Clever Quantum Tricks for Detecting Dark Matter Waves

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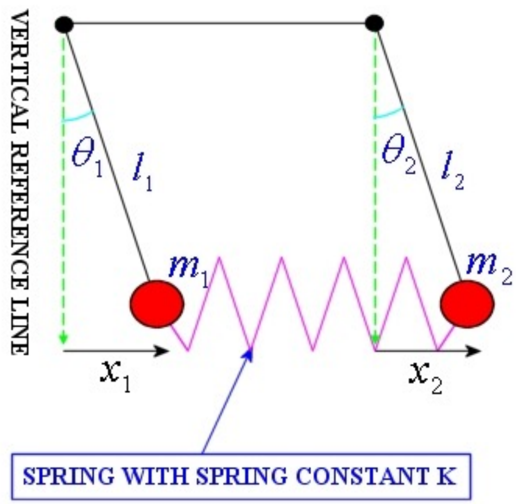
QSFP School

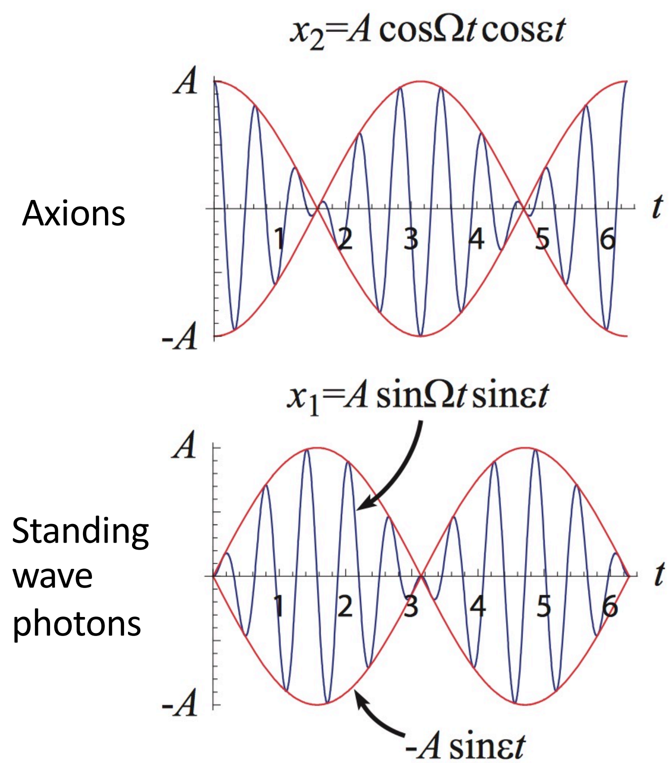
Lecture 3

September 10, 2021

Lecture 2 review:

Transfer of energy from the axion DM to the RF cavity is the same as that of a system of two pendula coupled by a weak spring





$$H = \begin{pmatrix} \omega_1 & g \\ g & \omega_2 \end{pmatrix}$$

Mixing angle=45° on resonance when w1=w2:

Mixing frequency = energy “g” stored in spring.
In the limit of infinite coherence time, **all** of the dark matter would be converted into photons. In finite coherence time, only get a few signal photons.

Lecture 2 review:

Creation/annihilation operators are just translation operators in phase space

$$p = i\frac{d}{dx}$$

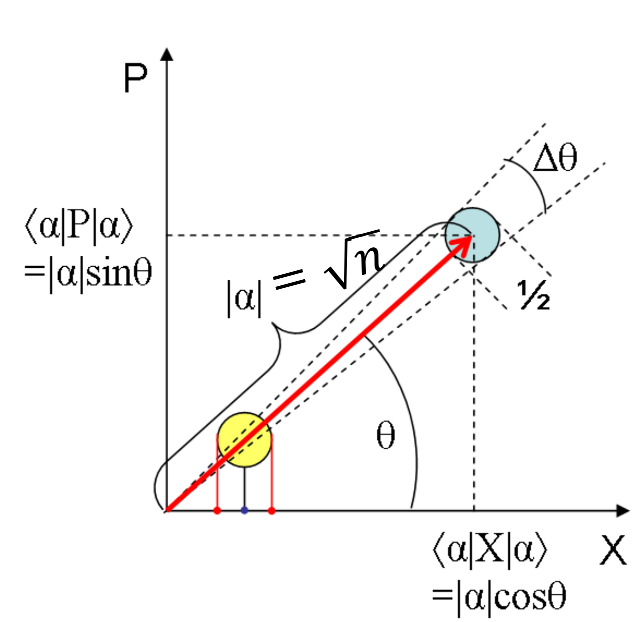
Generates translations in position

$$x = i\frac{d}{dp}$$

Generates translations in momentum

$$a^\dagger = x + ip$$
$$a = x - ip$$

Generate translations in an arbitrary direction in x-p phase space



Exponentiate differential operator to get finite translation α in complex plane:

$$\hat{D}(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)$$

Phasor of amplitude α is generated as:

$$D(\alpha)\left|0\right\rangle = \left|\alpha\right\rangle$$

"Coherent state" describing a classical sine wave

Lecture 2 review:

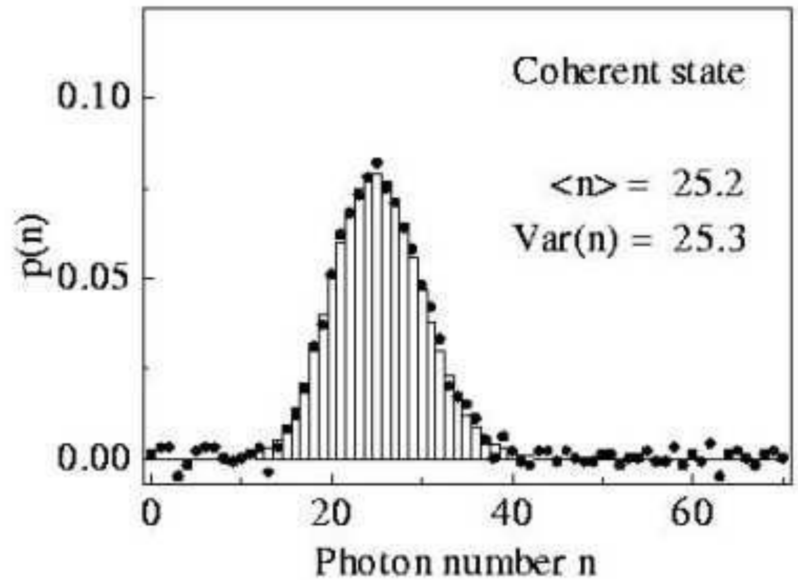
Classical sine waves have intrinsic Poisson noise

Coherent states form a Poisson distribution in the number state basis:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}^\dagger} |0\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle} \frac{\langle n\rangle^n}{n!}$$

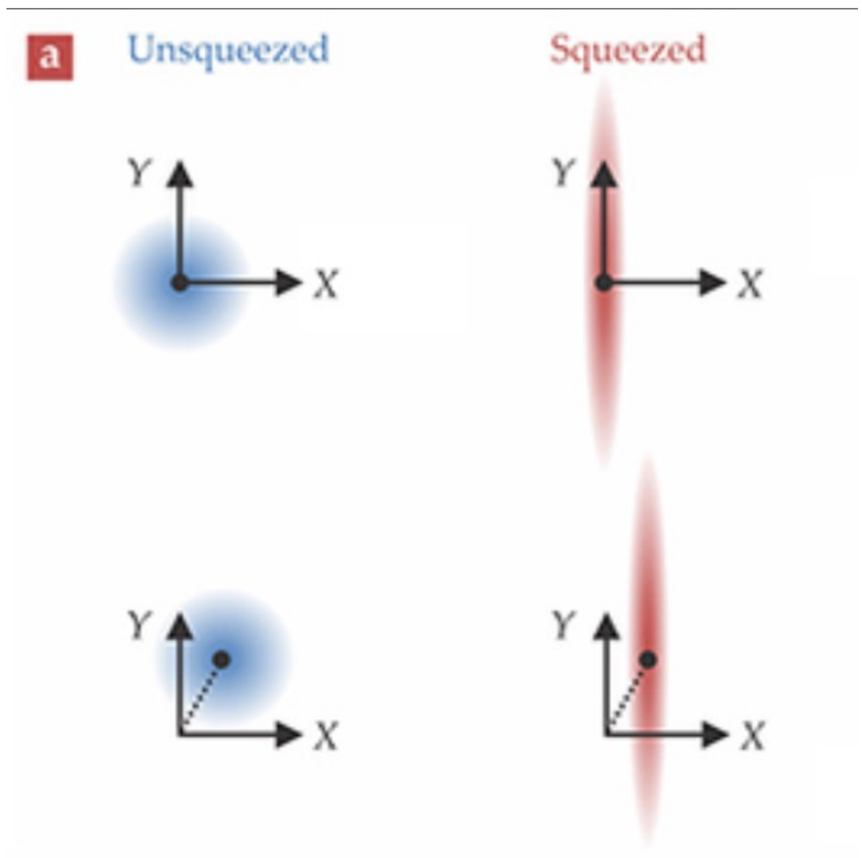
$$\langle n\rangle = \langle \hat{a}^\dagger \hat{a}\rangle = |\alpha|^2$$



Like the zero-point fluctuations, the Poisson shot noise in classical wave intensity is a consequence of the Heisenberg uncertainty principle.

