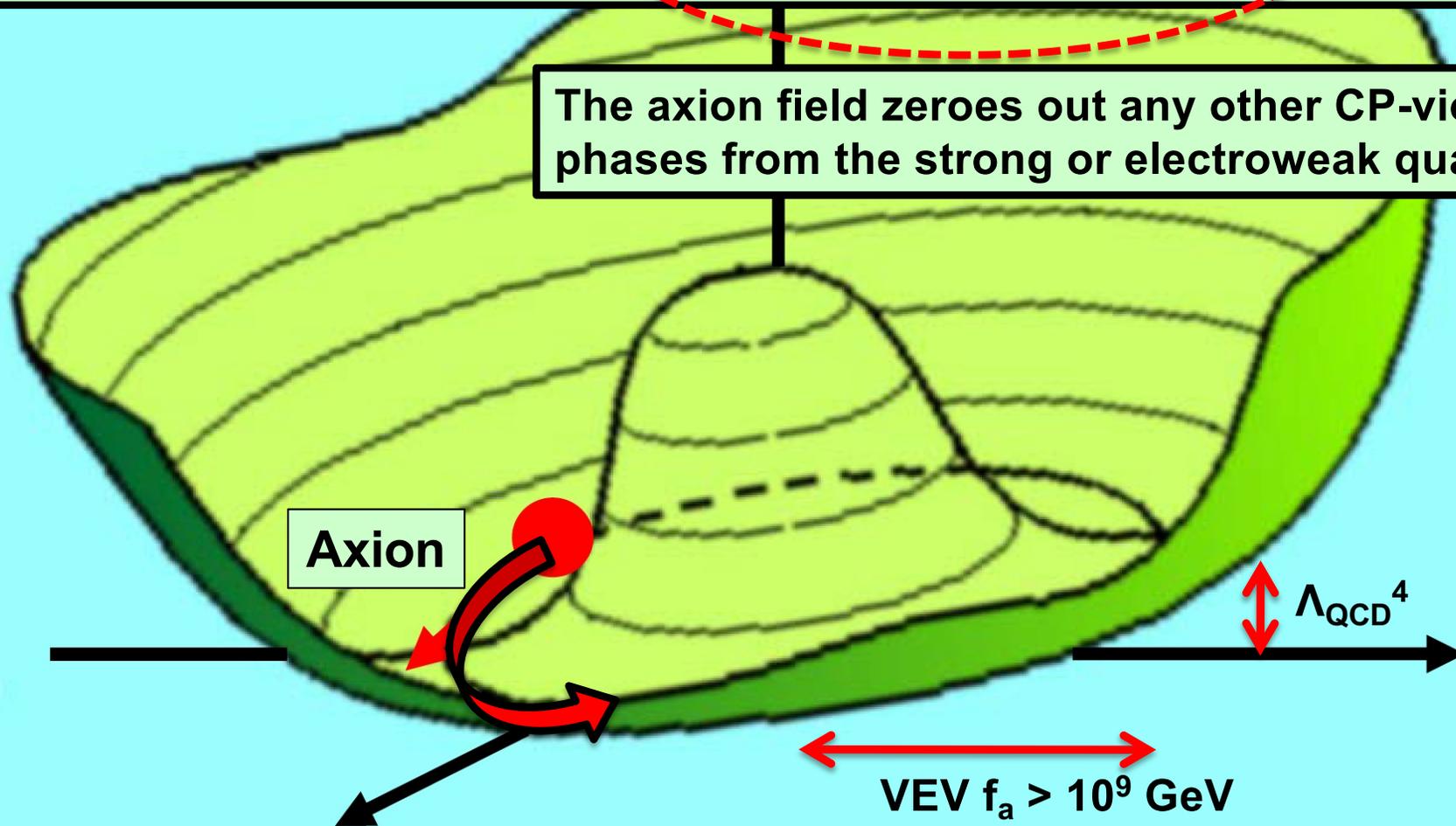
$$V(A) = -f_a^2 A^2 + \frac{\lambda}{4!} A^4 + \left( \frac{g^2}{32\pi^2} \arg(A) - \frac{\alpha_s}{8\pi} (\theta_{QCD} + \theta_{quark}) \right) \langle G\tilde{G} \rangle$$



# Natural potential energy function

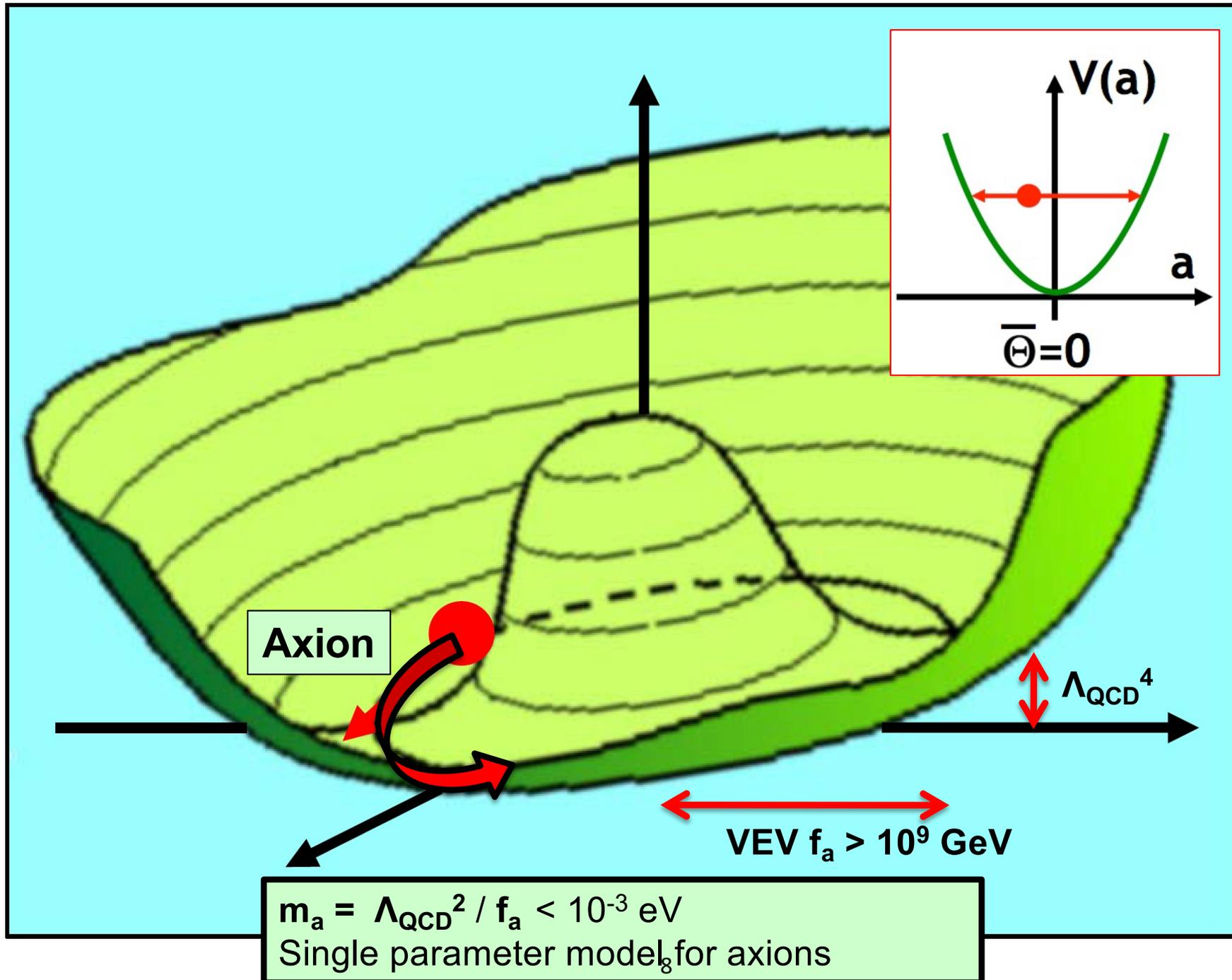
$$V(A) = -f_a^2 A^2 + \frac{\lambda}{4!} A^4 + \left( \frac{g^2}{32\pi^2} \arg(A) - \frac{\alpha_s}{8\pi} (\theta_{QCD} + \theta_{quark}) \right) \langle G\tilde{G} \rangle$$

The axion field zeroes out any other CP-violating phases from the strong or electroweak quark sector.



