# Understanding the Higgs mass in string theory

**Steve Abel *(IPPP)* 12/03/21**

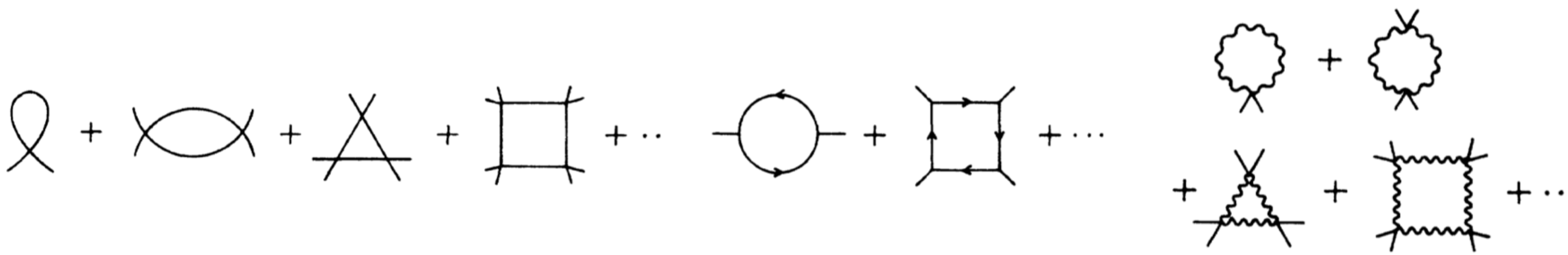
Mainly based on work with Keith Dienes arXiv:2106.04622 and related to …

- w/ Dienes+Mavroudi *Phys.Rev. D* 91, (2015) 126014, arXiv:1502.03087
- SAA JHEP 1611 (2016) 085, arXiv:1609.01311
- Aaronson, SAA, Mavroudi, *Phys. Rev. D* 95, (2016) 106001, arXiv:1612.05742
- w/ Stewart, *Phys.Rev.D* 96 (2017) 10, 106013 arXiv:1701.06629
- w/ Dienes+Mavroudi *Phys.Rev.D* 97 (2018) 12, 126017 arXiv: 1712.06894

# Motivation: key questions for the UV completion

**Effective field theories leave many unsolved problems for scales like the Higgs:**
**e.g. hierarchy problem (essentially the statement that an EFT doesn't make sense)**

Coleman-Weinberg



Potential by doing one loop momentum integrals with a cut-off is

$$\Lambda(\phi) = \frac{M_{UV}^2}{32\pi^2}\underset{\text{eff}}{\text{Str}} M^2 - \frac{1}{64\pi^2}\underset{\text{eff}}{\text{Str}} M^4 \log\left(c\frac{M^2}{M_{UV}^2}\right)$$

where masses *M*($\phi$) are themselves functions of the field $\phi$ *and the cut off* $M_{UV}$ *is …???*

The Higgs is maximally sensitive to both UV and IR: *think of it less as a problem and more as a "**canary in the coal mine**"*

# Motivation: key questions for the UV completion

$$m_\phi^2 \;=\; \frac{M_{\rm UV}^2}{32\pi^2}\,\underset{\rm eff}{\rm Str}\;\partial_\phi^2 M^2 \;-\; \underset{\rm eff}{\rm Str}\;\partial_\phi^2\left[\frac{M^4}{64\pi^2}\log\left(c\,\frac{M^2}{M_{\rm UV}^2}\right)\right]$$

- Is there any meaning at all to the oft-considered Veltman condition? My guess is not: note that the above supertrace is over the EFT only. But why should the whole UV complete theory care about just that?! → $\mathrm{Str}_{\rm eff}\,\partial_\phi^2 M^2 \ll 32\pi^2\,\frac{m_h^2}{M_{UV}^2}$

- What determines if a field is light enough to be called part of the EFT and appear in the log? e.g. GUT states do not contribute in the effective SM even though their mass is much less than the cut-off?!

- Where does the Higgs mass run *to* in the IR? Where and how do we stop it?

- What is the real effect of the UV regulation? — e.g. dimensional regularisation doesn't give a leading quadratically UV sensitive piece. CW called the quadratic piece precisely zero based on some weird argument about it "vanishing at the origin of field space" that I don't understand. Also arguments based on scale invariance — can these arguments have any meaning for a quadratic ***UV divergence***??

- Note the mass is both UV hypersensitive and IR divergent if there are massless states — almost the only operator that is. How can this operator be regulated at both ends at the same time? (UV/IR mixing?)

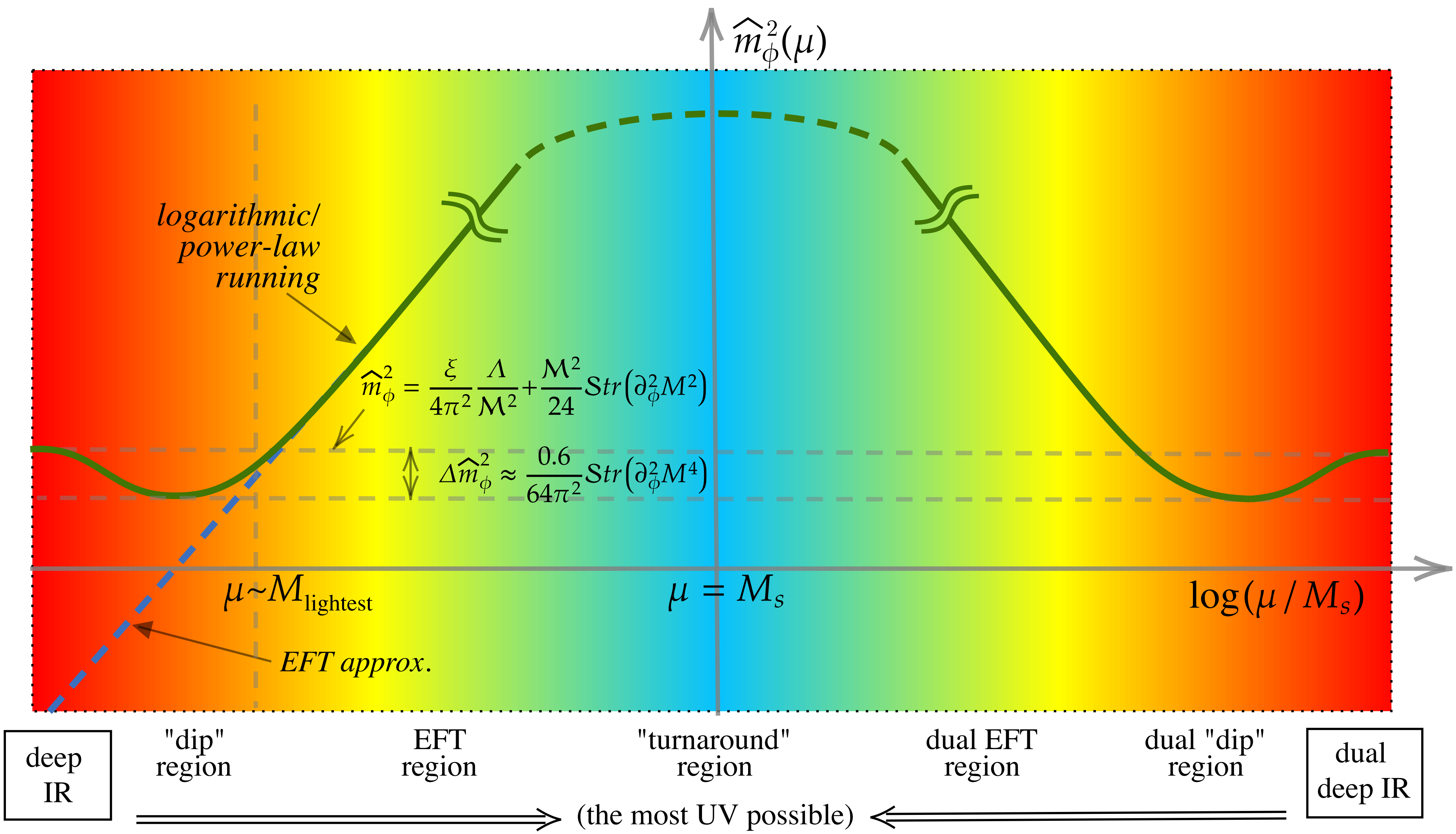
# Motivation: key questions for the UV completion

**Q** (2017): "Suppose nature is a closed string theory. It is finite entirely because of its special symmetries (modular invariance) and that would be true — even today!! Surely this can tell us something about the Higgs mass emerges in such a theory? A truly UV complete theory should answer ALL these questions."

