Electroweak precisin calculations for future lepton colliders

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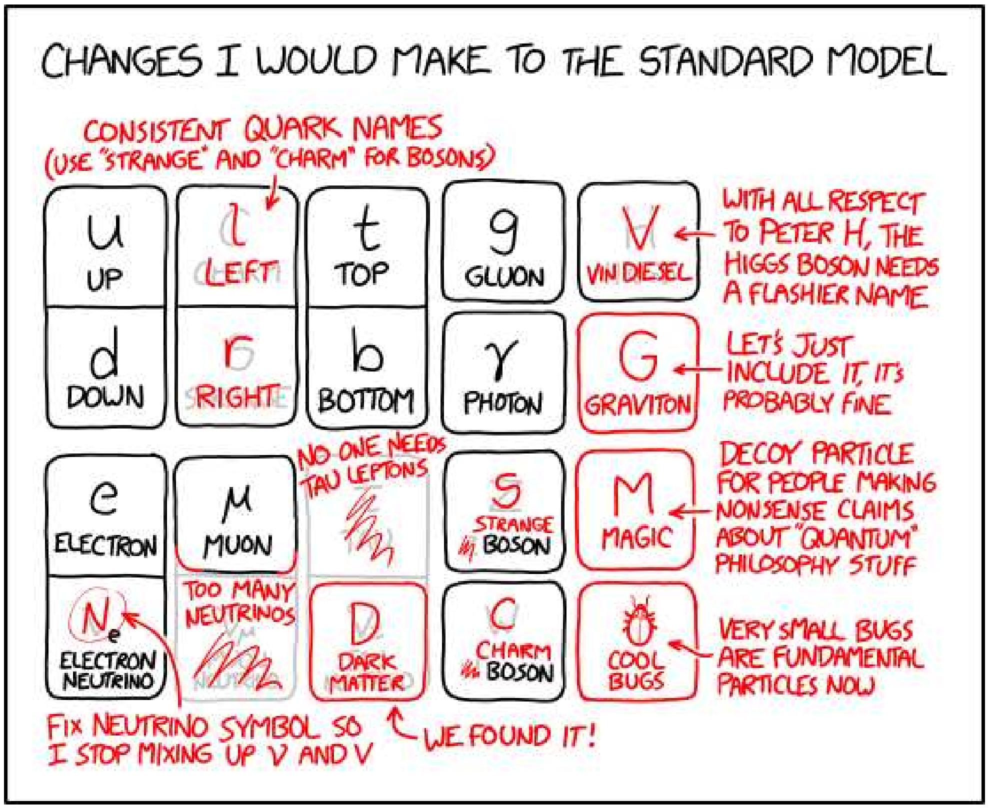


(from xkcd.com)

1. Electroweak precision observables
2. Real observables vs. pseudo-observables
3. Theoretical predictions in the SM
4. Electroweak precision physics with future e^+e^- colliders
5. New physics reach: Effective theory
6. New physics reach: Models

Status of electroweak Standard Model

Structure of $SU(2)\times U(1)$ interactions well understood

Open questions of the Standard Model: → lecture by S. Heinemeyer

- Is the Higgs boson part of a more complex sector?
- Is there an extended/unified symmetry group?
- What is dark matter?
- Why is there more matter than anti-matter in the universe?
- ...

→ **Physics beyond the Standard Model**

→ Direct searches at high-energy colliders (LHC)

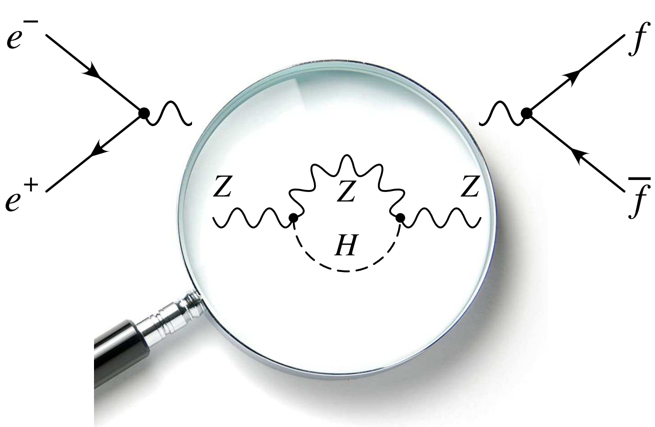
→ Astro-physics searches (e.g. DM direct / indirect detection)

→ Indirect evidence from precision measurements

Quantum effects

Quantum fluctuations in Quantum Field Theory:
Virtual emission and re-absorption of **all** physical particles

→ Inference of information about **heavy SM particles** and **new physics** from precision measurements without direct observation



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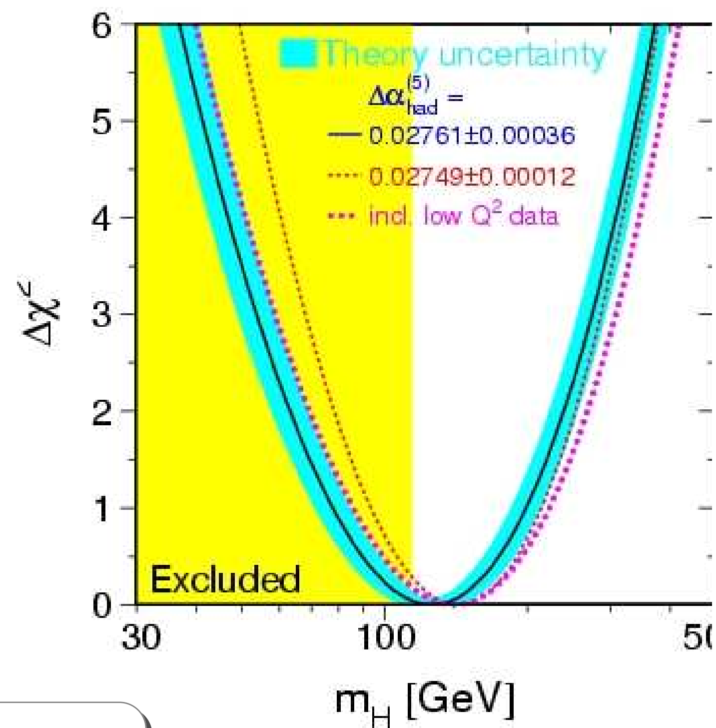
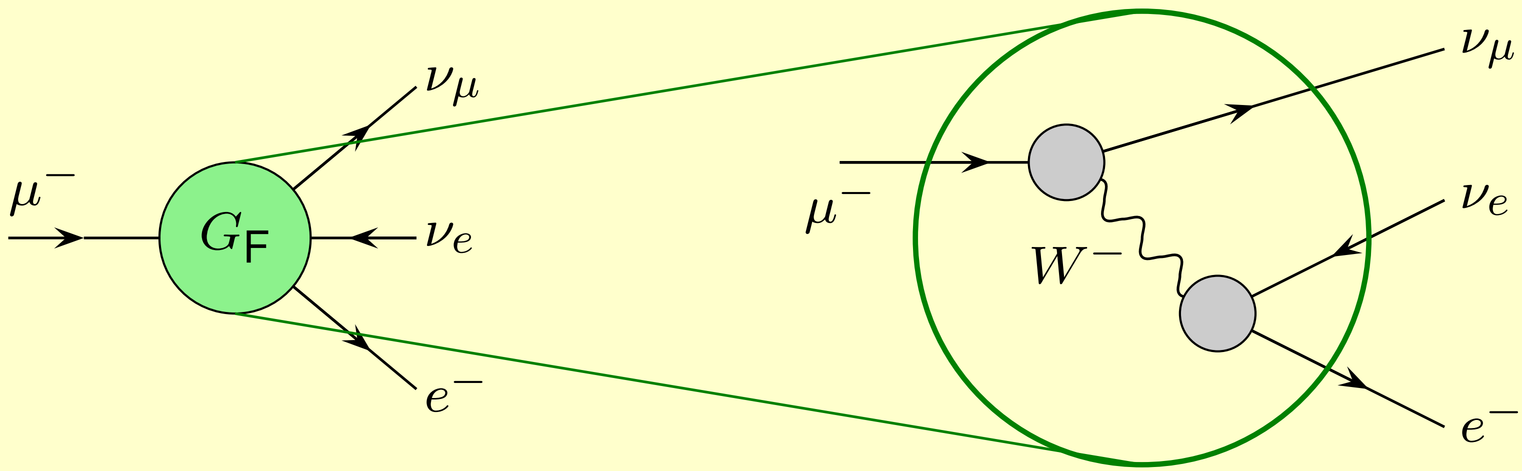


typical corrections of order $\mathcal{O}(\%)$
experimental precision up to $\sim 10^{-3}$

Fermi constant / W mass

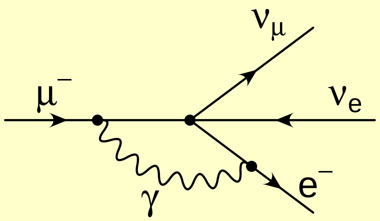
W-boson mass can be calculated from muon decay rate:



μ decay in Fermi Model

$$\Gamma_\mu = \frac{G_F m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q)$$

QED corrections (2-loop)



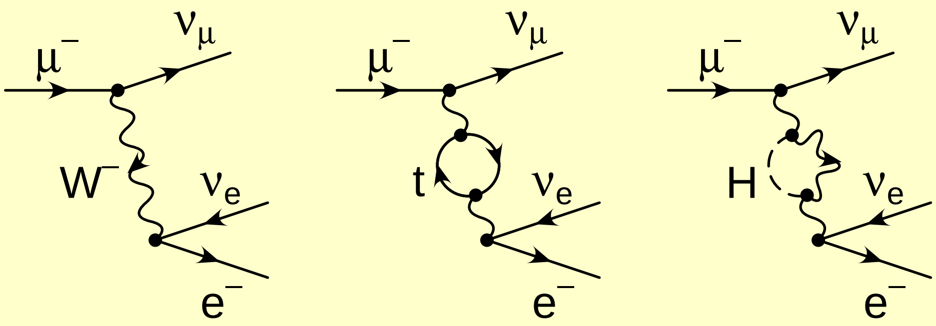
Ritbergen, Stuart '98
Pak, Czarnecki '08