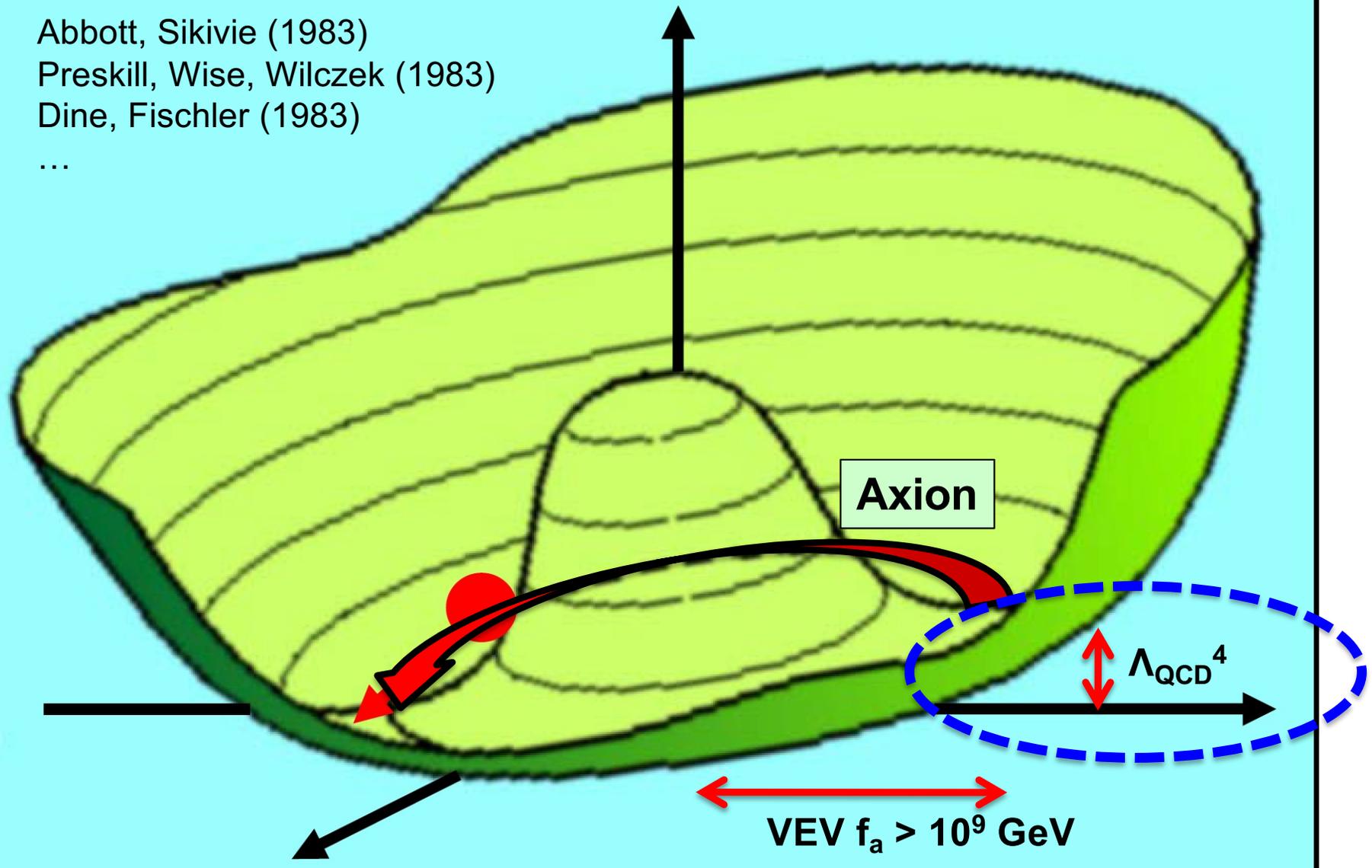
The neutron EDM vanishes, solving the **strong CP fine-tuning problem**.

# Axion mass = harmonic oscillator frequency



# The initial potential energy density is released as ultracold dark matter

Abbott, Sikivie (1983)  
Preskill, Wise, Wilczek (1983)  
Dine, Fischler (1983)  
...



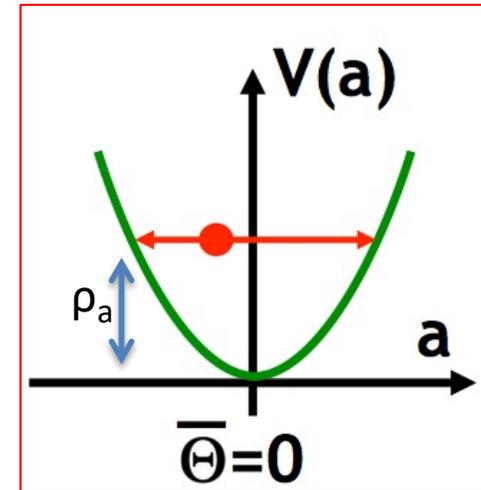
The initial azimuthal angle  $\theta_0$ , determines the available potential energy to be released.  $\mathcal{O}(1) \times \Lambda_{\text{QCD}}^4$  of potential energy density is converted into dark matter.

# Axion dark matter = waves of oscillating $\theta_{CP}$

Locally coherent oscillation of the QCD  $\theta$  angle about its CP-conserving minimum:

$$\theta(x, t) = \theta_{\max} e^{i(kx - m_a t)}$$

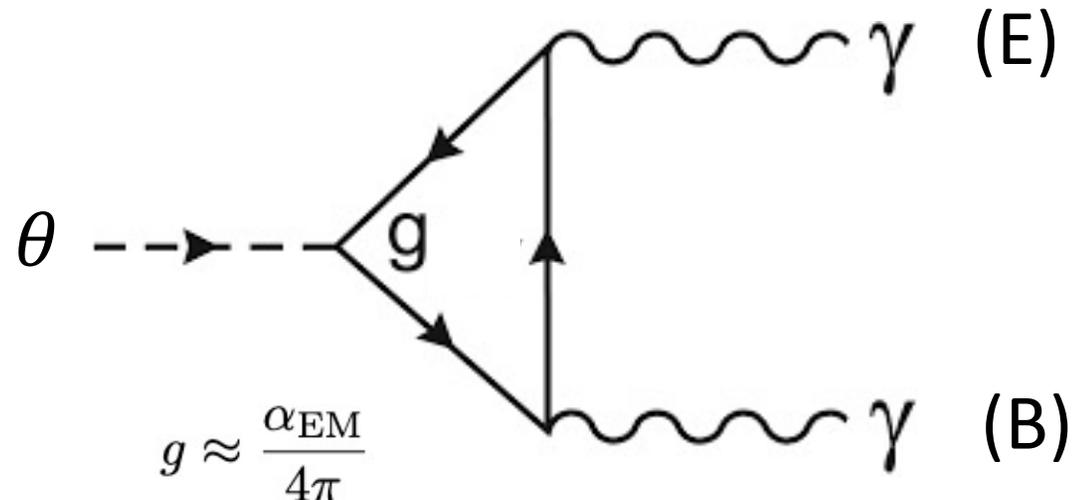
where 
$$\theta_{\max} = \sqrt{\frac{2\rho_a}{\Lambda_{\text{QCD}}^4}} \approx 3.7 \times 10^{-19} \text{ radians}$$



DM oscillations partially undo the Peccei-Quinn mechanism by enabling the coherent field to climb out of the potential minimum. **Signal strength depends only on local dark matter density, and is independent of DM mass and phase transition scale  $f_a$**

Phenomenology based on a classically oscillating CP-violating angle which:

- Rotates B-fields into E-fields
- Creates AC nucleon EDMs
- Creates AC torques on fermion spins



Bucket of dark matter is dumped into the red-shifting photon bath at time  $1/H \approx 1/m_a$



Non-DM  
density  
redshifting  
away

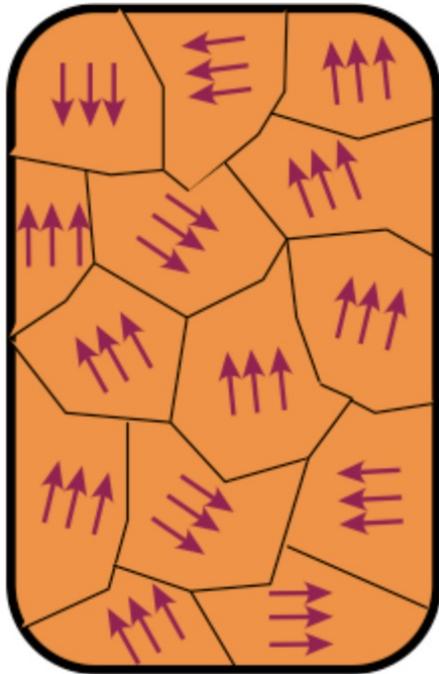


For a bucket filled to the level  $\langle \sin^2 \theta_0 \rangle \times \chi$  of fish, dumping it too late creates an improper balance of fish/water.

If you are going to procrastinate and dump it late, you better not have too many fish in that bucket since there is not a lot of water left!

→ **Small  $m_a$  requires small  $\langle \sin^2 \theta_0 \rangle$  to avoid overproducing dark matter.**