- ***In short***: *the problem has been that we are trying to guess how a parameter might behave because of the UV completion, when we don't know the UV completion!*
- *Non-SUSY strings are an interesting laboratory to address these questions.*
- *Note that in most "string phenomenology" (which starts supersymmetric and then jumps to the EFT) you are frankly "blind to the beauties of number theory".*
- *(Note also the world today cannot be blind to the beauties of number theory because it is UV complete)*
- *It seems that no one before us wrote down the string equivalent to the CW effective potential!*
- ***Warning***: *in this talk (much as in CW) I will not favour any particular model. I just draw general conclusions about the properties the Higgs mass must have (even today) due to the theory's finiteness.*

# Take away picture of Higgs mass in strings:

**A** (2021): “The Higgs mass begins in the UV with a value we can calculate, has RG running, maybe GUT breaking, EW and QCD phase transition, yada yada yada. But then it must eventually wind up at the exact same value in the IR. And everything is finite. Like this…”

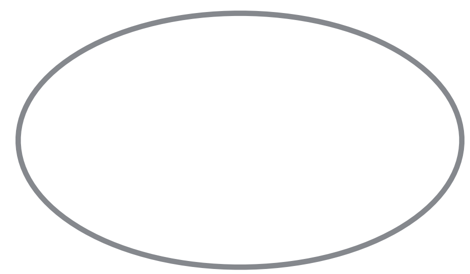


# Layout

- Background - the effective potential in a stringy way
- Modular invariance — the ultimate UV/IR mixer
- The Higgs mass set-up: relation between the Higgs mass and the Cosmological constant
- Regulating the Higgs mass: see how it runs!

# 1. Background: the effective potential in a stringy way

Let's look at the one-loop cosmological constant (a.k.a. effective potential). Simplest way to derive it is as a trivial loop of massive propagators of mass *M(ϕ)* as follows:



For our discussion this can be written in a "stringy way" using a Schwinger worldline parameter, $t$ :

$$\Lambda = \sum_i \int \frac{d^4k}{(2\pi)^4}(-1)^F \log(k^2+M_i^2) = \sum_i \int \frac{d^4k}{(2\pi)^4} \int \frac{dt}{t}(-1)^F e^{-t(k^2+M_i^2)}$$

$$= \sum_i \int_{M_{UV}^{-2}}^{\mu_{UV}^{-2}} \frac{dt}{t^3}(-1)^F e^{-t\,M_i^2}$$

***Can identity a partition function as a weighted sum over spectral density:***

$$\mathcal{Z}(t) = \mathrm{Str}\left(t^{-2}e^{-tM^2}\right)$$

**Performing the integral indeed gives effective potential:**

$$\Lambda(\phi) = \frac{M_{UV}^2}{32\pi^2}\mathrm{Str}M^2 - \frac{1}{64\pi^2}\mathrm{Str}M^4\log\left(c\frac{M^2}{M_{UV}^2}\right)$$

From which we can indeed infer a running Higgs mass-squared from the double derivative:

$$m_\phi^2 \;=\; \frac{M_{\mathrm{UV}}^2}{32\pi^2}\,\underset{\mathrm{eff}}{\mathrm{Str}}\;\partial_\phi^2 M^2 \;-\underset{\mathrm{eff}}{\mathrm{Str}}\;\partial_\phi^2\left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{M_{\mathrm{UV}}^2}\right)\right]$$

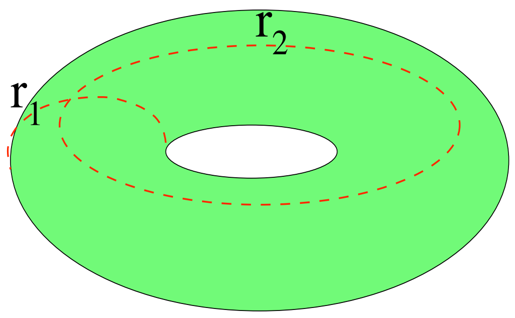
# 2. Modular invariance

## Or: the ultimate UV/IR mixer

***Let's understand how string theory does this but at the same time gets to be finite:***

Revisit the cosmological constant but now in string theory

Closed string theory instead maps out a torus:



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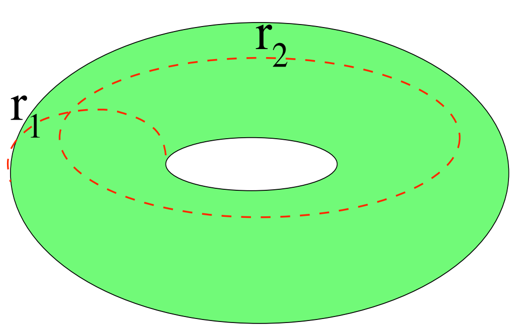
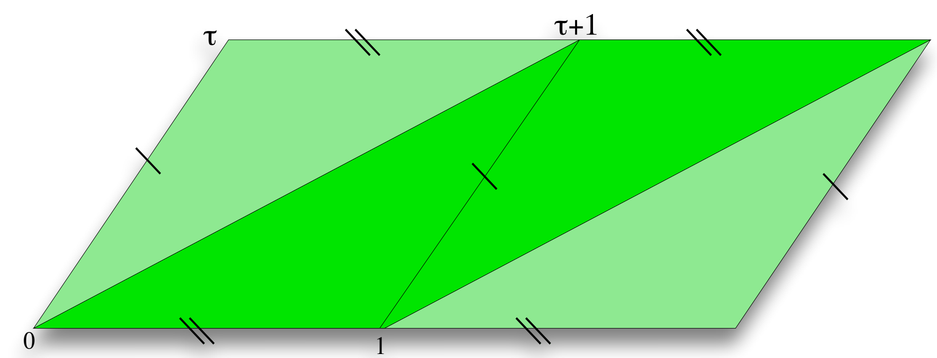



can be mapped to complex plane, $\tau$

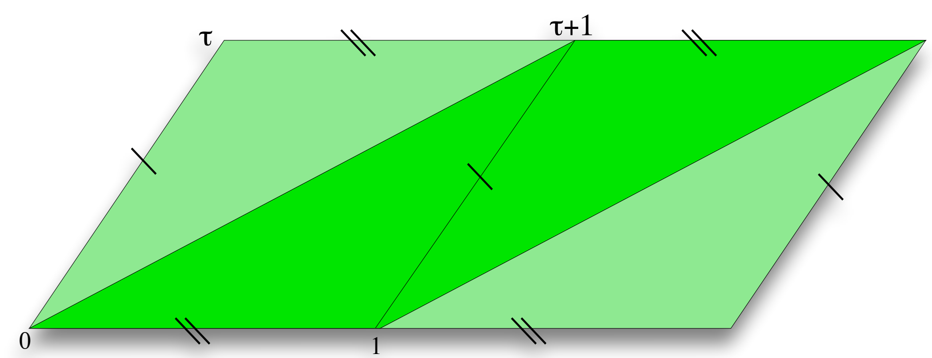
but theory invariant under modular

transformations:




$\tau \to \tau + 1$ redefines torus :

