Lecture 2: Clever Quantum Tricks for Detecting Dark Matter Waves

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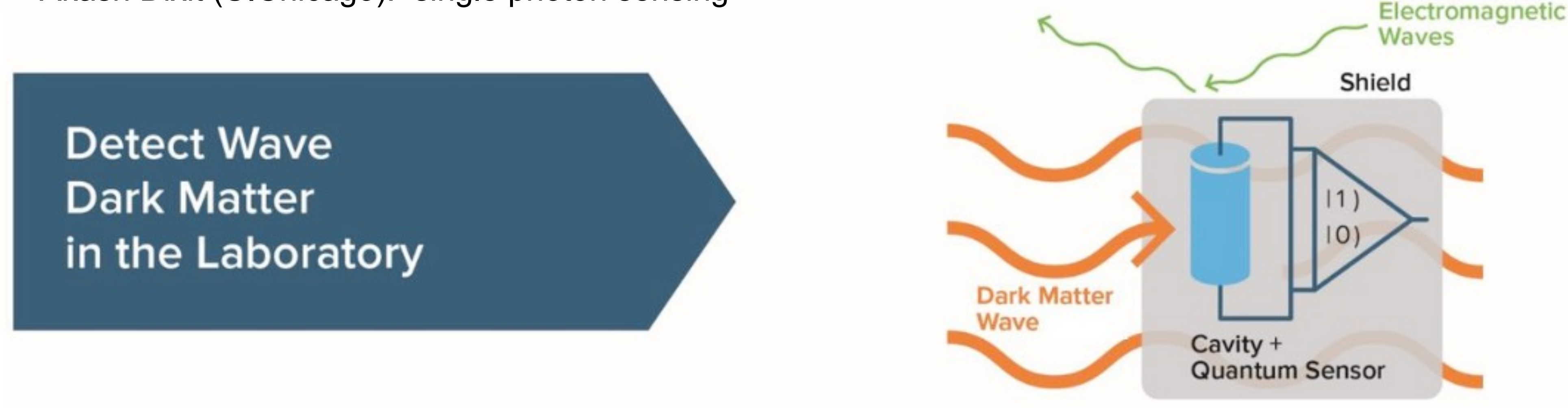
QSFP School

Tutorials (repeated today and Thursday):
Chelsea Bartram (U.Washington): axion experimental techniques
Samantha Lewis (Fermilab): axions and microwave cavities
Akash Dixit (U.Chicago): single photon sensing



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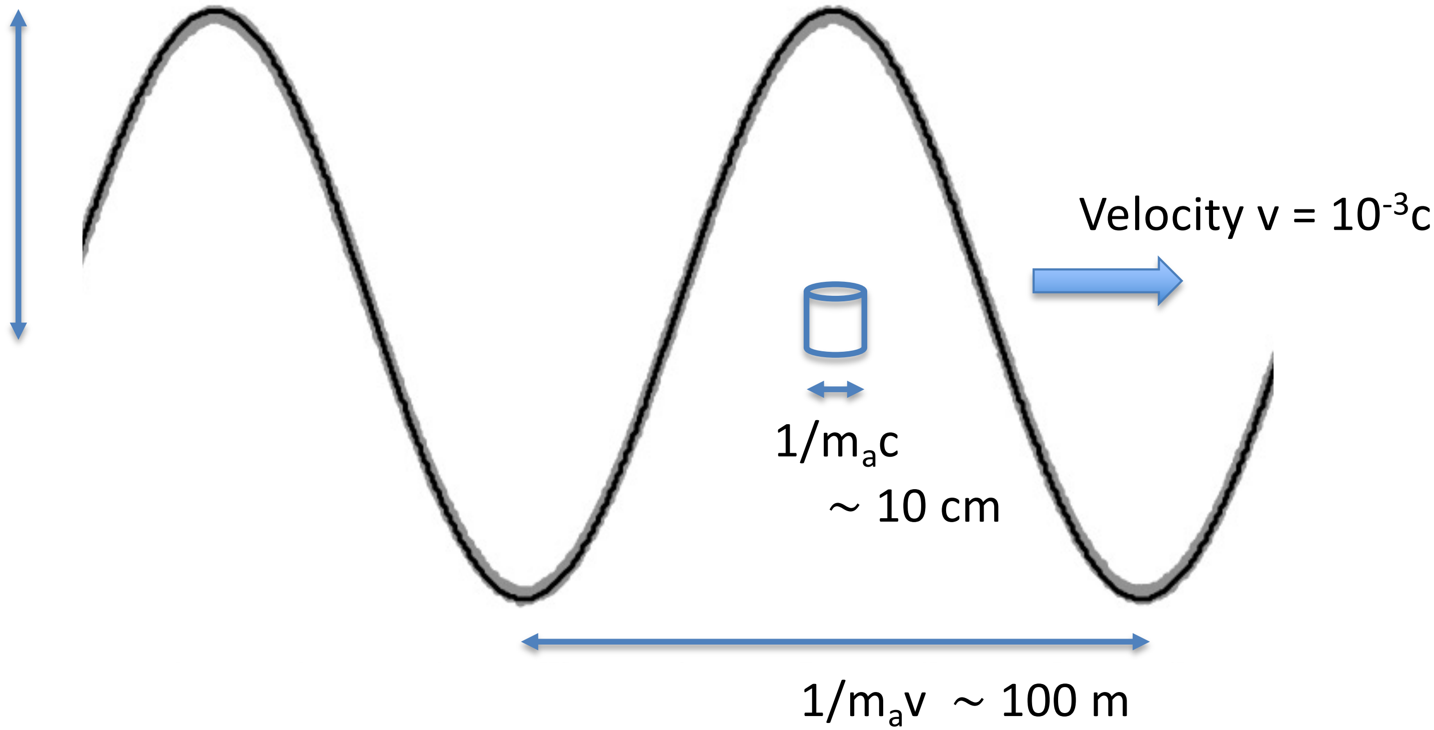
Summary of lecture 1:
Axion dark matter forms a slow classical wave

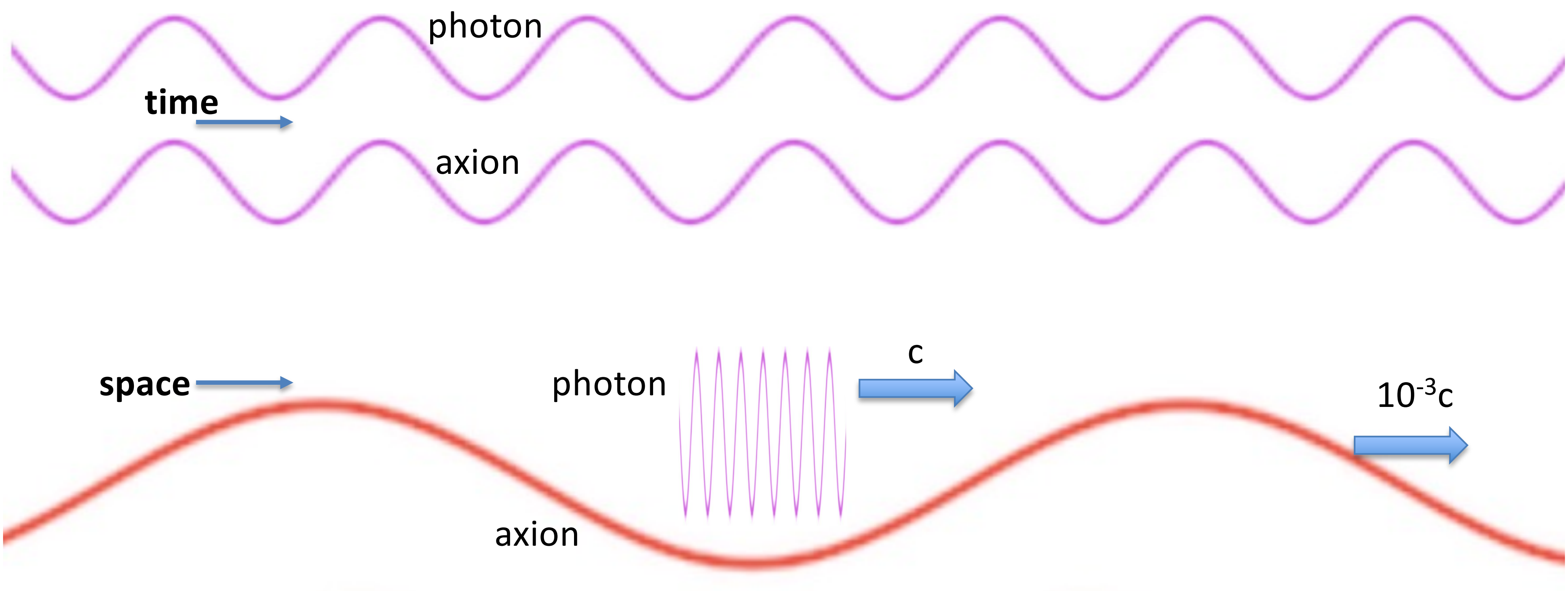
$$\theta_{\max} = \sqrt{\frac{2\rho_a}{\Lambda_{\mathrm{QCD}}^4}}$$

$\sim 10^{-19}$ radians



Mode volume V = $(1/m_a v)^3$
Occupation number N = $(\rho_a/m_a)V = \rho_a/m_a^4 v^3 \sim 10^{23}$

Both axion and photon waves oscillate in time at the same frequency m_a



In space, the axion wave is 1000x longer and 1000x slower,
so it can coherently drive the same photon wave through $Q_a=10^6$ temporal oscillations.

In real life, the cavity has losses and so the photon might not live as long as 10^6 oscillations.

Usually $Q_c < Q_a$

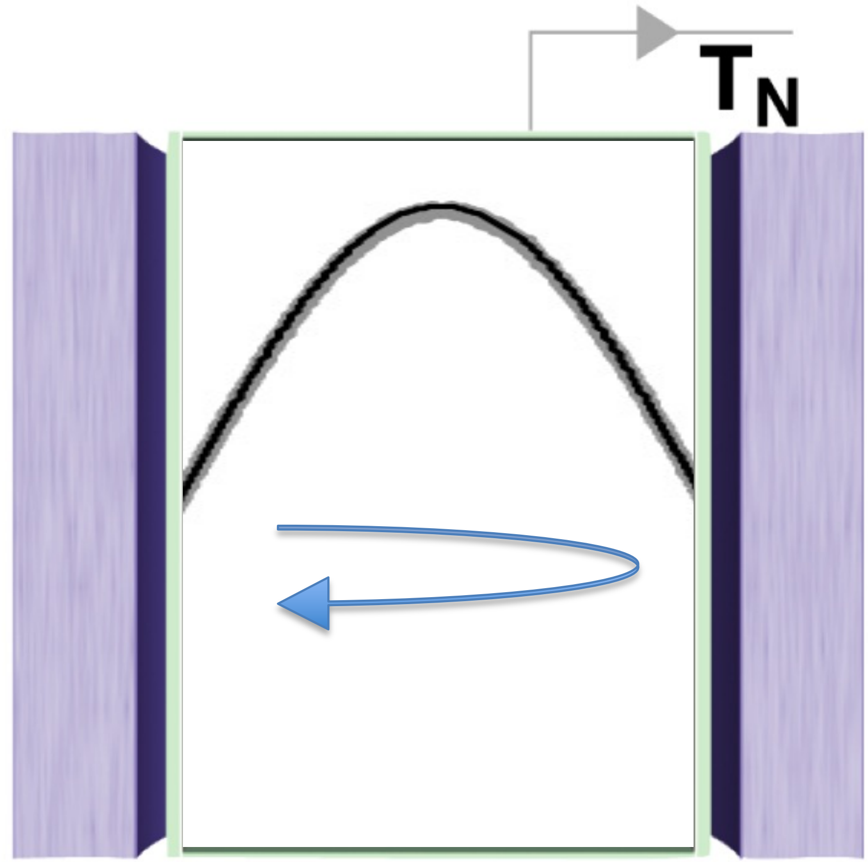
Periodic boundary conditions keep the photon wave in phase with the 10^6 axion oscillations

- In a constant background B_0 field, the oscillating axion field acts as an exotic, space-filling current source

$$\vec{J}_a(t) = -g\theta\vec{B}_0 m_a e^{i m_a t}$$

which drives E&M via Faraday's law:

$$\vec{\nabla} \times \vec{H}_r - \frac{d\vec{D}_r}{dt} = \vec{J}_a$$



D = gθB   for one oscillation…

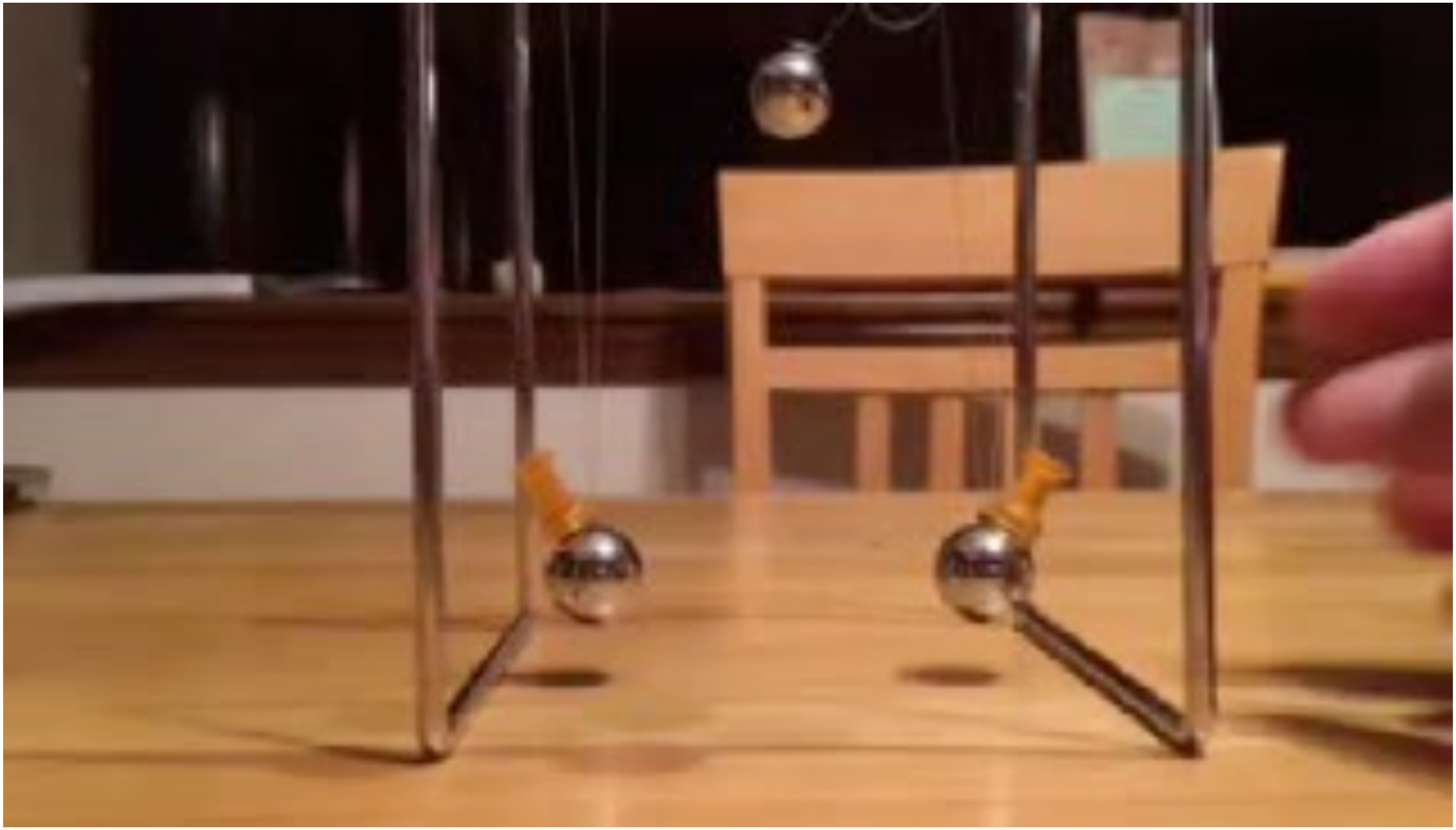
D = gθBQ   for Q coherent oscillations

Stored energy $U = D^2 V/\varepsilon$, cavity lifetime $\tau = Q/m_a$,

Signal power $P = U/\tau \sim (g\theta B)^2\, V\, m_a Q/\varepsilon$

ε = permittivity

Energy transfer between axion and photon



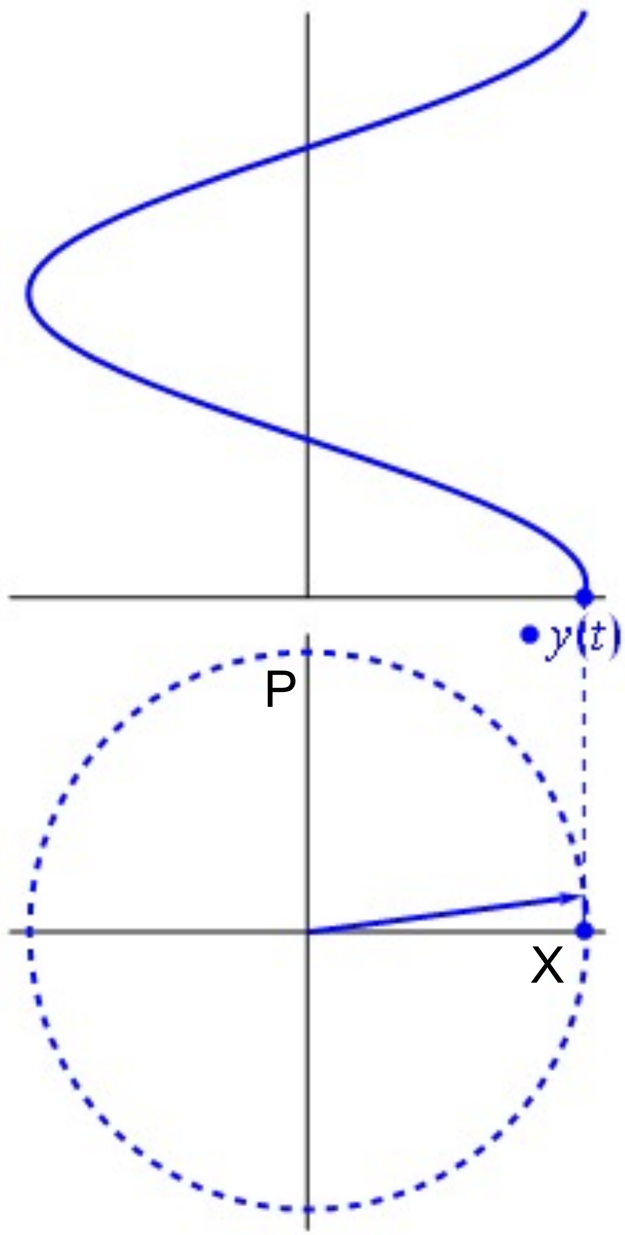
Weak coupling -- takes many swings to fully transfer the wave amplitude.
In real life, Q = number of useful swings is limited by coherence time.

A classical sine wave is described by a rotating phasor:

The energy oscillates between potential energy and kinetic energy, as parameterized by the **position X** and **momentum P**.

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

$$= \hbar\omega(a^\dagger a + 1/2)$$

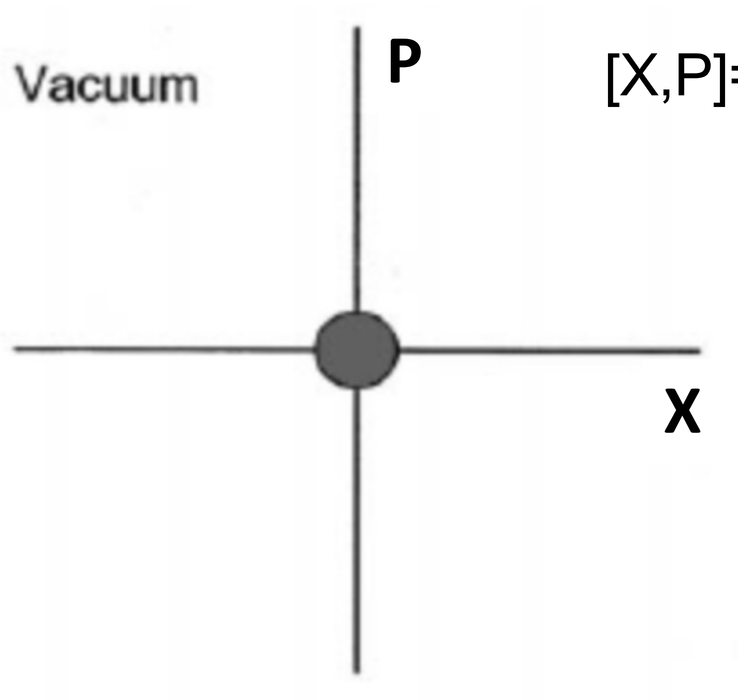


For photons, “X” and “P” are the cosine and sine quadratures of the electric field oscillation.

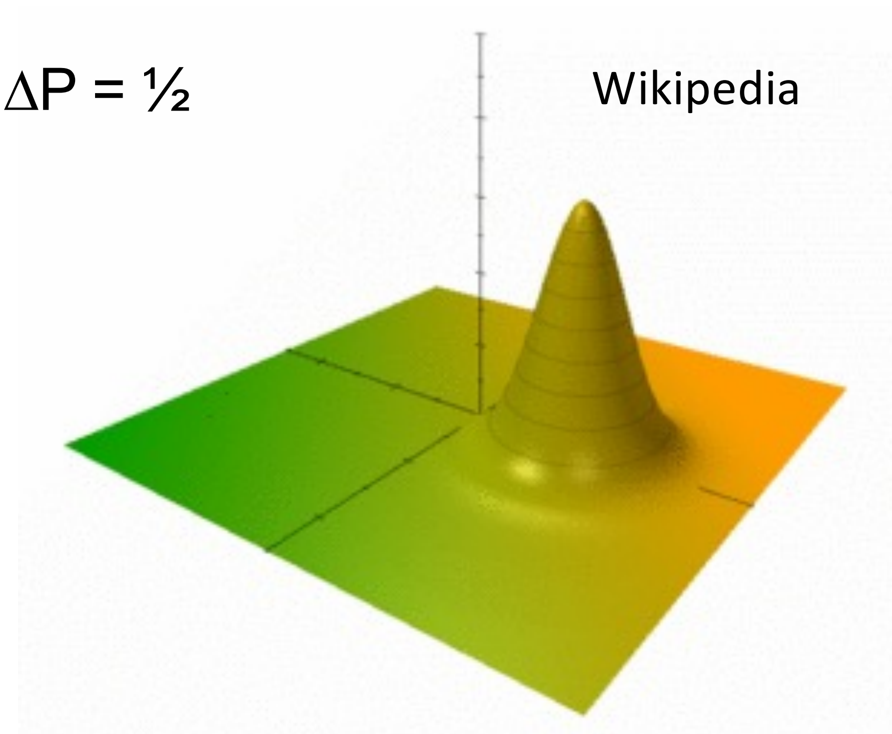
2nd quantization: [X,P]=i even for internal field quadratures (since these can drive mechanical oscillators.)

Heisenberg uncertainty principle = quantization of (internal) phase space area

Wigner pseudo-probability distributions for the endpoint of the phasor:



$[X,P]=i$ so $\Delta X \cdot \Delta P = ½$



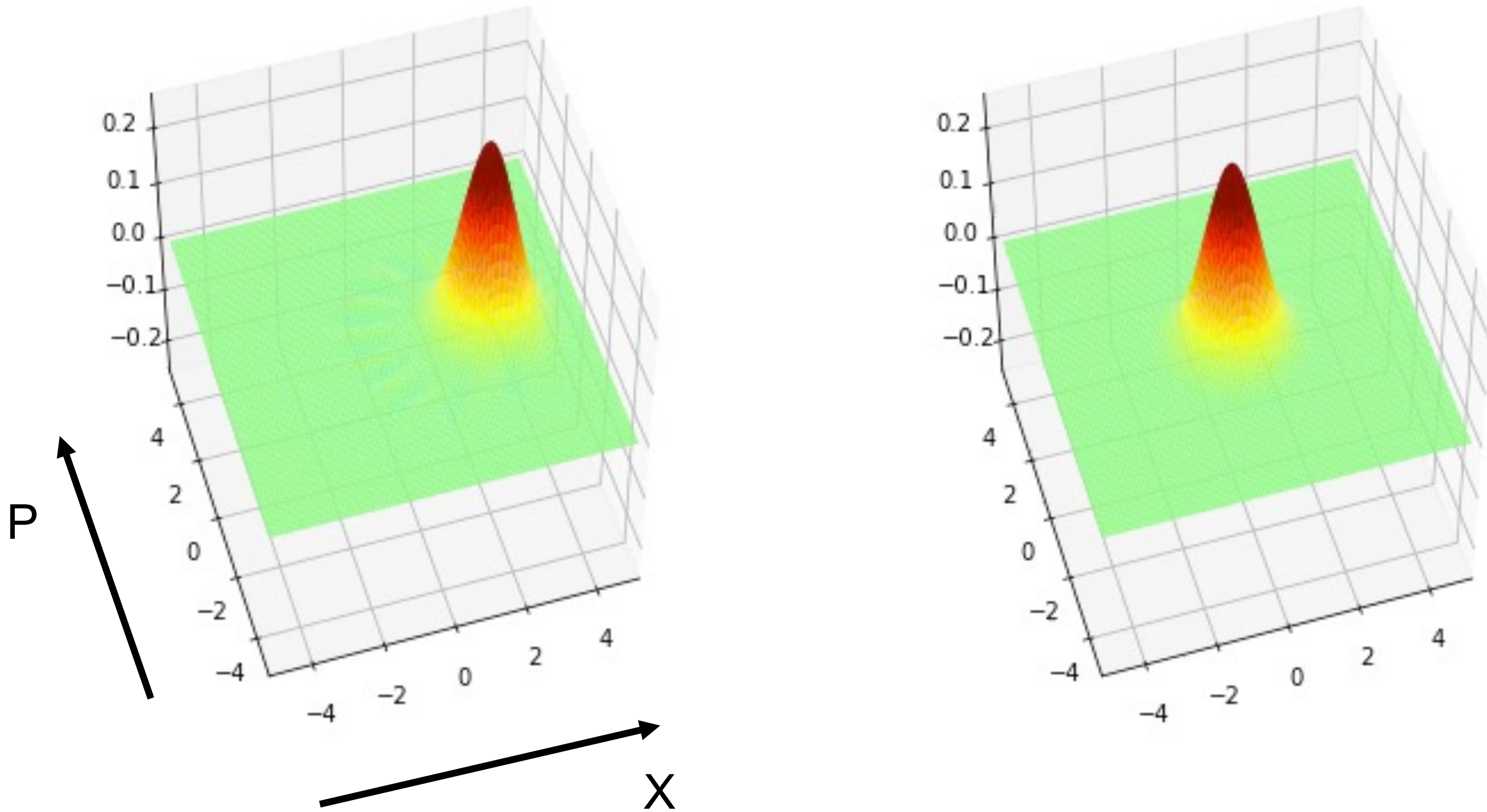
The vacuum state of the oscillator is a zero length phasor which still exhibits **zero-point noise**.

Sine wave with quantum uncertainty included. The Gaussian width in the radial direction manifests as Poisson **shot noise**.

In polar coordinates, Heisenberg becomes number-phase uncertainty: $\Delta N \times \Delta \varphi \geq ½$

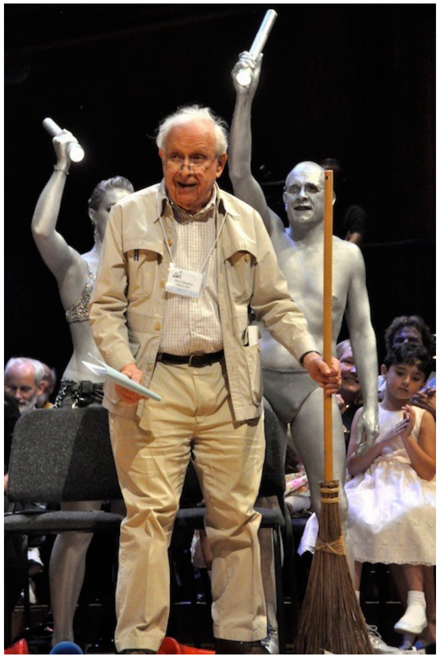
Classical pendulum system: $|\alpha = 3\rangle \otimes |\alpha = 0\rangle$

Time evolution of Wigner distributions in X-P “phasor” space.
Each gaussian blob of phase space area satisfies $\Delta X \cdot \Delta P = \frac{1}{2}$



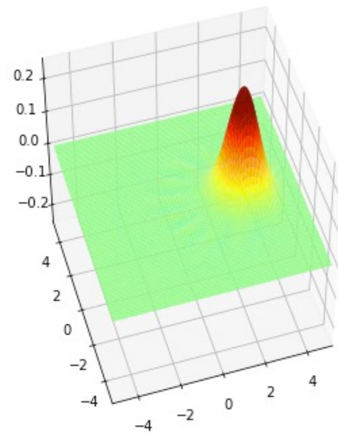
The two pendula swap their coherent states.