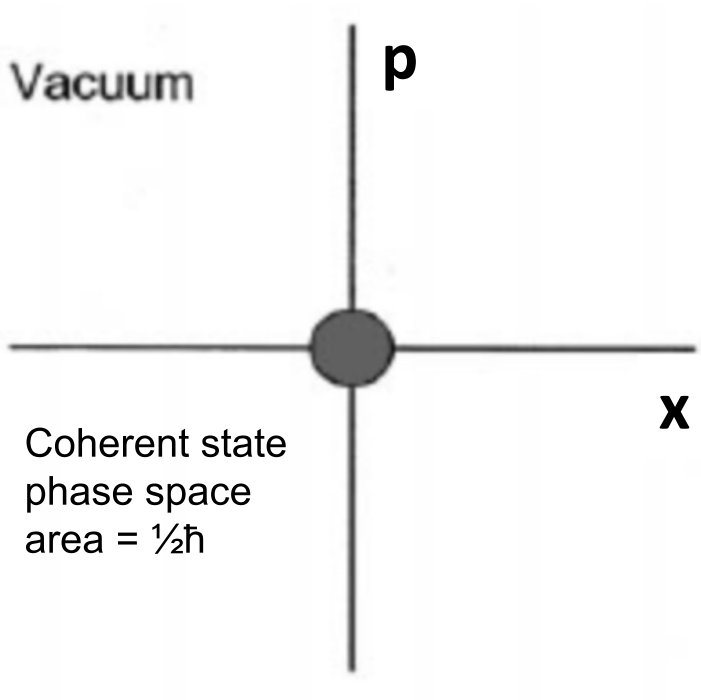
Lecture 2 review:

The resolution of a probe to displacement signals is given by its phase space distribution

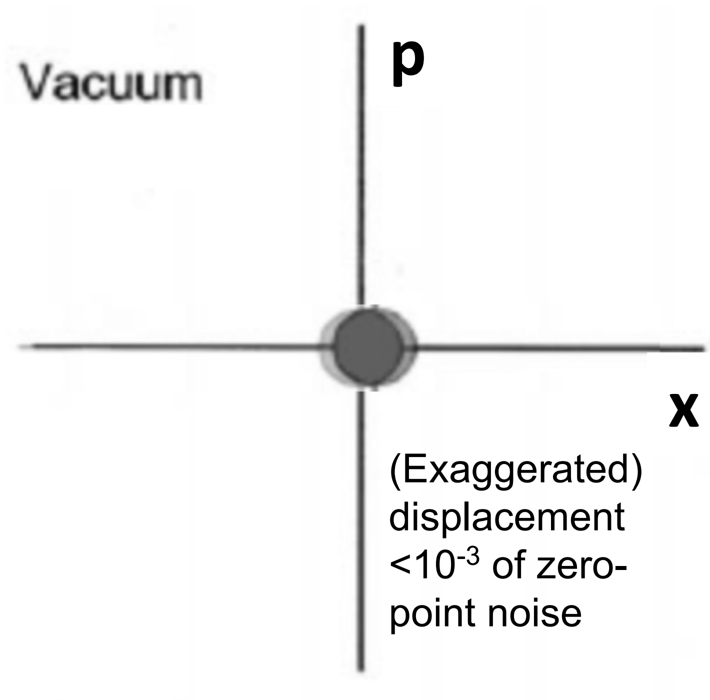


Lecture 2 review:

The displaced vacuum state is usually measured using a quantum-limited amplifier whose phase-space resolution function satisfies the standard quantum limit



Axion-photon mixing induces tiny coherent state displacement $D(g\theta B_0 Q_{cav}/m_a^2)|0\rangle$



Blob represents variance from:

- ½ photon from the zero-point noise of the signal oscillator
- ½ photon from the zero-point noise of the idler mode required by the amplifier to conserve energy when converting pump quanta into signal quanta.

More stupid qubit tricks: Stimulated emission of photons from DM axions

Start the photon wave swinging so it can more easily accept energy from the axions.

Good!

Waiting…

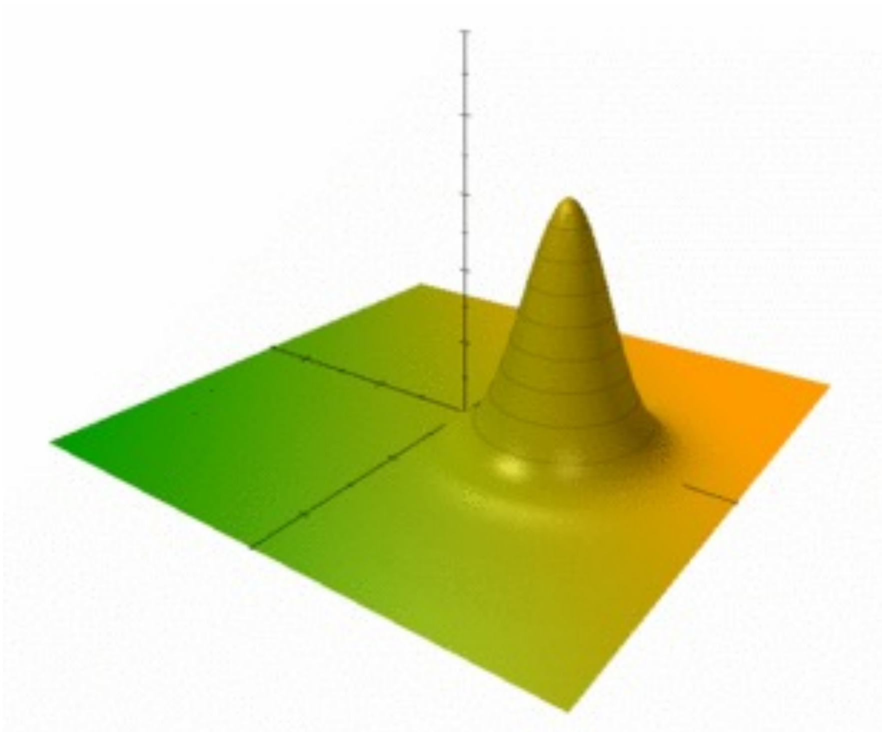
Oops, wrong phase

Power = $\overrightarrow{Force} \cdot \overrightarrow{velocity}$

Phase offset determines the direction of energy flow.

But the axion wave is a coherent state of unknown phase which changes every millisecond… How do we prepare the cavity oscillator???

A sinusoidally swinging oscillator actually has exactly the same resolution as the vacuum state



Resolution is just the size of the Gaussian blob in phase space.

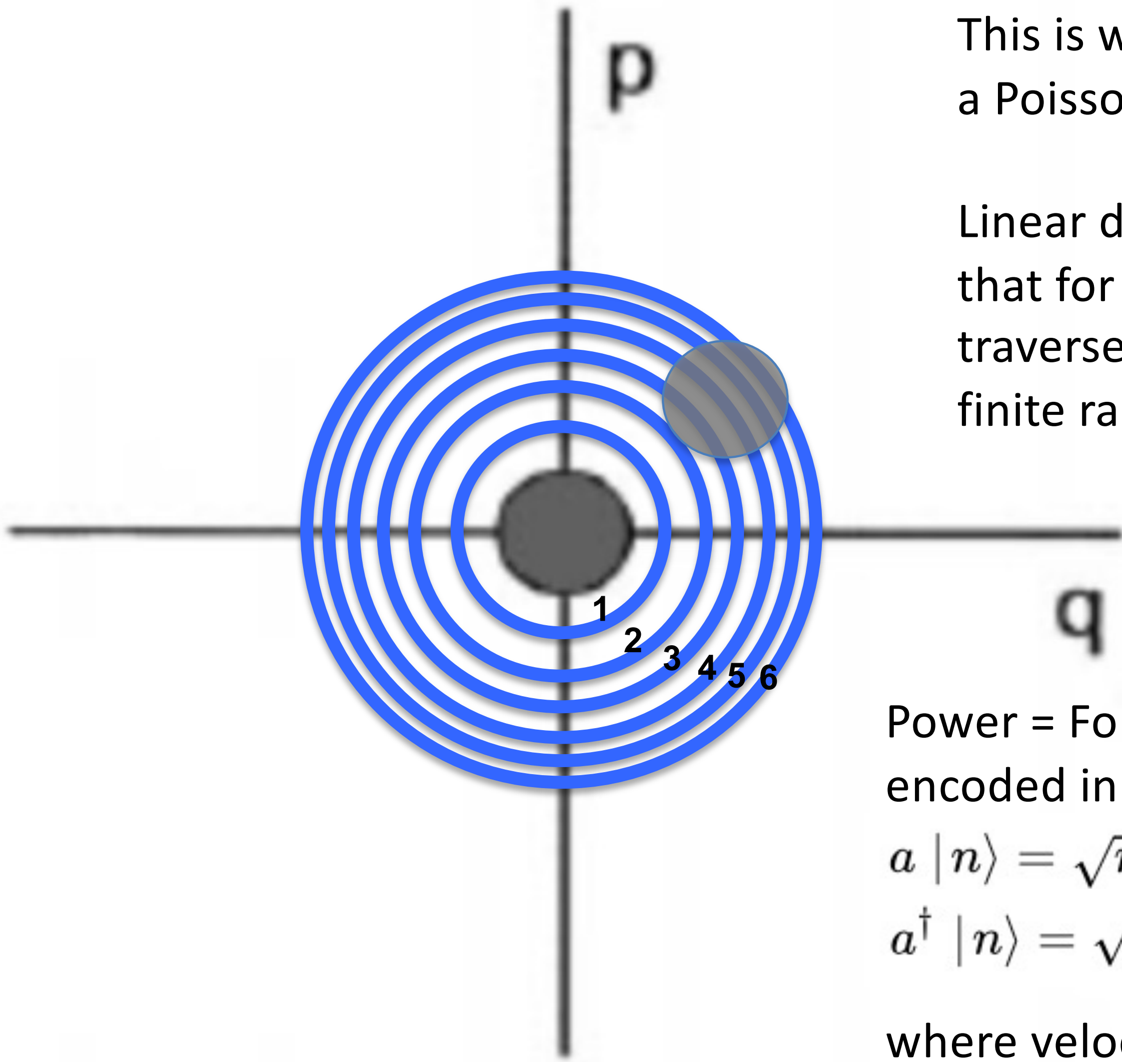
Displacements from forces just act linearly on this phase space distribution:

$$\hat{D}(\alpha)\hat{D}(\beta) = e^{(\alpha\beta^* - \alpha^*\beta)/2}\hat{D}(\alpha + \beta)$$

It doesn't matter whether you prepared the state at finite amplitude β or if you started with β=0. The resolution on α is the same.

Annuli corresponding to integer occupation number become more closely spaced for larger n

$$\langle n \rangle = \langle \hat{a}^\dagger \hat{a} \rangle = |\alpha|^2$$



This is why a displaced Gaussian blob produces a Poisson distribution.