$\tau \to -1/\tau$ swops $\sigma_1$ and $\sigma_2$ and just reorients torus

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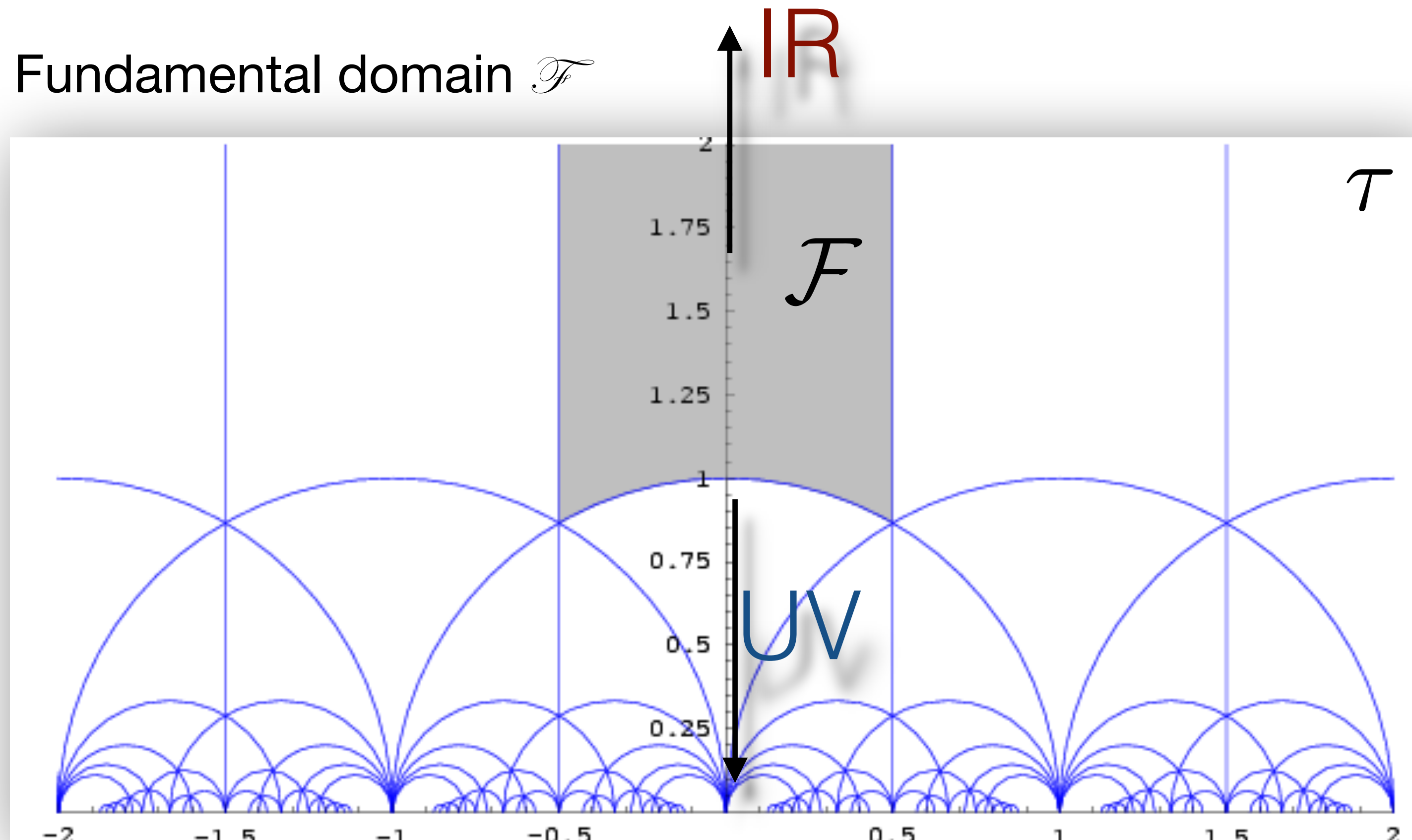
Fundamental domain $\mathscr{F}$

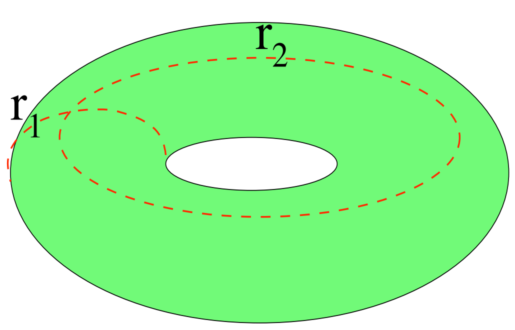


$$\tau = \tau_1 + i\tau_2$$

$$\left(\mathcal{M}^2 = \frac{1}{4\pi^2\alpha'} = \frac{M_s^2}{4\pi^2}\right)$$

$$\Lambda \equiv -\tfrac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^2} Z(\tau) \qquad q = e^{2\pi i\tau}$$

$$= -\frac{1}{2}\mathcal{M}^D \int_{\mathcal{F}} \frac{d^2\tau}{{\tau_2}^{\frac{D}{2}+1}} \sum_{m,n} a_{mn}\overline{q}^m q^n$$




Counts physical (level matched) states weighted by statistics at each level

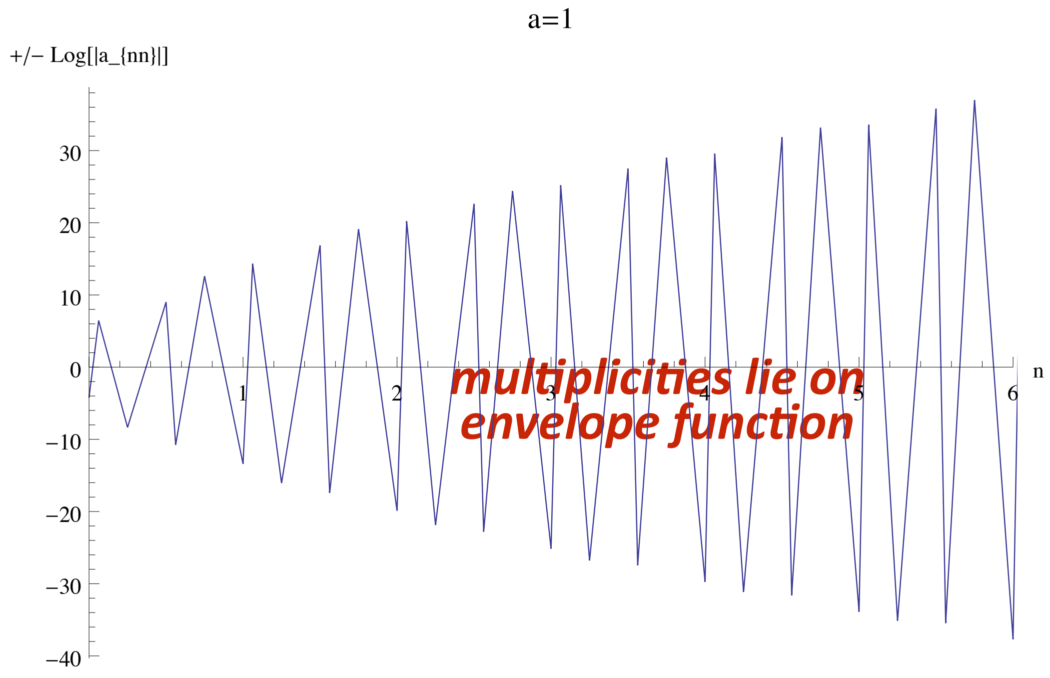
$$\approx -\frac{1}{2}\mathcal{M}^D \int_{M_{UV}^{-2}}^{\mu_{IR}^{-2}} \frac{d\tau_2}{{\tau_2}^{\frac{D}{2}+1}} \sum_n a_{nn}\, e^{-\pi\tau_2\alpha' M_n^2}$$

***Due to modular invariance: there's an important way to understand this as a supertrace relation over the infinite tower of physical states. Much more natural and general for what we want to do. Superficially even looks similar to the field theory:***

$$\Lambda = \frac{1}{24}\mathcal{M}^2 \underset{\text{phys}}{\mathrm{STr}} M^2$$

- Dienes, Misaligned SUSY, 1994
- Kutasov, Seiberg, 1994
- Dienes, Moshe, Myers 1995

But note this definitely is *not* a field theory object — this supertrace is over the *infinite* string tower of states!!