Simulated with QuTIP

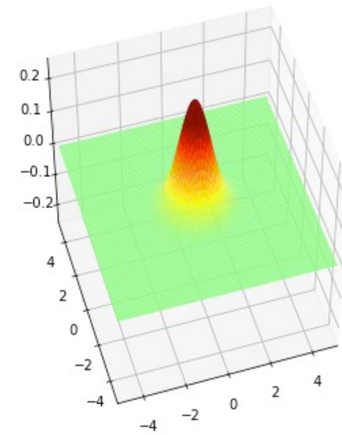


Classically-driven quantum harmonic oscillator

Roy Glauber
Nobel Prize
2005,
"Keeper of the
Broom"



Osc1 is classical sine wave
$f(t) = f_0 e^{-i\omega t}$



Osc2 is 2nd quantized
$a + a^\dagger$

$$\hat{H} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + i\hbar\left(f(t)a^\dagger - f^*(t)a\right)$$

$$U_\mathrm{I}(t) = \exp\left[(f_0 a^\dagger - f_0^* a)t\right]$$ Time evolution operator

$$|\psi(t)\rangle_\mathrm{I} = \exp[(f_0 a^\dagger - f_0^* a)t]|0\rangle = e^{-|f_0|^2 t^2/2} e^{f_0 a^\dagger t}|0\rangle$$

$\equiv D(f_0 t)|0\rangle$ **Displacement operator**

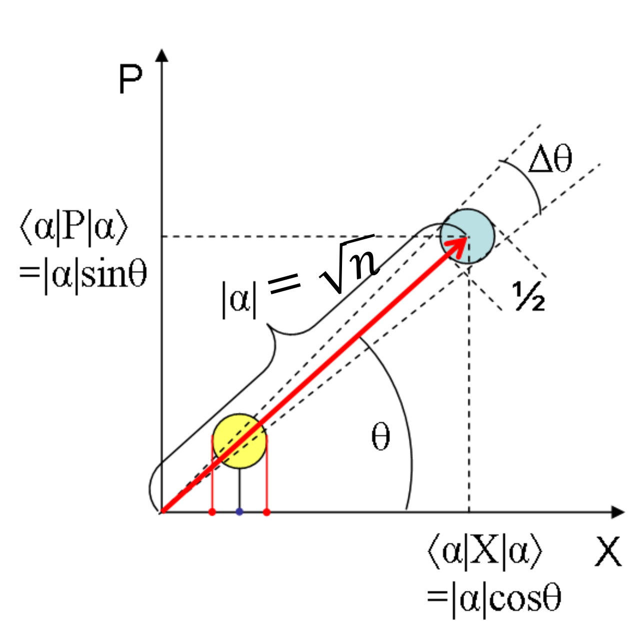
$\equiv |\alpha = f_0 t\rangle$ **Coherent state:** quantum description of a classical sine wave with amplitude $\alpha = \sqrt{\langle n \rangle}$

The Glauber displacement operator and coherent states

$$p = i\frac{d}{dx}$$ Generates translations in position

$$x = i\frac{d}{dp}$$ Generates translations in momentum

$$a^\dagger = x + ip$$
$$a = x - ip$$
Generate translations in an arbitrary direction in x-p phase space



Exponentiate differential operator to get finite translation α in complex plane: $$\hat{D}(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)$$

Phasor of amplitude α is generated as: $$D(\alpha)\,|0\rangle = |\alpha\rangle$$ Classical sine wave

This is an eigenstate of the annihilation operator: $$a\,|\alpha\rangle = \alpha\,|\alpha\rangle$$ Prove this!

Classical sine waves have intrinsic Poisson noise

Coherent states are eigenstates of the annihilation operator:

$$a\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

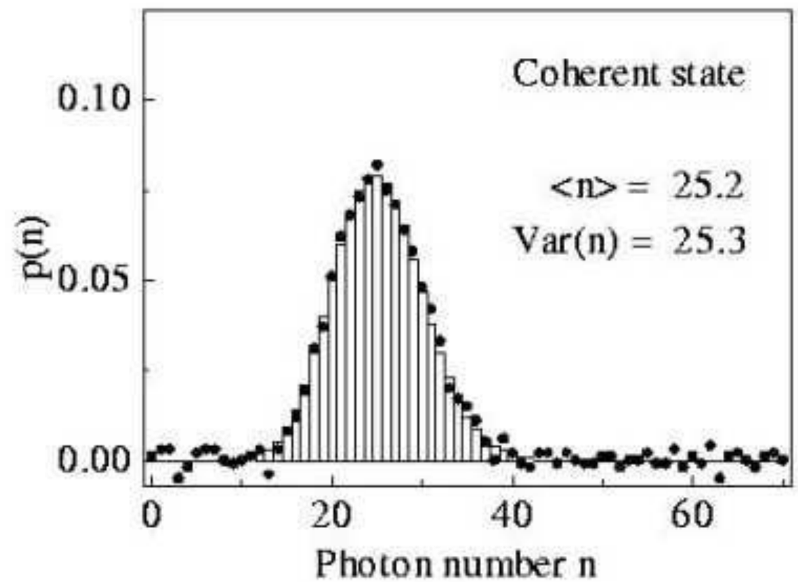
They form a Poisson distribution in the number state basis:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha\hat{a}^\dagger}|0\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

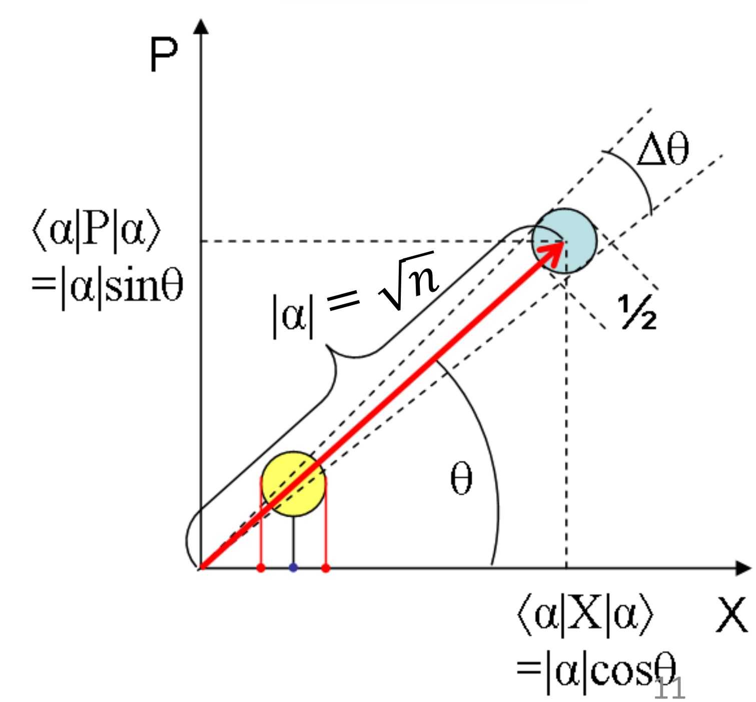
$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle}\frac{\langle n\rangle^n}{n!}$$

$$\langle n\rangle = \langle \hat{a}^\dagger \hat{a}\rangle = |\alpha|^2$$

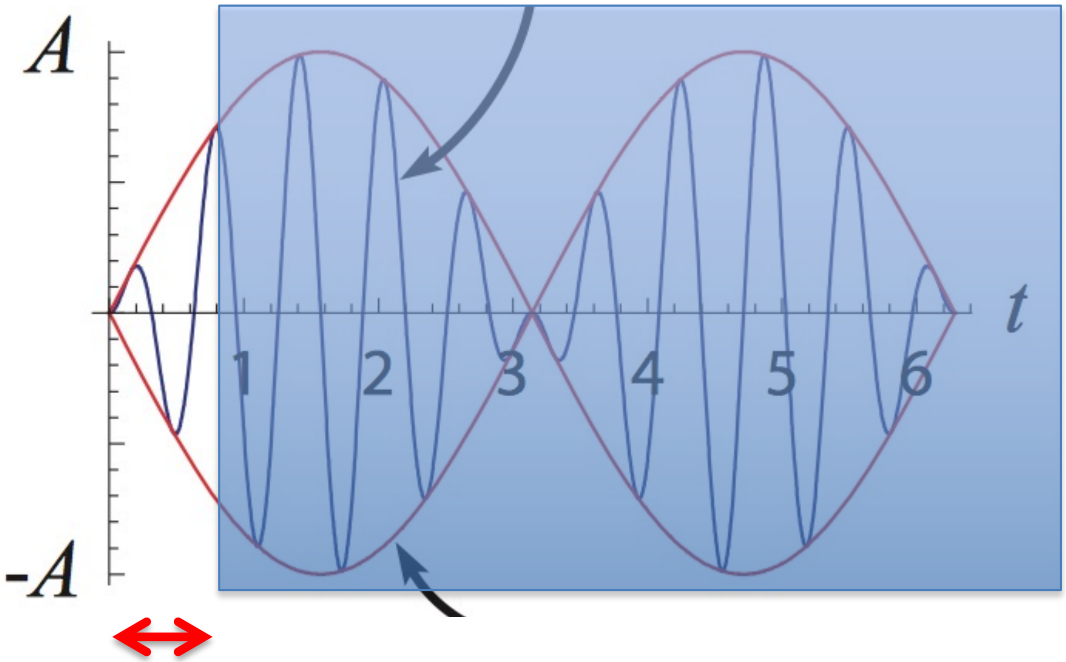
$$(\Delta n)^2 = \mathrm{Var}\left(\hat{a}^\dagger \hat{a}\right) = |\alpha|^2$$



Like the zero-point fluctuations, the Poisson shot noise in classical wave intensity is a consequence of the Heisenberg uncertainty principle.



Only a small amplitude displacement of the photon field can be accumulated over the cavity or axion coherence time



$t_{cav} = 1/df_{cav} = Q_{cav}/f$

Beat period =
1/(Interaction Energy)

>> cavity coherence time

The signal will be tiny!

Need 10^5 seconds to completely convert the axion wave into a photon wave. But only have 10^{-4} s of cavity time…

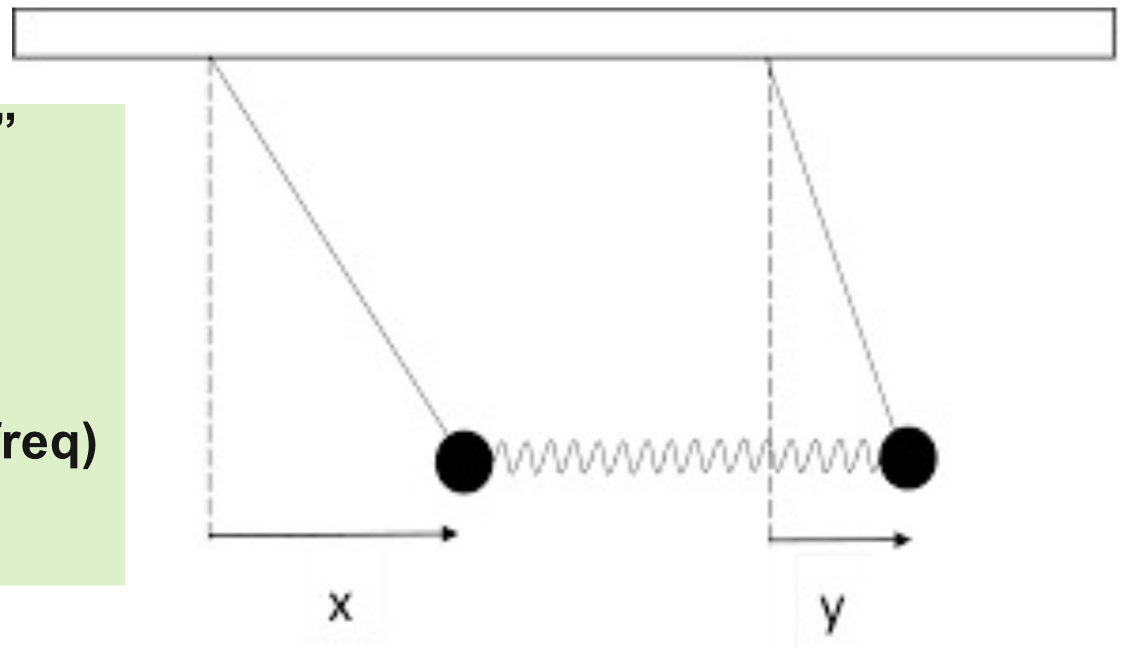
"Real world values"

$|\alpha\rangle$ = 10^{11}

$\frac{\omega}{2\pi} = 10^{10}$ Hz

$\frac{2g}{2\pi} = 10^{-5}$ Hz (beat freq)

$t_{coherence} = 10^{-4}$ s



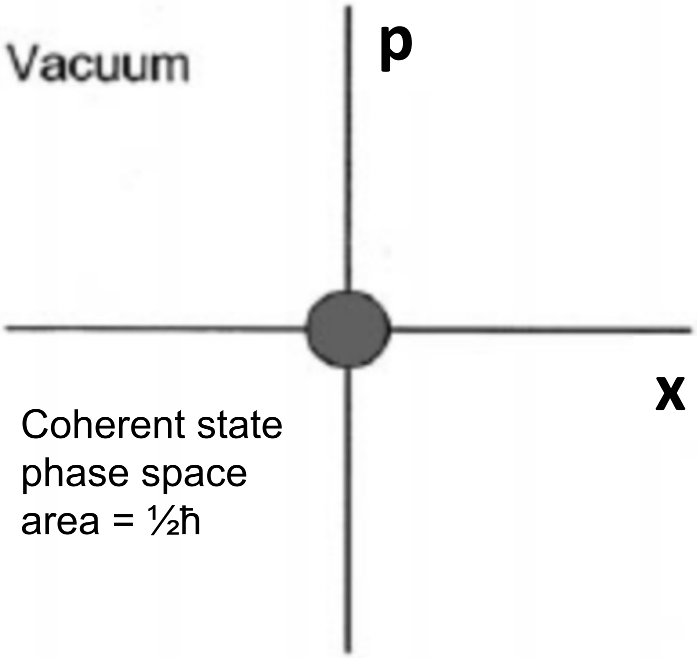
Axion = classical wave

$$\theta(t) = \sqrt{\frac{2\rho_a}{\Lambda_{\rm QCD}^4}}\, e^{im_a t}$$

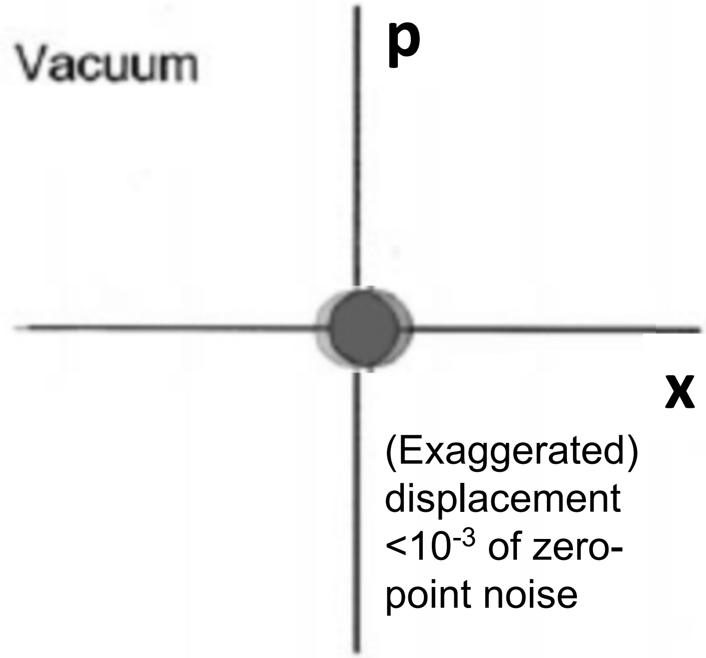
$$H_{\rm I} = igB\sqrt{\omega V}\,\left(\theta(t)a^\dagger - \theta(t)^* a\right)$$