# Fullness of bucket depends on whether the axion phase transition happens before or after cosmic inflation



Ferromagnetic spin domains

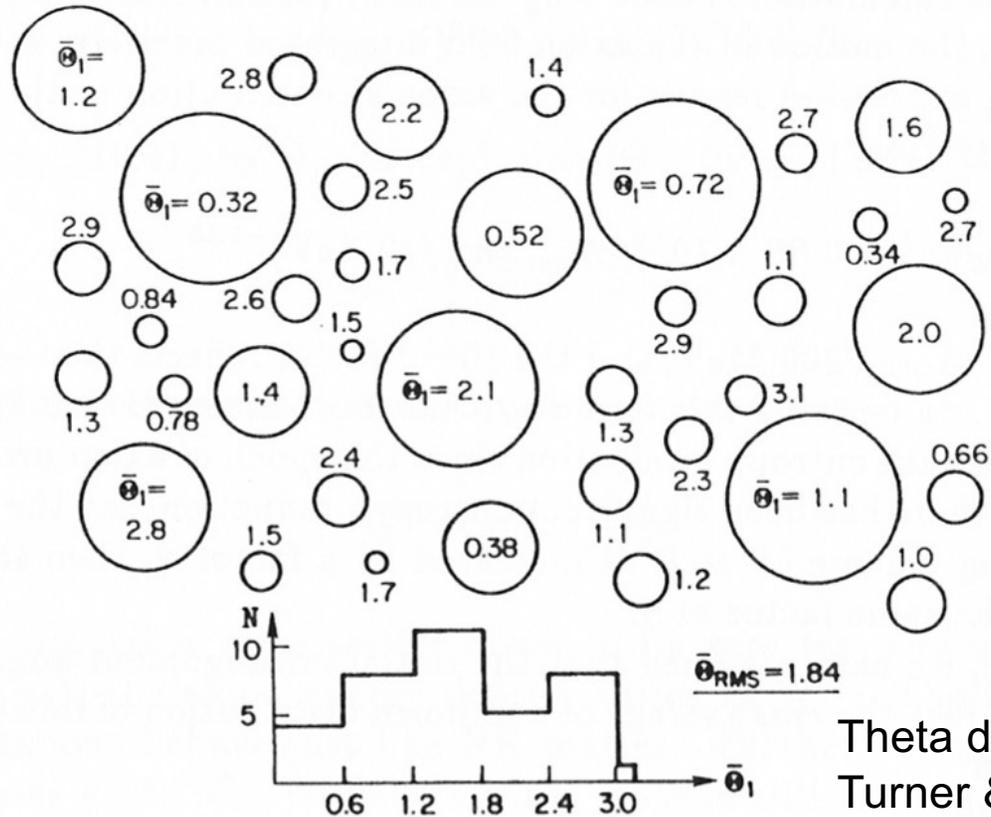
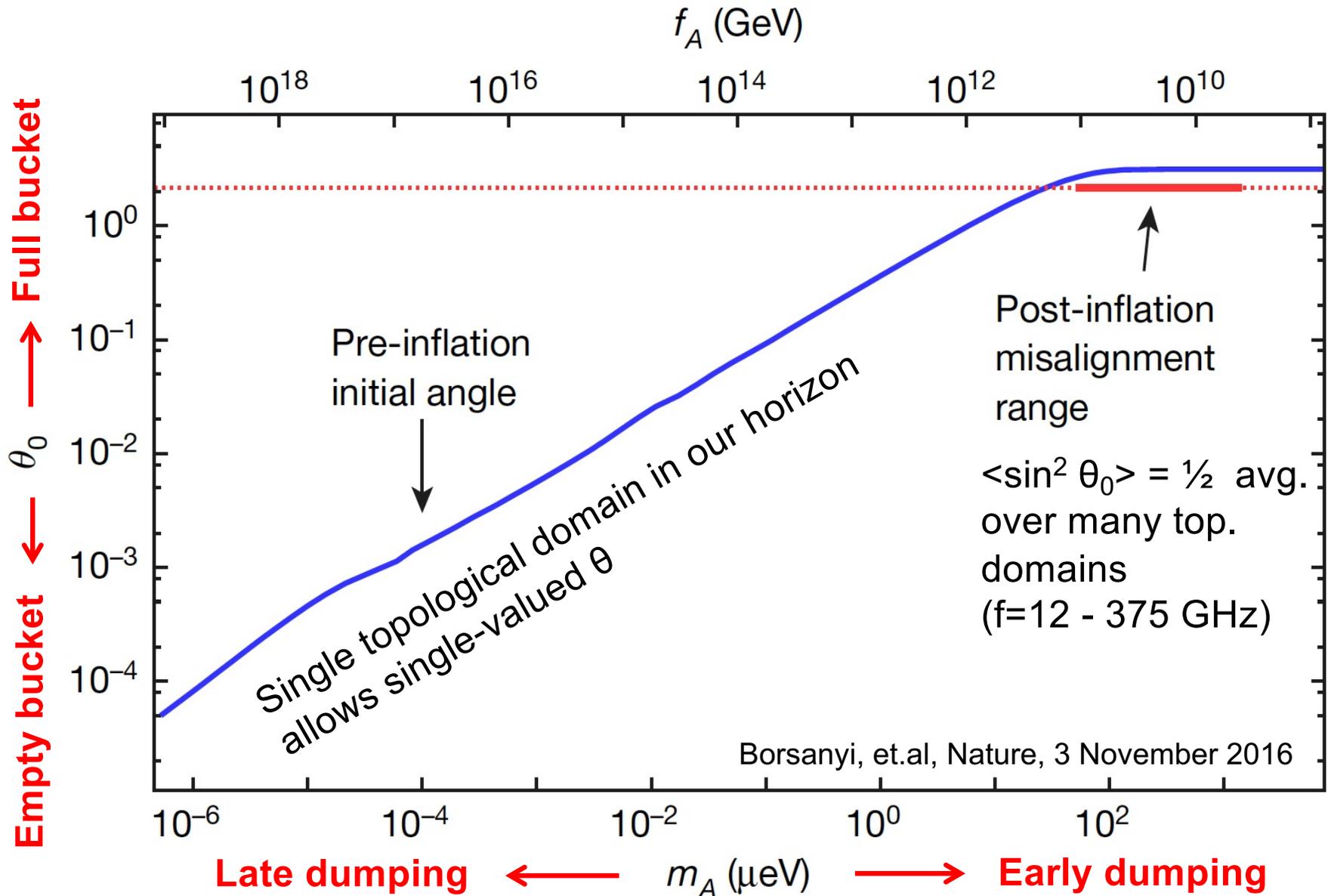


Fig. 10.9: Distribution of  $|\bar{\theta}_1|$  in an inflationary Universe.

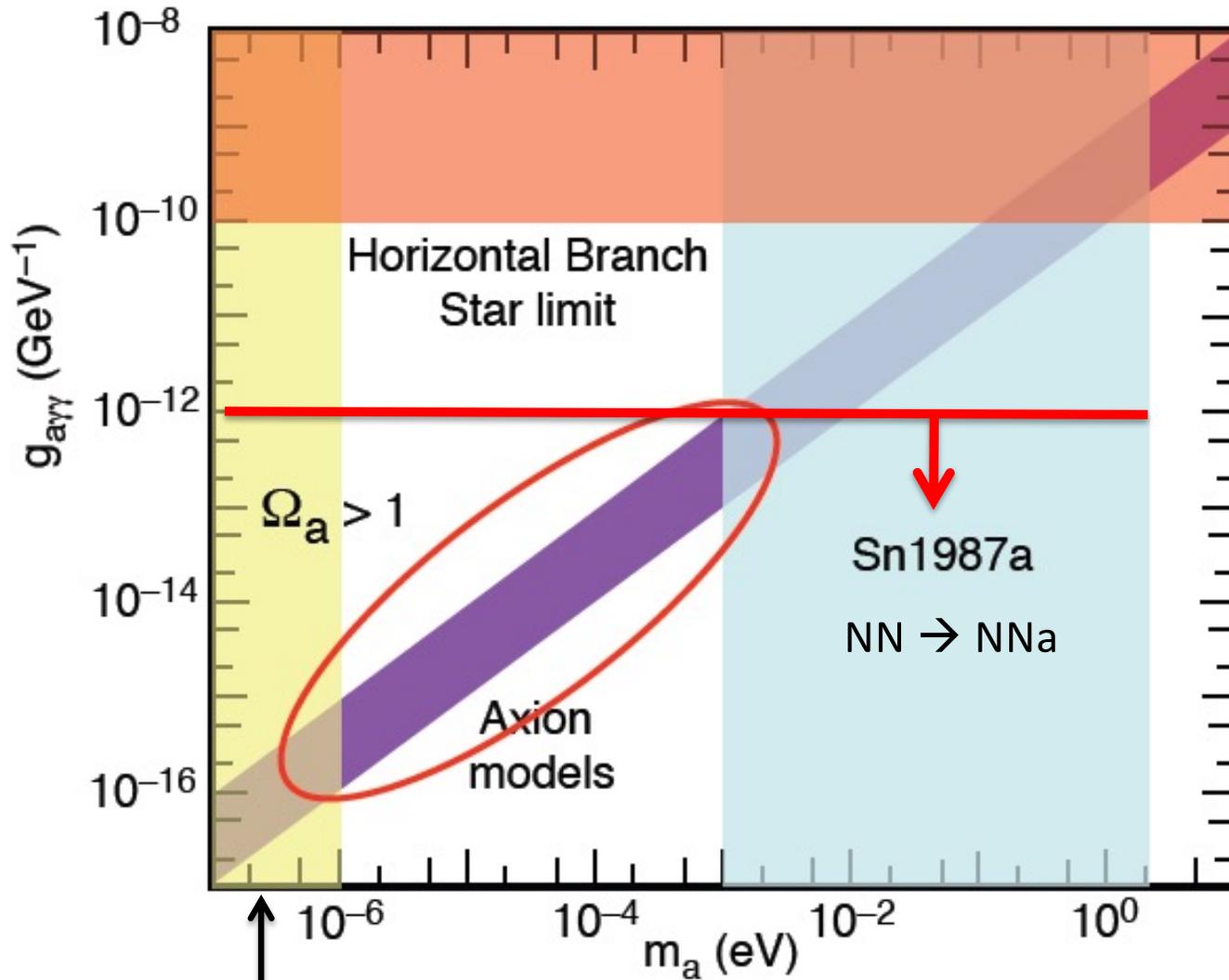
- If axion phase transition occurs **pre-inflation**, bubbles are inflated, and we live in a single bubble which by chance can have  $\theta < 1$ .
- If axion phase transition occurs **post-inflation**, many bubbles are contained within our horizon, and so we get average value  $\langle \sin^2 \theta \rangle \times \Lambda_{\text{QCD}}^4$  of dark matter.

New lattice result gives dividing line at  $m_a \approx 50 \mu\text{eV}$  between pre- vs post-inflationary axion phase transition



Borsanyi, et.al, Nature, 3 November 2016

# The classic axion window



$$\Omega_a \approx \left( \frac{6 \mu\text{eV}}{m_a} \right)^{\frac{7}{6}}$$

or even more due to cosmic string decay.