- This crazy spectrum has finite $\Lambda$

**Let's see how to derive**: the integral we need to do in 4D is:

$$\Lambda \quad = -\frac{\mathcal{M}^4}{2}\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\,\mathcal{Z}(\tau)$$

Want to write this in terms of physical (level-matched) states whose nett spectral density is:

$$\begin{aligned} g(\tau_2) &= -\frac{\mathcal{M}^4}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \mathcal{Z}(\tau) \\ &= -\frac{\mathcal{M}^4}{2}\tau_2^{-1}\underset{\text{phys}}{\text{Str}}e^{-\pi\tau_2\alpha' M^2} \end{aligned}$$

Rankin-Selberg: unfold integral to the “critical strip” by convoluting it with an Eisenstein function:

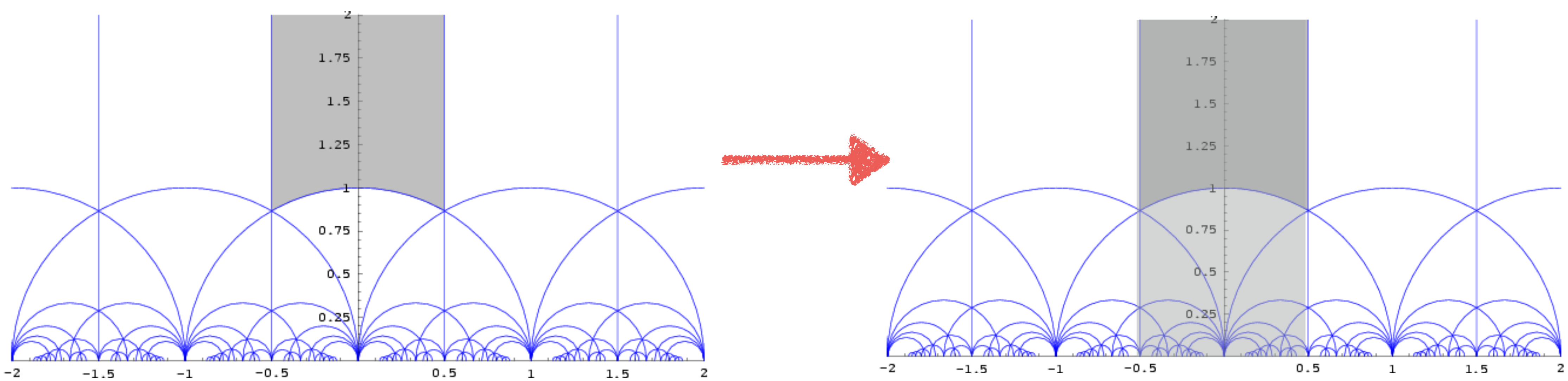
- Rankin, (1940), Selberg (1940), Zagier (1981)
- Angelantonj, Cardella, Elitzur, and Rabinovici
- Angelantonj, Florakis, and Pioline

$$\Lambda \quad = \quad 2\,\mathrm{Res}_{s=1}(\mathcal{R}^\star(F,s))$$

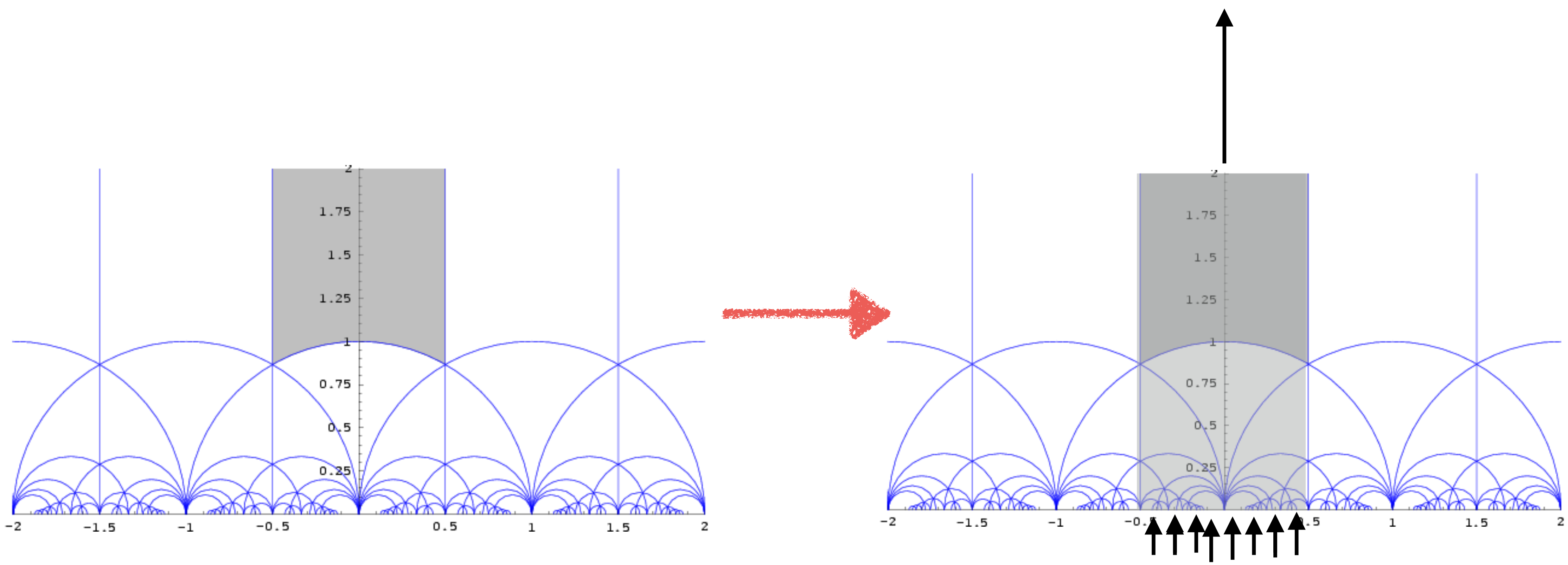
where $\mathcal{R}^\star$ is the Rankin-Selberg (Mellin) transform:

$$\begin{aligned}\mathcal{R}^\star(F,s) \;&=\; \int_0^\infty \frac{d^2\tau}{\tau_2^2}\tau_2^s\pi^{-s}\Gamma(s)\zeta(2s)\,g(\tau_2)\\ &=\; -\frac{\mathcal{M}^4}{2\pi^2}\,\Gamma(s)\Gamma(s-2)\,\zeta(2s)\,\underset{\text{phys}}{\mathrm{STr}}(\pi^2\alpha' M^2)^{2-s}\end{aligned}$$

The whole integral here including the projection to physical states is now looking like:



***Note the important difference from the naive string-theory-textbook picture. There is now clearly no single “IR cusp”. All cusps contribute equally to the integral:***




All cusps are equivalent under modular transformations. In a modular invariant integral there is only IR: there is no “ultra UV” anywhere.

The incredible fact that this infinite supertrace is finite can then be put down to the fact that the spectral density function, which recall is this thing that counts the physical states …

$$
\begin{aligned}
g(\tau_2) &= -\frac{\mathcal{M}^4}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \mathcal{Z}(\tau) \\
&= -\frac{\mathcal{M}^4}{2}\tau_2^{-1}\mathrm{Str} e^{-\pi\tau_2\alpha' M^2}
\end{aligned}
$$

… behaves as follows in the UV (i.e. as $\tau_2 \to 0$ ):

$$
g(\tau_2) \;\sim\; \tau_2^{1-D/2}\underset{\text{phys}}{\mathrm{Str}}\,(e^{-\tau_2 M^2}) \;\longrightarrow\; c_0
$$

In other words *Str(1)=0*. In other words the nett spectrum "behaves" like a ***2 dimensional theory in the UV.*** *Unlike* supersymmetry however there is no level by level cancellation and the nett (Boson-Fermion) numbers of states in each level are completely crazy!