μ decay in Standard Model

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 s_w^2 M_W^2}(1+\Delta r)$$

electroweak corrections

μ decay in Standard Model:



$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 s_\mathrm{w}^2 M_\mathrm{W}^2}\Big(1 + \Delta r(M_\mathrm{Z}, M_\mathrm{H}, m_\mathrm{t}, \ldots)\Big)$$

electroweak corrections (few %)

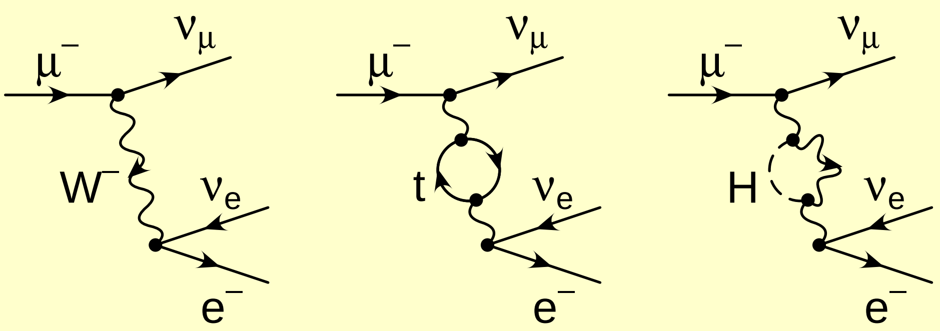
Can solve for

$$M_\mathrm{W} = M_\mathrm{W}(G_F, M_\mathrm{Z}, M_\mathrm{H}, m_\mathrm{t}, \ldots)$$

Although $m_\mu \ll m_\mathrm{t}, M_\mathrm{H}, \ldots$, the muon decay rate is sensitive to $m_\mathrm{t}, M_\mathrm{H}, \ldots$ through **quantum corrections**

W mass

μ decay in Standard Model:



$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}\left(1+\Delta r(M_Z, M_H, m_t, \ldots)\right)$$

electroweak corrections (few %)

Can solve for

$$M_W = M_W(G_F, M_Z, M_H, m_t, \ldots)$$

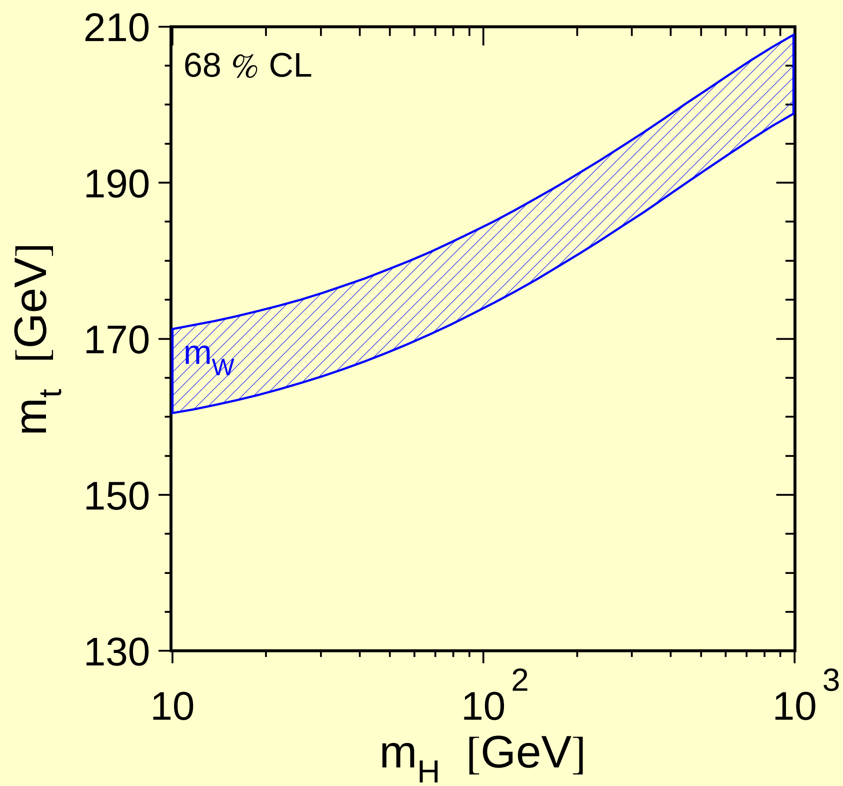
Experiment: Particle Data Group '18

$G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}

$M_W = 80.376(33)$ GeV (LEP)

$80.387(16)$ GeV (TEV)

$80.370(19)$ GeV (LHC)

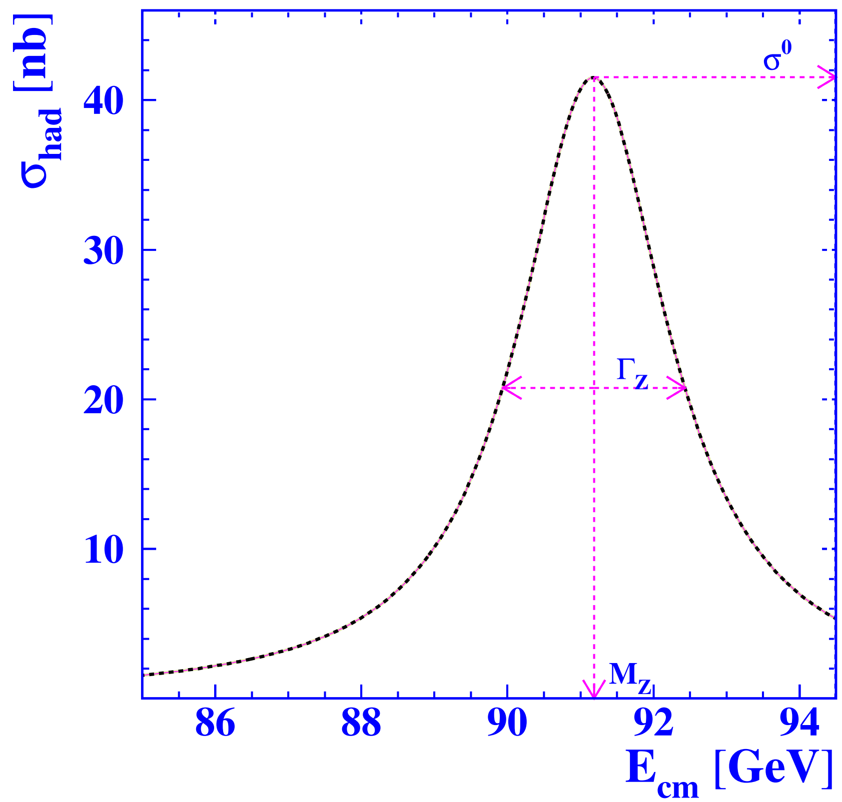


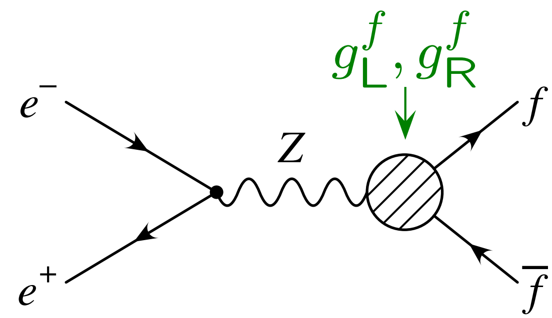
Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\mathrm{CM}} \sim M_{\mathrm{Z}}$:

- Mass M_{Z}
- Width $\Gamma_{\mathrm{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\mathrm{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\mathrm{Z}}^2)^2+M_{\mathrm{Z}}^2\Gamma_{\mathrm{Z}}^2} = \frac{12\pi}{M_{\mathrm{Z}}^2}R_e R_f$

$$\Gamma_{ff} = C\left[(g_{\mathrm{L}}^f)^2 + (g_{\mathrm{R}}^f)^2\right]$$





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LEP (CERN)

SLC (SLAC)

Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_Z$:

- Mass M_Z
- Width $\Gamma_Z = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_Z$
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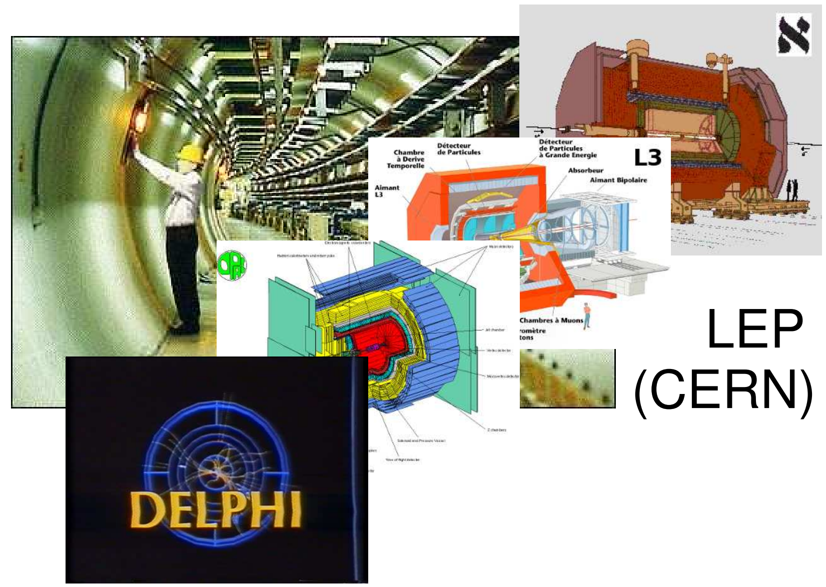
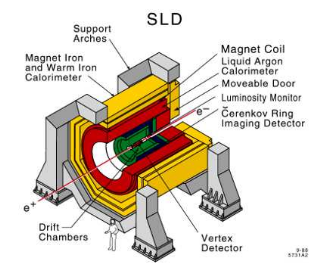


LEP (CERN)



SLC (SLAC)

Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_{\text{Z}}$:

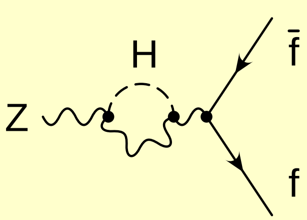
- Mass M_{Z}
- Width $\Gamma_{\text{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\text{Z}}$
- $\sigma^0 \approx \frac{12\pi\,\Gamma_{ee}\Gamma_{ff}}{(s-M_{\text{Z}}^2)^2+M_{\text{Z}}^2\Gamma_{\text{Z}}^2} = \frac{12\pi}{M_{\text{Z}}^2} R_e R_f$

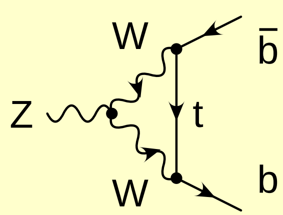
$$\Gamma_{ff} = C\left[(g_{\text{L}}^f)^2 + (g_{\text{R}}^f)^2\right]$$

$$g_{\text{L}}^f = \frac{e}{s_{\text{W}}c_{\text{W}}}\left(I_f^3 - s_{\text{W}}^2 Q_f\right) + \Delta g_{\text{L}}^f$$

$$g_{\text{R}}^f = -eQ_f\frac{s_{\text{W}}}{c_{\text{W}}} + \Delta g_{\text{R}}^f$$

rad. corr.