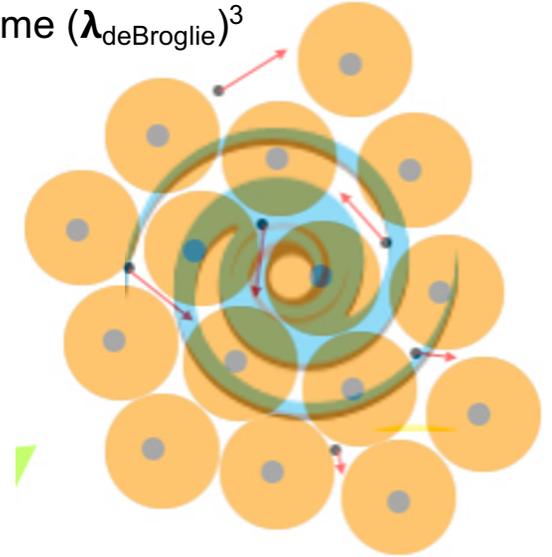
PQ model + local energy conservation **guarantees** the existence of dark matter axions in the last place we haven't looked!

Disfavored by "naturalness." Requires small initial  $\theta$  to avoid DM overproduction.

# Low mass dark matter generically takes the form of classical bosonic sine waves

For mass  $< 70$  eV, Pauli exclusion principle causes dark matter clumps to swell up to be larger than the size of the smallest dwarf galaxies. (Randall, Scholtz, Unwin 2017)

Fermions: 1 DM particle per mode volume  $(\lambda_{\text{deBroglie}})^3$



→ If lower mass, dark matter must be coherent bosonic sine waves with **macroscopic mode occupation number  $\gg 1$**



Not billiard balls.



Need coherent wave detector.

# Ultracold low mass dark = classical wave

Described by a **Glauber** coherent state with properties similar to a modern laser.



Huge number density  $10^{14}/\text{cc}$ ,  
Linewidth  $\approx \text{kHz}$

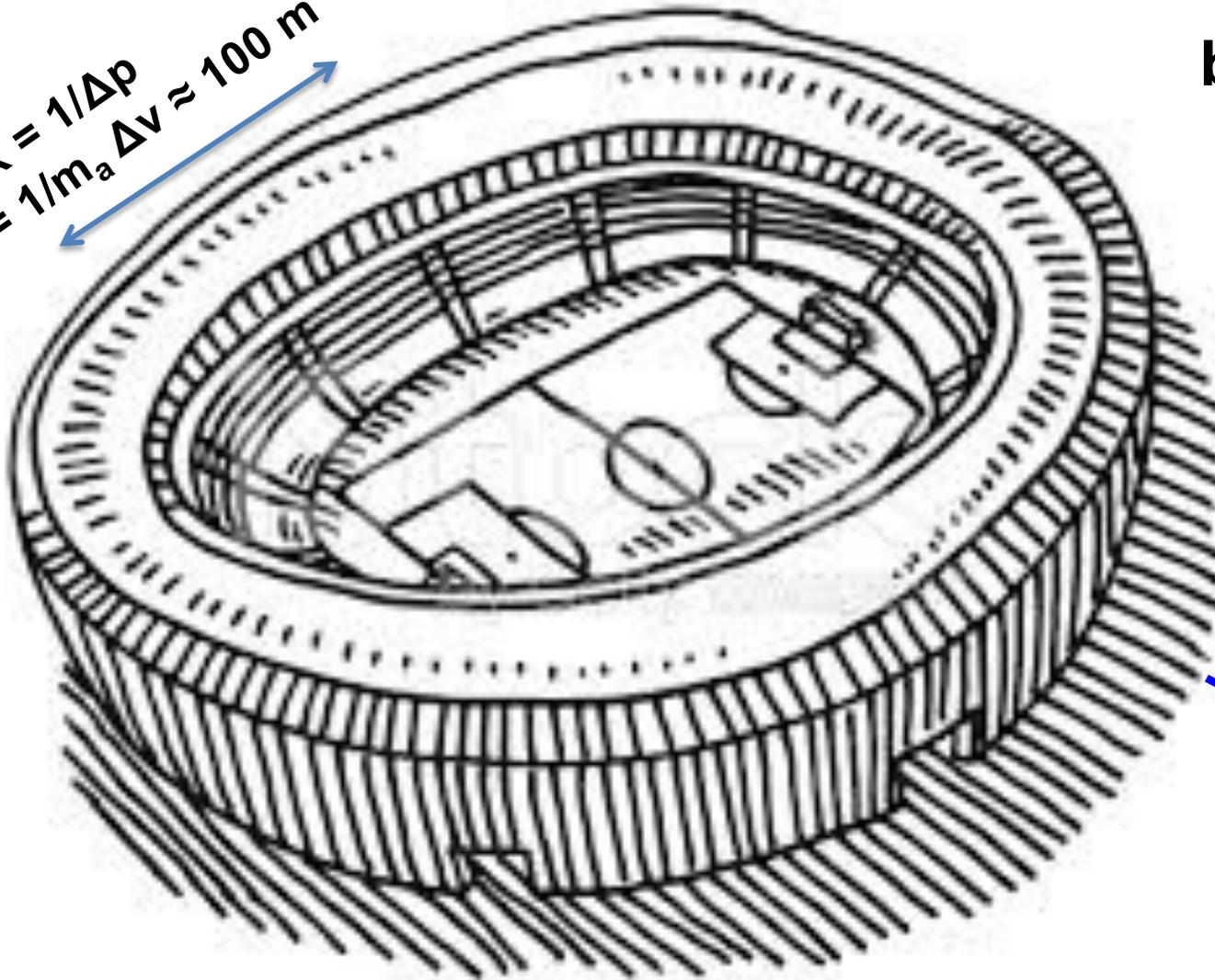


$$a \approx \frac{\sqrt{2\rho_a}}{m_a} e^{i(\vec{k}\cdot\vec{x}-m_a t+\phi)}$$

$$|\vec{A}| \approx \frac{\sqrt{Z_0 P_{\text{laser}} / \text{Area}}}{\omega} e^{i(\vec{k}\cdot\vec{x}-\omega t+\phi)}$$

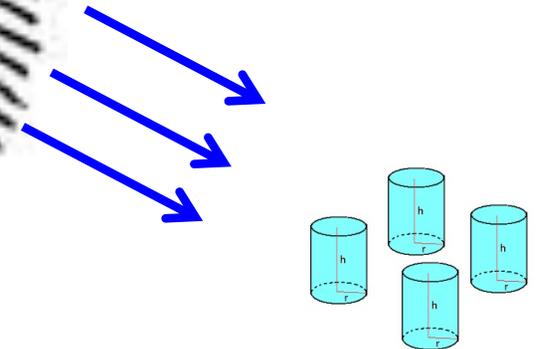
e.g.  $10^{-5}$  eV = GHz dark matter

$$\Delta x = 1/\Delta p \\ = 1/m_a \Delta v \approx 100 \text{ m}$$



**Non-relativistic bosonic DM is like a slow CW laser with  $f=m_a/2\pi$**

$v \approx \Delta v \approx 300$  km/s  
(galactic escape velocity)



Football stadium-sized regions of coherently oscillating **classical sine waves** slowly drifting through detectors. Mean DM occupation number  $N > 10^{22}$  per mode.

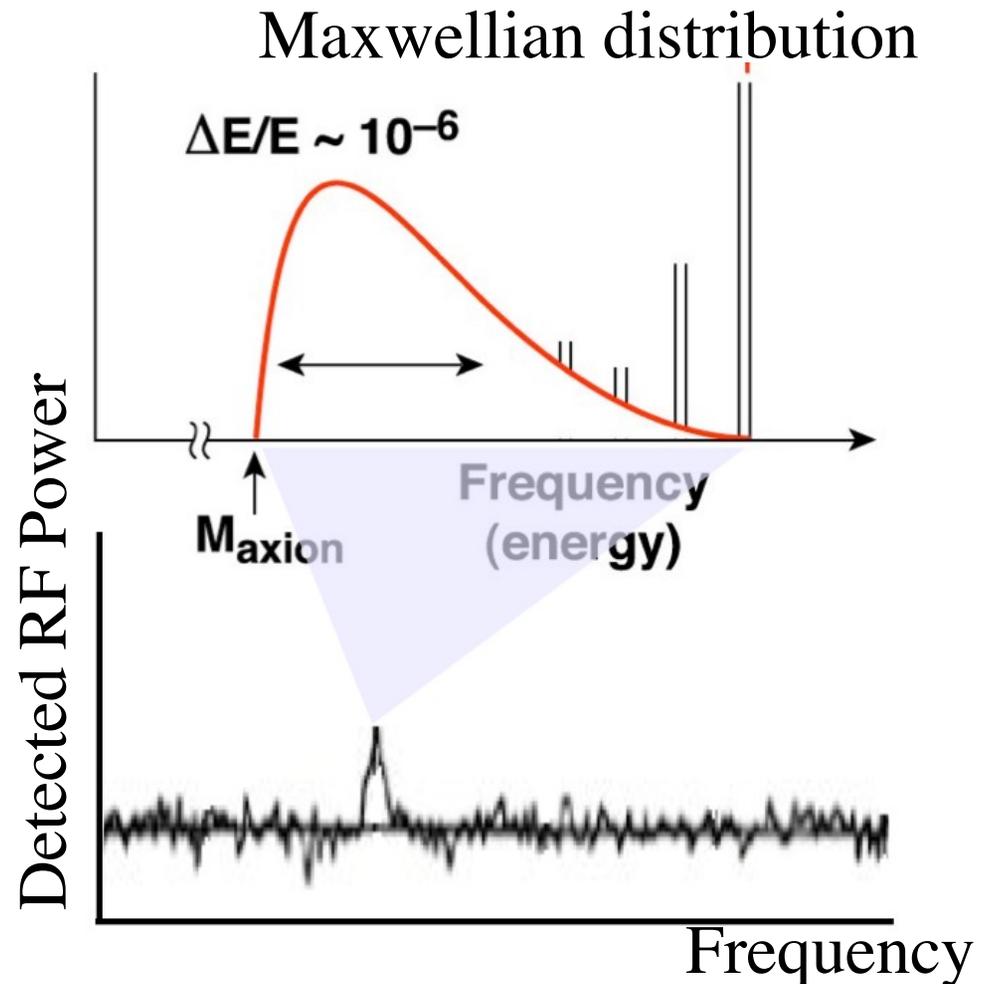
Accumulate oscillatory signals in various kinds of laboratory oscillators which are weakly coupled to the DM wave