Beat frequency **(derive this!)**

Due to limited coherence time << mixing period, the axion wave displaces the cavity vacuum state by an amount much smaller than the zero-point vacuum noise



Axion-photon mixing induces tiny coherent state displacement $D(g\theta B_0 Q_{cav}/m_a^2)|0\rangle$

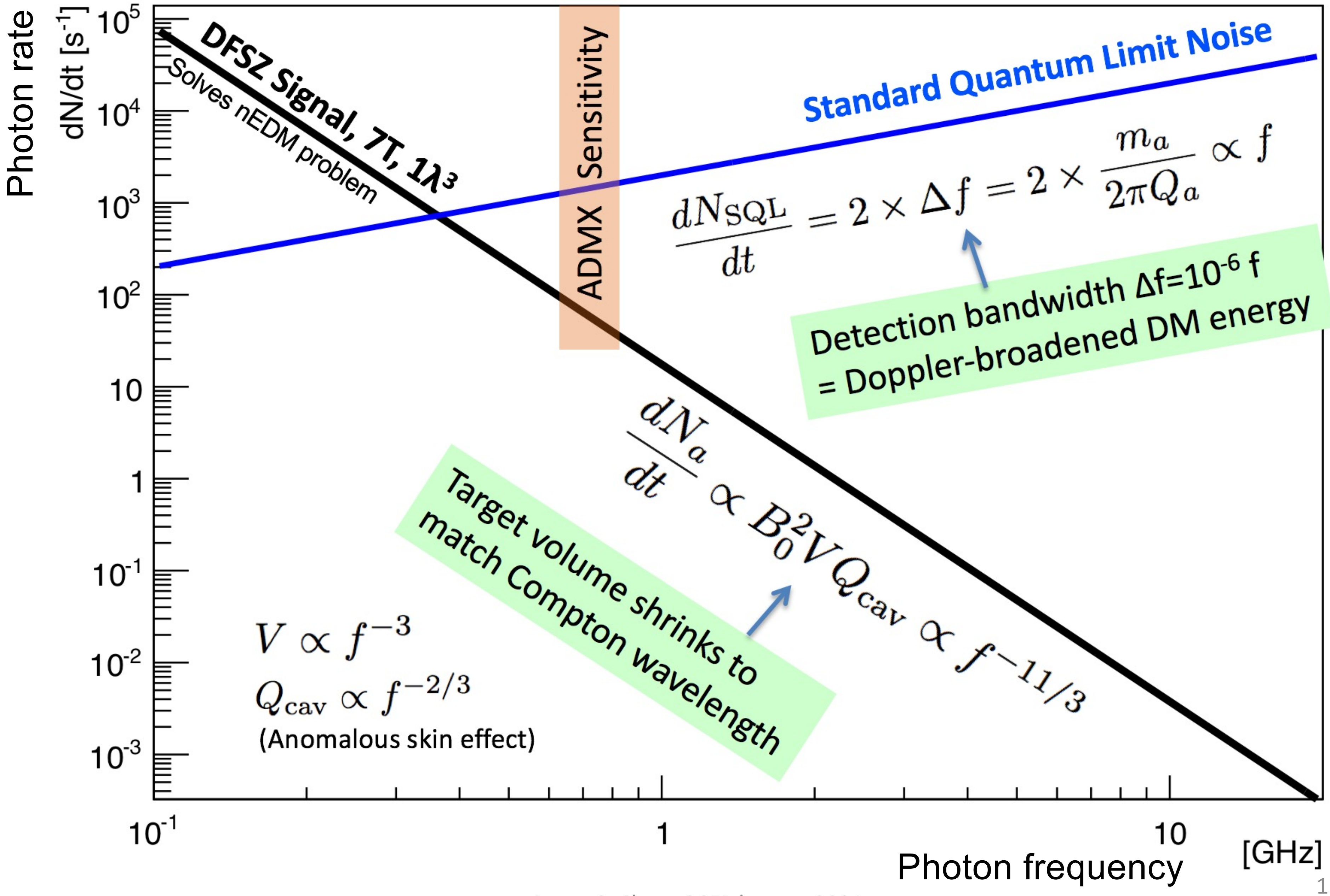


Standard quantum limit: As T→0, even the best phase-preserving amplifiers have an irreducible zero-point noise floor of +/-1 photon/mode (Carlton Caves, 1982)

Simultaneous measurement of non-commuting observables N and φ incurs the Heisenberg uncertainty principle $\Delta N \times \Delta\varphi \geq ½$. The blob is effectively the probe resolution.

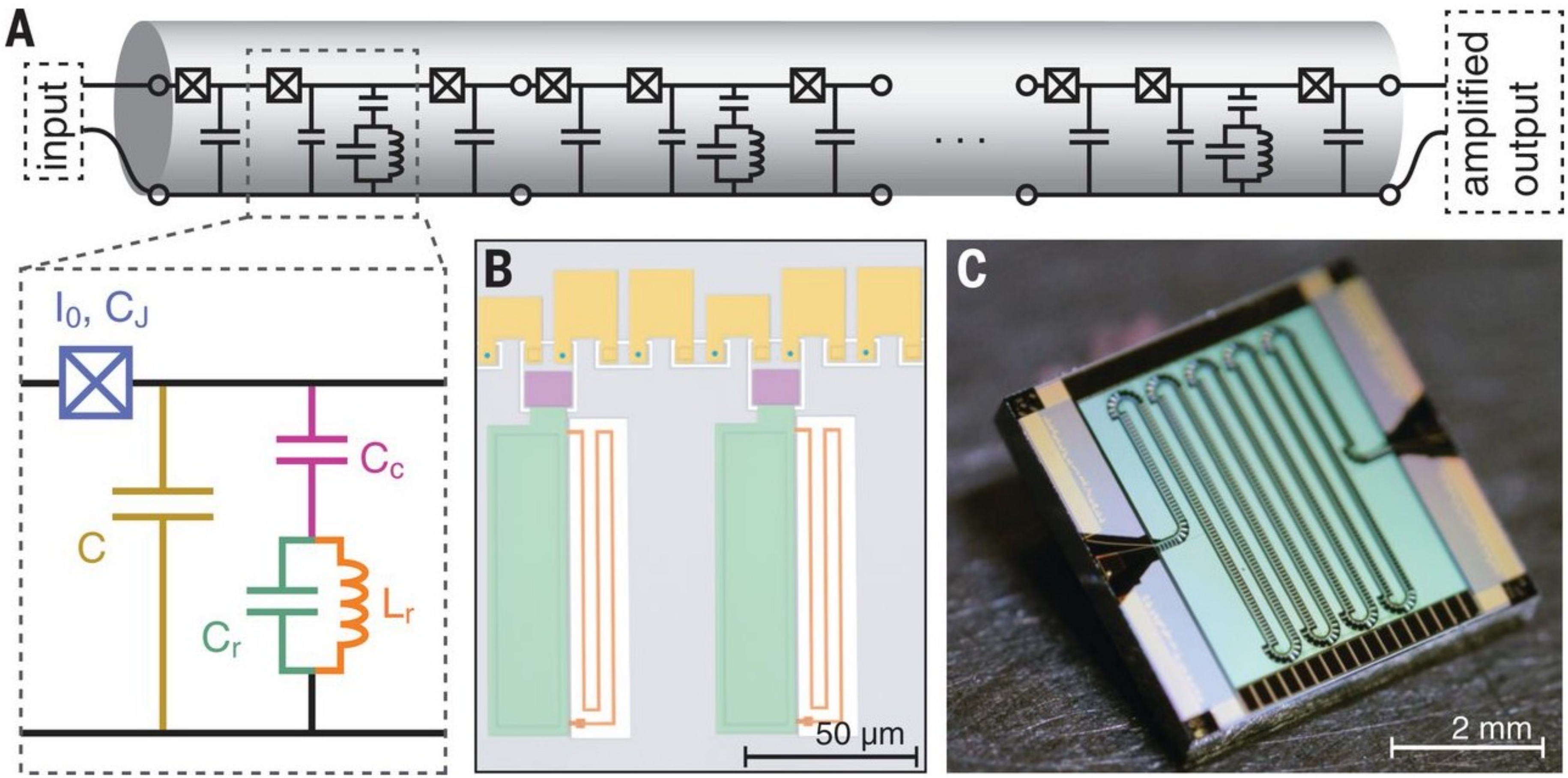
Need millions of power spectrum measurements to average away the zero-point noise.

The predicted axion DM signal/noise ratio plummets as the axion mass increases → SQL readout is not scalable.

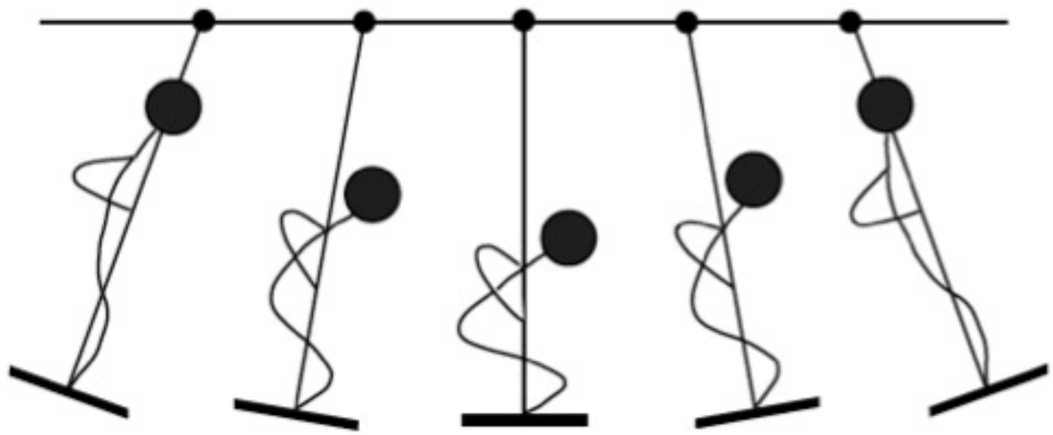


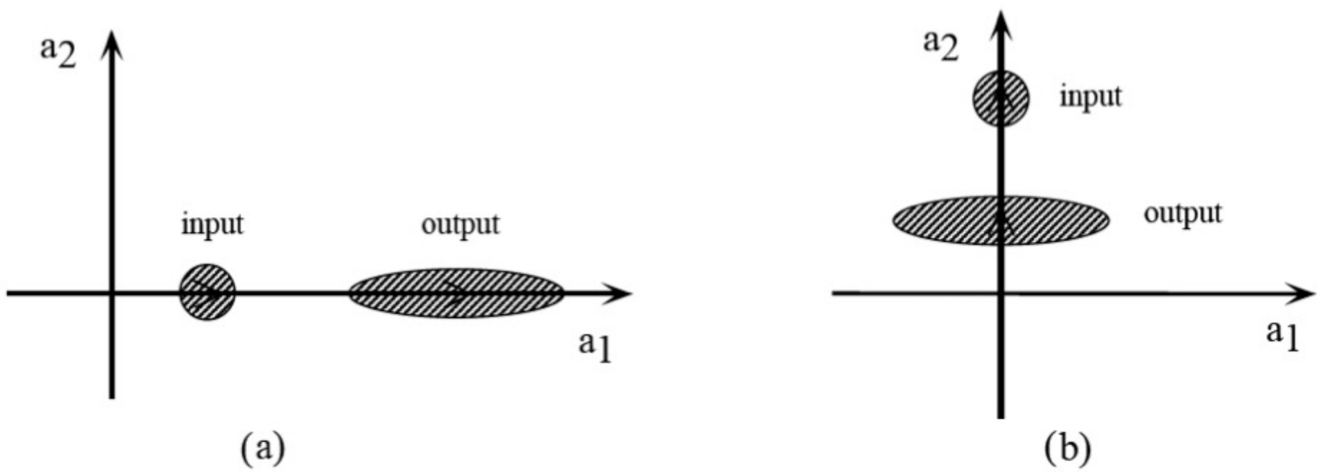
Amplifiers = scattering process via nonlinear 4-*wave mixing*