Linear displacement already encodes the fact that for a displacement D(α), more annuli are traversed if the starting point is already at finite radius β

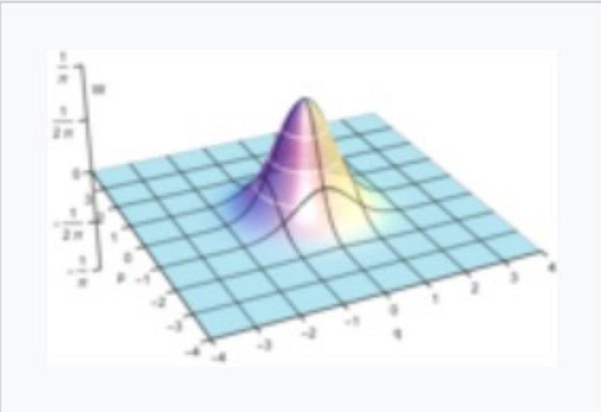
Power = Force x velocity is also already encoded in the operator normalizations:

$$a\,|n\rangle = \sqrt{n}\,|n-1\rangle$$

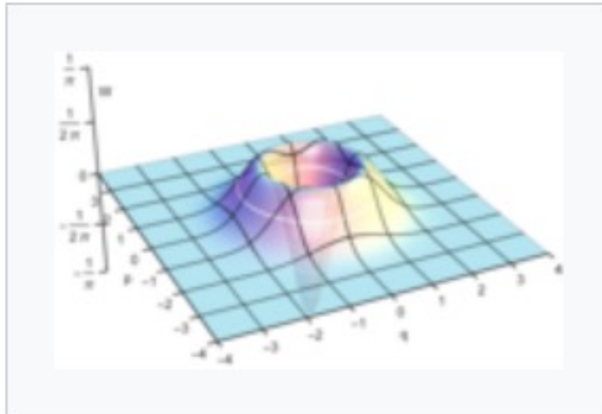
$$a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle.$$

where velocity is proportional to wave amplitude

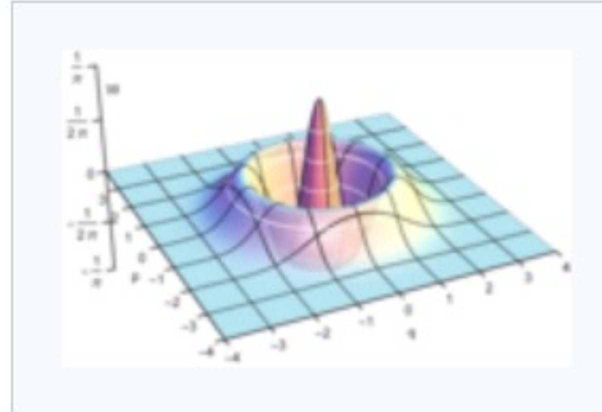
The actual Fock states are Laguerre-Gauss functions



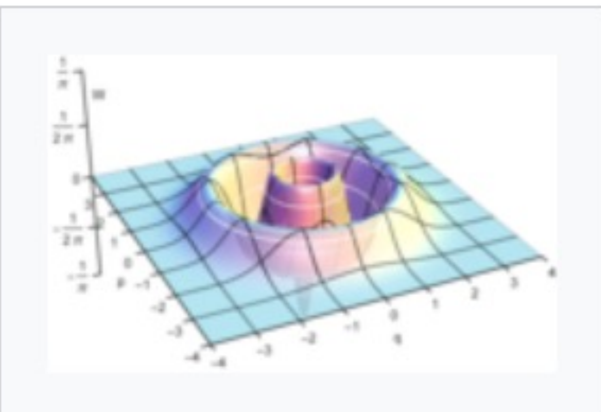
Wigner function of $|0\rangle$



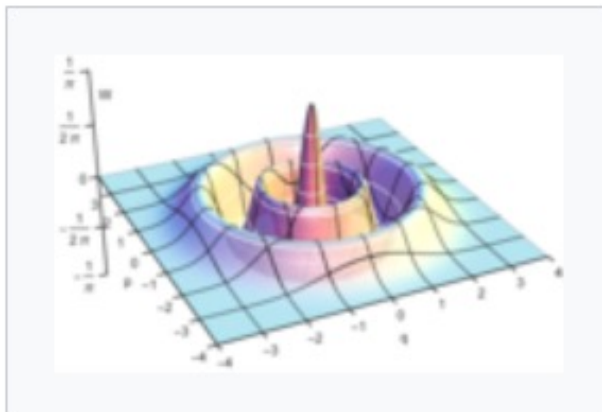
Wigner function of $|1\rangle$



Wigner function of $|2\rangle$



Wigner function of $|3\rangle$



Wigner function of $|4\rangle$

Wikipedia

Just like Hermite-Gauss eigenfunctions, but in cylindrical coordinates. The Hamiltonian can be viewed as a 2-dim harmonic potential with linear restoring force for excursions in x or p.

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

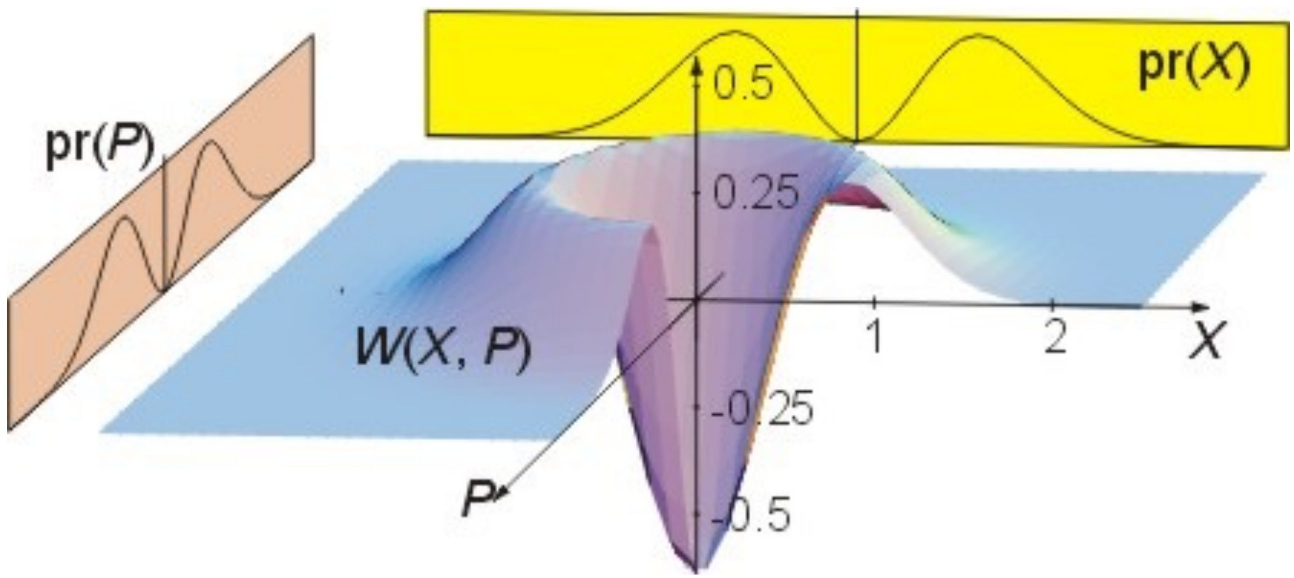
Wait! Wouldn't a Fock state be perfect for measurements of tiny displacements where the phase is unknown?

The Fock state is symmetric in phase angle
→ **responds equally well to pushes at any time.**
It also has definite occupation number N
→ **no Poisson noise!**

$H_I = g(a^\dagger b + a b^\dagger) \rightarrow \langle \alpha, N+1 | H_I | \alpha, N \rangle = g\alpha\sqrt{N+1}$

Konrad Lehnert
(Colorado)

N=1 Fock state $\Delta N \times \Delta \varphi = ½$

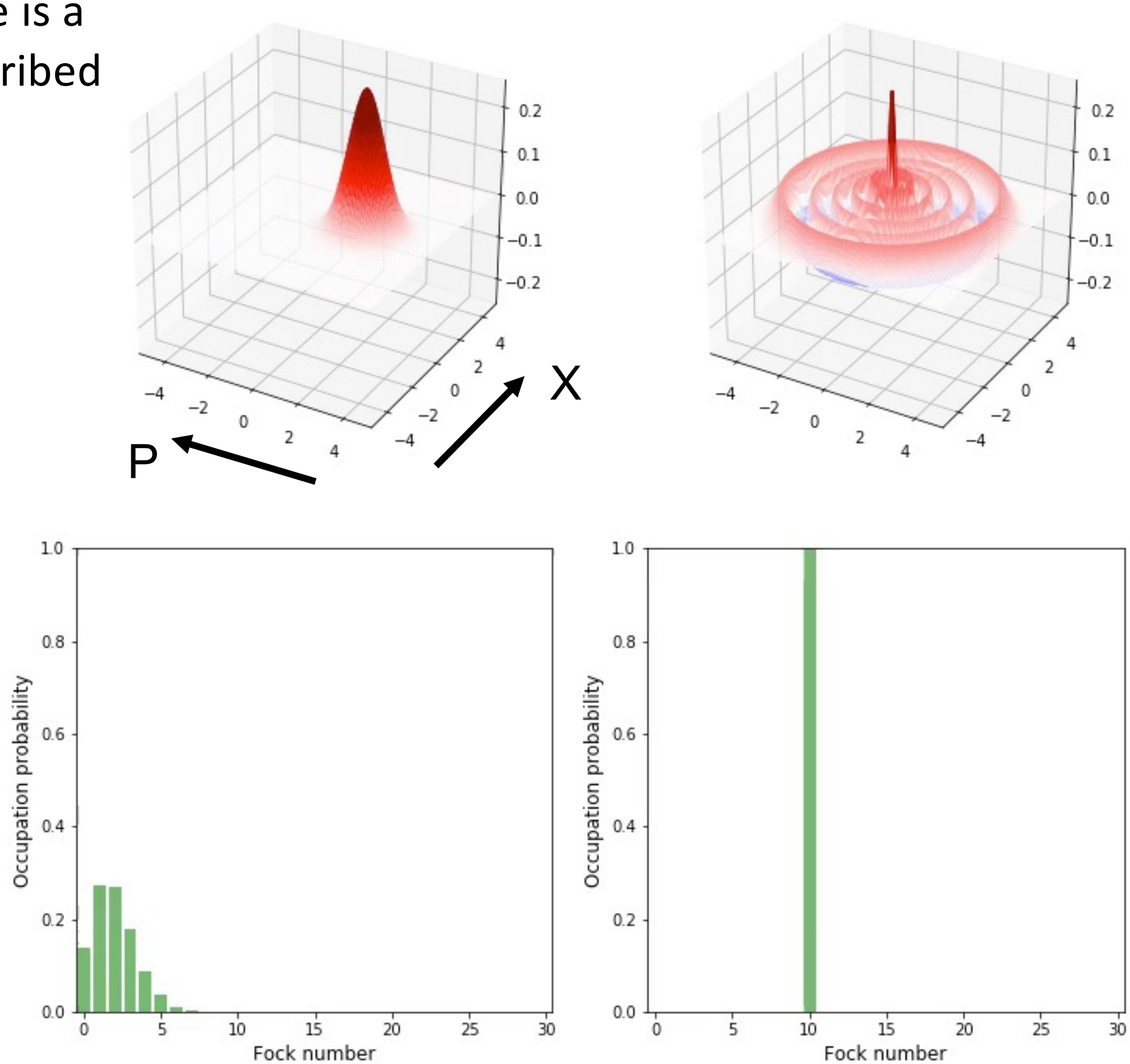


A Fock state is a superposition of an oscillator in all possible phases of its sinusoidal motion