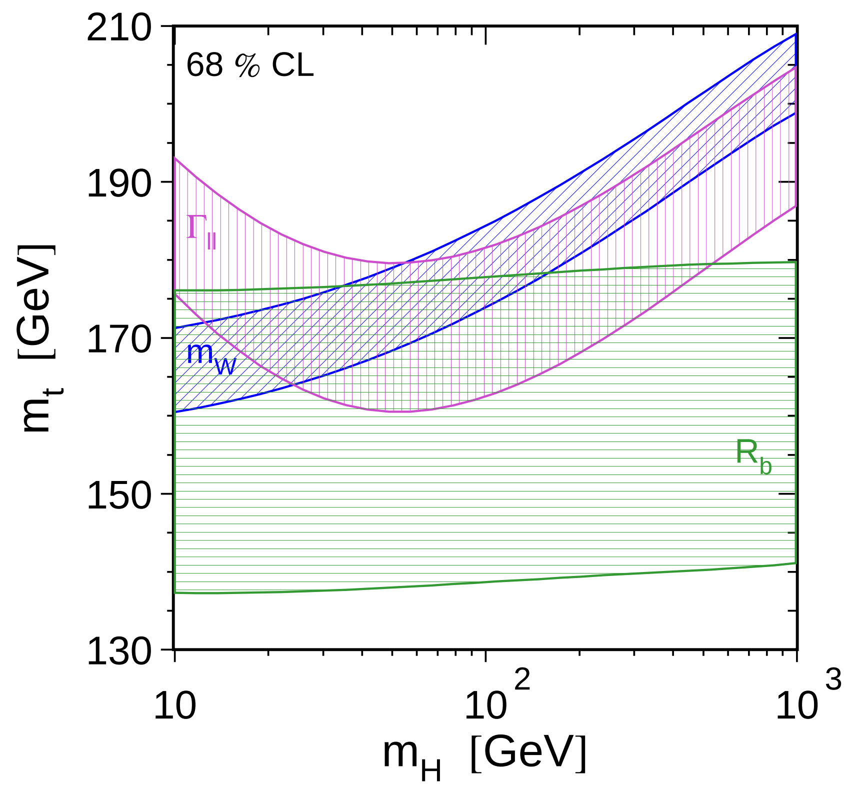


Comparison with experiment:

$\Gamma_{ll} = 83.984(86)$ MeV

$R_b = 0.2163(7)$

Particle Data Group '18



Z-pole asymmetries

Parity violation in $Zf\bar{f}$ couplings:

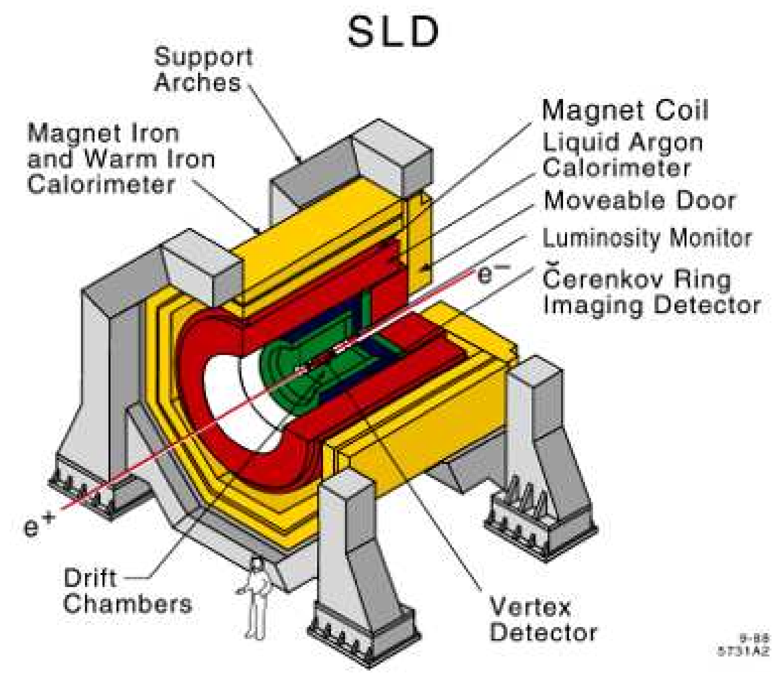
$g^f_{\mathrm{L}} \neq g^f_{\mathrm{R}}$

Left-right asymmetry:

$$A_{\mathrm{LR}} \equiv \frac{1}{P_{e^-}} \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e$$

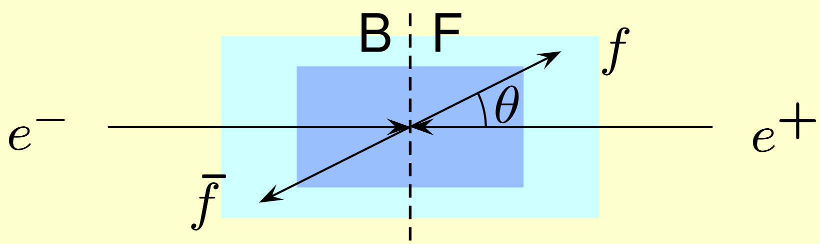
$$\mathcal{A}_f = \frac{2(1 - 4\sin^2\theta^f_{\mathrm{eff}})}{1 + (1 - 4\sin^2\theta^f_{\mathrm{eff}})^2}$$

$$\sin^2\theta^f_{\mathrm{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}$$



Forward-backward asymmetry:

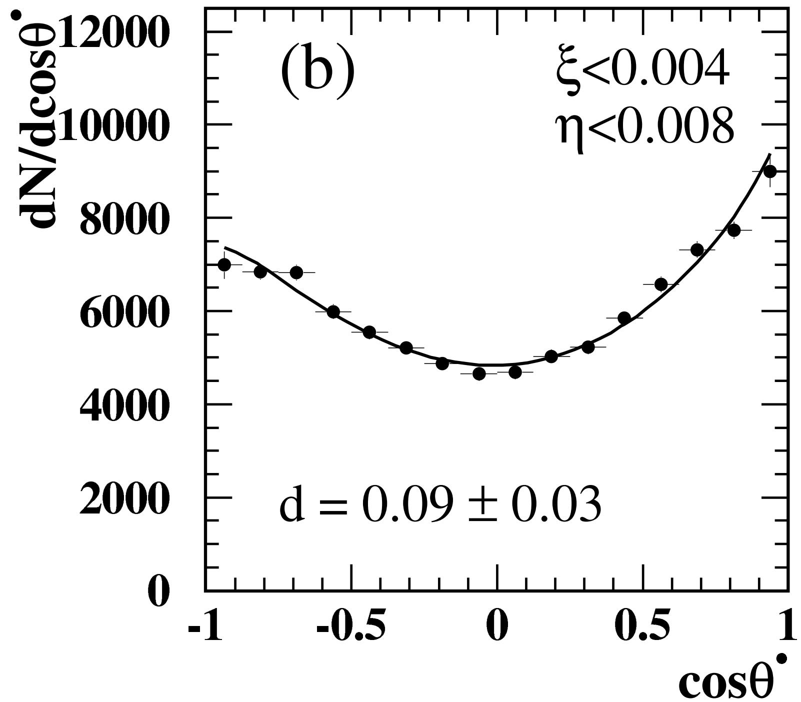
$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$



Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$

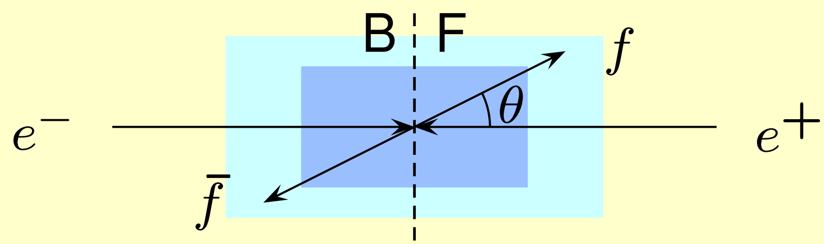
$$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$



OPAL collaboration '01

Forward-backward asymmetry:

$$A_{\text{FB}} \equiv \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$



Polarization asymmetry:

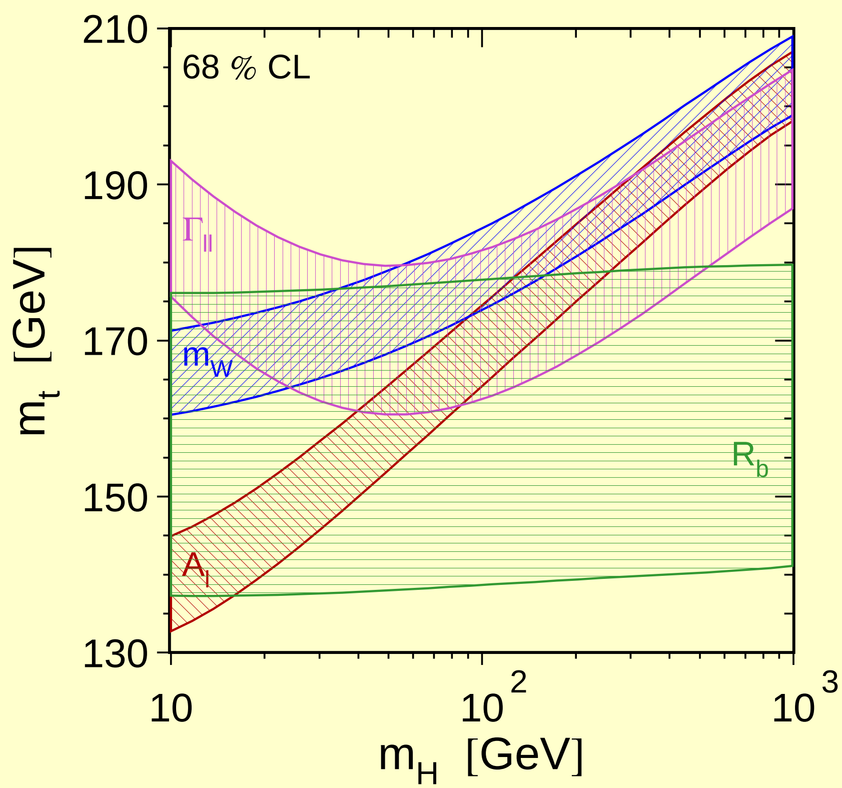
Average τ pol. in $e^+e^- \to \tau^+\tau^-$

$$\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$$

Comparison with experiment:

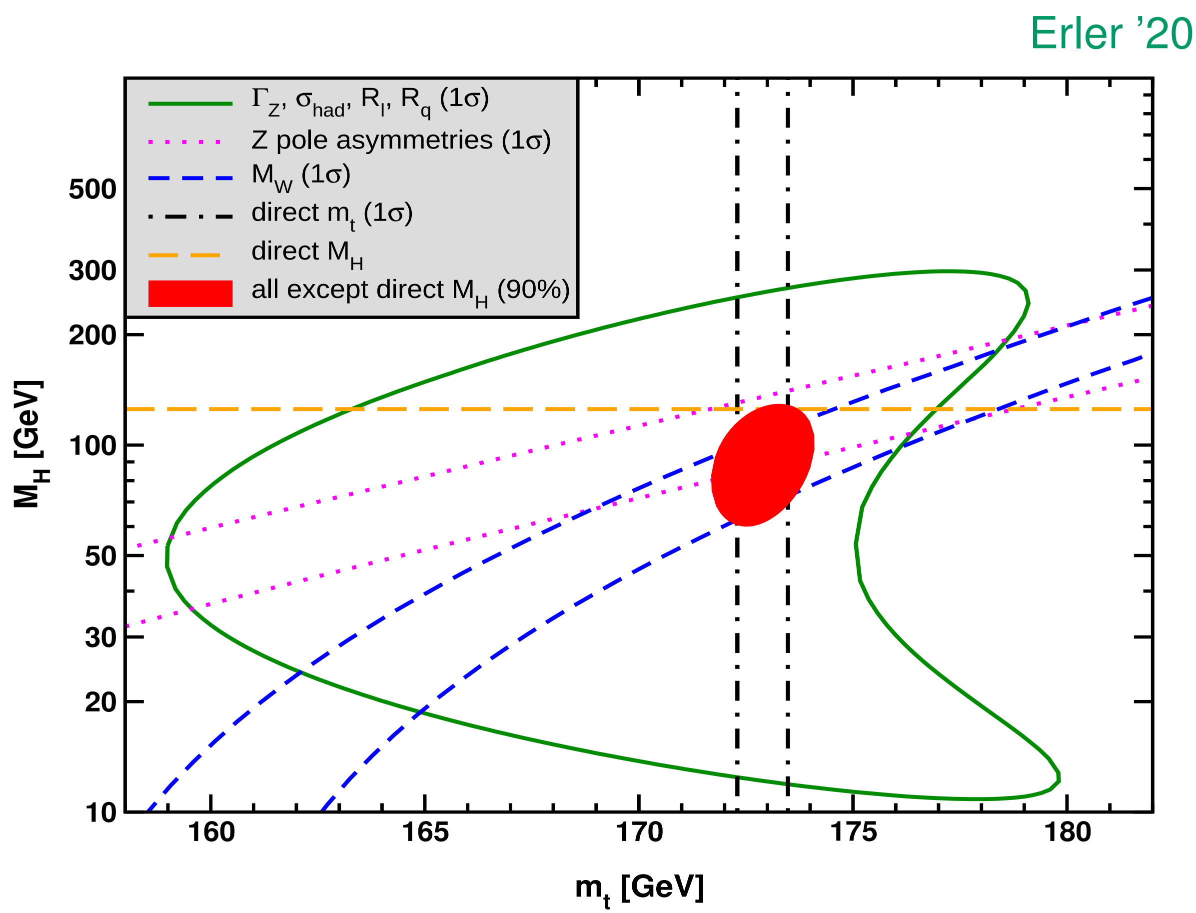
$\mathcal{A}_l = 0.1475(10)$

Particle Data Group '12



Combination and SM fit

Constraints from fit of SM to *all* electroweak precision observables:



Direct measurements:

$M_H = 125.30 \pm 0.13$ GeV

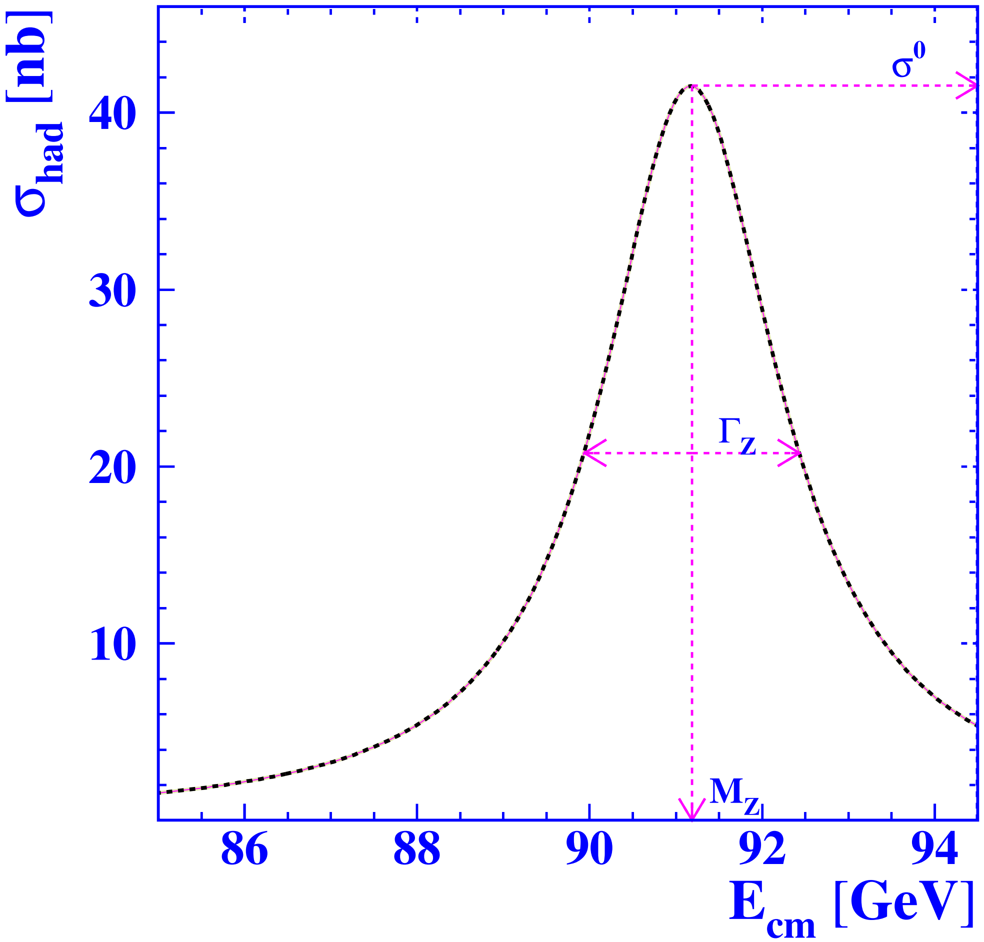
$m_t = 172.89 \pm 0.28$ GeV

Indirect prediction:

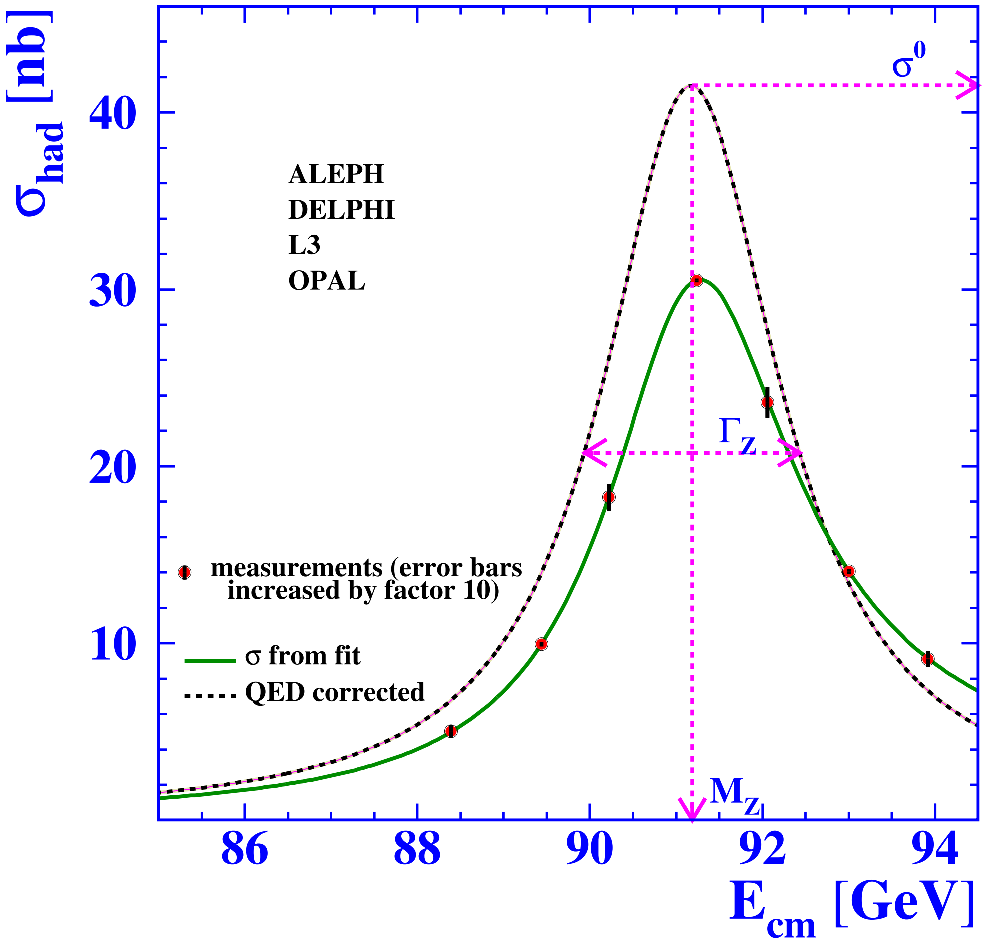
$M_H = 90^{+18}_{-16}$ GeV

$m_t = 176.3 \pm 1.9$ GeV

Real observables vs. pseudo-observables



Real observables vs. pseudo-observables



LEP EWWG ’05

→ Large effects from initial-state QED radiation

→ Theory input necessary to extract relevant EWPOs (“pseudo-observables”)