Ex. Josephson Traveling Wave Parametric Amplifier uses Josephson waveguide

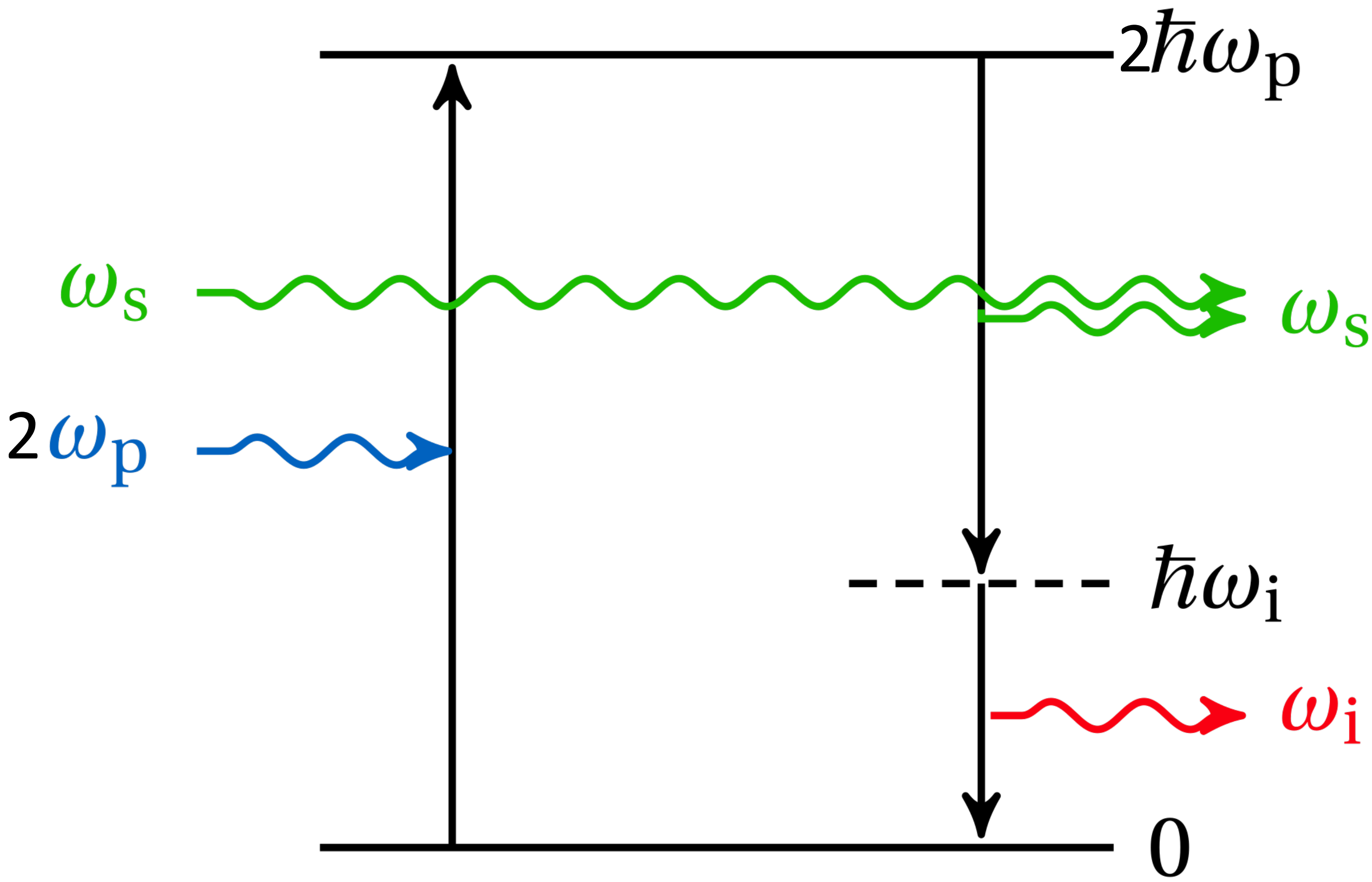


Parametric amplification:
Kicking at twice the swing frequency **amplifies** the in-phase quadrature but ***deamplifies*** the 90-degree phase quadrature





For signal frequencies offset from the pump frequency, amplifiers mix signal and idler waves and amplify both



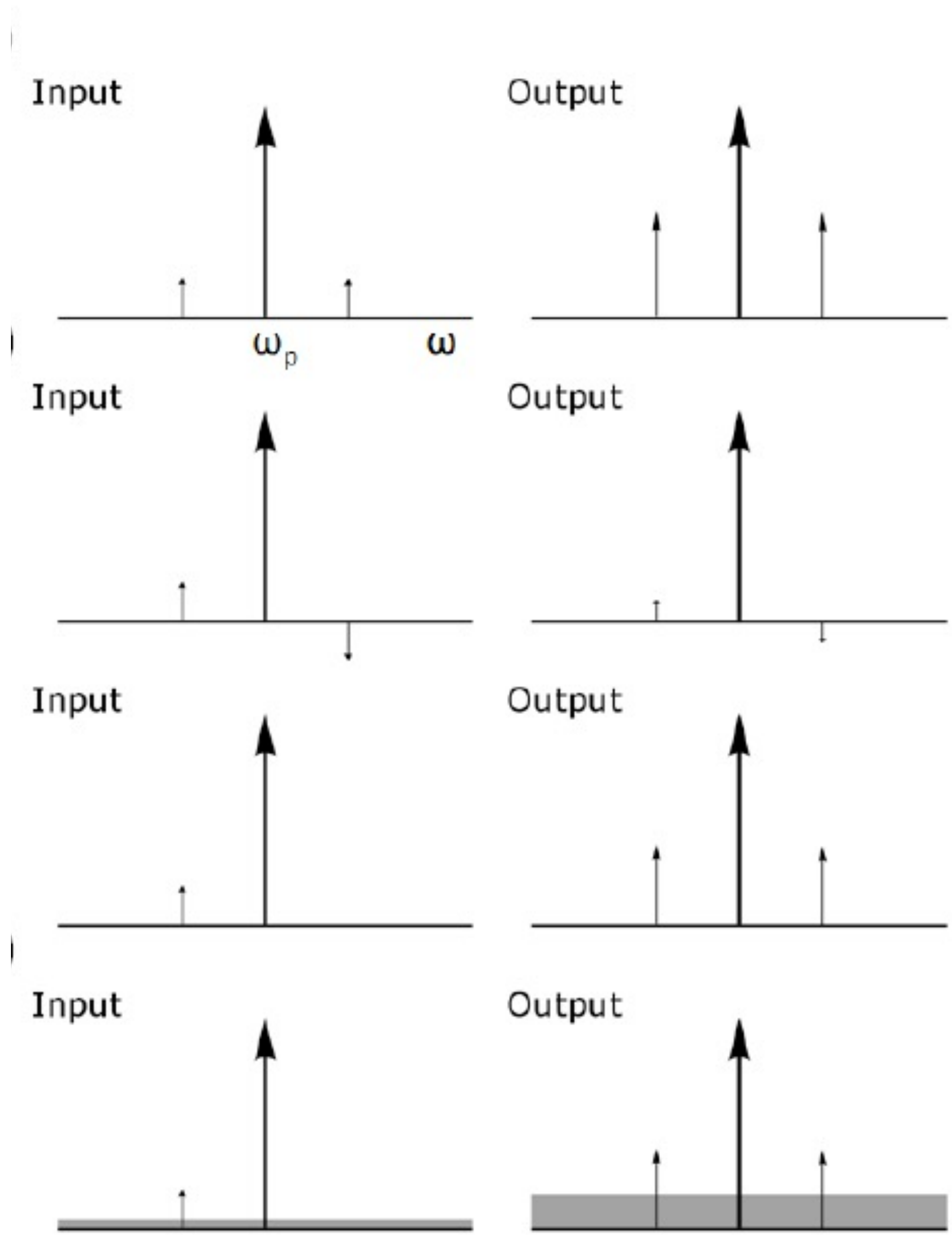
Signal + Idler

In-phase combination of signal and idler is amplified

Out-of-phase combination of signal and idler is deamplified

By superposition of above, signal in only 1 sideband generates output at both sidebands

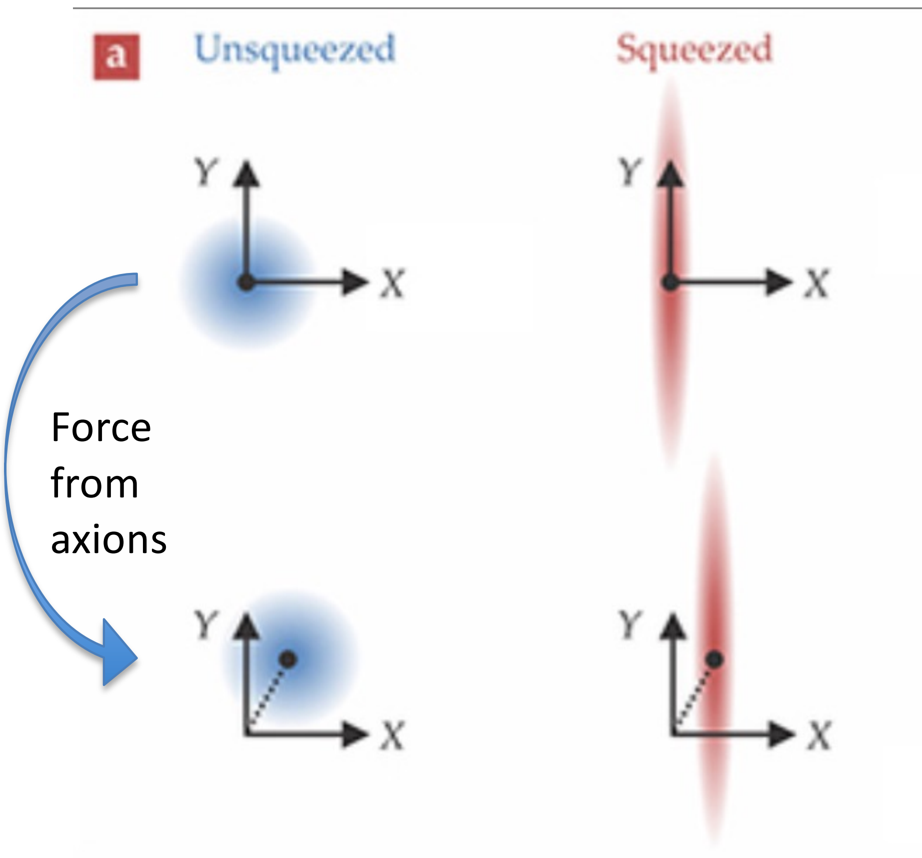
Noise at all frequencies is amplified and doubled as it is mirrored across all sideband pairs.

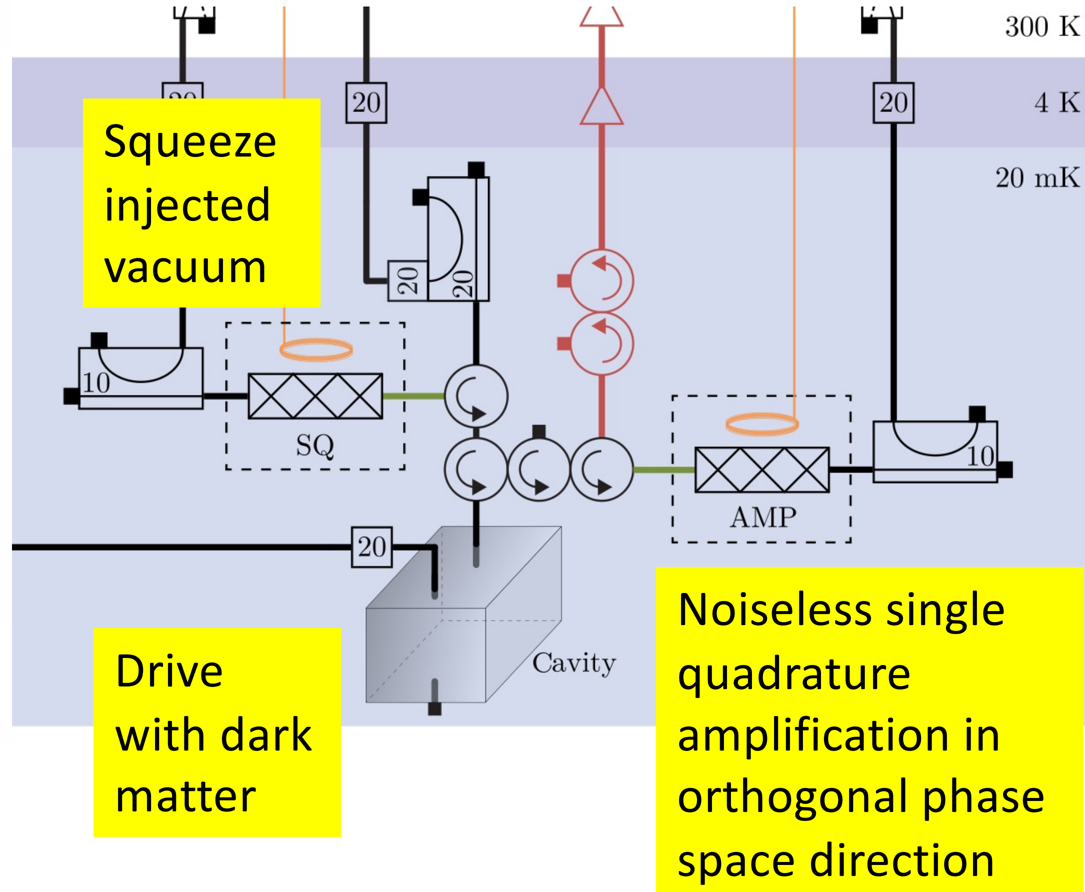


Ben Brubaker thesis

When signal and idler frequencies coincide → noiseless single quadrature amplification → squeezed states.
Inject squeezed vacuum state into the open port of the cavity.

HAYSTAC Collaboration, Nature 590 (2021)





Again, think of the phase space distribution of the probe as its resolution function.

When squeezing the amplifier noise, the effective filter bandwidth of the cavity can be increased to many linewidths while maintaining constant Signal/(Cavity Noise) ratio

Cavity filters both signal and its own noise by the same Lorentzian transfer function.

