Tutorial 1: Neutrino detection

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Quantum Sensors for Fundamental Physics — QSFP 2021
13 September 2021

β-decay: Fermi Theory (1)

We can derive the electron spectrum of the process $n \to p + e^- + \bar{\nu}_e$ from first principles in Quantum mechanics:

- Assume free particle states:

$$|i\rangle = |n\rangle \qquad \Rightarrow \qquad \psi_i(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_n \cdot \vec{x}_1}$$

$$|f\rangle = |p, e^-, \bar{\nu}_e\rangle \qquad \Rightarrow \qquad \psi_f(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_p \cdot \vec{x}_2}\, e^{i\,\vec{p}_e \cdot \vec{x}_3}\, e^{i\,\vec{p}_\nu \cdot \vec{x}_4}$$

- We can assume the interaction to be point-like:

$$V_W(x_1, x_3, x_3, x_4) = G_F\ \delta^{(3)}(\vec{x}_1 - \vec{x}_2)\ \delta^{(3)}(\vec{x}_2 - \vec{x}_3)\ \delta^{(3)}(\vec{x}_3 - \vec{x}_4)$$

New interaction constant

β-decay: Fermi Theory (2)

The matrix element for this process is given by

$$\langle i | V_W | f \rangle = \int d^3x_1 \ldots d^3x_4 \; \psi_i^* \; V_W(x_1, x_2, x_3, x_4) \; \psi_f$$

Inserting the expression and integrating over x_1, x_2, x_3 yields

$$\langle i | V_W | f \rangle = G_F \int d^3x^4 \; e^{i\,(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n)\cdot \vec{x}_4} = (2\pi)^3 \; G_F \; \delta^{(3)}(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n)$$

With the matrix element we can compute the decay rate of the process using **Fermi's golden rule**:

$$d\Gamma_{if} = 2\pi \; |V_{if}|^2 \; \rho(E_i)$$

With the density of states

$$\rho(E_i) = \delta(Q - E_e - E_\nu) \frac{d^3p_p}{(2\pi)^3} \frac{d^3p_e}{(2\pi)^3} \frac{d^3p_\nu}{(2\pi)^3} \qquad \textbf{with} \quad Q = m_n - m_p$$

β-decay: Fermi Theory - Exercise

Exercise 1. Using **Fermi's golden rule** find an expression for the electron spectrum in the beta decay, *i.e.* $d\Gamma_{if}/dE_e$

Hint 1: Remember that $\int dx\ \delta(x-a)\, f(x) = f(a)$

Hint 2: Integrate first over the proton momentum p_p and use the relation

$$\int d^3p_p\ \delta^{(3)}(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n)\ \delta^{(3)}(\vec{p}_p + \vec{p}_e + \vec{p}_\nu - \vec{p}_n) = \ \delta^{(3)}(0) = \frac{V}{(2\pi)^3},$$

where V is a normalisation volume that can be set to 1.