Mixing between a coherent state and a Fock state

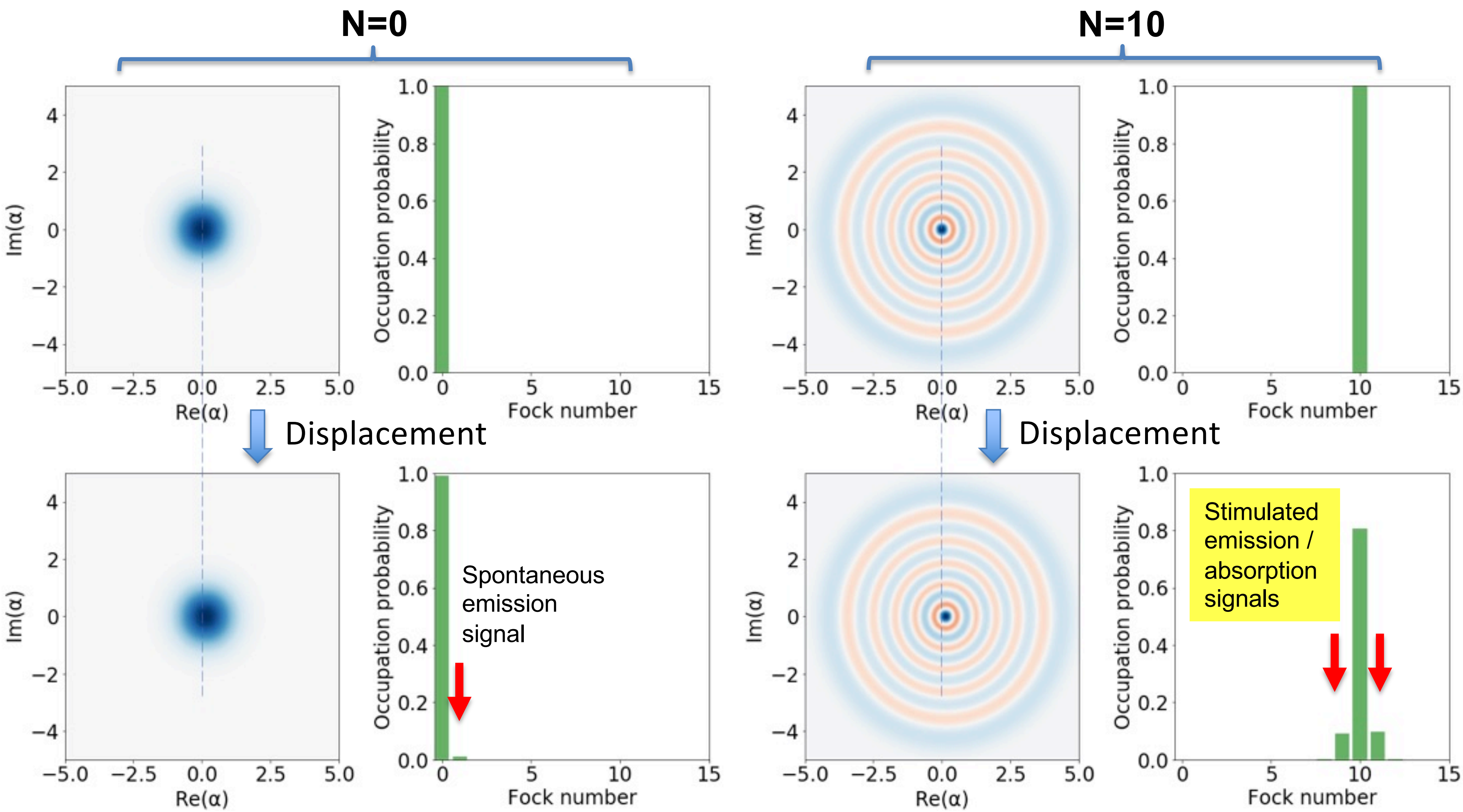
The axion wave is a sine wave described by a Glauber coherent state

The cavity photon mode has been prepared as a N=10 Fock state



QuTiP simulation

For finite coherence time << mixing time, the transfer of quanta from axions to photons is enhanced by a factor (N+1)

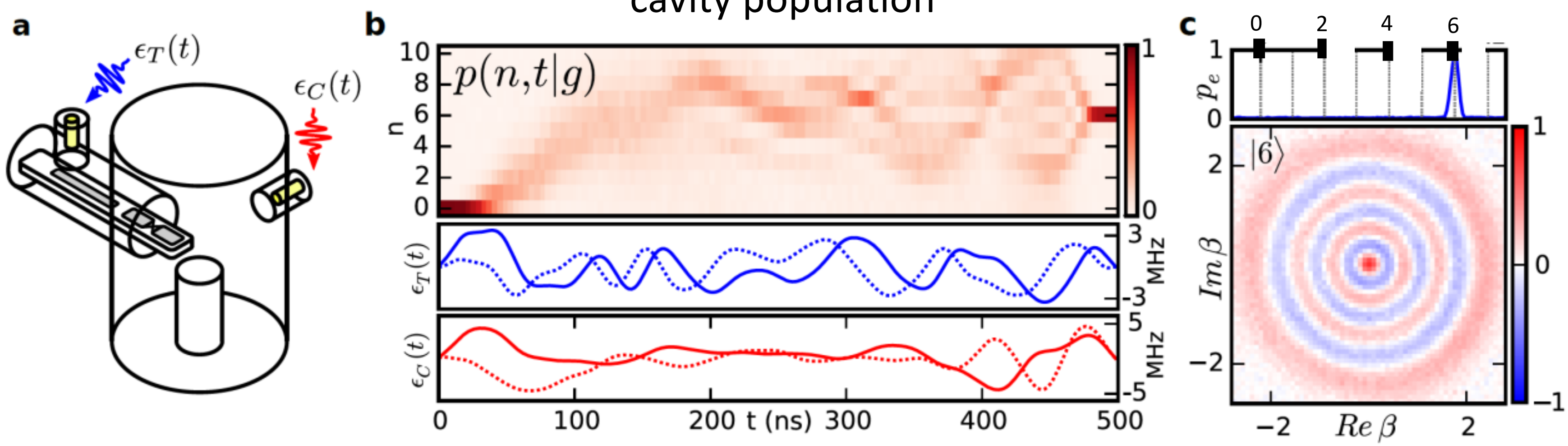


QuTiP simulation

Universal, optimal control is used to initialize a cavity photon mode to an arbitrary initial quantum state

Example: Preparation of photon Fock state $\left|n=6\right\rangle$

cavity population



ancilla and cavity quadrature drives

Measured Wigner function

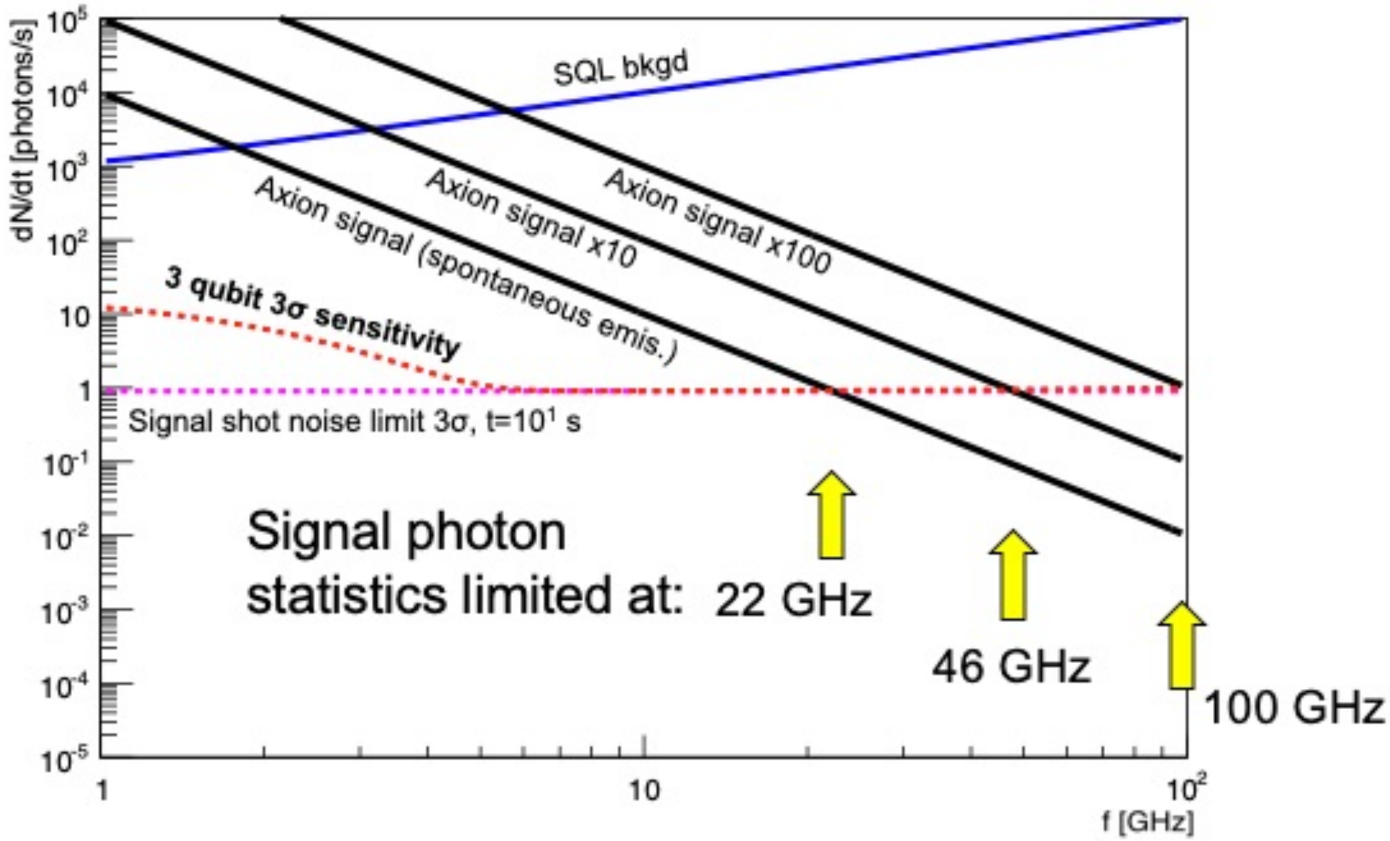
Heeres et al. *Nature Communications* 8, 94 (2017)

Requires nonlinear oscillator (qubit) to enable this nonlinear transformation.