# Axion energy is kinetically broadened by virial velocity

Non-relativistic DM:  
 $v \sim \Delta v \sim 10^{-3} c$   
(galactic escape velocity)

Kinetic energy =  $\frac{1}{2} m_a v^2$   
 $\Delta E = m_a v \Delta v$   
 $E_{\text{rest}} = m_a c^2$

$\Delta E/E = v \Delta v / c^2 \sim 10^{-6}$



**Very narrowband line, but can reconfirm signal in minutes once found.**

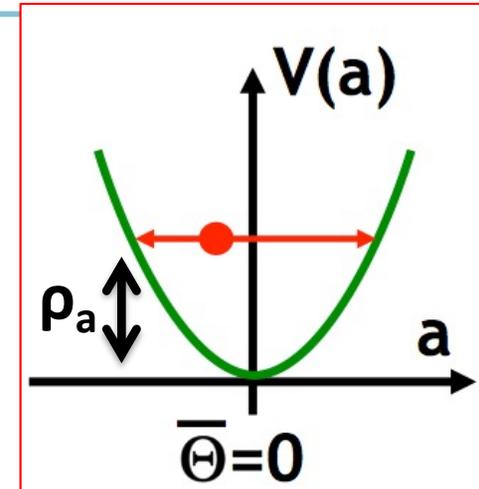
Most of search time spent slowly stepping through frequency space, one frequency tuning at a time.

# Signal strength is independent of $m_a$ , $f_a$

Locally coherent oscillation of the QCD  $\theta$  angle about its CP-conserving minimum:

$$\theta(x, t) = \theta_{\max} e^{i(kx - m_a t)}$$

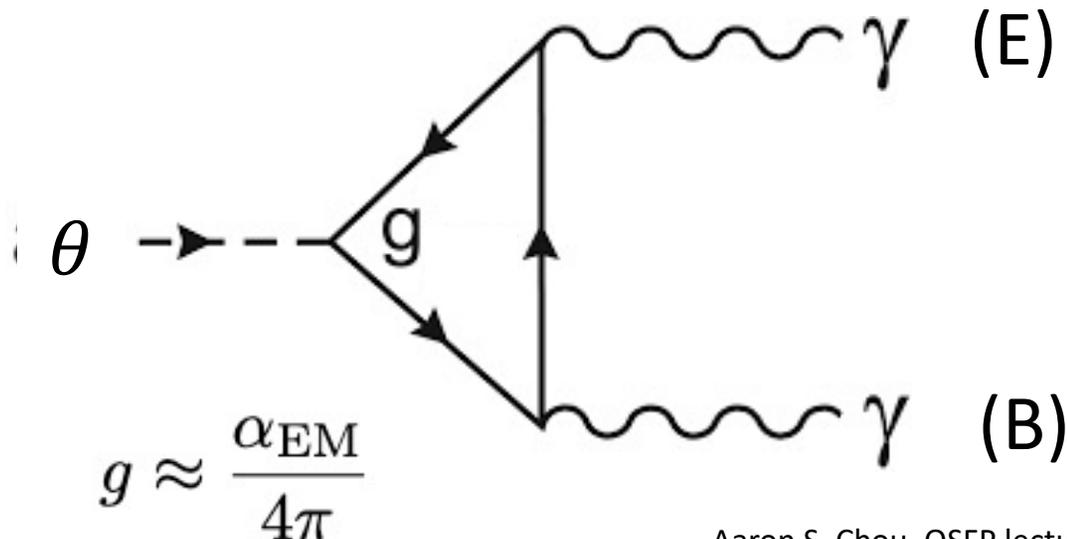
where 
$$\theta_{\max} = \sqrt{\frac{2\rho_a}{\Lambda_{\text{QCD}}^4}} \approx 3.7 \times 10^{-19} \text{ radians}$$



DM oscillations partially undo the Peccei-Quinn mechanism by enabling the coherent field to climb out of the potential minimum.

**Wave amplitude and hence signal strength depends only on local dark matter density  $\rho_a$  !**

Experimental goal:  
Determine frequency of the signal and hence the axion mass





Pierre Sikivie,  
Sakurai Prize 2019

# The Dark Matter Haloscope: Classical axion wave drives RF cavity mode

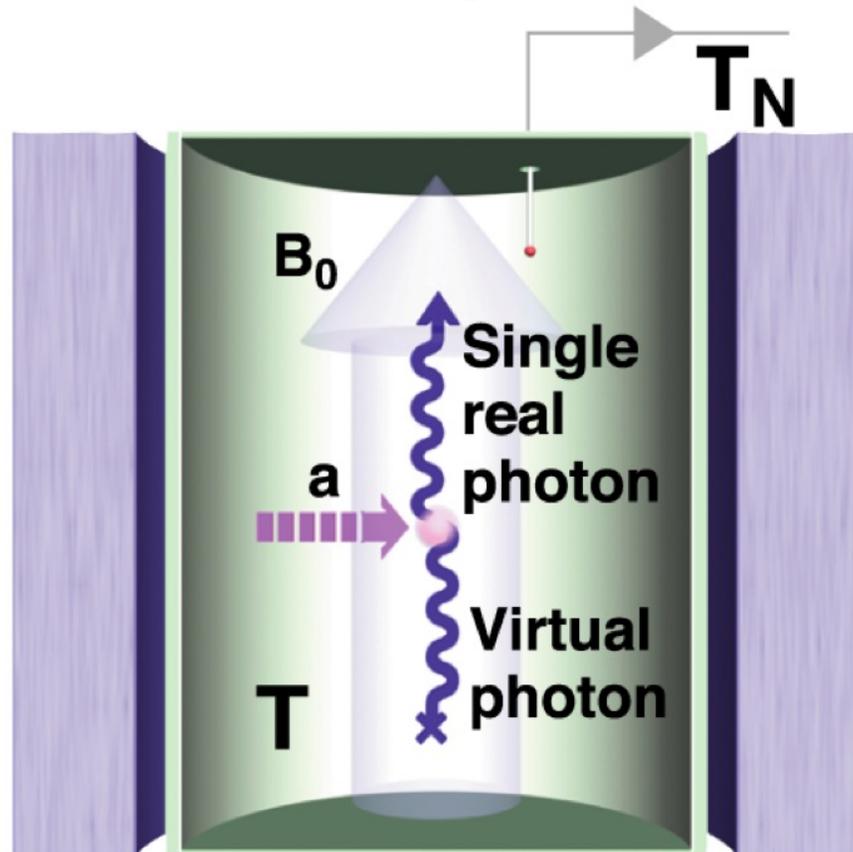
- In a constant background  $B_0$  field, the oscillating axion field acts as an exotic, space-filling current source

$$\vec{J}_a(t) = -g\theta\vec{B}_0 m_a e^{im_a t}$$

which drives E&M via Faraday's law:

$$\vec{\nabla} \times \vec{H}_r - \frac{d\vec{D}_r}{dt} = \vec{J}_a$$

- Periodic cavity boundary conditions extend the coherent interaction time (**cavity size**  $\approx 1/m_a$ )  $\rightarrow$  the exotic current excites standing-wave RF fields.