Hint 3: Integrate over p_ν and substitute p_e using the relations

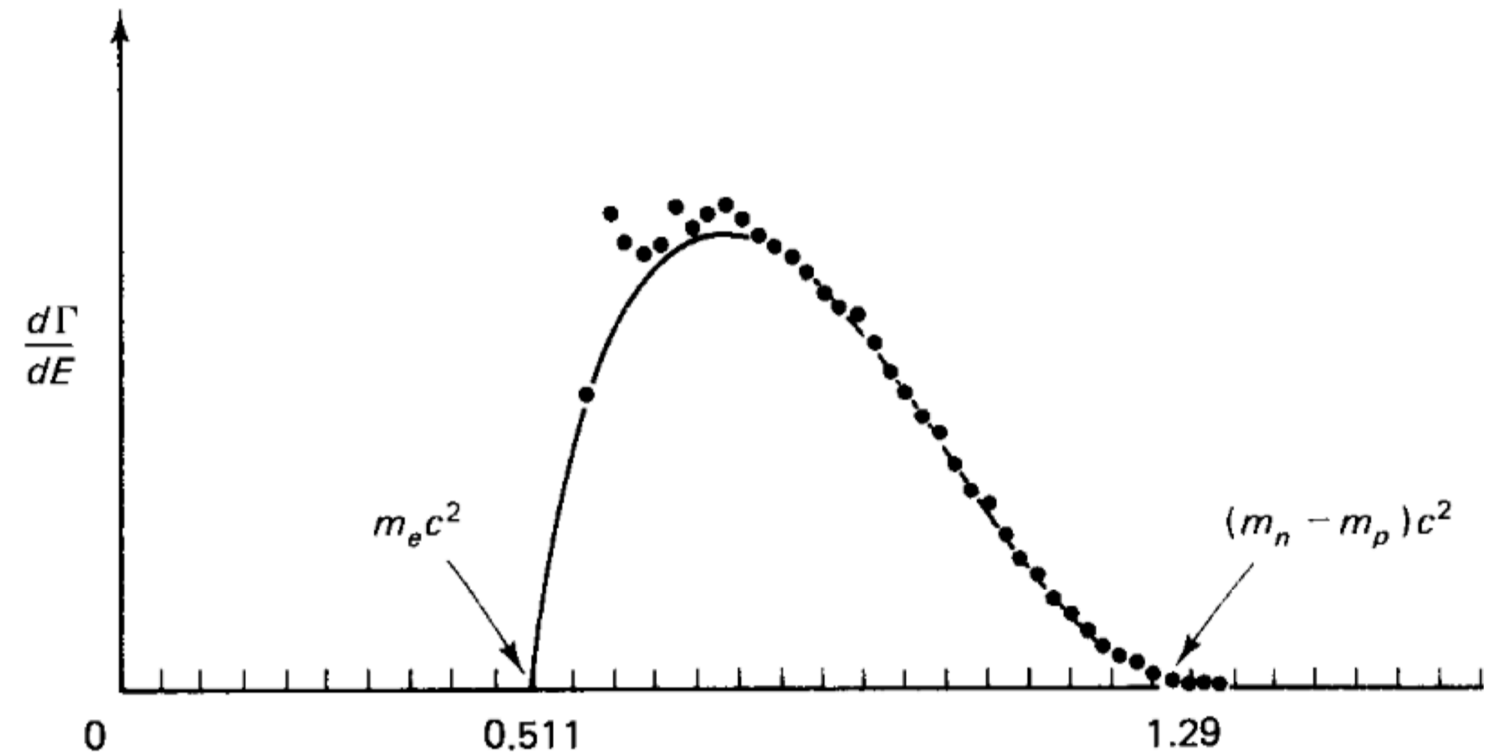
$$d^3p_\nu = 4\pi\ E_\nu^2\ dE_\nu$$

$$d^3p_e = 4\pi\ p_e^2\ dp_e = 4\pi\ \sqrt{E_e^2 - m_e^2}\ E_e\ dE_e$$

β-decay: Fermi Theory (3)

The expression we obtained from our quantum mechanics calculation exactly describes the measured β-decay electron spectrum

$$\frac{d\Gamma_{if}}{dE_e} = \frac{G_F^2}{2\pi^3}(Q - E_e)^2 \sqrt{E_e^2 - m_e^2}\; E_e$$



This allows to extract the **Fermi constant** G_F from the spectrum and determine it to

$$G_F \approx 1.15 \times 10^{-5}\ \text{GeV}^{-2}$$

Inverse β-decay

With the methods developed before we can also find an expression for the **inverse β-decay**, or neutrino-proton scattering $\bar{\nu}_e + p \rightarrow n + e^+$.