# 3. Setting up the Higgs mass itself …

## Or: a connection between the cosmological constant and the Higgs

First assume that the partition function is a function of the higgs. Then begin with the naive expression:

$$m_\phi^2 \equiv \left. \frac{d^2\Lambda(\phi)}{d\phi^2} \right|_{\phi=0}$$

where $\Lambda(\phi) \equiv -\frac{\mathcal{M}^4}{2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \mathcal{Z}(\tau, \overline{\tau}, \phi)$

Let's look a little at the form of the partition function $\mathcal{Z}$. It has to be a combination of modular functions. These are all famous objects in their own right. Often this can be written as a sum over lattice vectors:

$$\mathcal{Z}(\tau, \overline{\tau}, \phi) = \tau_2^{-1} \frac{1}{\overline{\eta}^{12} \eta^{24}} \sum_{\mathbf{Q}_L, \mathbf{Q}_R} (-1)^F \overline{q}^{\mathbf{Q}_R^2/2} q^{\mathbf{Q}_L^2/2}$$

So naively we need to do a double derivative to get a modular integral of the following sort of object:

$$\frac{\partial^2 \mathcal{Z}}{\partial \phi^2} = \tau_2^{-1} \frac{1}{\overline{\eta}^{12}\eta^{24}} \sum_{\mathbf{Q}_L,\mathbf{Q}_R\in L} (-1)^F \, X \, \overline{q}^{\mathbf{Q}_R^2/2} q^{\mathbf{Q}_L^2/2}$$

where everything hinges on the summand insertion *X,* coming from the derivatives: explicitly

$$X \equiv \pi i \frac{\partial^2}{\partial \phi^2}(\tau \mathbf{Q}_L^2 - \overline{\tau}\mathbf{Q}_R^2) - \pi^2 \left[\frac{\partial}{\partial \phi}(\tau \mathbf{Q}_L^2 - \overline{\tau}\mathbf{Q}_R^2)\right]^2$$

This in turn requires us to work out the most general shifts of the following form that maintain modular invariance:

$$\begin{aligned} \mathbf{Q}_L &\to \mathbf{Q}_L + \sqrt{\alpha'}\phi\mathbf{Q}_a + \frac{1}{2}\alpha'\phi^2\mathbf{Q}_b + \ldots \\ \mathbf{Q}_R &\to \mathbf{Q}_R + \sqrt{\alpha'}\phi\tilde{\mathbf{Q}}_a + \frac{1}{2}\alpha'\phi^2\tilde{\mathbf{Q}}_b + \ldots , \end{aligned}$$

Turns out the allowed shifts can be decomposed as gauge generators acting on $\mathbf{Q} = \{\mathbf{Q}_L\,, \mathbf{Q}_R\,\}$ that take the following form:

$$\mathcal{T} \sim \left(\begin{array}{cccccc|cc} & & & & & \mathbf{t} & 0 & \tilde{\mathbf{t}} \\ & & & & & \mathbf{0} & 0 & \tilde{\mathbf{0}} \\ & & \mathbf{0}_{5\times 5} & & & \mathbf{0} & 0 & \tilde{\mathbf{0}} \\ & & & & & \mathbf{0} & 0 & \tilde{\mathbf{0}} \\ & & & & & \mathbf{0} & 0 & \tilde{\mathbf{0}} \\ -\mathbf{t}^t & \mathbf{0}^t & \mathbf{0}^t & \mathbf{0}^t & \mathbf{0}^t & \mathbf{t}_{11} & \mathbf{0}^t & \mathbf{t}_{12} \\ \hline 0 & 0 & 0 & 0 & 0 & \mathbf{0} & 0 & \tilde{\mathbf{0}} \\ -\tilde{\mathbf{t}}^t & \tilde{\mathbf{0}}^t & \tilde{\mathbf{0}}^t & \tilde{\mathbf{0}}^t & \tilde{\mathbf{0}}^t & -\mathbf{t}_{12}^t & \tilde{\mathbf{0}}^t & \mathbf{t}_{22} \end{array}\right) \qquad \text{where e.g.} \qquad \begin{pmatrix} \mathbf{Q}_a \\ \tilde{\mathbf{Q}}_a \end{pmatrix} = \mathcal{T}\cdot\mathbf{Q}$$

and where the *t* are row vectors.

But then at the end of the day the relevant part of the allowed summand *X* is (**almost**) given by

$$X \;=\; -\,\pi\alpha'\tau_2\,\partial_\phi^2 M^2 + (\pi\alpha'\tau_2)^2\,\left(\partial_\phi M^2\right)^2$$

***Almost but not quite:*** the shifts in $\mathbf{Q}$ induced by the Higgs correspond to coordinate shifts of the modular forms (actually the Higgs *is* a linear combination of these coordinates). For the Higgs derivatives to be modular *covariant* derivatives we require a modular completion which is found to be universal:

$$X \longrightarrow X + \frac{\xi}{4\pi^2 \mathcal{M}^2} \qquad \xi = -\mathrm{T}r(\mathcal{T}_{21}\mathcal{T}_{12})$$

Note that this cosmological constant contribution is due to the modular anomaly of the original naive ***X***. This universal term would in most practical cases be identified as a Higgs dependent shift in the volume of the compactification space (e.g. 10D —> 4D compactification).

So ***finally*** putting this all into Rankin-Selberg we get … ta da !

$$m_\phi^2 \; = \frac{\xi}{4\pi^2}\frac{\Lambda^{(1)}}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \mathrm{Str}\partial_\phi^2 M^2 +$$

$$\mathrm{STr}_{M=0}(\partial_\phi M^2)^2 \times \infty + \mathrm{STr}_{M>0}(\partial_\phi M^2)^2 \times 0$$

**Wait. What?!**

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**Wait. What?!**

# 4. Regularisation and renormalisation

**Or: see how it runs!**

The quartic terms are precisely those terms that should be logarithmically dependent on RG scale. But we didn't yet put in any physical RG scale! So at the moment they can only return infinity if the state is massless (or zero if it is massive).

Generally need to find a way to regulate the theory at some IR scale $\mu$ to extract a physical "running"

Typically to do RG in string theory we subtract the logarithmically divergent states (i.e. the massless "cusp" contribution to the partition function).

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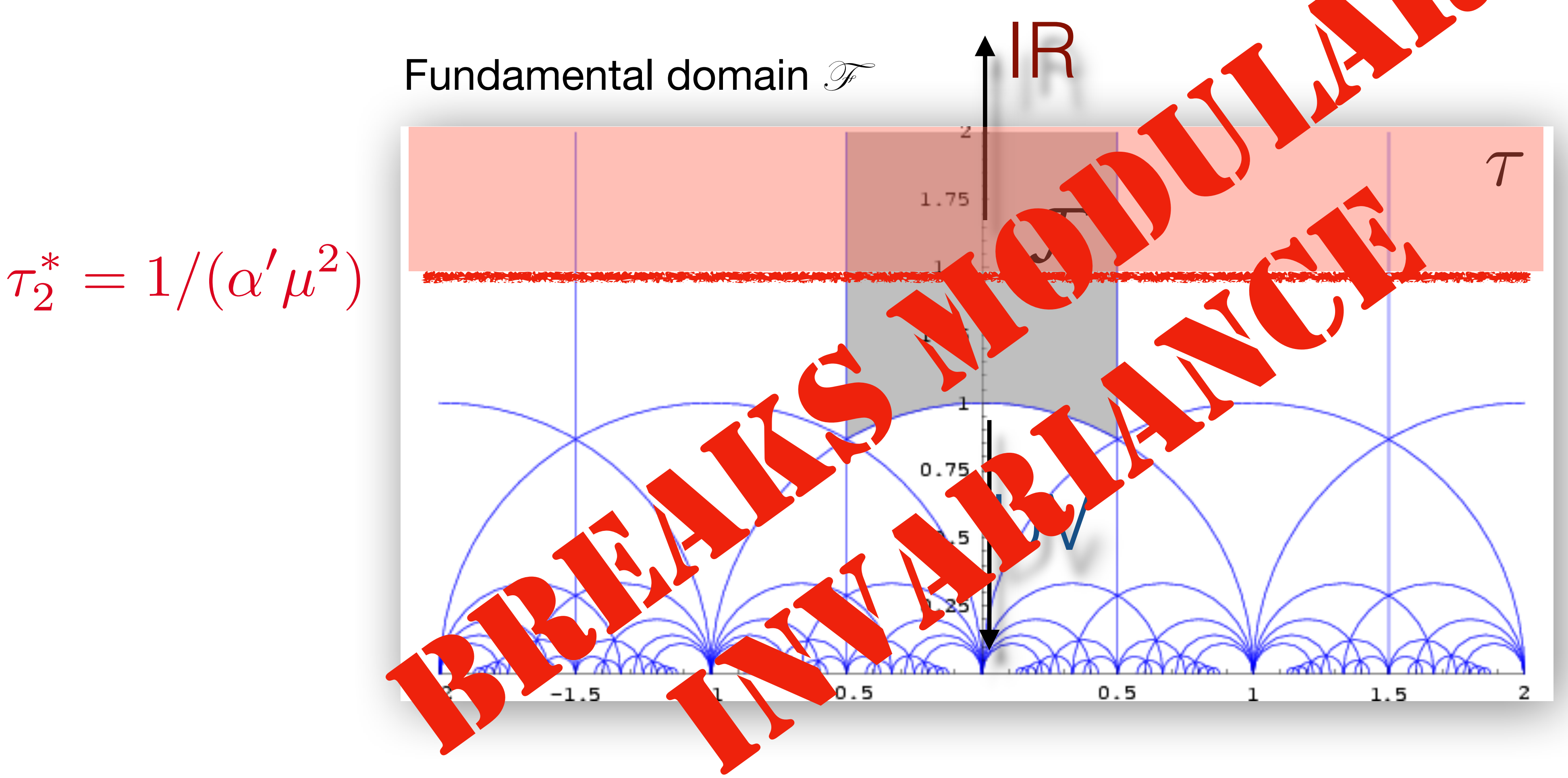
How should we do this? Let's just think about the general modular integral: $I \equiv \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} F(\tau,\overline{\tau})$