Sequence of waveforms determined by computer. No known general recipe.
Used in quantum computing to load arbitrary initial states into the qRAM.

Using Q=10^8 cavities, stimulated emission boosts axion signal by factors up to 100.

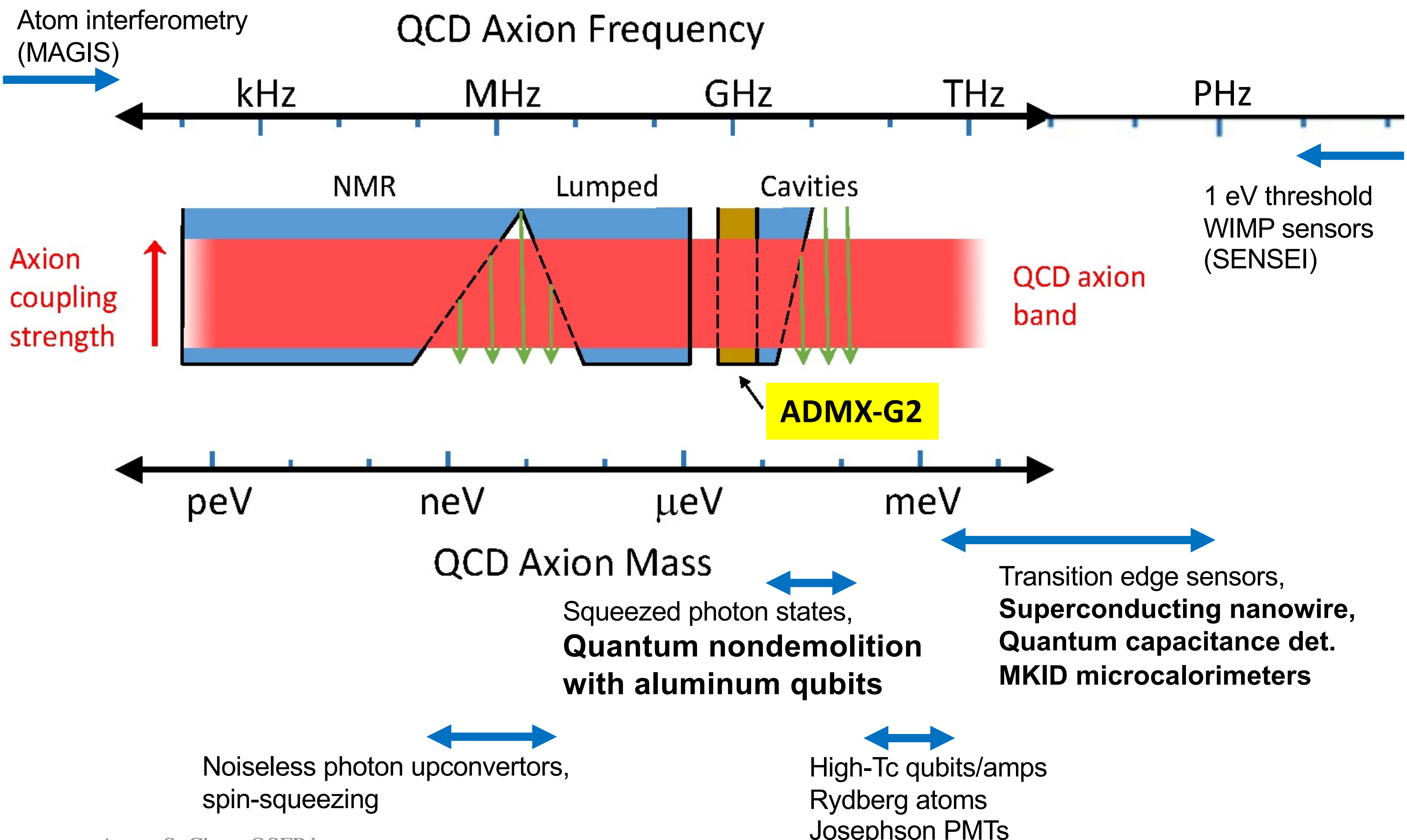
Not only increases SNR at lower masses, but also extends range to higher masses!



Only works up to around 30 GHz where we get too close to the Josephson plasma frequency where resistive heating in the junction is enough to break Cooper pairs

So what do we do for even higher frequencies?

Figure adapted from DOE "Small Dark Matter" BRN Study, 2018



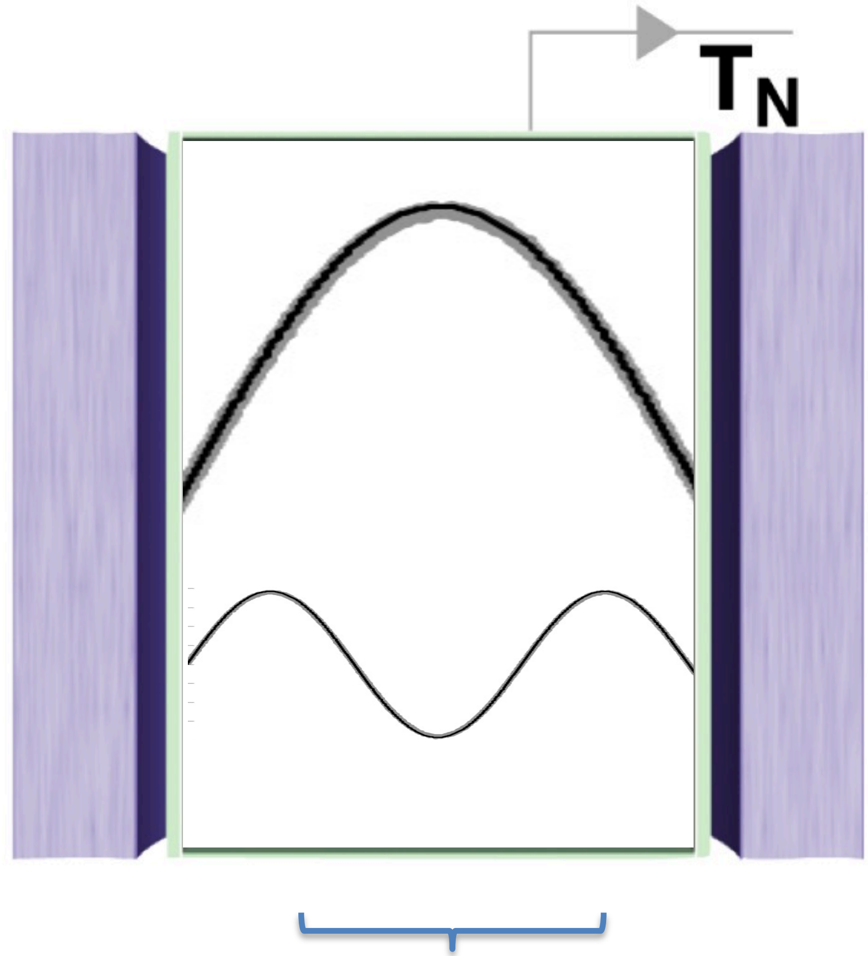
Actually, only the cavity volume within one Compton wavelength from the wall matters

Nothing can possibly happen in the empty space far from a cavity wall since empty space is translation symmetric. The extra interior wiggles in the higher frequency mode all cancel each other out in this semi-classical power calculation:

$$P_a(t) = \int \vec{J}_a(t) \cdot \vec{E}_r(t) \; dV$$

J is spatially uniform on laboratory scales and points in the direction of the applied B field

The direction of E oscillates up/down



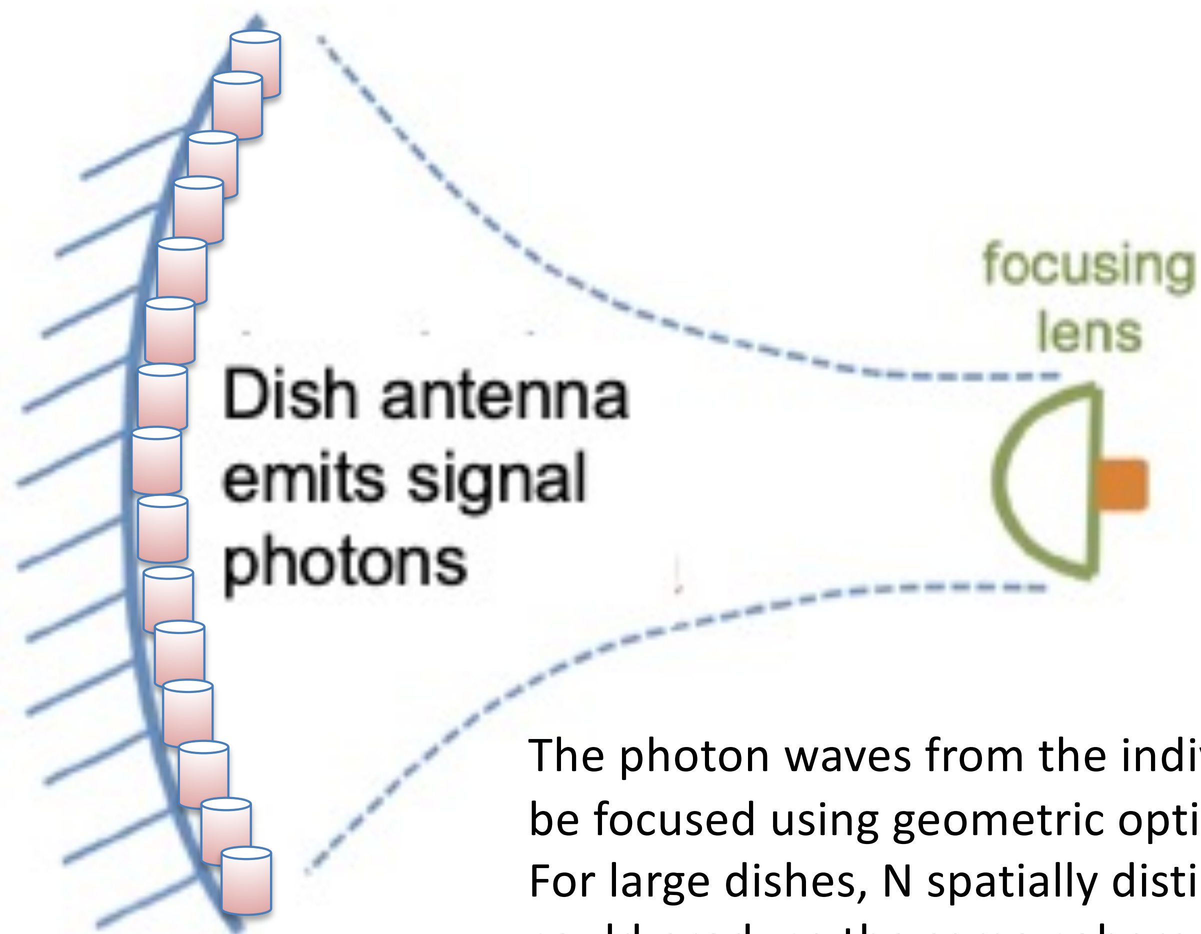
The interior region integrates to zero

One can instead use a huge dish antenna whose area contains many wavelength-squared pixels