We proceed exactly analogous, but with the initial and final states as

$$|i\rangle = |\bar{\nu}_e, p\rangle \qquad \Rightarrow \qquad \psi_i(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_\nu \cdot \vec{x}_1}\, e^{i\,\vec{p}_p \cdot \vec{x}_2}$$

$$|f\rangle = |n, e^+\rangle \qquad \Rightarrow \qquad \psi_f(x_1, x_2, x_3, x_4) = e^{i\,\vec{p}_n \cdot \vec{x}_3}\, e^{i\,\vec{p}_e \cdot \vec{x}_4}$$

Assuming the proton to be at rest $\vec{p}_p = 0$, we can derive the cross section from

$$d\sigma = 2\pi\, |V_{if}|^2\, \delta(E_\nu + m_p - E_n - E_e)\, \frac{d^3p_e}{(2\pi)^3}\, \frac{d^3p_n}{(2\pi)^3}$$

Inserting the expression for V_{ij} from before and integrating over p_n yields the simple expression for the total cross section (neglecting m_e and taking $m_n \approx m_p$)

$$\sigma_{\text{tot}} \approx \frac{G_F^2}{\pi}\, \frac{E_\nu^2\, m_p}{2E_\nu + m_p}$$

Inverse β-decay - Exercise

Exercise 2. Using the expression for the neutrino-proton scattering cross section

$$\sigma_{\rm tot} \approx \frac{G_F^2}{\pi} \frac{E_\nu^2 m_p}{2E_\nu + m_p}$$

give a rough estimate for the expected cross section of reactor neutrinos with an energy of $E_\nu \approx 1$ MeV. Express your results in units of barn and cm^2.

Compare your result to Thomson scattering of a photon on a non-relativistic electron, $\sigma_{\rm Thomson} \approx 10^{-24}$ cm^2.

Hint: 1 barn $\equiv 10^{-24}$ cm$^2 = 2.57 \times 10^3$ GeV^{-2}

Neutrino detection

Exercise 3. Consider a research nuclear reactor with a power of $P_{\text{reactor}} = 20 \text{ MW}$ which mainly has ${}^{235}_{92}U$. If an average of 6 neutrinos are emitted from each fission, what would be the neutrino flux at a location 150 m away from the core?

Hint: The reaction is ${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{55}Cs + {}^{90}_{37}Rb + 2{}^{1}_{0}n$,
and the atomic masses for each element are:

$$m({}^{235}U) = 235.044u,$$
$$m({}^{144}Cs) = 143.932u,$$
$$m({}^{90}Rb) = 89.915u,$$
$$m({}^{1}_{0}n) = 1.009u.$$

$$1u = 931.5 \text{ MeV}$$

Hint 2: $1 \text{ MW} \approx 6.25 \cdot 10^{18} \text{ MeV/s}$

Neutrino detection

Exercise 4. If we wanted to detect these reactor neutrinos through inverse beta decay (IBD), how much mass of the liquid scintillator $C_{18}H_{30}$ would we need to have an event rate of about 10 events/day?

Hint1: Remember that the IBD cross section on free protons is $\sigma_{\text{IBD}} \sim 10^{-44} cm^2$, and the event rate would be given by

$$N_{\text{IBD}}/\Delta t = \phi_\nu \sigma_{\text{IBD}} N_{\text{targets}}.$$

Hint2: Assume that the number of free protons per molecule is given by the number of H atoms

The molar mass for $C_{18}H_{30}$ is $M_{C_{18}H_{30}} = 246.4 \text{ g/mol}$

Avogadro's number is $N_A = 6.022 \times 10^{23}/\text{mol}$

Neutrino detection

Exercise 5. Apart from IBD, there are other processes through which neutrinos can interact with matter. A particularly interesting one is coherent neutrino nucleus scattering (CEvNS), in which low energy neutrinos interact with the whole atomic nucleus instead of interacting with individual nucleons.

How large would a detector made of CsI(Na) need to be to observe an event rate of about 10 events/day?

Compare this to the previous detector relying on IBD.

Hint: The CEvNS cross section on Cs for neutrinos with $E_\nu \sim 2\ \mathrm{MeV}$ is $\sigma_{\mathrm{CE}\nu\mathrm{NS}} \sim 10^{-40}\ \mathrm{cm}^2$.

The molar mass of CsI is $M_{\mathrm{CsI}} = 259.8\ \mathrm{g/mol}$