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97
Ablinger, Blümlein, De Freitas, Schönwald '20

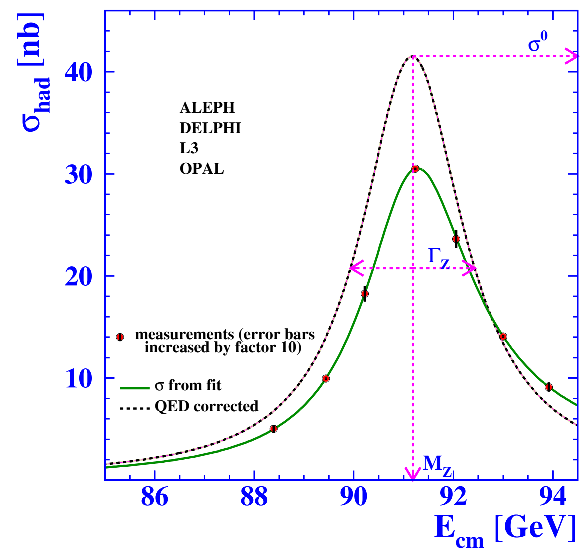
Soft photons (resummed) + collinear photons

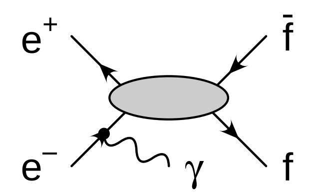
$$\mathcal{R}_{\text{ini}} = \frac{\zeta(1-s'/s)^{\zeta-1}}{\Gamma(1-\zeta)} e^{-\gamma_{\text{E}}\zeta + 3\alpha L/2\pi} - \frac{\alpha}{\pi} L\left(1+\frac{s'}{s}\right) + \alpha^2 L^2 ... + \alpha^3 L^3 ...$$

$$\zeta = \frac{2\alpha}{\pi}(L-1)$$

$$L = \log\frac{s}{m_{\text{e}}^2}$$

LEP EWWG '05



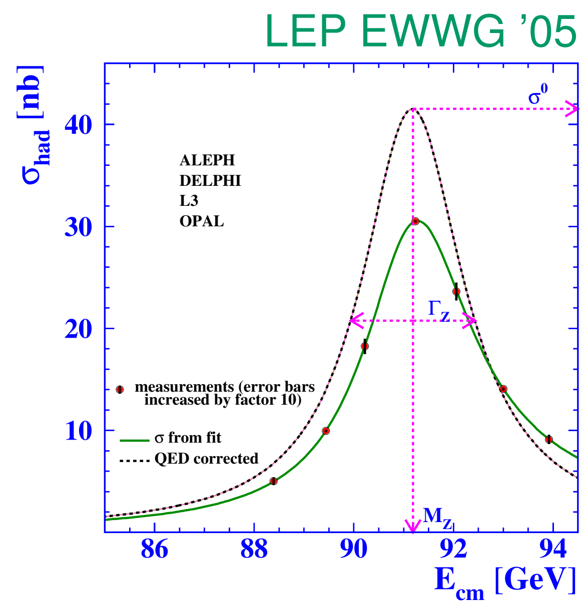


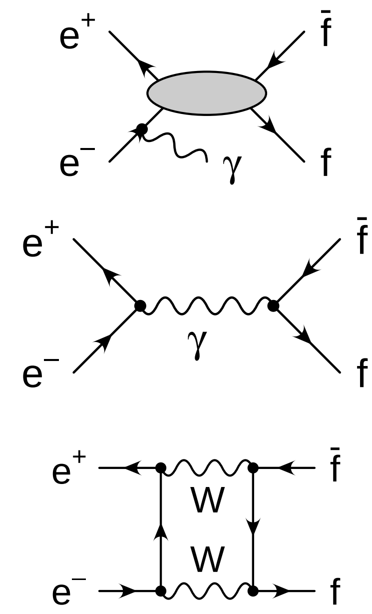
- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$





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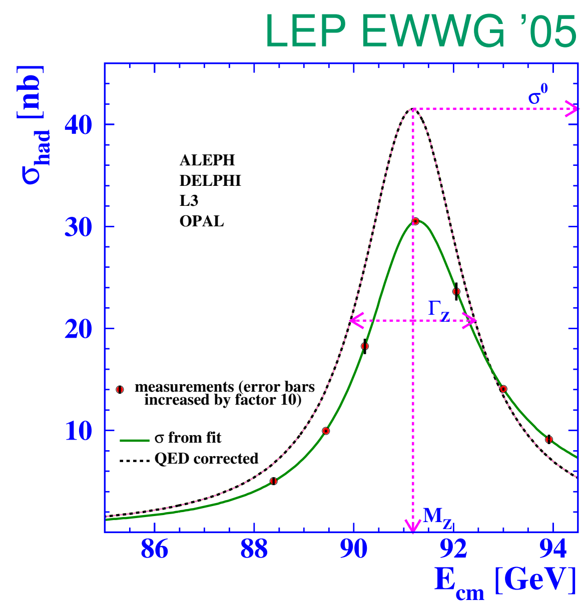
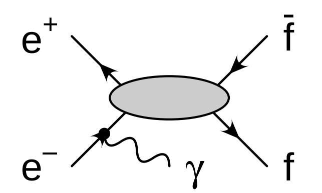
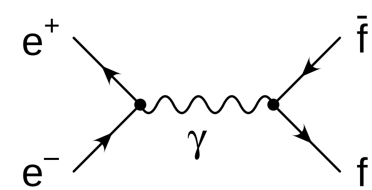
$$\sigma_{\mathrm{hard}} = \sigma_{\mathrm{Z}} + \sigma_{\gamma} + \sigma_{\gamma\mathrm{Z}} + \sigma_{\mathrm{box}}$$

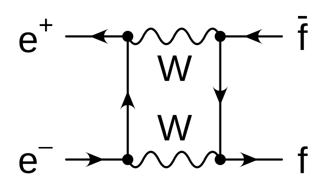
- Z-pole contribution:

$$\sigma_{\mathrm{Z}} = \frac{R}{(s - \overline{M}_{\mathrm{Z}}^2)^2 + \overline{M}_{\mathrm{Z}}^2 \overline{\Gamma}_{\mathrm{Z}}^2} + \sigma_{\mathrm{non-res}}$$









Z lineshape

- Deconvolution of initial-state QED radiation:

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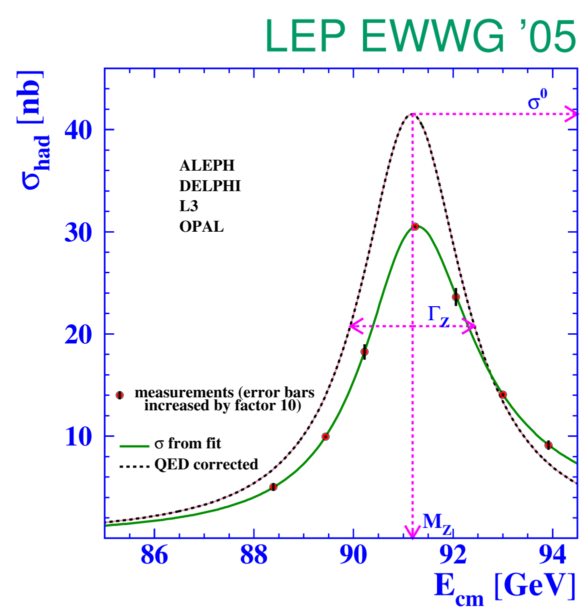
- Subtraction of γ-exchange, γ–Z interference, box contributions:

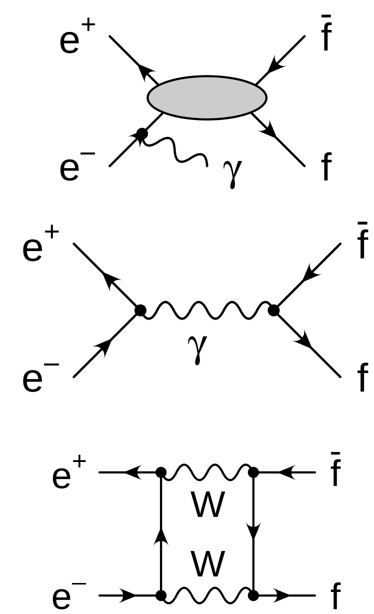
$$\sigma_{\mathrm{hard}} = \sigma_{\mathrm{Z}} + \underbrace{\sigma_\gamma + \sigma_{\gamma\mathrm{Z}} + \sigma_{\mathrm{box}}}$$

- Z-pole contribution:

computed in SM

$$\sigma_{\mathrm{Z}} = \frac{R}{(s-\overline{M}_{\mathrm{Z}}^2)^2 + \overline{M}_{\mathrm{Z}}^2\overline{\Gamma}_{\mathrm{Z}}^2} + \sigma_{\mathrm{non-res}}$$





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- Deconvolution of initial-state QED radiation:

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- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\rm hard} = \sigma_{\rm Z} + \sigma_\gamma + \sigma_{\gamma\rm Z} + \sigma_{\rm box}$$

- Z-pole contribution:

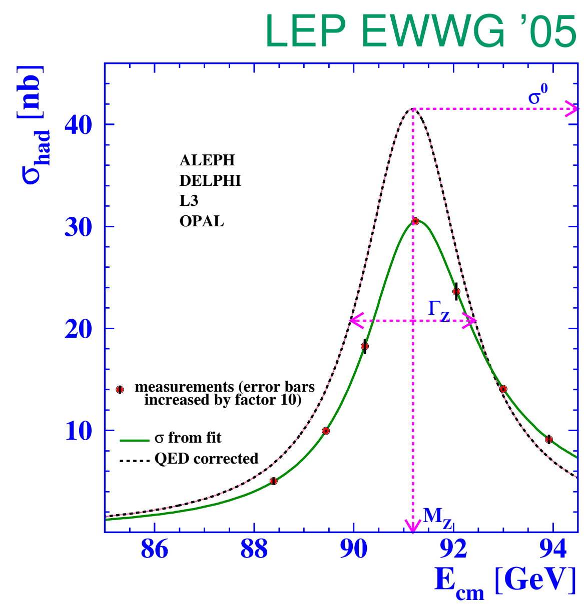
$$\sigma_{\rm Z} = \frac{R}{(s-\overline{M}_{\rm Z}^2)^2 + \overline{M}_{\rm Z}^2\overline{\Gamma}_{\rm Z}^2} + \sigma_{\rm non-res}$$

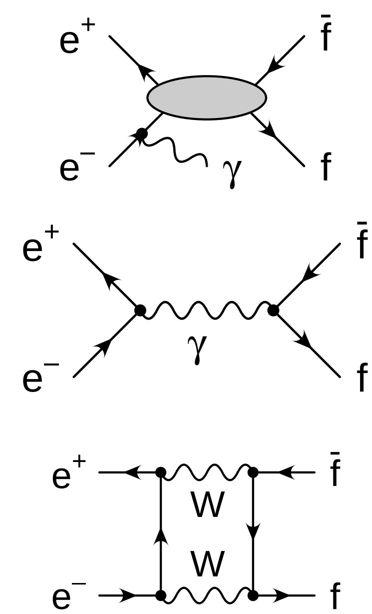
- In experimental analyses:

$$\sigma \sim \frac{1}{(s-M_{\rm Z}^2)^2 + s^2\Gamma_{\rm Z}^2/M_{\rm Z}^2}$$

$$\overline{M}_{\rm Z} = M_{\rm Z}\Big/\sqrt{1+\Gamma_{\rm Z}^2/M_{\rm Z}^2} \approx M_{\rm Z} - 34\ \text{MeV}$$

$$\overline{\Gamma}_{\rm Z} = \Gamma_{\rm Z}\Big/\sqrt{1+\Gamma_{\rm Z}^2/M_{\rm Z}^2} \approx \Gamma_{\rm Z} - 0.9\ \text{MeV}$$



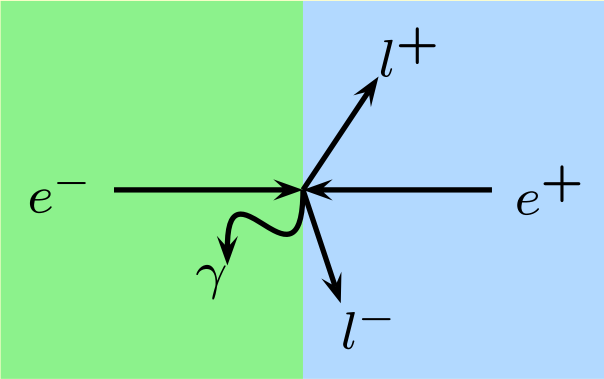


QED radiation in Z asymmetries

QED radiation in principle cancels in asymmetries, e.g. $A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$

Some effects from detector acceptance and cuts

Typical indfluence $< 10^{-3}$



Implementation of QED effects:

a) Analytical formulae, e.g. ZFITTER, TOPAZ0

Arbuzov, Bardin, Christova, Kalinovskaya, Riemann, Riemann, ...
Montagna, Nicrosini, Passarino, Piccinini, ...

b) Monte Carlo event generator, e.g. KORALZ

Jadach, Ward, ...

Theoretical predictions in the SM

Renormalization

- Loop integrals are divergent
- Cancel divergencies by matching input parameters to observable quantities (renormalization)

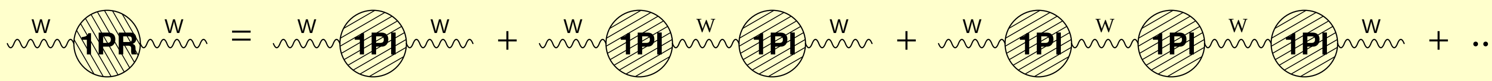
For EWPOs:

- **At Born level** three inputs, e.g. $\alpha,\ G_\mathsf{F},\ M_\mathsf{Z}$
- **Inside loops** additonal inputs: $M_\mathsf{H},\ m_\mathsf{t},\ \alpha_\mathsf{s},\ \ldots$
- Predictions: $M_\mathsf{W},\ \Gamma_{ff},\ \sin^2\theta^f_\mathsf{eff}$

Could choose different set of Born inputs, e.g. $\alpha,\ M_\mathsf{W},\ M_\mathsf{Z}$

Mass renormalization

Peak in cross-section / invariant mass $\quad\rightarrow\quad$ pole of propagator

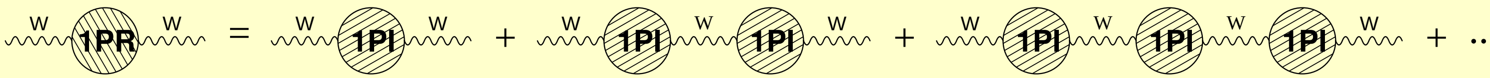


$$= \frac{-i}{q^2 - M_{\mathrm{W}}^2 + \Sigma_T^{\mathrm{W}}(q^2) - \delta M_{\mathrm{W}}^2}$$

Naive approach: denominator = 0 for $q^2 \rightarrow M_{\mathrm{W}}^2$

$\rightarrow \delta M_{\mathrm{W}}^2 = \quad \Sigma_T^{\mathrm{W}}(M_{\mathrm{W}}^2)$

Peak in cross-section / invariant mass $\rightarrow$ pole of propagator



$$= \frac{-i}{q^2 - M_W^2 + \Sigma_T^W(q^2) - \delta M_W^2}$$

Naive approach: denominator = 0 for $q^2 \rightarrow M_W^2$

$\rightarrow \delta M_W^2 = \text{Re}\, \Sigma_T^W(M_W^2)$

For $q^2 \approx M_W^2$: Expand real part of denom. in $q^2 - M_W^2$:

$$\underbrace{\text{Re}\,\Sigma_T^W(M_W^2) - \delta M_W^2}_{0} + (q^2 - M_W^2)\underbrace{(1 + \text{Re}\,\Sigma_T^{W\prime}(M_W^2))}_{\equiv Z} + \ldots$$

$$\frac{-i}{Z(q^2 - M_W^2) + i\,\underbrace{\text{Im}\,\Sigma_T^W(q^2)}_{\propto q^2}} \quad \overset{|\mathcal{M}|^2}{\Rightarrow} \quad \frac{Z^{-2}}{(q^2 - M_W^2)^2 + q^2 \frac{\Gamma_W^2}{M_W^2}}$$

Mass renormalization

Correct approach: physical (and gauge-invariant) pole is complex!

$$q^2 - \overline{M}_\mathrm{W}^2 + \Sigma_T^\mathrm{W}(q^2) - \delta\overline{M}_\mathrm{W}^2 = 0 \quad \text{for } q^2 \to \mu_\mathrm{W}^2 \equiv \overline{M}_\mathrm{W}^2 - i\overline{M}_\mathrm{W}\overline{\Gamma}_\mathrm{W}$$

$$\to \delta\overline{M}_\mathrm{W}^2 = \mathrm{Re}\,\Sigma_T^\mathrm{W}(\mu_\mathrm{Z}^2)$$

$$\approx \mathrm{Re}\,\Sigma_T^\mathrm{W}(\overline{M}_\mathrm{W}^2) + \overline{M}_\mathrm{W}\overline{\Gamma}_\mathrm{W}\,\mathrm{Im}\,\Sigma_T^{\mathrm{W}\prime}(\overline{M}_\mathrm{W}^2)$$

$$\overline{M}_\mathrm{W}\overline{\Gamma}_\mathrm{W} = \mathrm{Im}\,\Sigma_T^\mathrm{W}(\mu_\mathrm{Z}^2)$$

Expand denom. in $q^2 - \mu_\mathrm{Z}^2$:

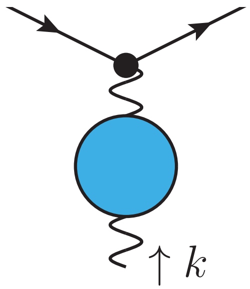
$$\underbrace{\mathrm{Re}\,\Sigma_T^\mathrm{W}(\mu_\mathrm{W}^2) - \delta M_\mathrm{W}^2}_{0} + \underbrace{i\,\mathrm{Im}\,\Sigma_T^\mathrm{W}(\mu_\mathrm{W}^2) - i\overline{M}_\mathrm{W}\overline{\Gamma}_\mathrm{Z}}_{0} + (q^2 - \mu_\mathrm{W}^2)\underbrace{(1 + \Sigma_T^{\mathrm{W}\prime}(\mu_\mathrm{W}^2))}_{\equiv Z} + \ldots$$

$$\frac{-i}{Z(q^2 - \mu_\mathrm{W}^2)} \quad \overset{|\mathcal{M}|^2}{\Rightarrow} \quad \frac{Z^{-2}}{(q^2 - \overline{M}_\mathrm{W}^2)^2 + \overline{M}_\mathrm{W}^2\overline{\Gamma}_\mathrm{W}^2}$$

Charge renormalization

Counterterm for el. charge related to photon vacuum pol. Π through Ward id.:

At 1-loop: $\delta e = \frac{e}{2}\,\Pi(k^2{=}0)$ $\left[\Pi(k^2) = \frac{\Sigma_T^\gamma(k^2)}{k^2}\right]$