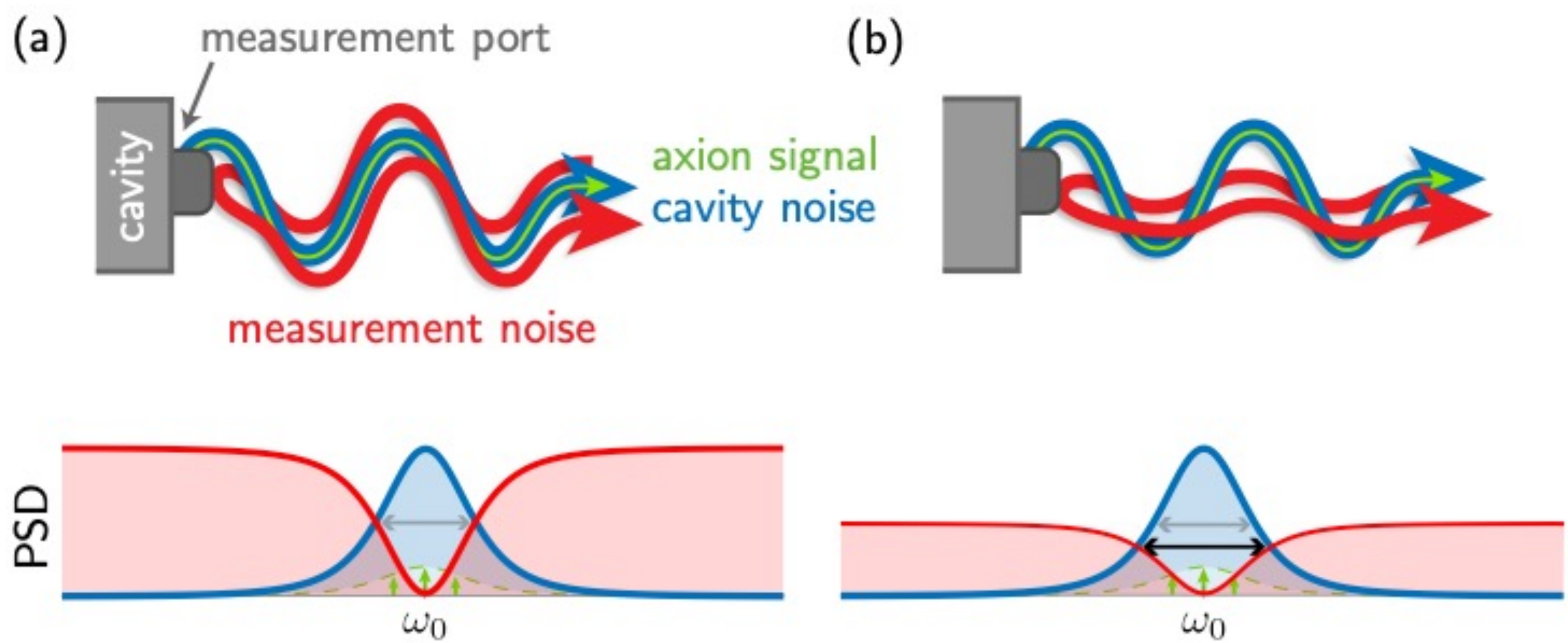


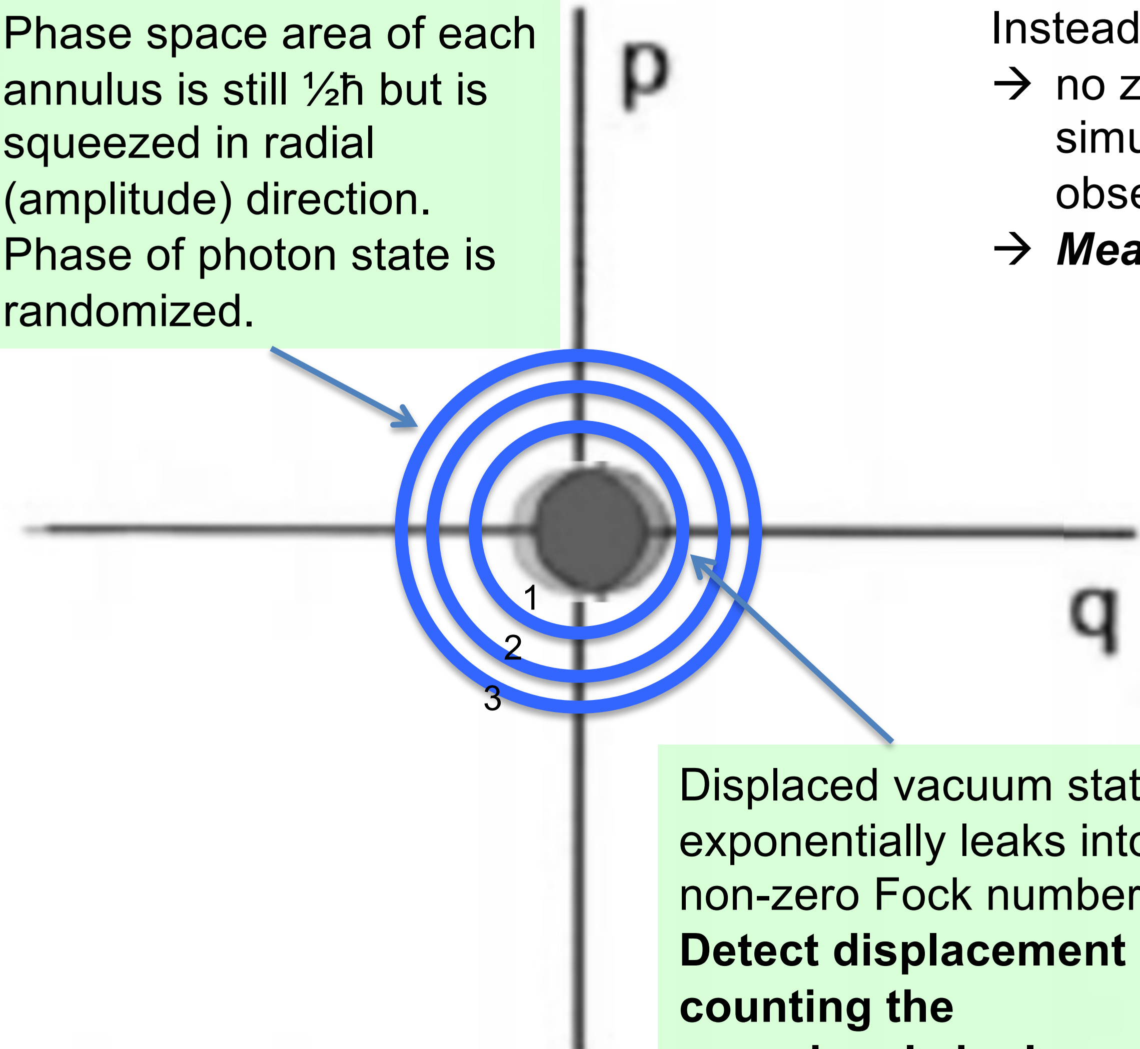
Figure from K.Wurtz, et.al, arXiv:2107.04147

Speeds up the radio scan rate since more frequencies can be simultaneously checked.

To further reduce readout noise, use photon counting to measure displacement using the Fock basis, i.e. number eigenstates

Previously we measured *both amplitude and phase*, but this is dumb since the axion phase is randomized every coherence time. Useless information obtained at high cost!

Phase space area of each annulus is still ½ħ but is squeezed in radial (amplitude) direction. Phase of photon state is randomized.

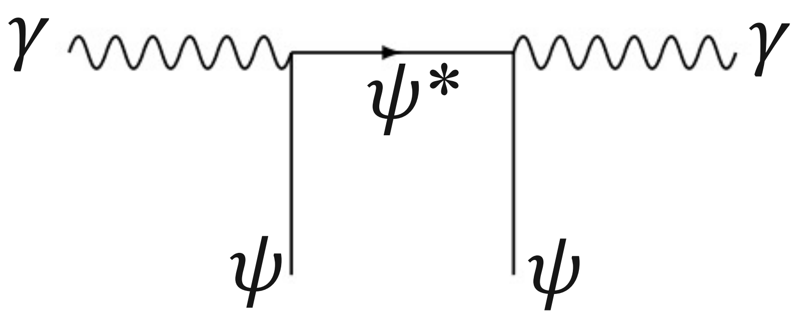


Instead, measure only displacement amplitude
→ no zero-point noise since we are not simultaneously measuring non-commuting observables
→ ***Measurement noise can be arbitrarily low***

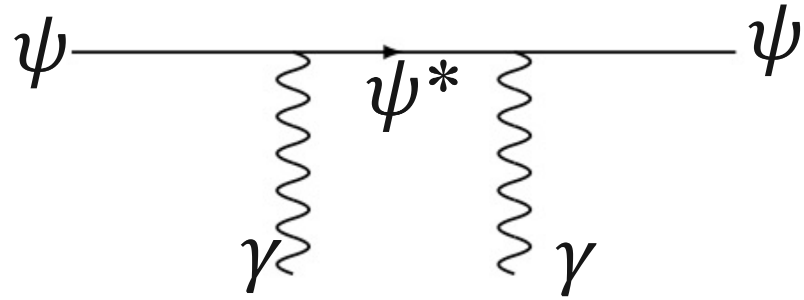
Displaced vacuum state exponentially leaks into non-zero Fock number. **Detect displacement by counting the occasional single photon.**

Quantum non-demolition "off-shell" sensors transduce photon occupation numbers into atomic frequency shifts

Index of refraction diagrams:



Photons slow down when passing through a dielectric medium



Atomic clocks slow down when interacting with a bath of background photons

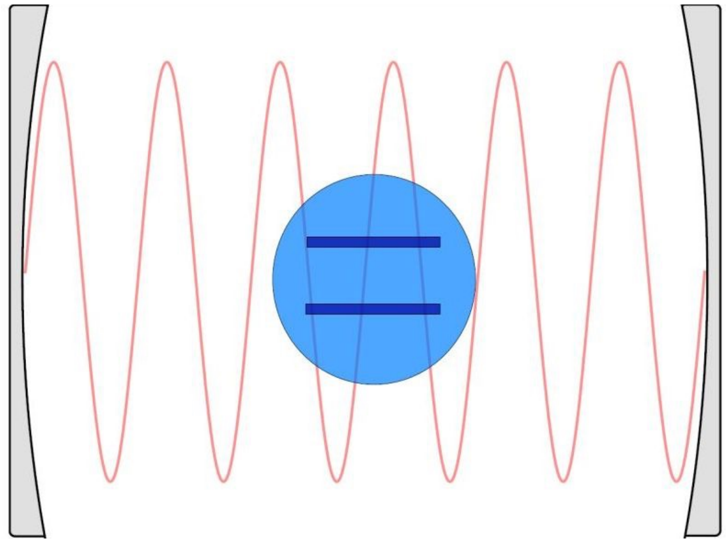
The photon occupation number of the cavity mode is encoded as a frequency shift of the probe atom.

Being far off-resonance of ψ^* results in **no net absorption** of photons.

Quantum non-demolition: indirectly measure the same photon many times (via atom's frequency shift) to achieve higher measurement fidelity.

Cavity QED:
Use 2-level atom to measure cavity photon population

Linear cavity
Bosonic oscillator,
Number operator = $a^\dagger a$



2-level "atom"
Fermionic oscillator,
Number operator = σ_z

The 1st order non-linearity in (number operator)^2 in the undiagonalized Hamiltonian is:

$$H \approx \hbar\omega_{\rm r}\left(a^\dagger a + 1/2\right) + \frac{\hbar}{2}\left(\omega_{\rm a} + \frac{2g^2}{\Delta}a^\dagger a + \frac{g^2}{\Delta}\right)\sigma_z$$

$$g \approx \vec{d}\cdot\vec{E}_0 \approx d\sqrt{\omega/V}$$

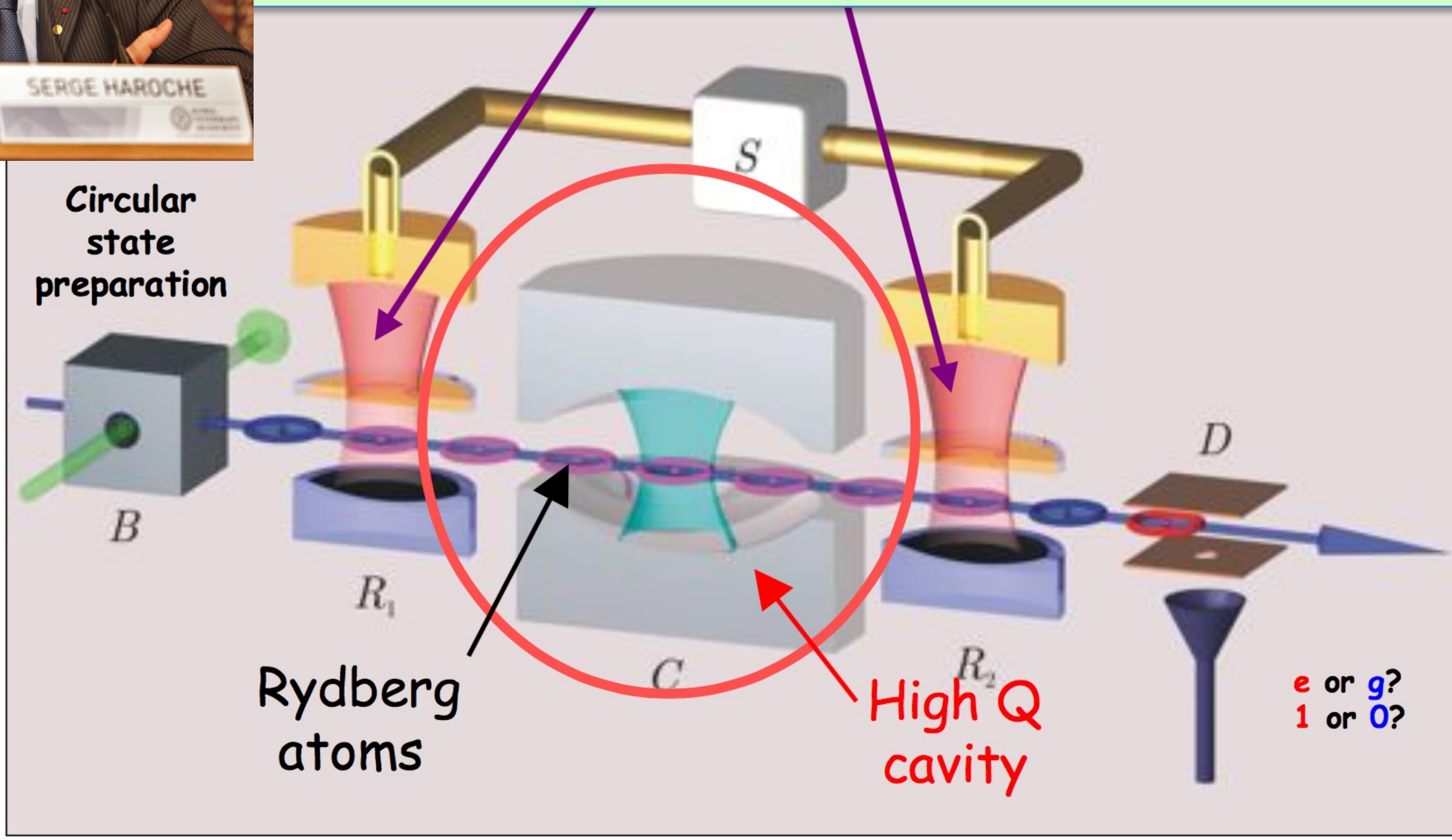
$\Delta = \omega_r - \omega_a$

The atom frequency depends on the cavity resonator's occupation number!
Frequency shift of $2\chi = 2g^2/\Delta$ per photon in the cavity mode.
This product of number operators commutes with H and allows QND measurement.

Quantum non-demolition measurements using atoms

Electric field of even a single trapped photon can “stretch” the atom and change its frequency. Just like tightening a violin string.