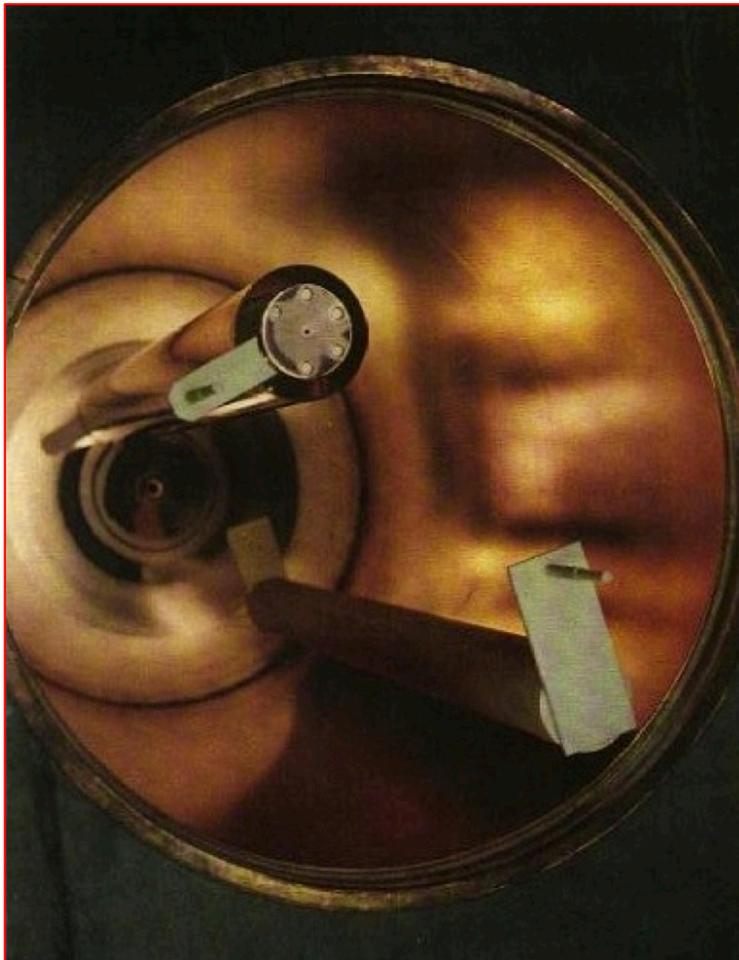


A spatially-uniform cavity mode can **optimally** extract power from the dark matter wave

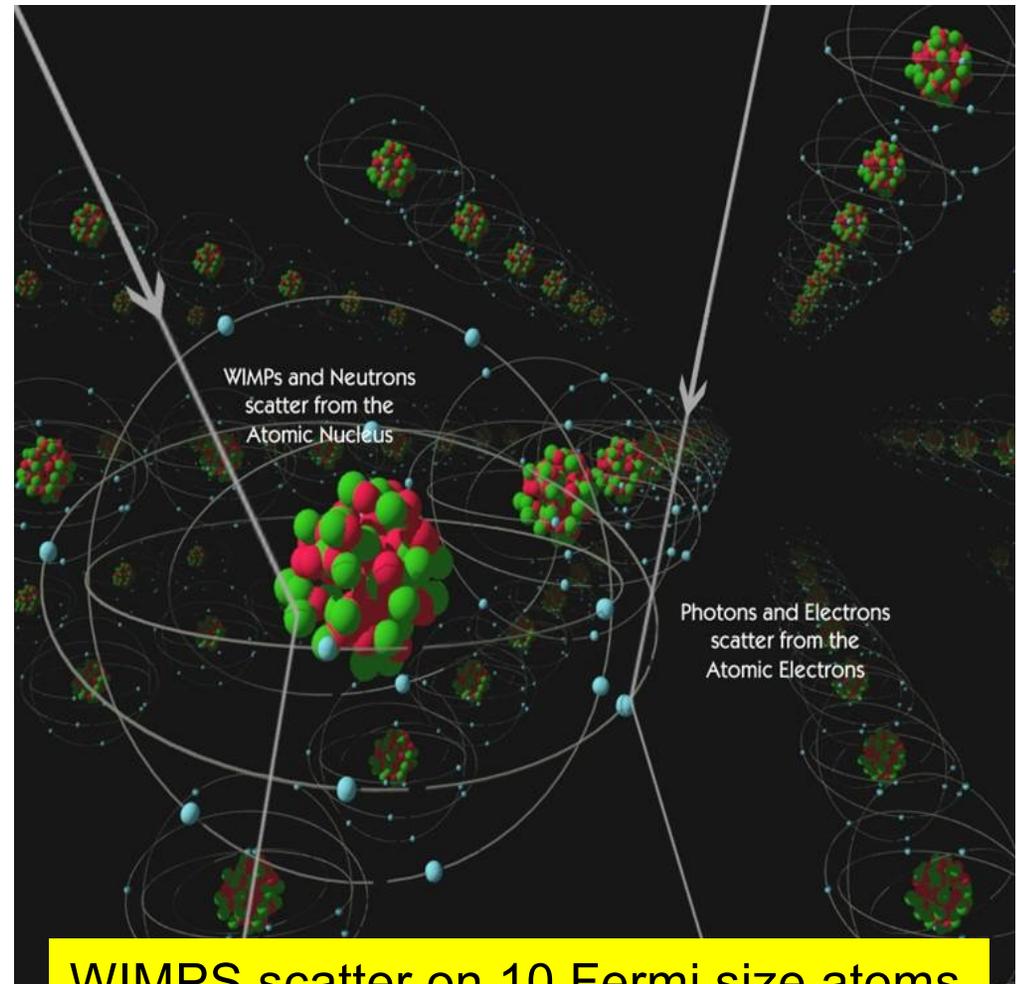
$$P_a(t) = \int \vec{J}_a(t) \cdot \vec{E}_r(t) dV$$

# Axions vs WIMPs:

Resonant scattering requires size of scattering target =  $1/(\text{momentum transfer})$