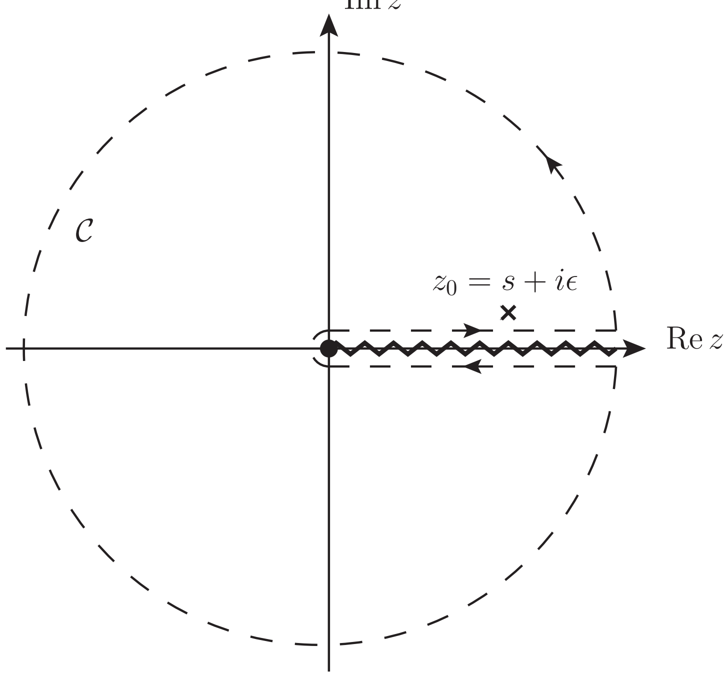


$$\Pi(0) = \sum_f N_C^f Q_f^2 \frac{\alpha}{3\pi}\left(\frac{2}{4-d} - \gamma_E - \ln\frac{m_f^2}{4\pi\mu^2}\right)$$

- m_f not well-defined for $f = u, d, s, (c)$
- At $k^2{=}0$ we have hadrons, not quarks

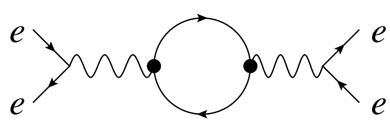
Use **dispersion relation**:

$$\mathrm{Re}\,\Pi(s) = \frac{1}{\pi}\int_0^\infty ds'\,\frac{\mathrm{Im}\,\Pi(s')}{s' - s - i\epsilon}$$



Charge renormalization

$$\text{Im}\,\Pi_{\text{had}}(s') = \frac{1}{e^2}\ \text{Im}\,\mathcal{M}\left\{ \text{[diagram]} \right\}_{\theta=0}$$



$$= \frac{s'}{e^2}\sum_q \sigma[e^+e^- \to q\bar{q}]$$ [optical theorem]

$$= \frac{s'}{e^2} R(s')\ \underbrace{\sigma[e^+e^- \to \mu^+\mu^-]}_{4\pi\alpha^2/(3s')}$$

$$R(s) = \frac{\sigma[e^+e^- \to \text{hadrons}]}{\sigma[e^+e^- \to \mu^+\mu^-]}$$ [from data]

Finite combination

$$\Delta\alpha_{\text{had}} \equiv \Pi_{\text{had}}(0) - \Pi_{\text{had}}(M_Z^2) = -\frac{\alpha}{3\pi}\int_0^\infty ds'\,\frac{R(s')}{s'(s' - M_Z^2 - i\epsilon)}$$

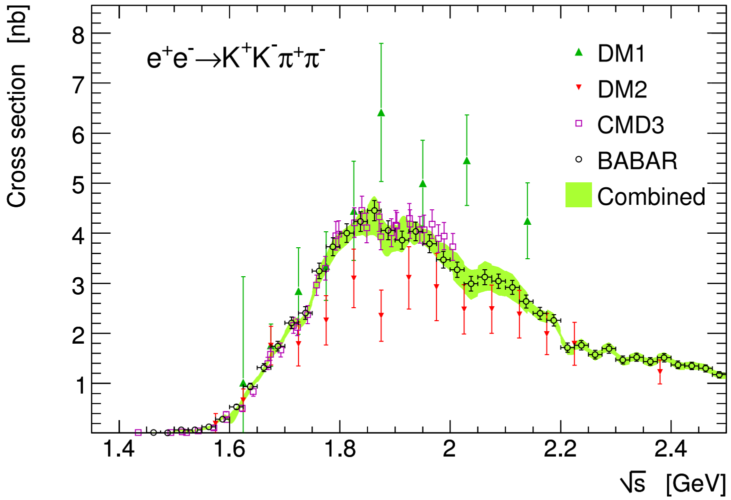
$$\delta e = \frac{e}{2}\Big[\underbrace{\Pi(M_Z^2)}_{\text{can be computed in perturb. theory}} + \Delta\alpha\Big]$$

can be computed in perturb. theory

a) $\Delta\alpha_{\text{had}}$ from $e^+e^- \to$ had.
using dispersion relation

- New data from BaBar, VEPP, BES, KLOE

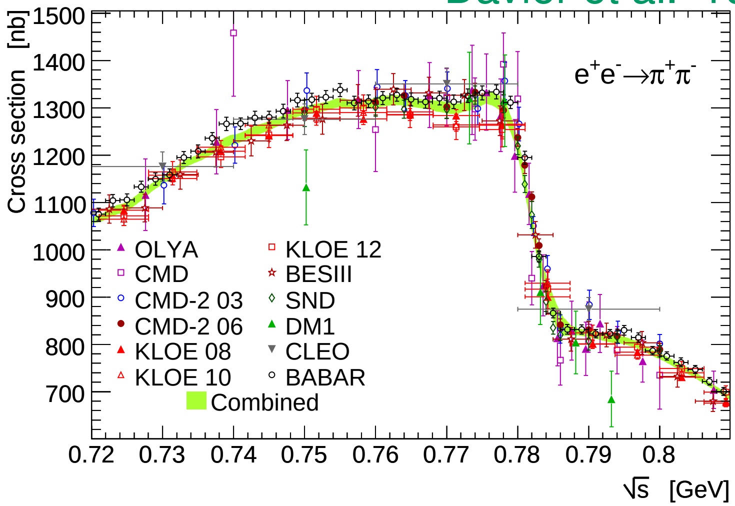
Davier et al. '17



a) $\Delta\alpha_{\text{had}}$ from $e^+e^- \to$ had.
using dispersion relation

- New data from BaBar, VEPP, BES, KLOE
- Discrep. between BaBar & KLOE has small impact

Davier et al. '19

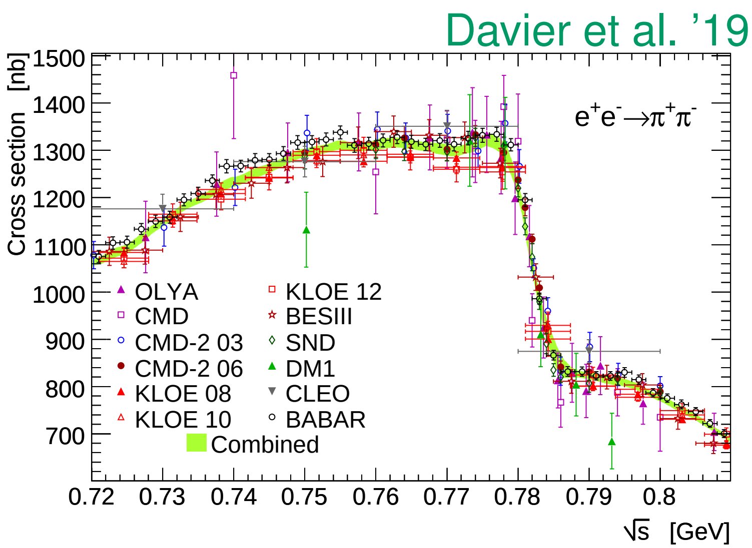


$\to$ Consistent results $\Delta\alpha_{\text{had}} \approx 0.0276 \pm 0.0001$

Davier et al. '19; Jegerlehner '19; Keshavarzi, Nomura, Teubner '19

Charge renormalization

Davier et al. ’19



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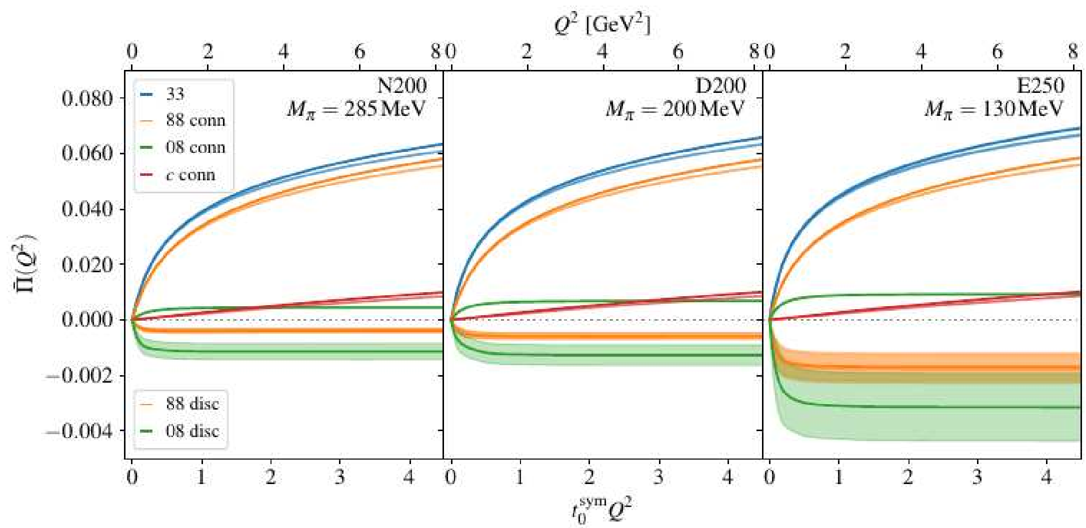
b) $\Delta\alpha_{\rm had}$ from Lattice QCD
(work in progress)