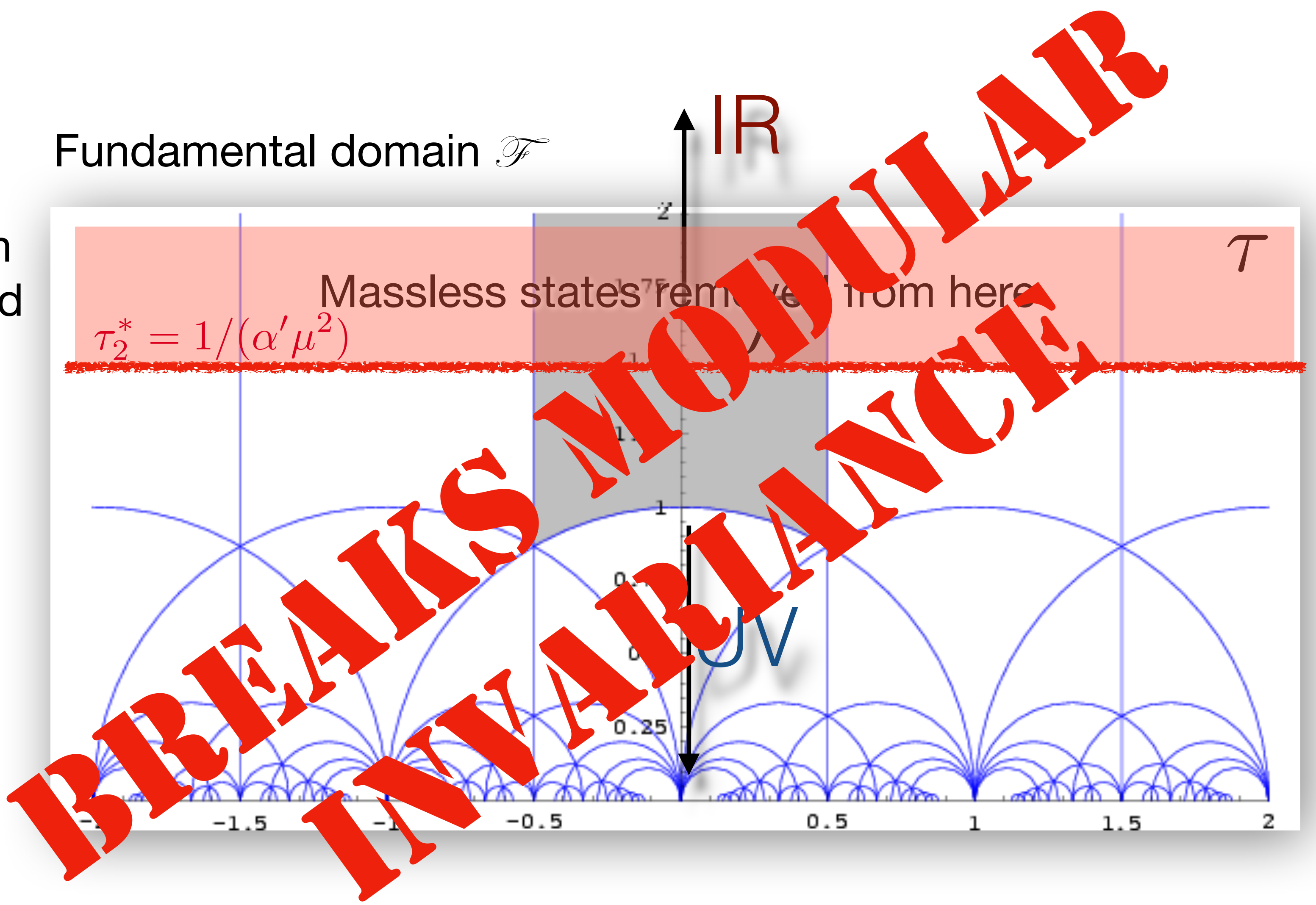
Could consider this: a simple cut-off:



What about modified regulator: subtract just the massless contribution in the IR sector — closest to the traditional RG method that is used in almost all string phenomenology (Kaplunovsky)

$$\widehat{I}(\tau_2^*) = \int_{\mathcal{F}_{\tau_2^*}} \frac{d^2\tau}{\tau_2^2} F + \int_{\mathcal{F}-\mathcal{F}_{\tau_2^*}} \frac{d^2\tau}{\tau_2^2} (F - c_1\tau_2)$$

In a tour-de-force in 1981 Zagier showed this can also be written as a Str formula …



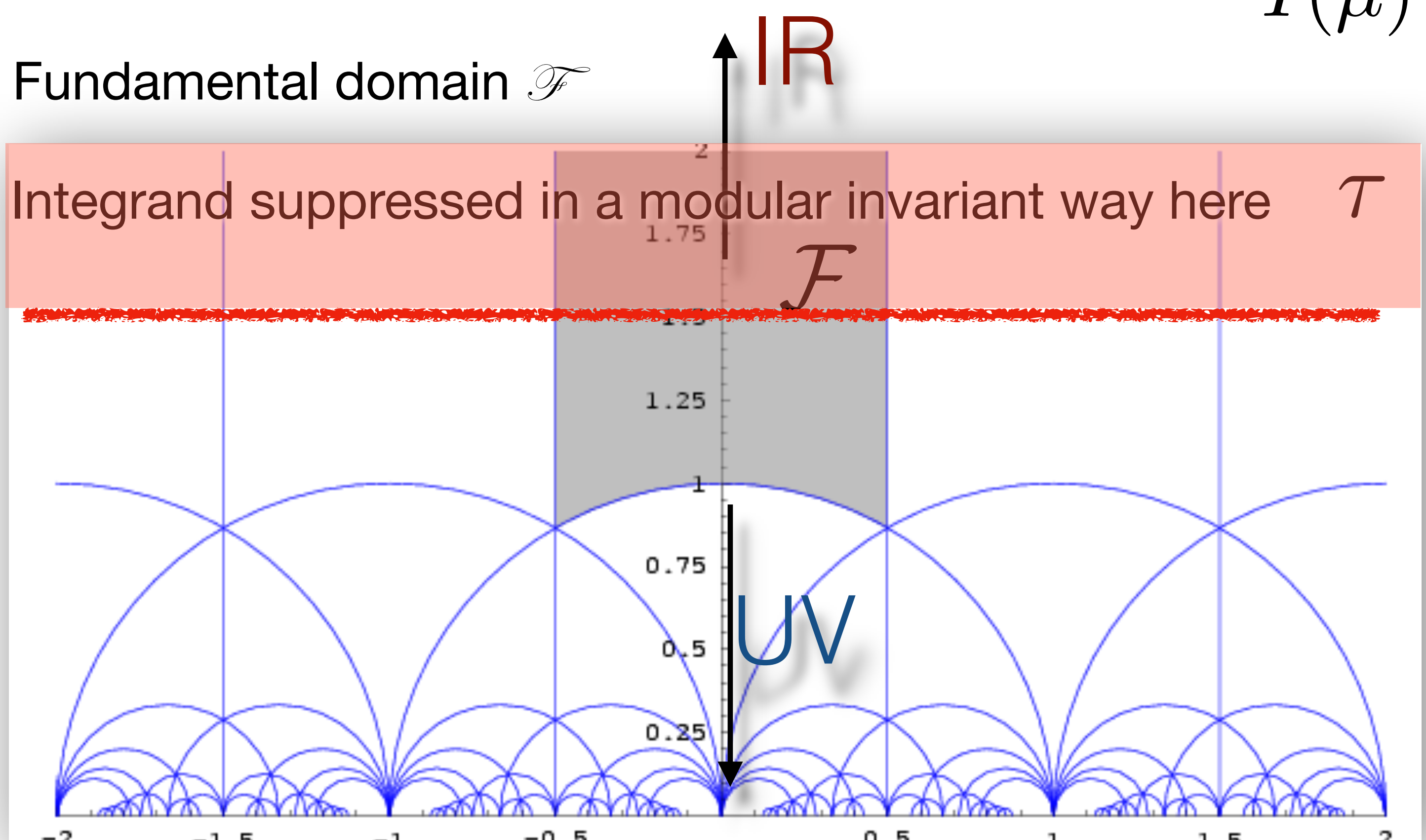
So far, and traditionally, we always think of stringy “threshold corrections” and match them to an effective field theory (EFT). But arguably this approach …