D. Horns, et.al, JCAP 1304, 016 (2013)

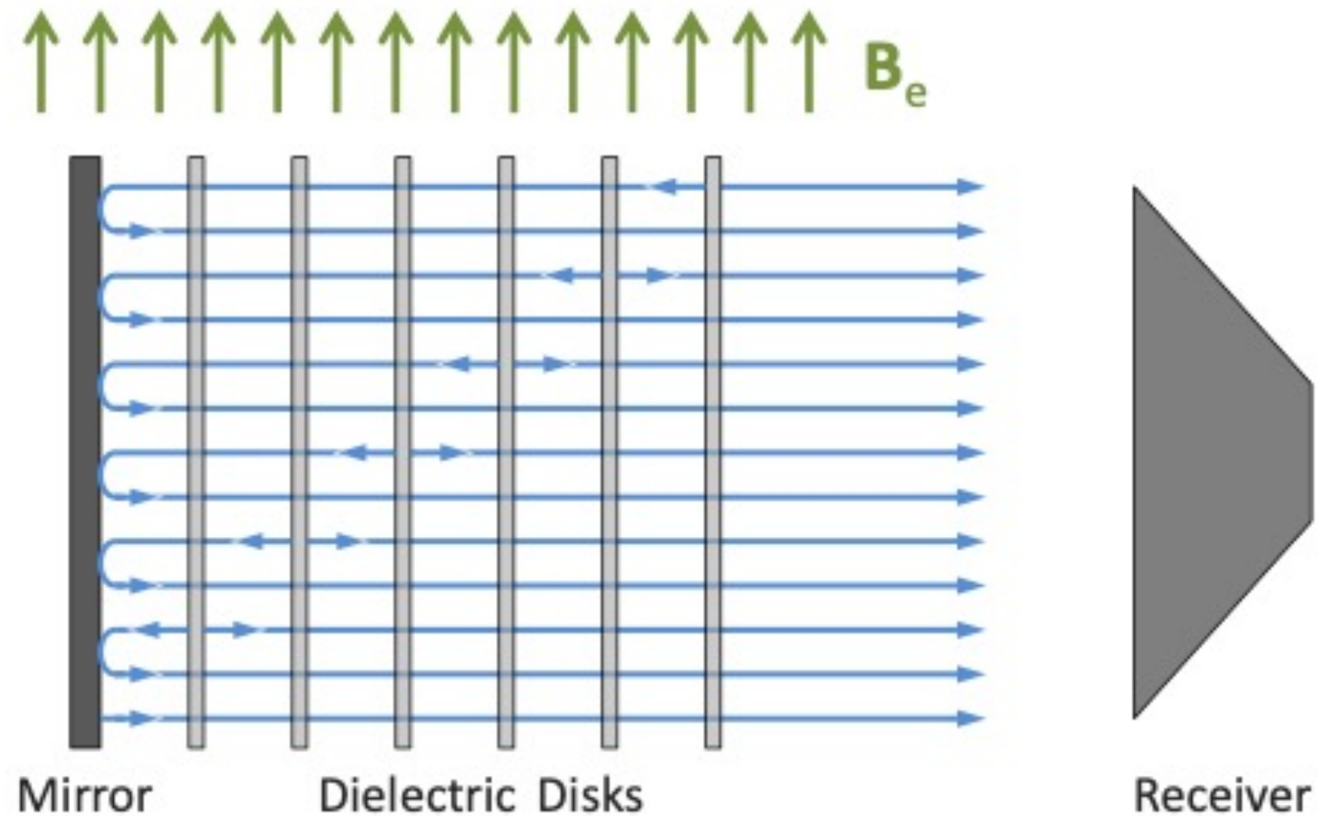
This is like having an array of N cavities, where each cavity only gets 1 bounce.

Axion emits transition radiation upon impedance mismatch.



The photon waves from the individual pixels can be focused using geometric optics.
For large dishes, N spatially distinct emitters could produce the same coherent sum as that of Q bounces inside a resonant cavity.

Since the axion can convert into photons when encountering any interface which breaks translation symmetry, we can also use many dielectric plates



MADMAX idea,
A. Caldwell, et.al, Phys. Rev. Lett. 118, 091801 (2017)

However, for achievable magnetic fields, the photon signal rate is low

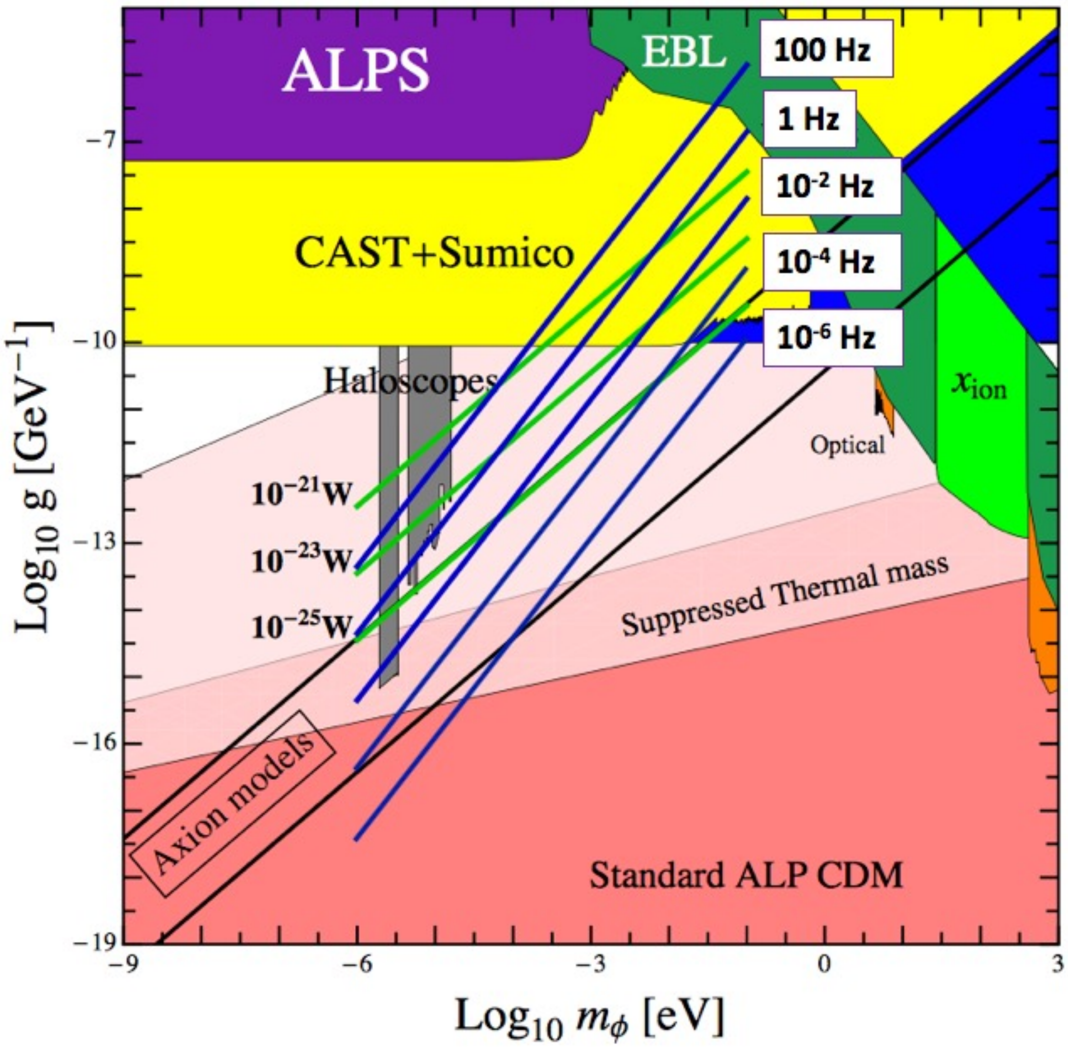


Figure adapted from Horns et.al (2012)

These will be long duration experiments, and we do not yet have the single photon detection technology with sufficiently low dark rates.

Cavity detectors (e.g. ADMX)

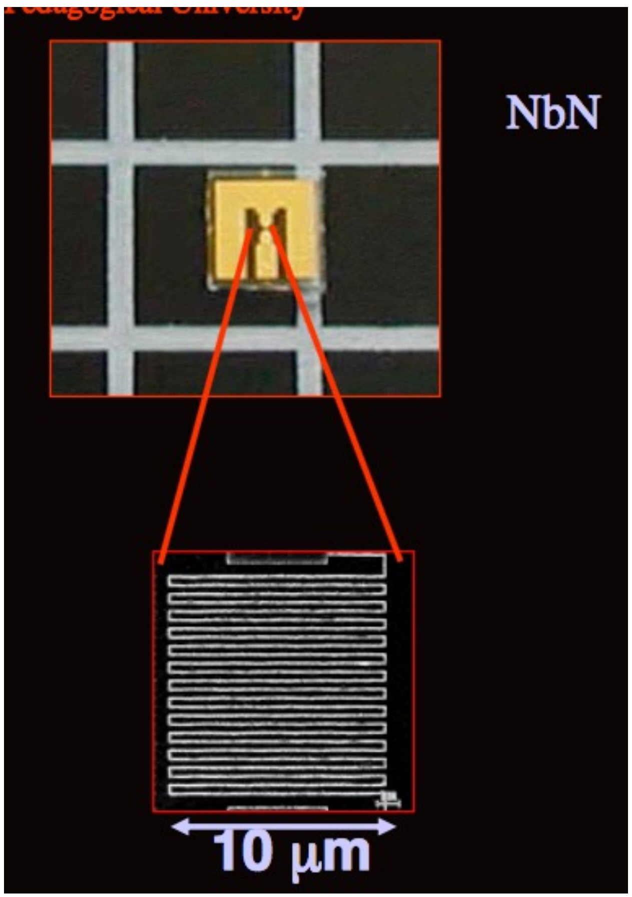
1 eV threshold particle detectors (e.g. SENSEI)