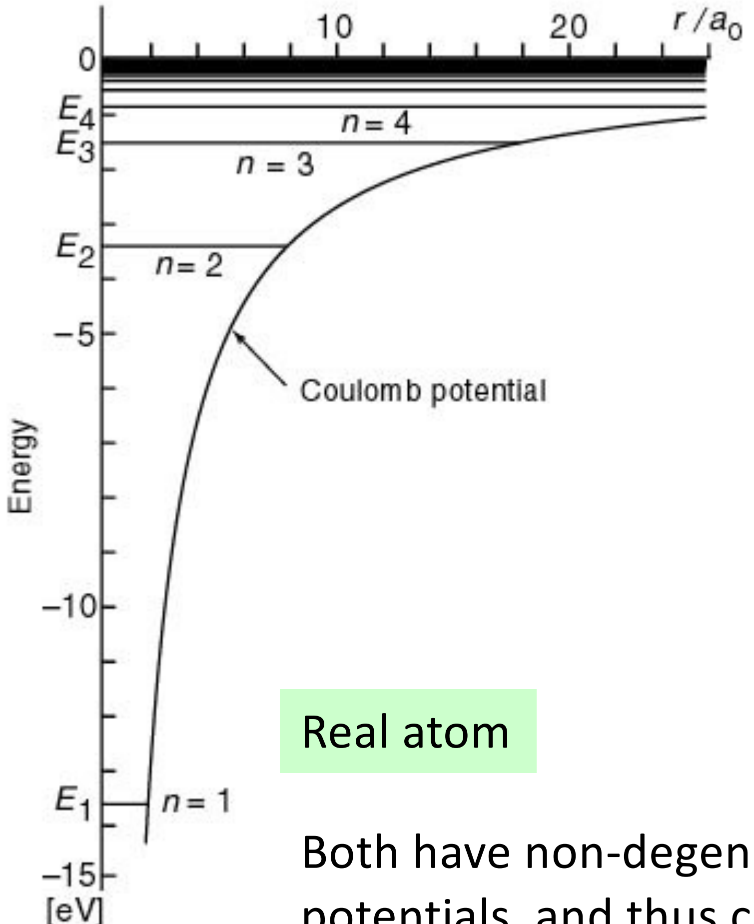
Serge Haroche 2012 Nobel prize: Measure the same photon with 100’s of atoms using Ramsey phase-shift interferometry



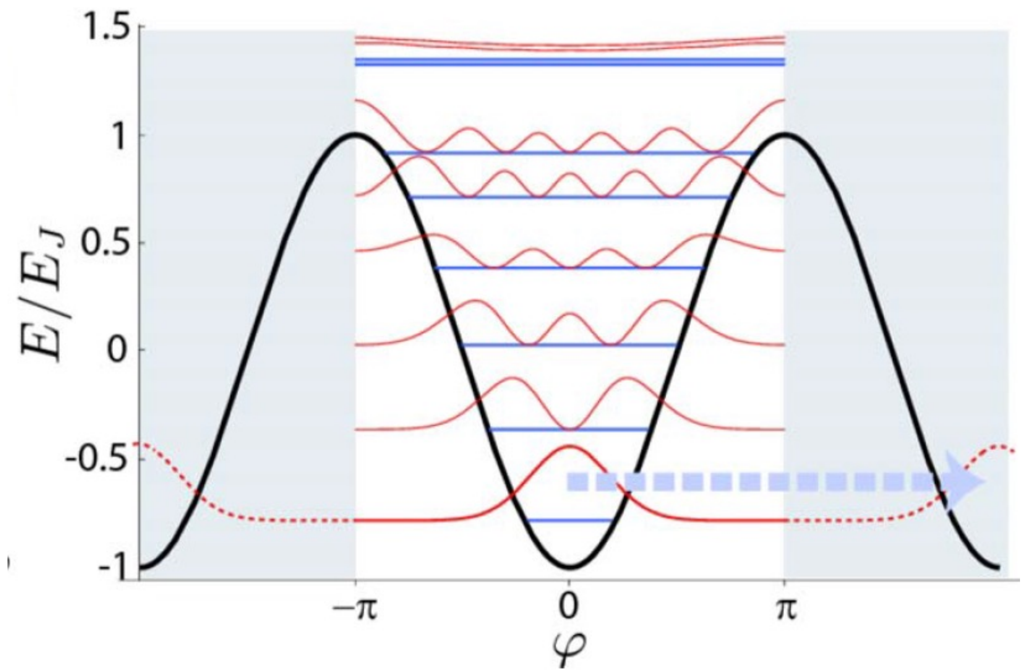
An atomic clock delayed by photons trapped inside

Quantum computing terminology: controlled phase (parity) gate

Any anharmonic oscillator exhibits 2-level system behavior and acts as a fermionic artificial atom



Real atom



Artificial atom (Josephson junction oscillator)

Both have non-degenerate energy level spacings due to the nonlinear potentials, and thus can act as fermionic 2-level systems.

Jostling of the nonlinear oscillator due to electric fields from background photon fields result in frequency shifts as the restoring force changes for larger amplitude motion

→ Lamb shift from zero-point fluctuations

→ **quantized AC Stark shift from finite background photon occupation number**

Use artificial atoms made of superconducting "transmon" qubits to nondestructively sense photons

A.S. Chou, Dave Schuster, Akash Dixit, Ankur Agrawal, …

$$H \approx \hbar\omega_r a^\dagger a + \frac{\hbar}{2}(\omega_a' + 2\chi a^\dagger a)\sigma_z$$

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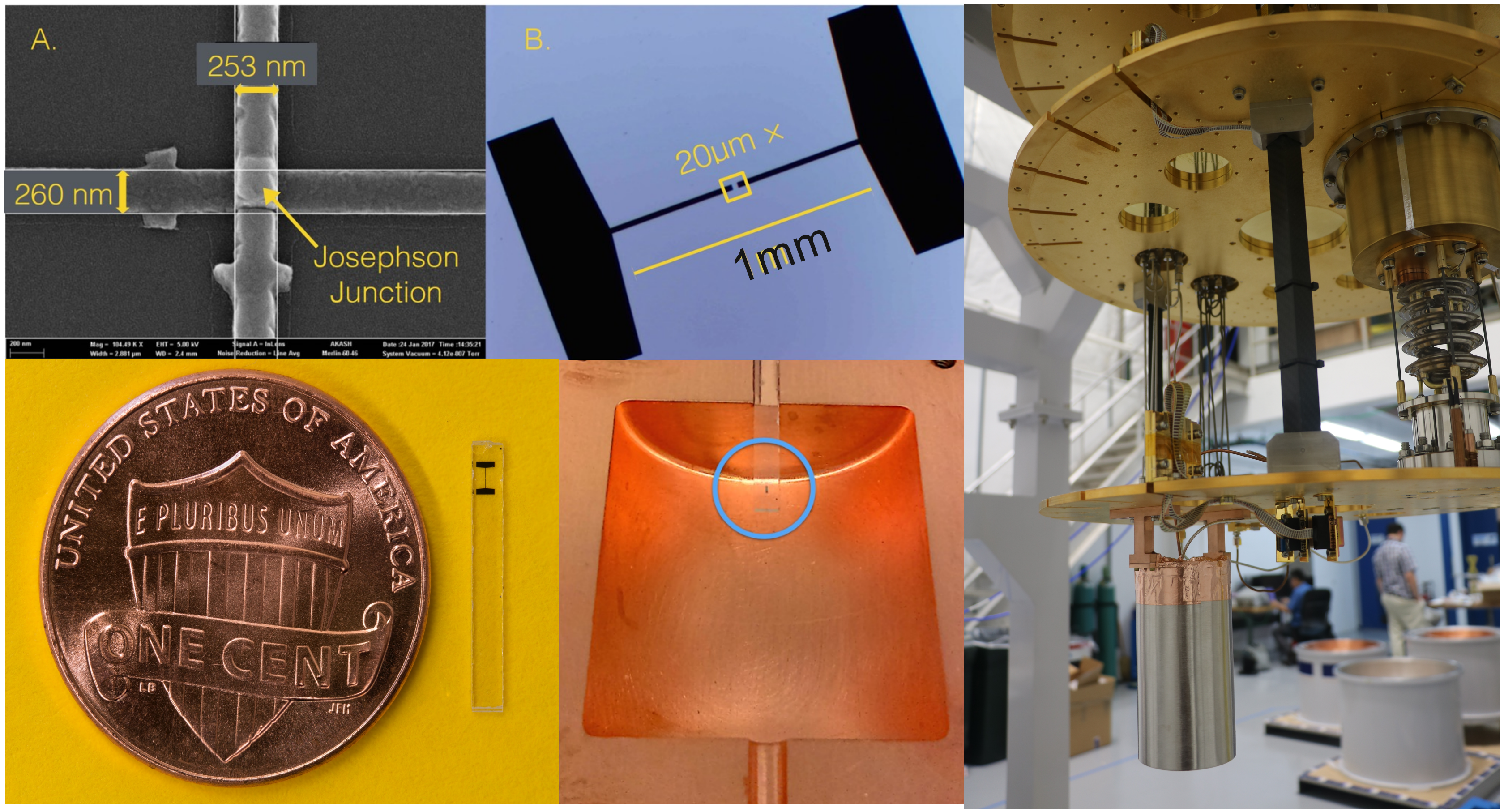
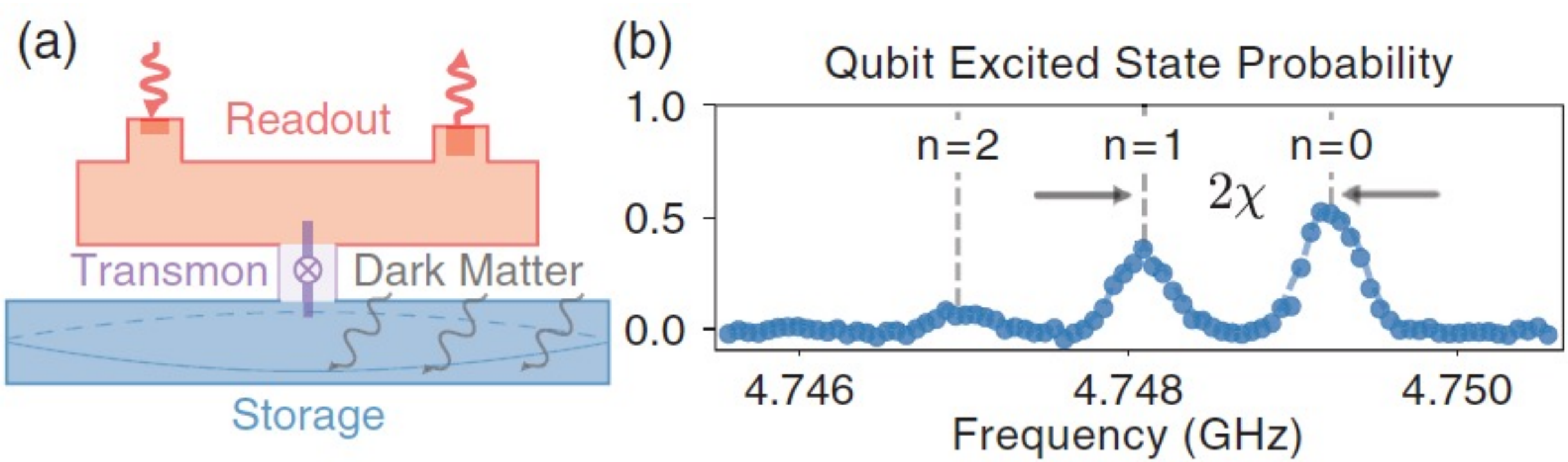


The electric field of individual photons exercises the nonlinear inductance of the Josephson junction. **Photon number is transduced into frequency shifts of the $|g\rangle \rightarrow |e\rangle$ transition.** Same as Lamb shift, but for finite photon number.

Single photon resolution:

Measure qubit $|g\rangle \rightarrow |e\rangle$ transition frequencies while weakly driving the primary cavity mode into a Glauber state with <n>=1



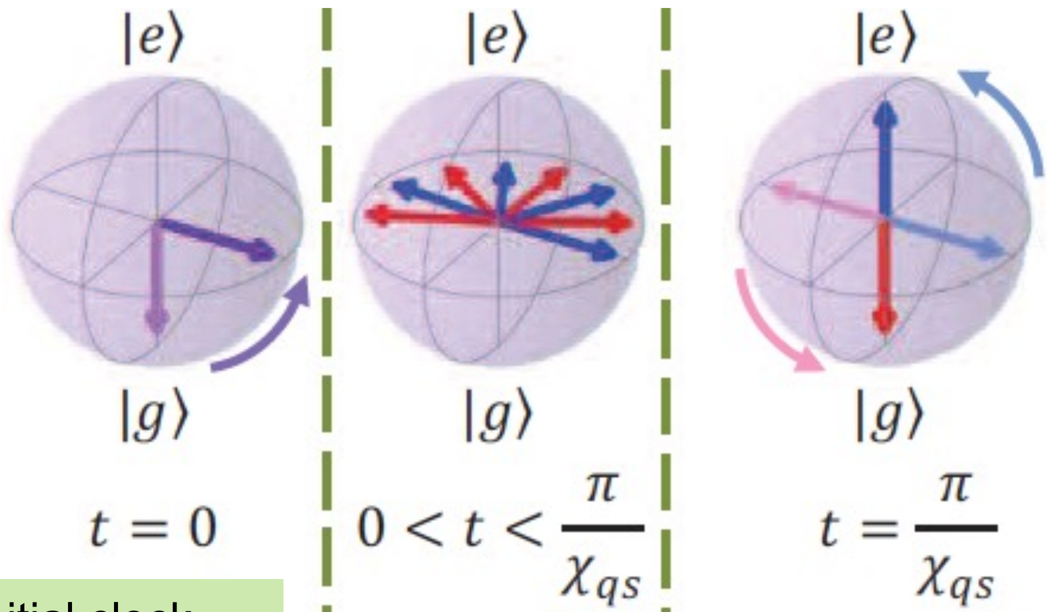
After displacing cavity with a sinusoidal drive, the measured qubit spectrum exhibits a distribution of resonances which are in 1-1 correspondence with the Poisson distribution of the cavity's coherent state.

Non-destructively count photons by measuring the qubit's quantized frequency shift.

Perform Ramsey interferometry with the oscillating qubit "clock" to measure cavity photon number parity

Just like asking in an oscillation experiment, do the neutrinos see "matter effects" or not?
If there is a photon, the clock runs slow. If no photon, the clock runs fast.

Bloch sphere:
Map qubit states
to spin ±1/2



Prepare initial clock state with $\pi/2$ pulse to give $|g\rangle + |e\rangle$ state

Evolve system to accumulate π phase difference over Stark period

Analyze with final $\pi/2$ pulse to map:
Even N → $|g\rangle$
Odd N → $|e\rangle$

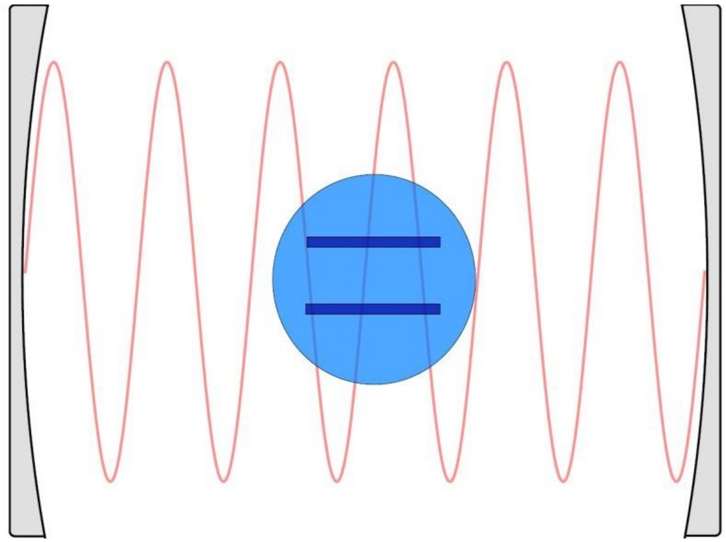
The qubit's "spin" flips only if a cavity photon is present.