- could never yield a fully modular invariant answer as the EFT is by definition not modular invariant
- cannot give “Wilsonian renormalisation”: my choice of whether the electron is light enough to be called massless and be subtracted is completely arbitrary and will always break modular invariance

**Instead we must abandon the idea of going to an EFT and introduce a modular invariant “Wilsonian” cut-off instead:**

$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$

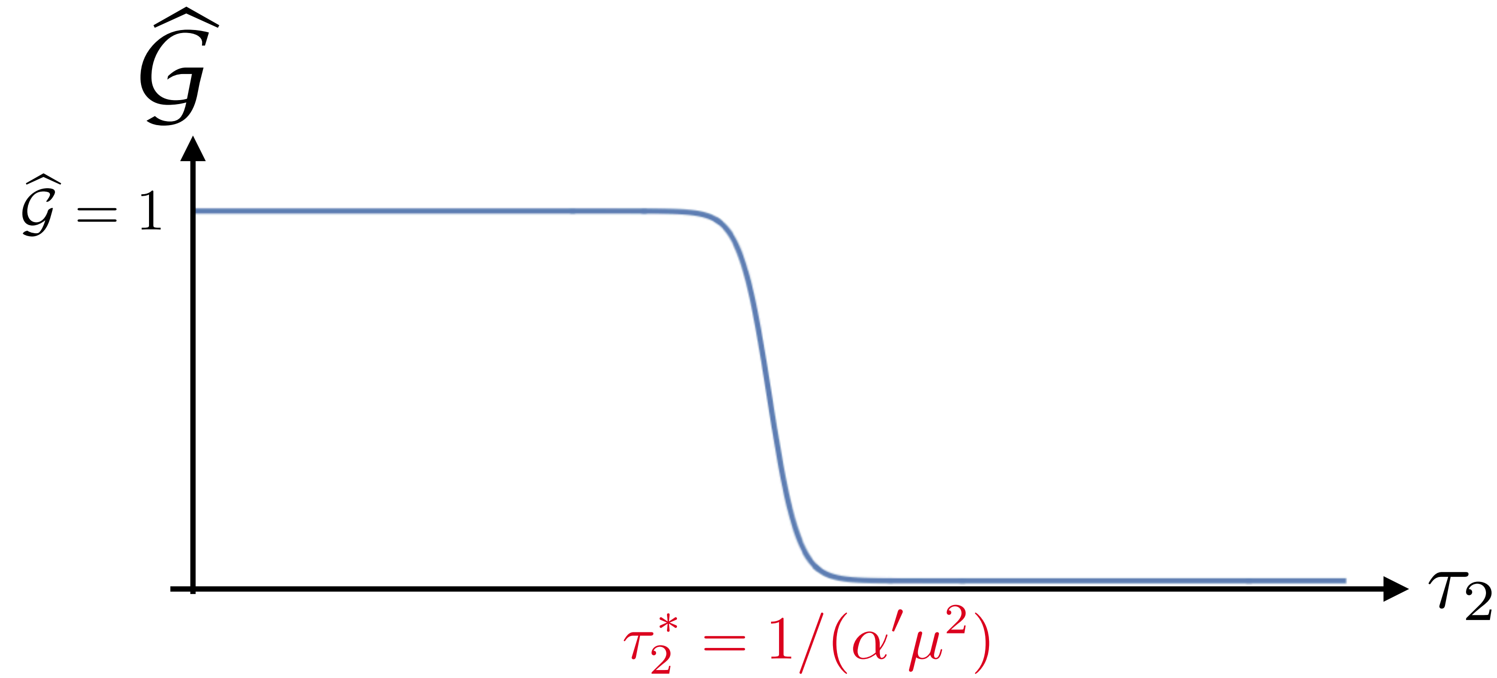
Fundamental domain $\mathscr{F}$



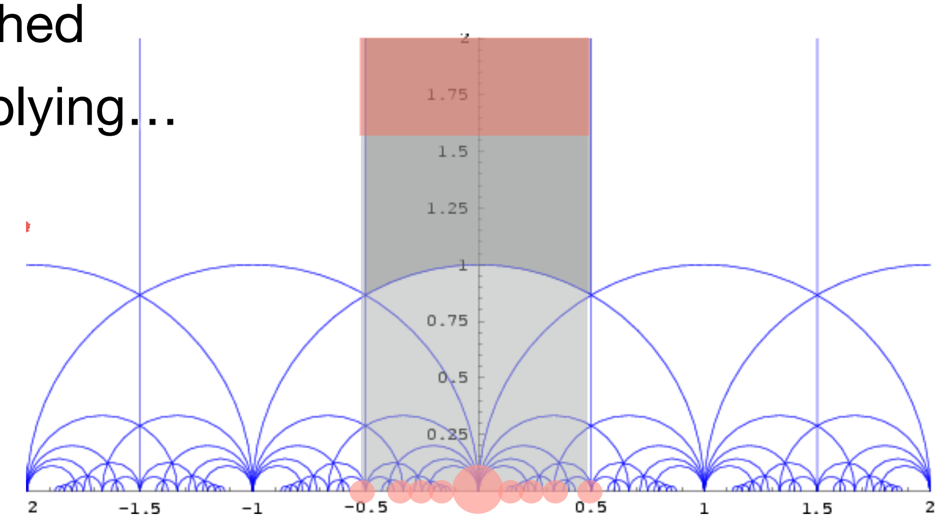
**Required properties of "Wilsonian" regulator, $\widehat{\mathcal{G}}$ :**

$$\widehat{I}(\mu) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau})\, F(\tau,\overline{\tau})$$

- a) Is itself a modular function
- b) Should look like this ….




