Tutorial 2: Neutrino Oscillations

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Derivation of oscillation probability

$$\mathscr{L}_{\mathrm{CC}} = \sum_{\alpha=e,\mu,\tau} \left[\frac{g}{\sqrt{2}} \overline{\nu_{\alpha L}} \gamma^\mu \ell_{\alpha L} W_\mu^+ + \frac{g}{\sqrt{2}} \overline{\ell_{\alpha L}} \gamma^\mu \nu_{\alpha L} W_\nu^- \right]$$

$$\nu_{\alpha L} = \sum_j U_{\alpha j} \nu_{jL}$$

← **Fields** with definite masses m_j

PMNS mixing matrix

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

← **States** with definite masses m_j

Let's treat $|\nu_j\rangle$ as plane waves. Then, at a distance L and time t after production

$$|\nu_\alpha(t, L)\rangle = \sum_j U^*_{\alpha j} e^{-iE_j t + i p_j L} |\nu_j\rangle$$

A neutrino detector measures the neutrino flavour β

$$\langle \nu_\beta | = \sum_j U_{\beta j} \langle \nu_j |$$

Derivation of oscillation probability

The amplitude for transition $\nu_\alpha \to \nu_\beta$

$$\mathscr{M}_{\alpha\beta} = \langle \nu_\beta | \nu_\alpha(t, L) \rangle = \sum_{j,k} U^*_{\alpha j} U_{\beta k}\, e^{-iE_j t + i p_j L} \underbrace{\langle \nu_k | \nu_j \rangle}_{\delta_{jk}} = \sum_j U^*_{\alpha j} U_{\beta j}\, e^{-iE_j t + i p_j L}$$

The oscillation probability

$$P_{\alpha\beta} = |\mathscr{M}_{\alpha\beta}|^2 = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\, e^{-i(E_j - E_k)t + i(p_j - p_k)L}$$

In all practical cases neutrinos travel almost at the speed of light, so

$$p_j \approx p_k \equiv p \approx E \qquad \text{and} \qquad t \approx L$$

$$E_j = \sqrt{p^2 + m_j^2} \approx p + \frac{m_j^2}{2E}$$

$$P_{\alpha\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k}\, e^{-i\frac{\Delta m^2_{jk} L}{2E}} \qquad \Delta m^2_{jk} \equiv m_j^2 - m_k^2$$

2-flavour case

Exercise 1. Using the obtained formula, *i.e.*

$$P_{\alpha\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \, e^{-i\frac{\Delta m^2_{jk} L}{2E}} \qquad \Delta m^2_{jk} \equiv m^2_j - m^2_k$$

and taking into account that in the case of two neutrino mixing

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

derive the expression for the oscillation probability $P_{e\mu}$ of $\nu_e \to \nu_\mu$

(Define $\Delta m^2 \equiv m_2^2 - m_1^2$)

Hint: Use the trigonometric identities

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = 2\cos(\theta)^2 - 1$$

Experimentally relevant units

Exercise 2. Express the argument of the second sine in terms of experimentally relevant units, *i.e.* find # in

$$\frac{\Delta m^2 L}{4E} = \# \frac{\Delta m^2 \left(\mathrm{eV}^2\right) \, L\,(\mathrm{m})}{E\,(\mathrm{MeV})}$$

Hint: Use the relation

$$\hbar c = 1 \approx 197 \ \mathrm{fm} \cdot \mathrm{MeV}$$

$$(1 \ \mathrm{fm} = 10^{-15} \ \mathrm{m} \ \ \text{and} \ \ 1 \ \mathrm{MeV} = 10^{6} \ \mathrm{eV})$$

Oscillation length

Exercise 3.

- Taking into account that the dependence of $P_{e\mu}$ on L/E is periodic, derive the expression for the oscillation length $L_{\rm osc}$ associated with Δm^2
- Express $L_{\rm osc}$ in meters for Δm^2 in $\rm eV^2$ and E in $\rm MeV$

$$P_{e\mu} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \approx \sin^2(2\theta)\,\sin^2\left(1.27\,\frac{\Delta m^2\left(\mathrm{eV}^2\right)\,L\,(\mathrm{m})}{E\,(\mathrm{MeV})}\right)$$

Typical oscillation lengths

Exercise 4. Using the obtained formula, *i.e.*

$$L_{\mathrm{osc}} \approx 2.48\ \mathrm{m}\ \frac{E\,(\mathrm{MeV})}{\left|\Delta m^2\left(\mathrm{eV}^2\right)\right|}$$

compute the oscillation length for

- reactor neutrinos with $E = 1\ \mathrm{MeV}$, assuming $\Delta m^2 = 7.4 \cdot 10^{-5}\ \mathrm{eV}^2$
- accelerator neutrinos with $E = 1\ \mathrm{GeV}$, assuming $\Delta m^2 = 2.5 \cdot 10^{-3}\ \mathrm{eV}^2$

What are representative neutrino oscillation experiments?

Where would you place the detector to measure oscillations driven by $\Delta m^2 = 10\ \mathrm{eV}^2$ with $E \sim 2.5\ \mathrm{GeV}$?

Oscillation maxima

Exercise 5. Assuming $\Delta m^2 = 2.5 \cdot 10^{-3}\ \mathrm{eV}^2$ and $L = 1300\ \mathrm{km}$, derive the energies of the first and second oscillation maxima

$$P_{e\mu} = \sin^2(2\theta)\ \sin^2\left(1.27\,\frac{\Delta m^2\left(\mathrm{eV}^2\right)\ L\left(\mathrm{km}\right)}{E\,(\mathrm{GeV})}\right)$$