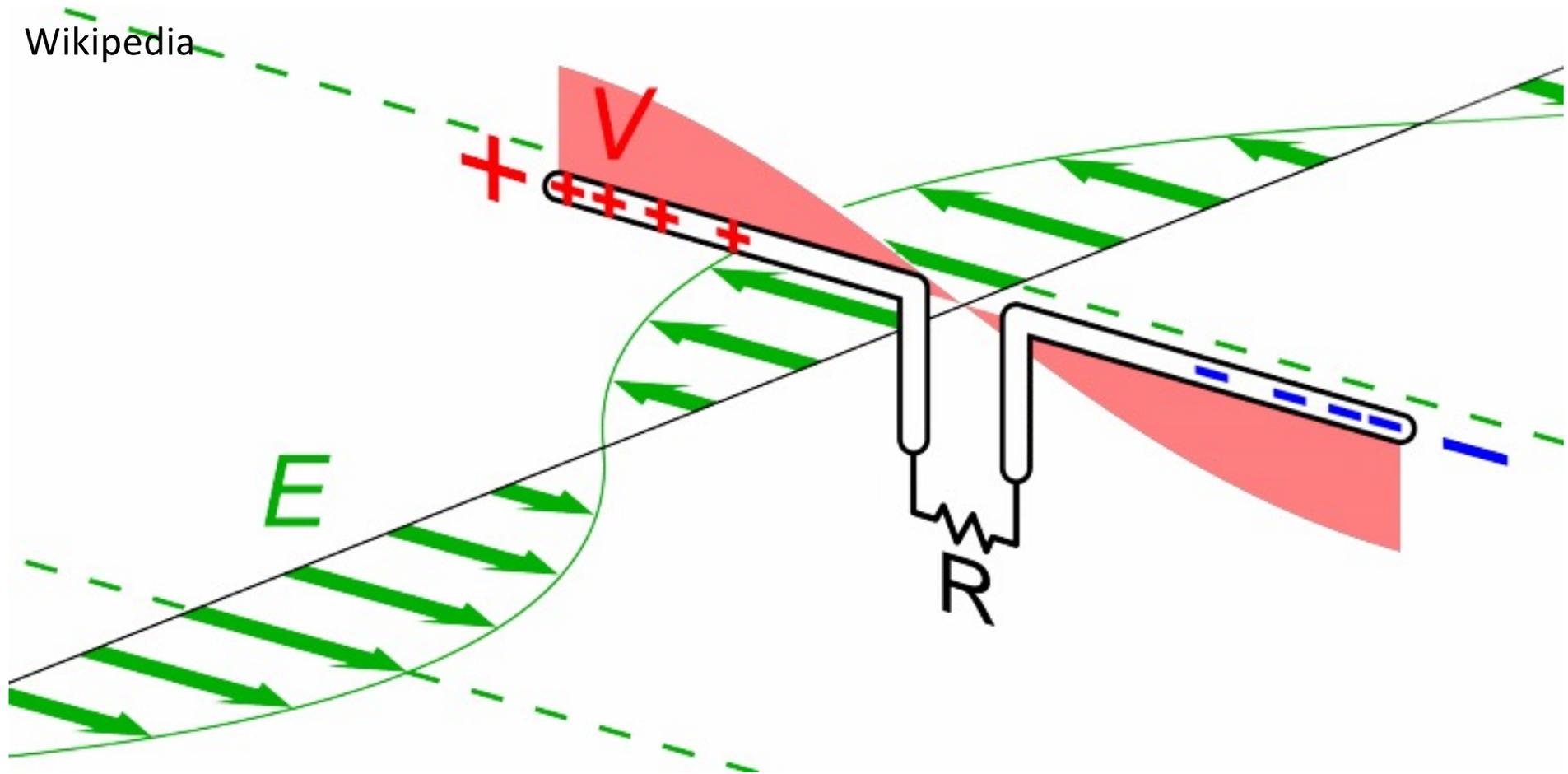


4  $\mu\text{eV}$  mass axions scatter on 50cm size microwave cavities



WIMPS scatter on 10 Fermi size atoms

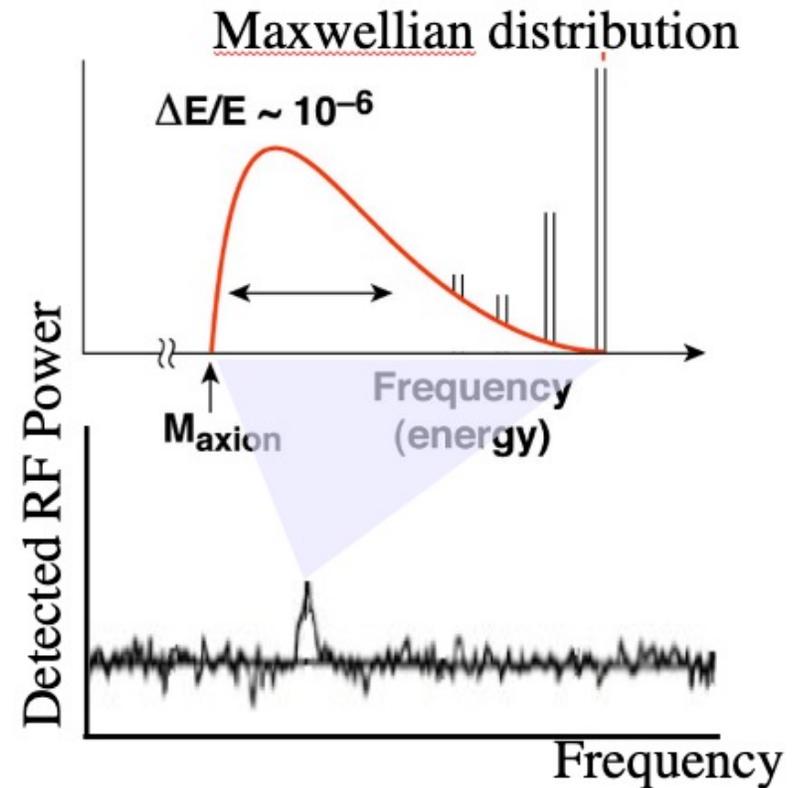
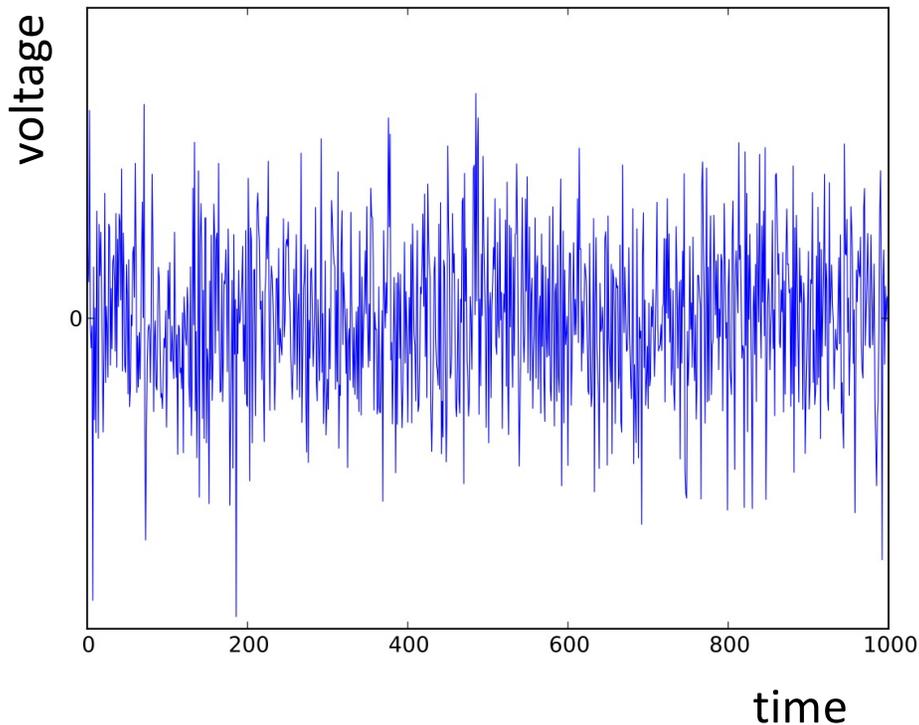
# Match size of antenna to wavelength of signal



Wave mechanics: scattering matrix element is proportional to spatial Fourier transform of the scattering potential, with respect to the momentum transfer

# Spectral analysis of output voltage time series

Discrete Fourier transform



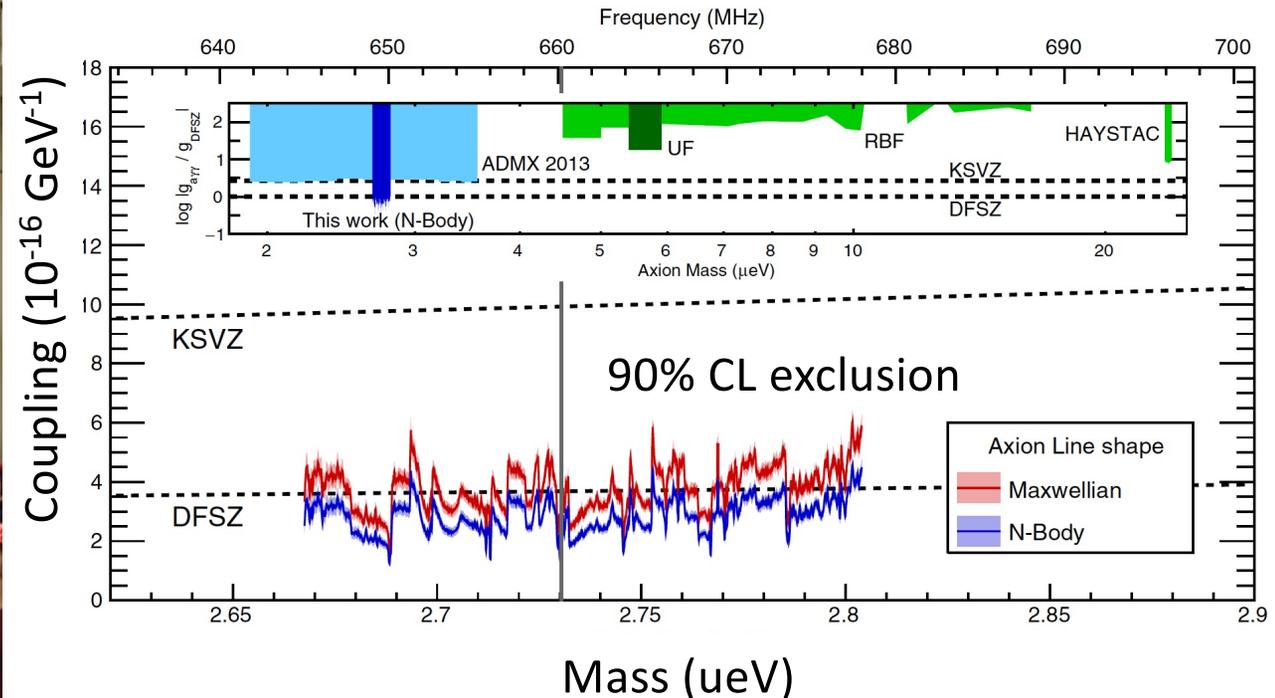
Digitization rate  $f_{\text{dig}}$  gives maximum resolvable “Nyquist” frequency  $f_{\text{dig}}/2$ .  
Duration  $\Delta t$  of acquired time series gives frequency resolution  $\Delta f = 1/2\Delta t$ .

Dark matter signal = excess above white noise backgrounds.

# 2017: 30-year axion R&D program culminates in first sensitivity to DFSZ axions

PRL 120, 151301 (2018)

ADMX at U.Washington,  
FNAL = DOE lead lab



Look for "spontaneous" emission from local axion dark matter into the empty cavity mode.

Signal power level =  $10^{-23} \text{ W}$

Need 15 minutes integration per radio tuning  
to beat thermal noise power at 500 mK.

Operate an ultrasensitive radio  
in a cold, RF-shielded box to  
tune in to the axion broadcast.

# How a theorist sees a spherical cow

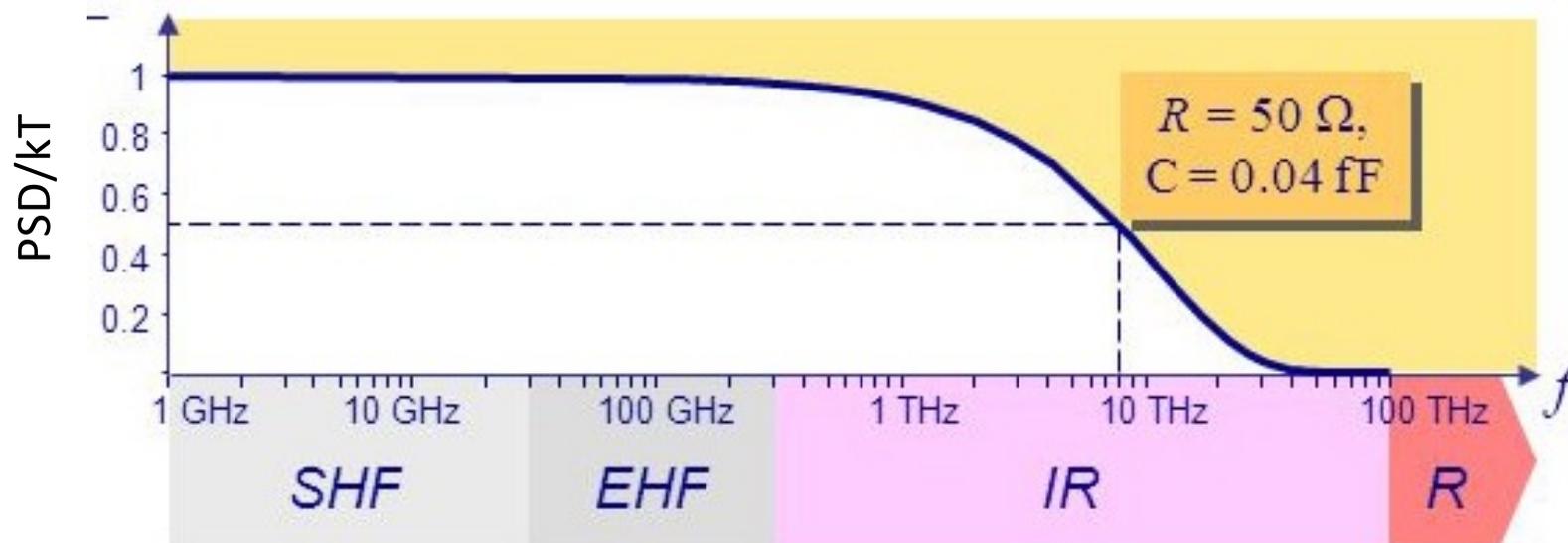


# How an experimentalist sees a spherical cow



Bose-Einstein occupancy of photon modes on a 1-dim transmission line, + zero point fluctuations:

$$n(\omega) = \frac{1}{2} \coth \frac{\hbar\omega}{2kT} = \frac{1}{e^{\hbar\omega/kT} - 1} + \frac{1}{2}$$



Energy/mode =  $kT$  ( $\frac{1}{2} kT$  for each quadrature).