- c) Remember, our goal is to write everything as a supertrace which ultimately means an integral over the critical strip …This only makes sense if all the cusps are crushed equally. In other words: all the cusps are equivalent IR cusps, implying…




$$\tau_2^* \equiv 1/\tau_2^* \implies \widehat{\mathcal{G}}(\mu,\tau,\overline{\tau}) = \widehat{\mathcal{G}}(M_s^2/\mu,\tau,\overline{\tau})$$

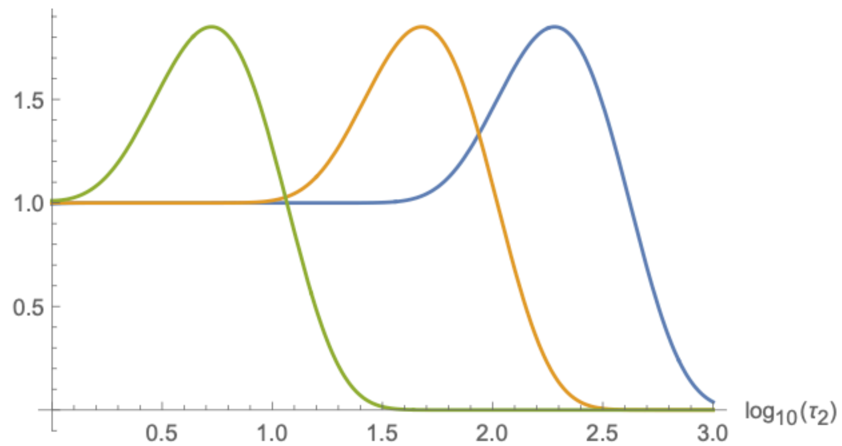
**Such a modular invariant regulator very nearly exists (modulo the last property)** (Kiritsis, Kounnas)

- Take the circle partition function with radius defined by parameter $a \equiv \sqrt{\alpha'}/R$ :

$$Z_{\rm circ}(a,\tau) = \sqrt{\tau_2} \sum_{m,n\in\mathbb{Z}} \overline{q}^{(ma-n/a)^2/4}\, q^{(ma+n/a)^2/4}$$

- Then a suitable cut-off function that obeys all these properties is …

$$\widehat{\mathcal{G}}_\rho(a,\tau) \;=\; \frac{\rho/(\rho-1)}{1+\rho a^2}\;\, a^2 \frac{\partial}{\partial a}\Big[Z_{\rm circ}(\rho a,\tau) - Z_{\rm circ}(a,\tau)\Big]$$




$$\mu^2(\rho,a) \;\equiv\; \frac{\rho a^2}{\alpha'} \qquad \Longrightarrow \;\tau_2^* = 1/\rho a^2$$

- The nice thing about this function is we can find the simpler as a supertrace by unfolding, and then take the $a$ derivative

$$P(a) = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, F(\tau)\, Z_{\rm circ}(a,\tau)$$

## The result is a smooth modular invariant answer with an IR cut-off

Complicated sum of Bessel functions, but it has the following magical behaviour …

$$\widehat{m}_\phi^2 = \frac{\xi}{4\pi^2}\frac{\widehat{\Lambda}(\mu)}{\mathcal{M}^2} + \partial_\phi^2 \widehat{\Lambda}(\mu)$$

$$\widehat{\Lambda}(\mu,\phi) = \frac{1}{24}\mathcal{M}^2 \,\mathrm{Str}\, M^2 - c' \underset{M \gtrsim \mu}{\mathrm{Str}}\, M^2\mu^2 \quad - \underset{0\leq M \lesssim \mu}{\mathrm{Str}} \left[\frac{M^4}{64\pi^2}\log\left(c\frac{M^2}{\mu^2}\right) + c''\mu^4\right]$$

$c = 2e^{2\gamma+1/2}$, $c' = 1/(96\pi^2)$, and $c'' = 7c'/10$

This is a fully UV complete effective potential the holds for any modular invariant theory!

Below the mass of all states (that couple to the Higgs) there is no further contribution

At some intermediate energy scale the result is a sum over all states *as **if they had all logarithmically run up from their mass***.

It is by construction symmetric around the string scale.

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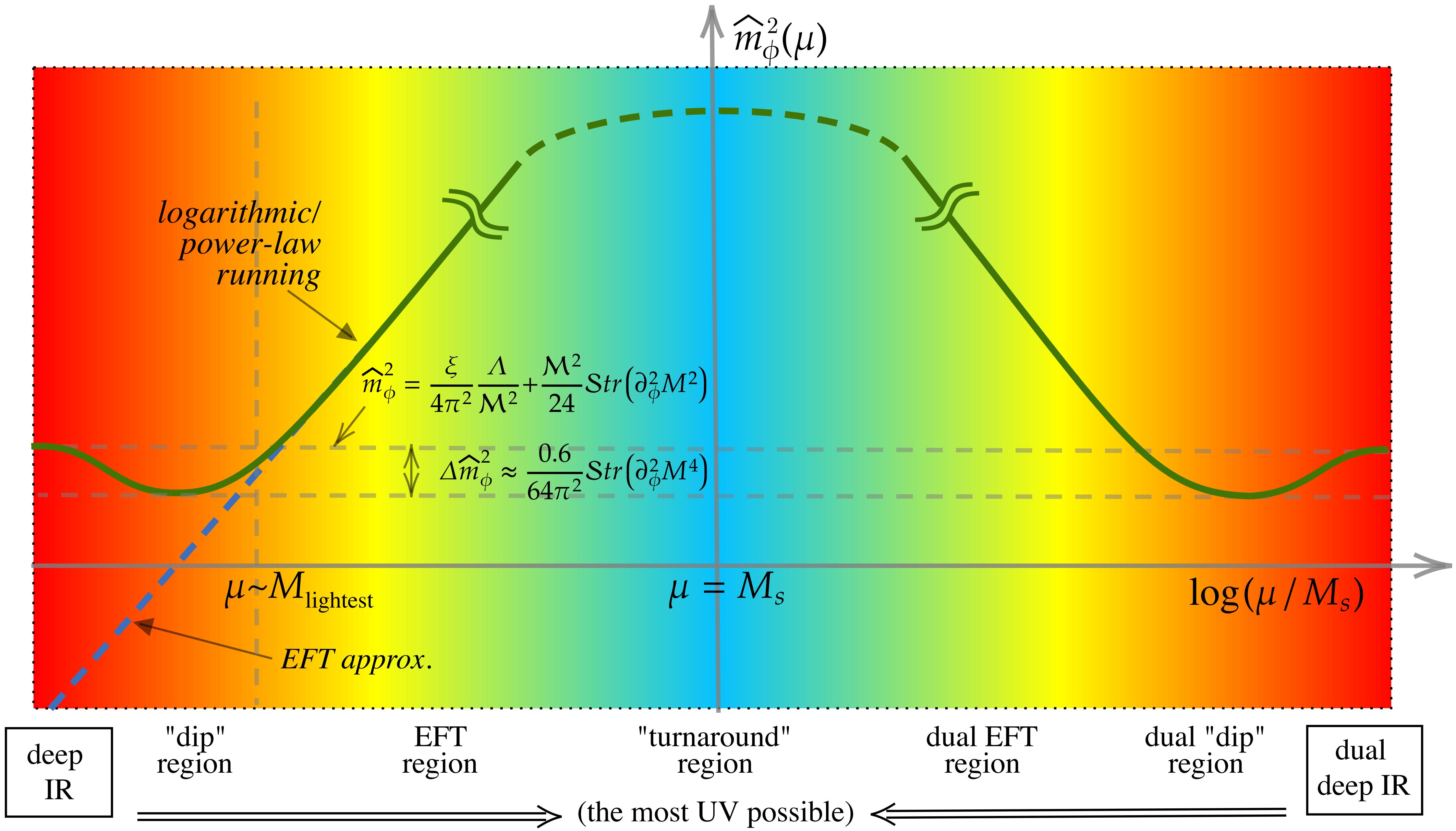


$$\lim_{\mu \to 0} \hat{m}_\phi^2(\mu) = \frac{\xi}{4\pi^2}\frac{\Lambda}{\mathcal{M}^2} + \frac{1}{24}\mathcal{M}^2 \mathrm{Str}\,\partial_\phi^2 M^2$$

# 5. Conclusions

- We have developed a general supertrace formula for the Higgs, that plays the role for all generic modular invariant theories that the effective potential plays in field theory.
- A modular invariant regulator provides a natural "Wilsonian energy cut-off" and a definition of RG scale. Gives meaning where the EFT fails, and retains the predictivity of the UV complete theory.
- Operators such as the Higgs mass can be thought of as "running" to its IR value: this is actually both the UV and IR asymptote.
- The Weak/Planck and cosmological constant hierarchy problems are connected in this one operator.
- Informs many old and new pheno ideas: e.g. a single stringy naturalness condition:

$$\mathrm{Str}\,\partial_\phi^2 M^2 \lesssim \frac{24}{\mathcal{M}^2} M_W^2$$