Measure final qubit state $|g\rangle$ *or* $|e\rangle$ via freq. shift of an auxiliary cavity mode.

Aaron S. Chou, QSFP lecture 2021

Cavity QED again:
Use another cavity mode to measure atom's final state

Linear cavity
Bosonic oscillator,
Number operator = $a^\dagger a$



2-level "atom"
Fermionic oscillator,
Number operator = σ_z

$$g \approx \vec{d} \cdot \vec{E}_0 \approx d\sqrt{\omega/V}$$

$\Delta = \omega_r - \omega_a$

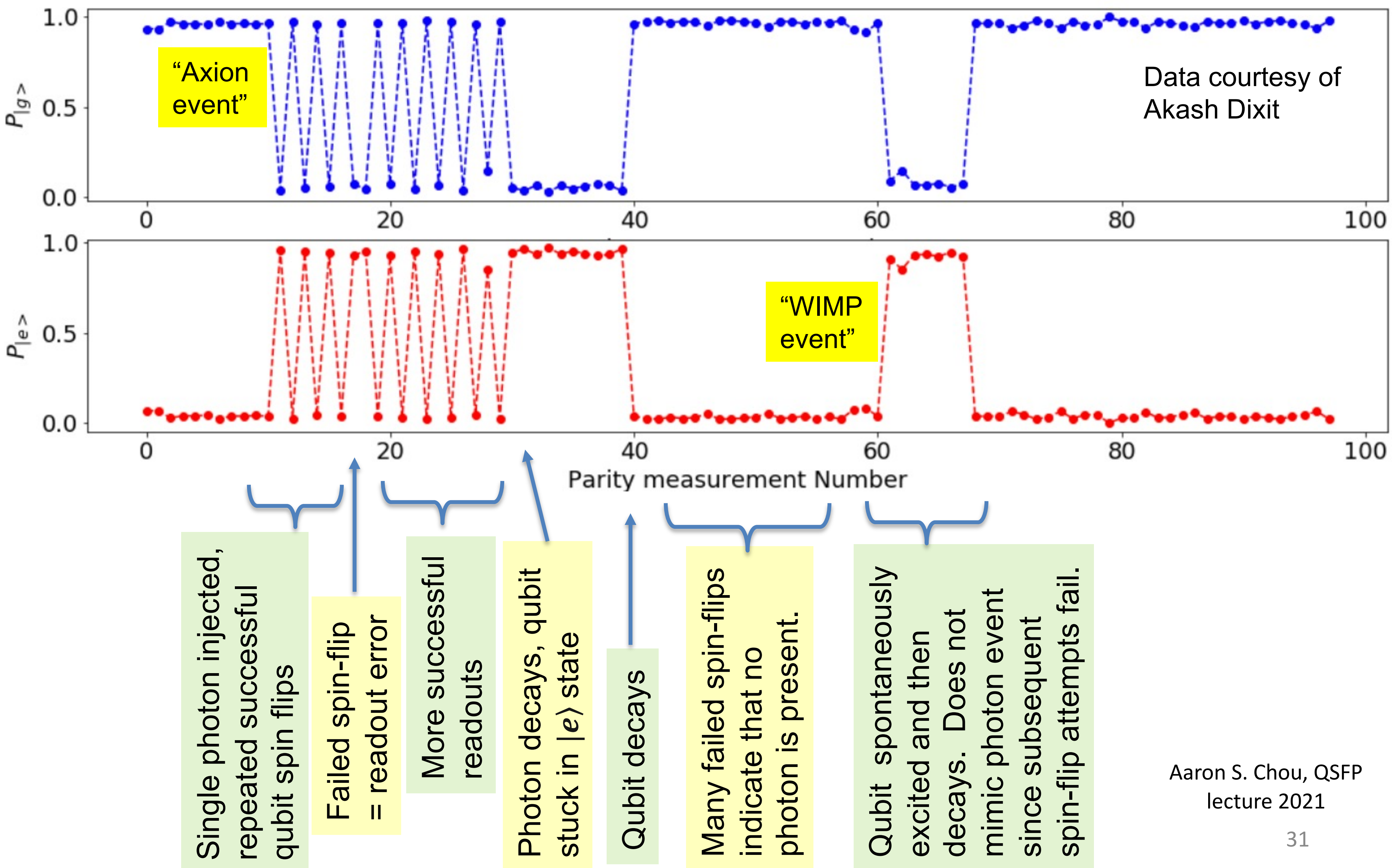
$$H \approx \hbar\omega_{\rm r}\left(a^\dagger a + 1/2\right) + \frac{\hbar}{2}\left(\omega_{\rm a} + \boxed{\frac{2g^2}{\Delta}a^\dagger a} + \frac{g^2}{\Delta}\right)\sigma_{\rm z}$$

Rewrite as:

$$H \approx \hbar\left(\omega_{\rm r} + \boxed{\frac{g^2}{\Delta}\sigma_{\rm z}}\right)\left(a^\dagger a + 1/2\right) + \frac{\hbar}{2}\left(\omega_{\rm a} + \frac{g^2}{\Delta}\right)\sigma_{\rm z}$$

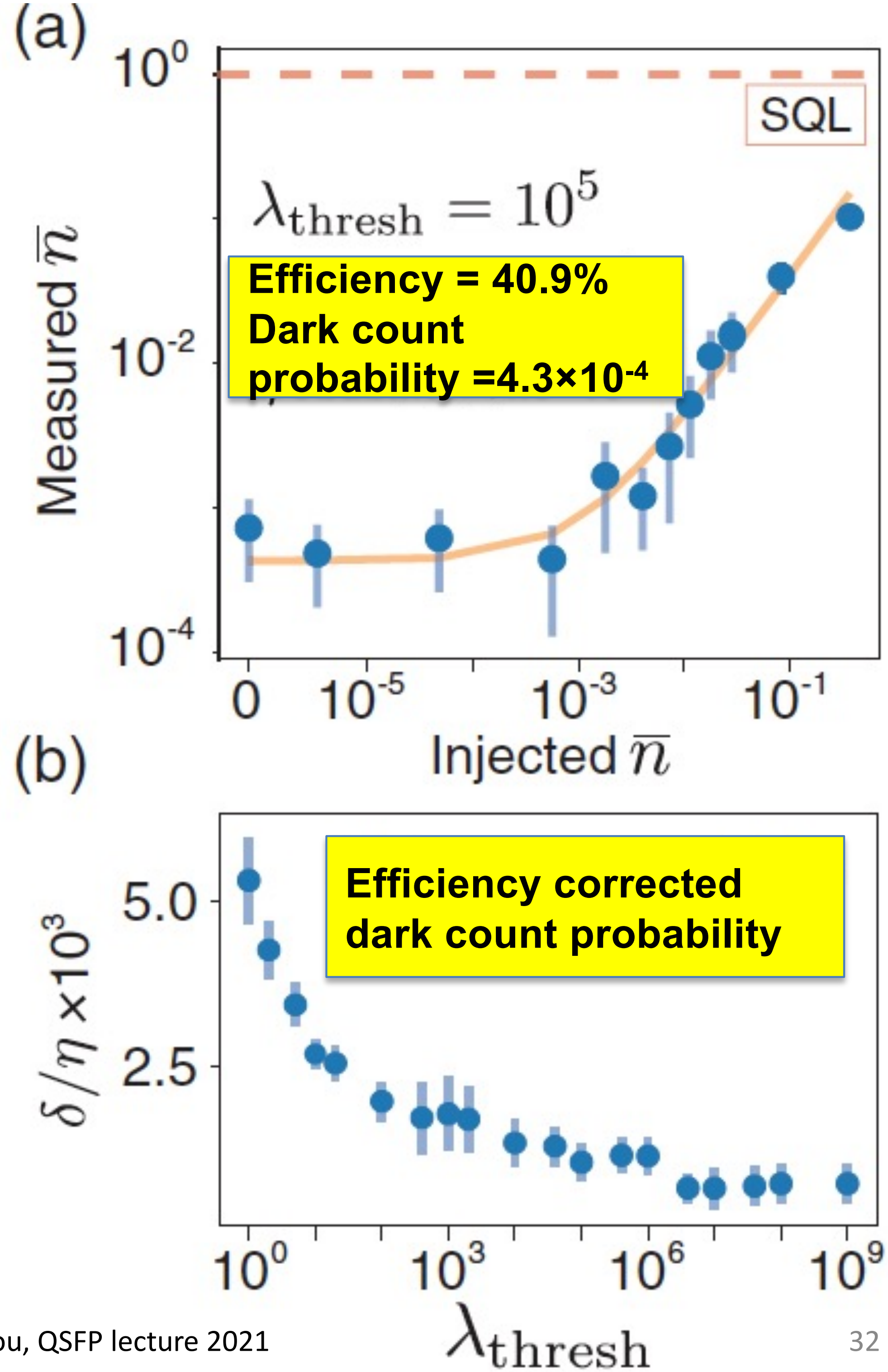
The cavity mode's frequency also depends on the atom's occupation number (0 or 1)!
Measure cavity's frequency shift with many probe photons without disturbing the atom.

Signature of a single signal photon is many sequential successful qubit “spin-flips” from $|g\rangle \leftrightarrow |e\rangle$



Trigger on photons by placing threshold on MCMC probability ratio Prob(γ)/Prob(no γ) for observed spin-flip sequence

Akash V. Dixit, et.al, Phys.Rev.Lett. 126, 141302 (2021)



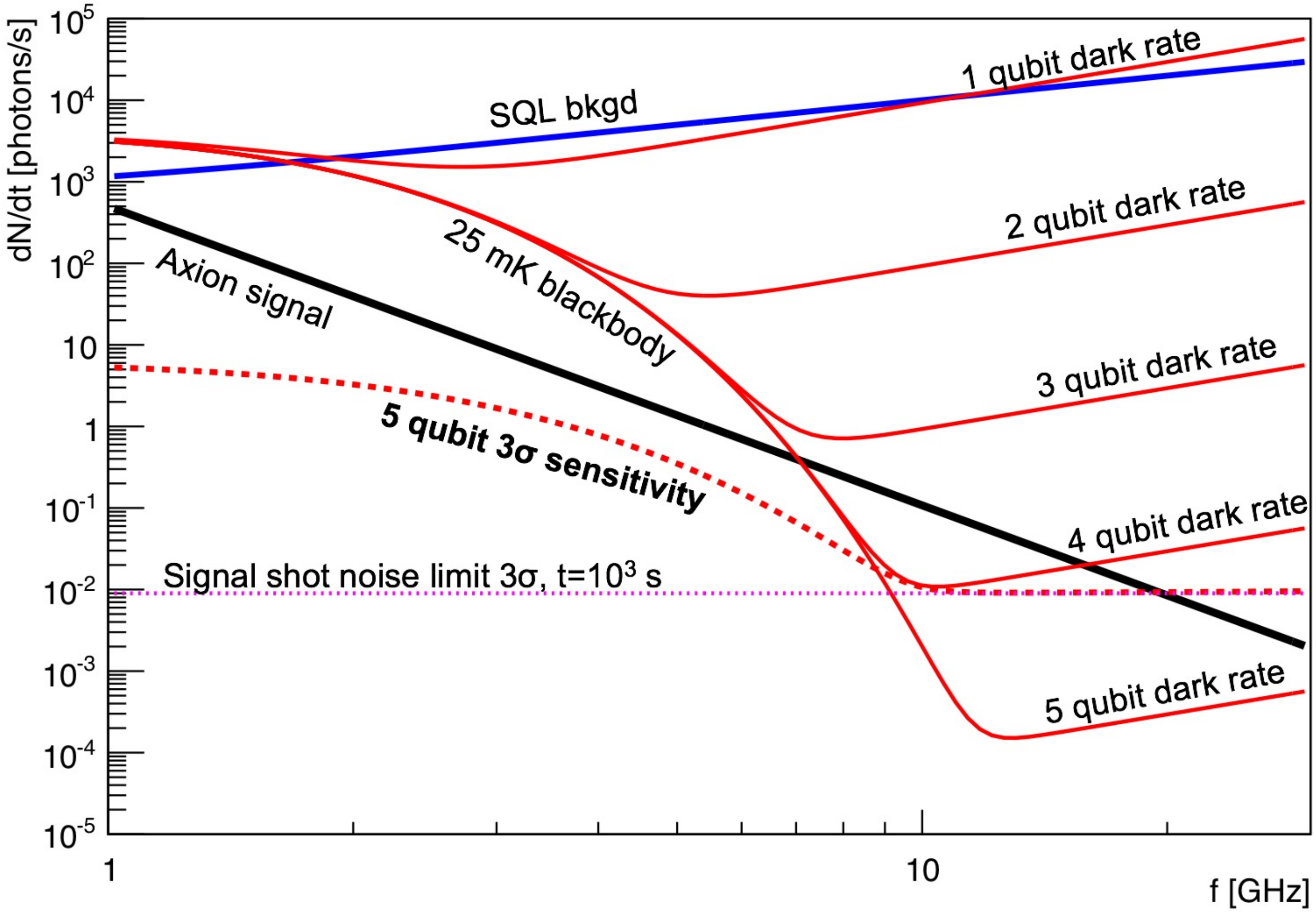
Background = few 10^{-3} of leakage photons per measurement.

Compare to amplifier readout which gives +/- 1 photon of zero-point variance per measurement.

Noise equivalent of 15.7 dB of squeezing!

No zero-point noise → no noise floor

If leaks can be sealed and qubit-error-induced dark counts are under control, then a background-free experiment is possible.



Good qubit single measurement error probability $p=10^{-2}$

With N QND measurements, **dark rate =** $\mathbf{(10^{-2})^N \Delta f}$

Assumes axion detection bandwidth $\Delta f \sim$ MHz

Sensitivity limited only by signal photon shot noise.

Serge Haroche used majority vote out of 8 atoms → Nobel prize 2012.
Here we assume unanimous vote amongst 5 qubit measurements to potentially **reduce dark rates by 10 orders of magnitude.**