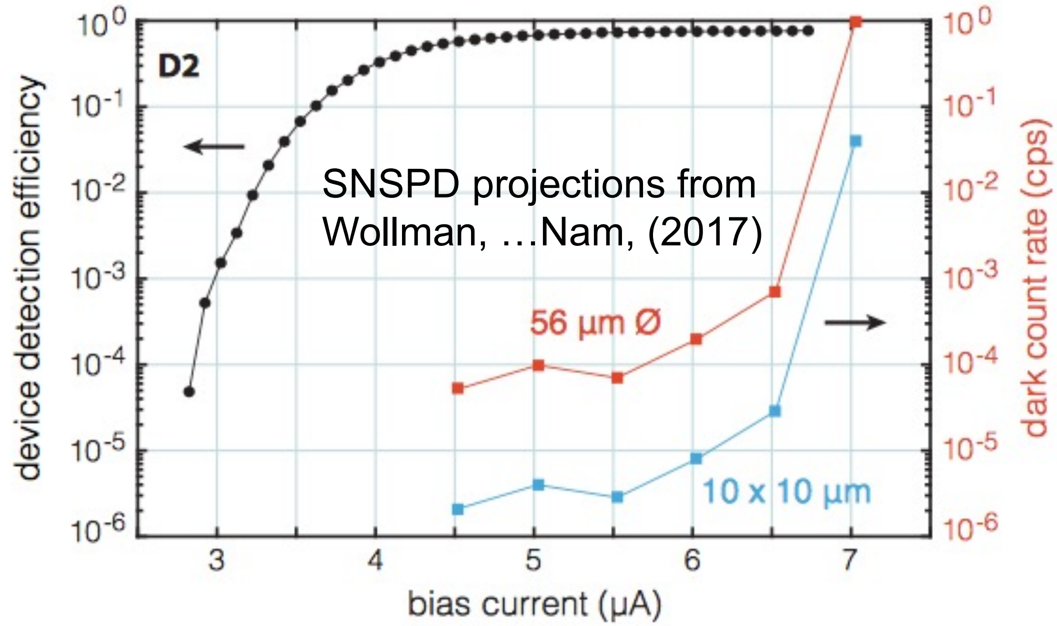
Need better sensors

Aaron S. Chou, QSFP lecture 2021

Superconducting nanowire single photon detector

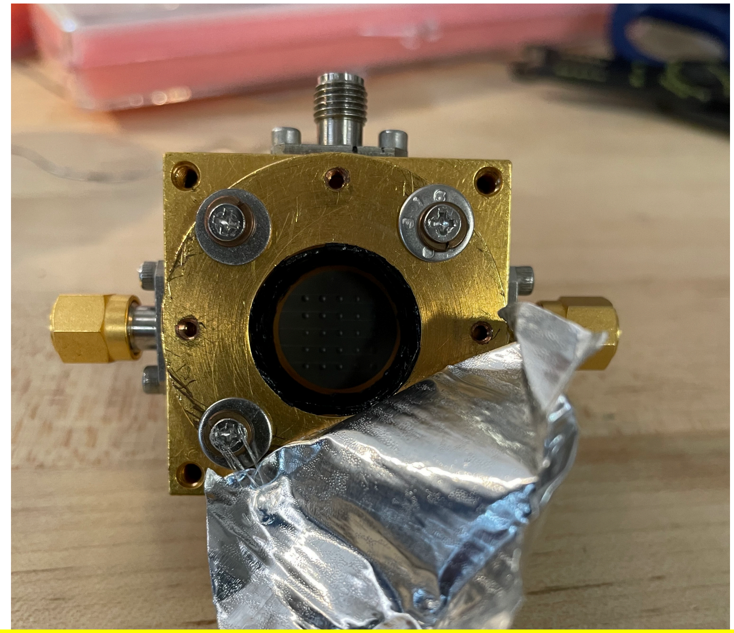
Create a large pixel with a meandering SC wire. Apply a current bias. Heating due to absorbing a photon will cause the wire to quench and become resistive, triggering a large voltage response.





- Deployed in Lamppost hidden photon search.
- **Dark rates <10^{-5} Hz already achieved for NIR photons.**
- Seem to work okay when placed parallel to high magnetic fields. (B. Lawrie, et. al, unpublished)
- Need to reduce threshold to detect FIR photons for axion dark matter.

Quantum Capacitance Detector

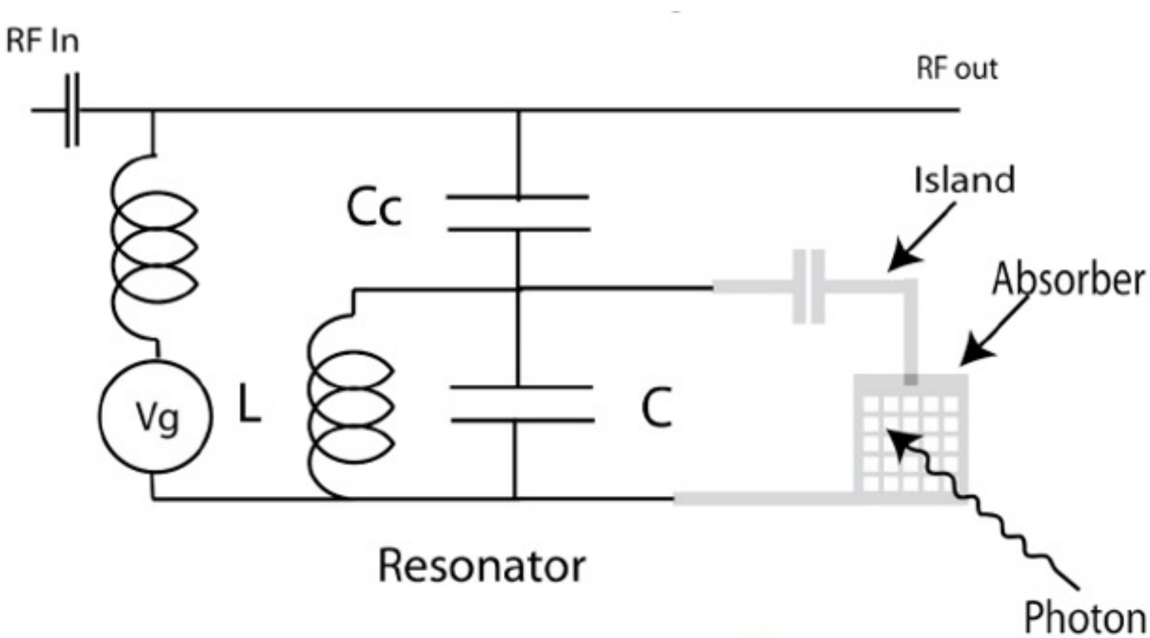


5x5 pixel array from Jet Propulsion Lab

Senses single photons which break even a single Cooper pair.

Based on the Cooper pair box, a.k.a. a charge qubit.

The effective capacitance of this Josephson oscillator has a large discrete change depending on whether the superconducting island has an even or odd number of charges on it.



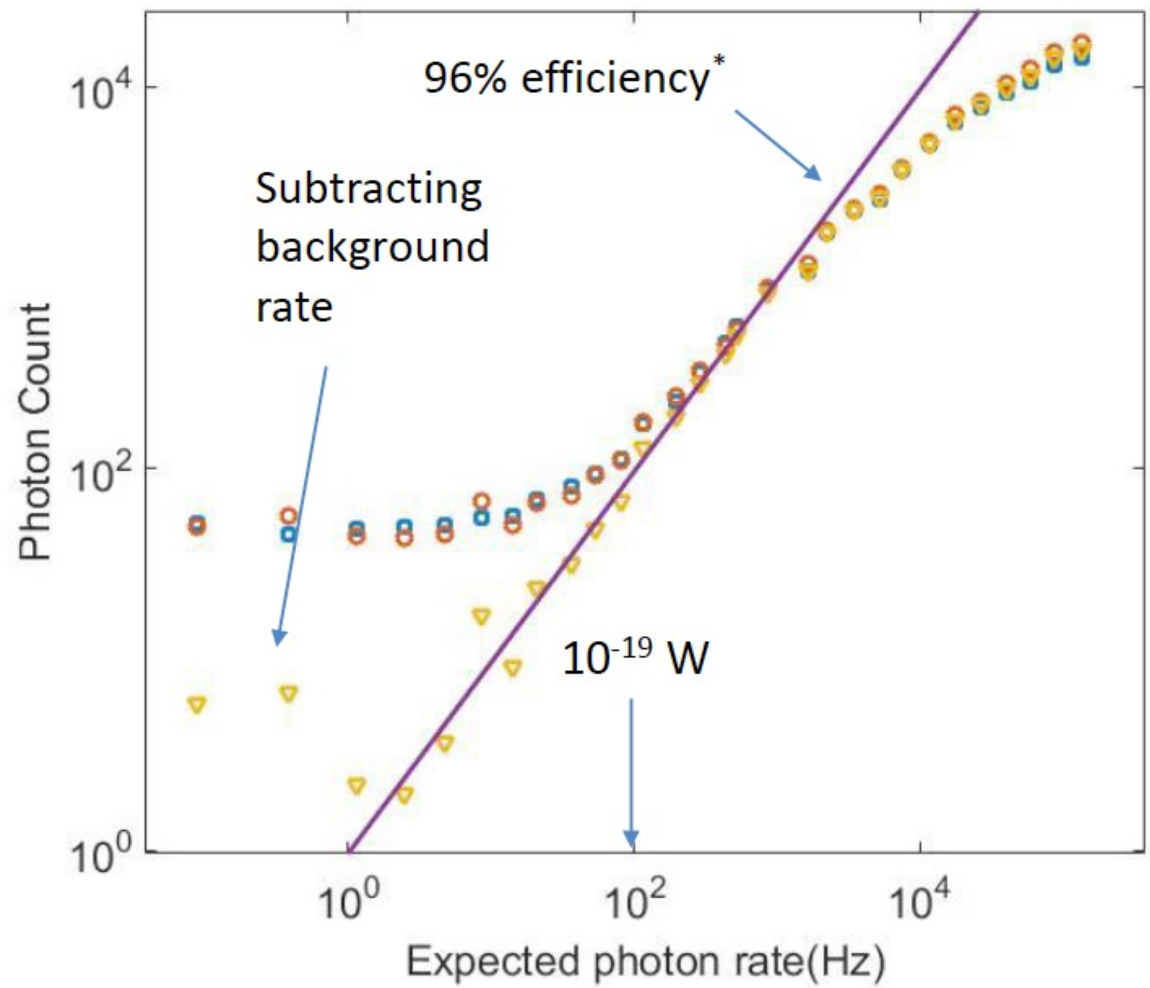
Connect the qubit in parallel to a regular LC oscillator and the resonant frequency of the combined circuit will have a large discrete shift when a photon is absorbed, creating excess charged quasiparticles

QCD detects single THz frequency photons

Lowest noise-equivalent power 10^{-21} W/rtHz of any FIR photon detector

P.M. Echternach, et.al, Nature Astronomy 2, 90-97 (2018)

However, the corresponding 100 Hz background rate is still too high for the axion dark matter detector



What could be causing these dark counts?