Modes of linewidth  $\Delta f$  are defined by the integration time  $\Delta t = 1/(2 \Delta f)$  required to resolve these modes.

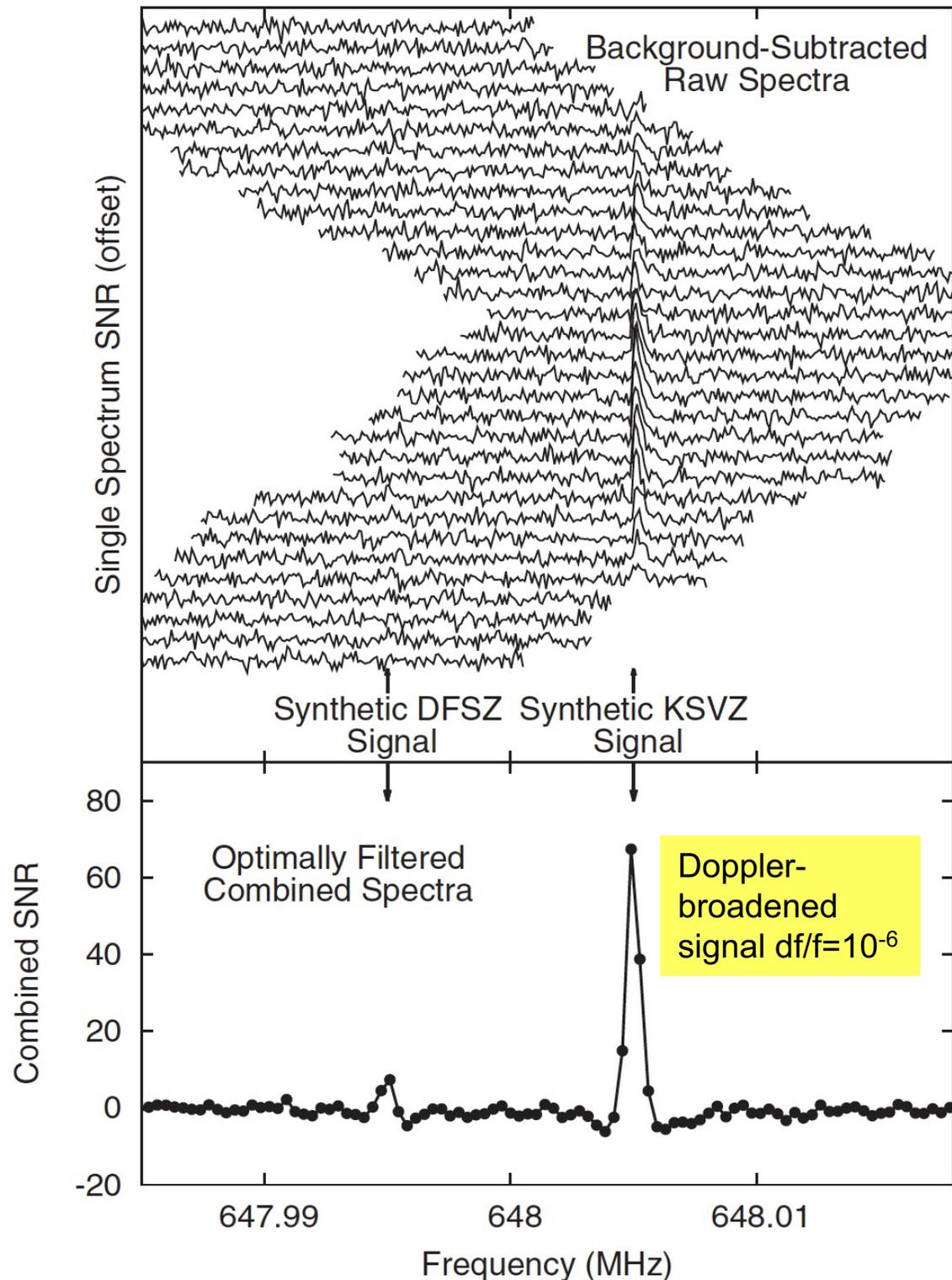
→ **Thermal noise power emitted from each mode  $P = kT \Delta f$ .**

Detect small signal  
excesses by averaging  
away the noise over  
many power spectrum  
measurements

$$SNR = \frac{P_{\text{signal}}}{\sqrt{2kT\Delta f}} \underbrace{\sqrt{2\Delta ft}}_{10^{-3} \text{ (for ADMX)}}$$

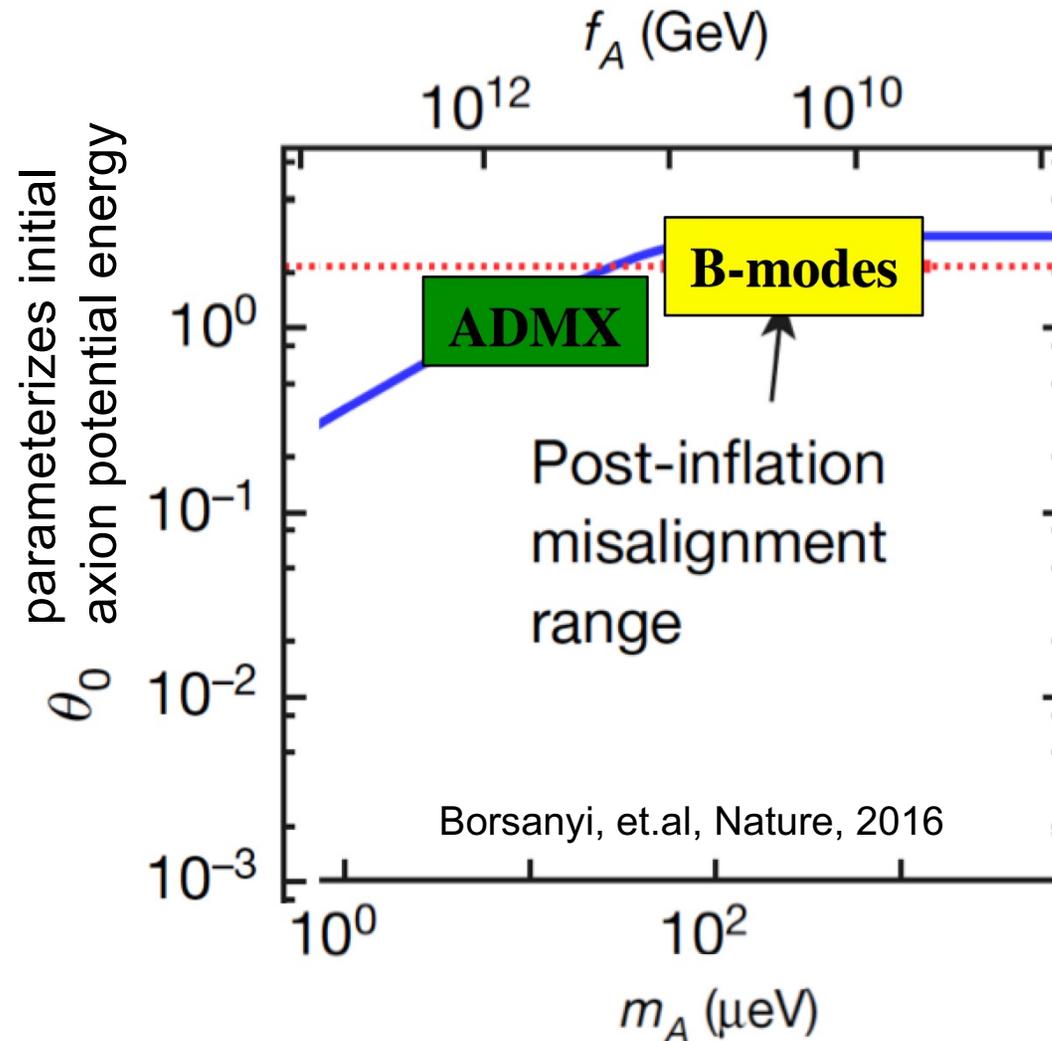
Require  $>10^6$   
averages!

Each measurement takes  $10^{-3}$  s to  
resolve a kHz-wide bin.  
→  $10^6$  measurements takes  $10^3$  s  
= 15 minutes.



# Targeting higher axion masses predicted in cosmological scenarios with *high energy scale* cosmic inflation

These simple inflation models also produce detectable primordial B-mode polarization patterns in the cosmic microwave background – science target for CMB-S4.



Higher axion mass allows early release of QCD vacuum energy. Avoids overproduction of dark matter.

**What prevents us from immediately going to higher frequencies?**

The predicted axion DM signal/noise ratio plummets as the axion mass increases → SQL readout is not scalable.