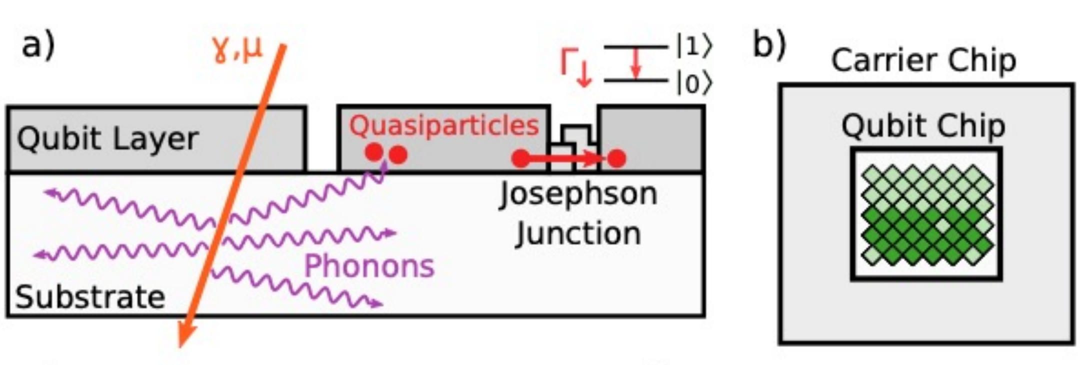
Dark counts probably not cosmic rays – observed 10^{-2} Hz rate in qubit CPU's is too low

Resolving catastrophic error bursts from cosmic rays in large arrays of superconducting qubits

Matt McEwen,[1,2] Lara Faoro,[3] Kunal Arya,[2] Andrew Dunsworth,[2] Trent Huang,[2] Seon Kim,[2] Brian

M. McEwen, et.al, arXiv:2104.05219

Google Sycamore chip already functions as a phonon detector with 100% chip-wide failure in response to ionizing radiation events which can be localized in both space and time



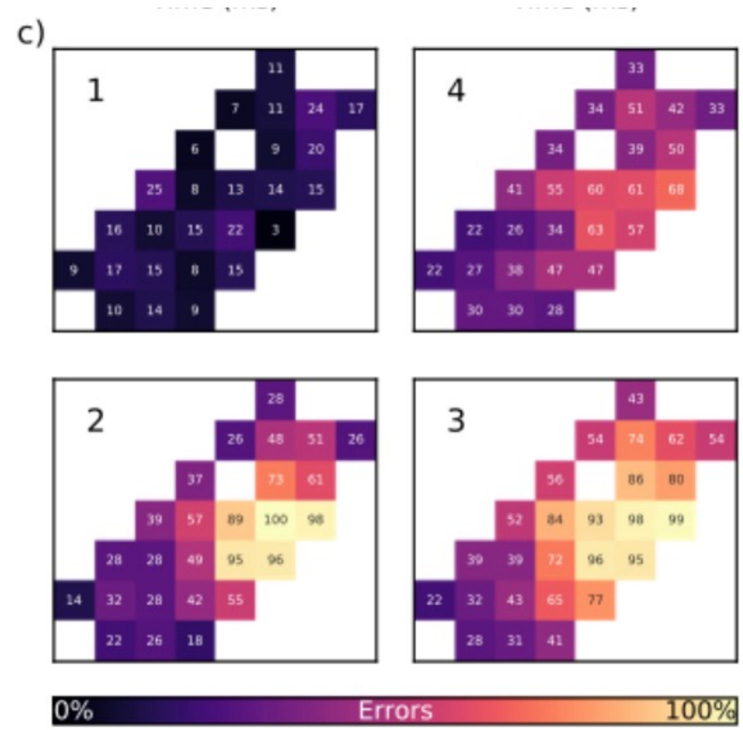


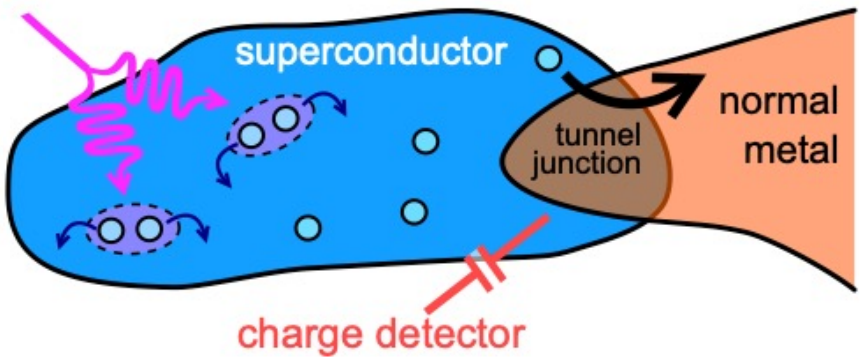
Figure 3. **Localization and spread of error.** (a-b) Time-

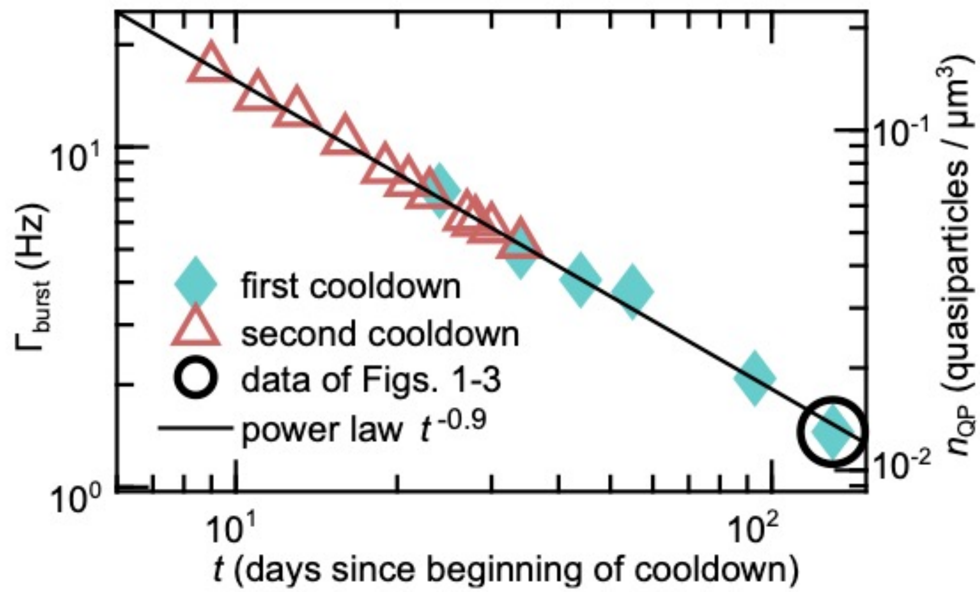
Eventually axion experiments will have to move underground just like WIMP experiments, but cosmic rays are not currently the dominant background.

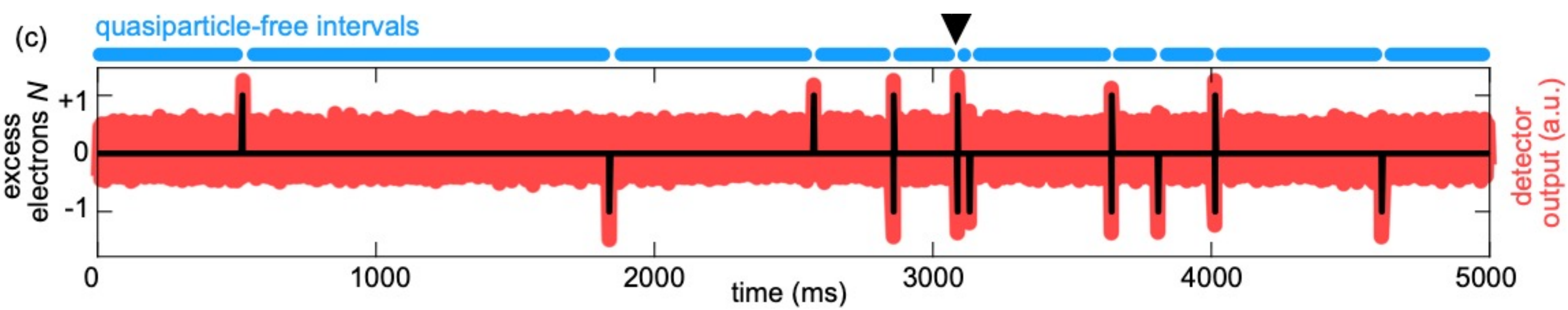
Superconducting devices all suffer from mysterious non-equilibrium quasiparticle population

These now appear to be created in discrete, time-resolved events.

E.T. Mannila, et.al, arXiv:2102.00484 *(2021)*







Origins of events still a 20-year-old mystery….

Study guide

- Low mass dark matter form classical bosonic wave which can drive quantum oscillators
- The signal wavefunction is squared in offline analysis to compare the signal variance to the noise variance
- Measurement of non-commuting observables incurs the penalty of zero-point noise, which can be squeezed
- Forces induce displacements in phase space
- “a” stands for “amplitude” in polar phase space coordinates, not “annihilation”
- The resolution of a probe is determined by its phase space distribution
- Better to measuring only a single observable – wave amplitude
- Quantum non-demolition measurements can drastically reduce noise
- Fock states are the ideal basis to measure amplitude displacements in arbitrary directions
- The signal from wave dark matter is secretly transition radiation on interfaces with mismatched electromagnetic response
- Large area reflector dishes can focus signals onto single photon detectors
- Existing detectors have demonstrated single photon sensitivity but dark rates are too high. Some clues coming from the quantum computing community.