Burger et al. ’15
Cè et al. ’19



Reviews: 1906.05379, 2012.11642

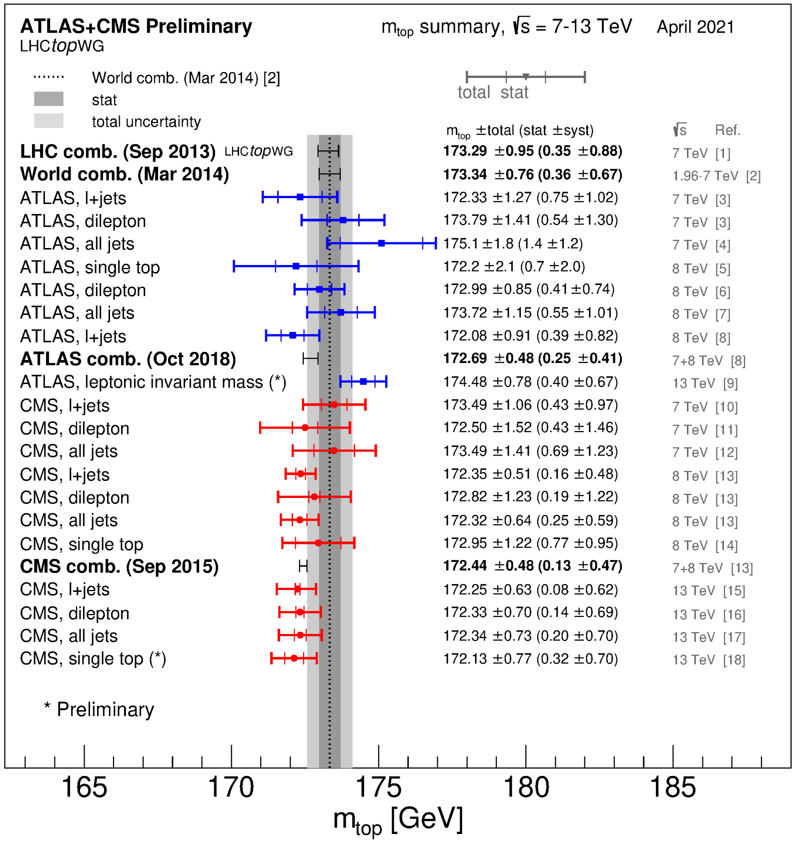
- $M_Z,\ \Gamma_Z$: From $\sigma(\sqrt{s})$ lineshape; $\delta M_Z,\ \delta\Gamma_Z \sim 0.1$ MeV at FCC-ee
  $\rightarrow$ Main theory uncertainties: QED ISR

- m_t: Most precise measurement
  at LHC: $\delta m_t \sim 0.3$ GeV PDG '20

  Theoretical ambiguity in mass def.:

  Hoang, Plätzer, Samitz '18

$$m_t^{\text{CB}}(Q_0) - m_t^{\text{pole}}$$
$$= -\tfrac{2}{3}\alpha_s(Q_0)\,Q_0 + \mathcal{O}(\alpha_s^2 Q_0)$$
$$\approx 0.5 \pm 0.2_{\text{pert.}} \pm 0.2_{\text{np.}}\,\text{GeV}$$



SM input parameters

Reviews: 1906.05379, 2012.11642

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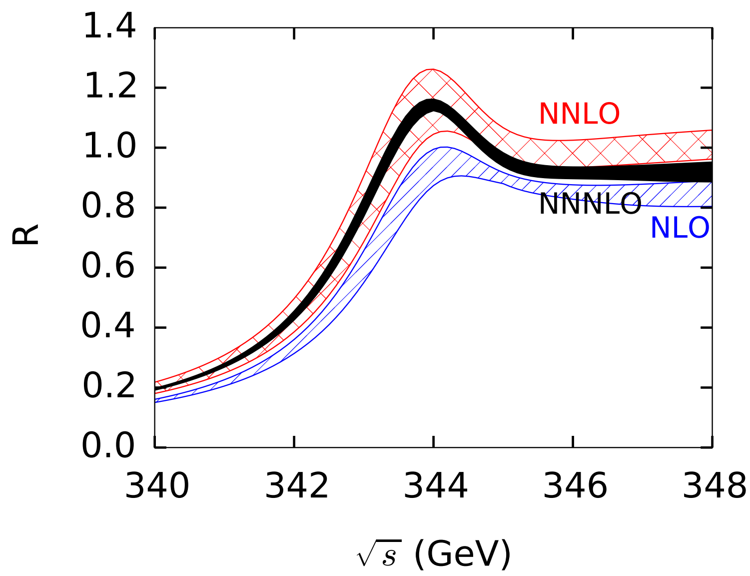
- m_t: Most precise measurement at LHC: $\delta m_t \sim 0.3$ GeV

  From $e^+e^- \rightarrow t\bar{t}$ at $\sqrt{s} \sim 350$ GeV:

  Impact of theory modelling:

$$\delta m_t^{\overline{\rm MS}} = [\quad]_{\rm exp} \oplus [50\ \text{MeV}]_{\rm QCD} \oplus [10\ \text{MeV}]_{\text{mass def.}} \oplus [70\ \text{MeV}]_{\alpha_s} > 100\ \text{MeV}$$



Beneke et al. '15

SM input parameters

Reviews: 1906.05379, 2012.11642

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  future improvements:

  $$[20\text{ MeV}]_{exp} \oplus [30\text{ MeV}]_{QCD} \oplus [10\text{ MeV}]_{\text{mass def.}} \oplus [15\text{ MeV}]_{\alpha_s} \lesssim 50\text{ MeV}$$

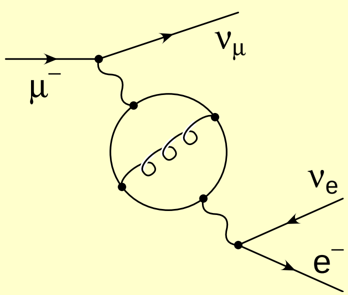
  $[30\text{ MeV}]_{QCD}$: (h.o. resummation)

  $[15\text{ MeV}]_{\alpha_s}$: ($\delta\alpha_s \lesssim 0.0002$)

- m_t:
  $\rightarrow$ Impact on EWPOs:

$$\delta m_t = 0.5 \text{ GeV} \quad \Rightarrow \quad \delta M_W \approx 3 \text{ MeV}$$
$$\delta \sin^2\theta^\ell_{eff} \approx 1.5 \times 10^{-5}$$

- α_s: $\rightarrow$ lecture by P. Skands
  Many methods, e.g. Lattice QCD ($\alpha_s \sim 0.118$),
  e^+e^- event shapes and DIS prefer ($\alpha_s \sim 0.114$)



$\rightarrow$ Impact on EWPOs:

$$\delta\alpha_s = 0.004 \quad \Rightarrow \quad \delta M_W \approx 3 \text{ MeV}$$
$$\delta \sin^2\theta^\ell_{eff} \approx 1.5 \times 10^{-5}$$

Currently not dominant, but also not negligible error

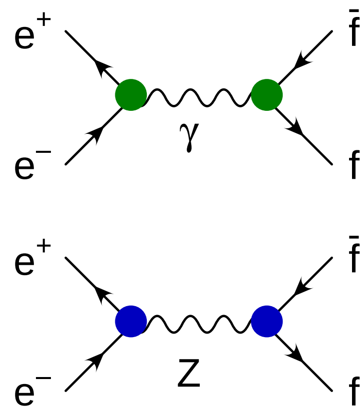
- $\Delta\alpha_{\text{had}}$:

a) From $e^+e^- \to$ had. using dispersion relation
Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely — Jegerlehner '19

b) Direct determination from $e^+e^- \to \mu^+\mu^-$ off the Z peak — Janot '15

$$|\mathcal{M}_{ij}|^2 \propto |g_i^\ell|^2 |g_j^\ell|^2 + \frac{s - M_Z^2}{s} \alpha(M_Z) |g_{i,j}^\ell|^2 + \ldots$$

↑ determined from Z pole



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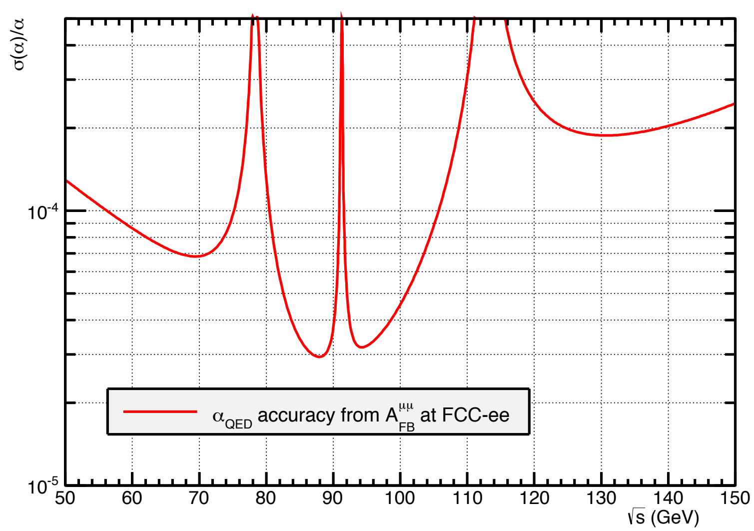
→ Use $A_{\text{FB}}^{\mu\mu}$ and two cms energies to reduce systematics

→ Sensitivity maximized for $\sqrt{s_1} \sim 88$ GeV, $\sqrt{s_2} \sim 95$ GeV

→ $\delta(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$ for $\mathcal{L}_{\text{int}} = 85$ ab^{-1}



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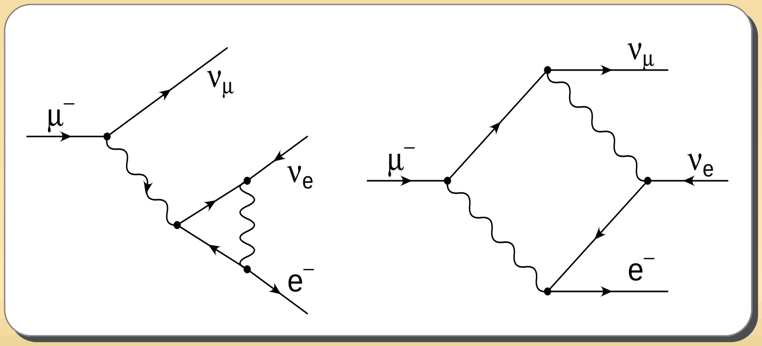
Requires theory input:
2-/3-loop corrections for $e^+e^- \to \mu^+\mu^-$

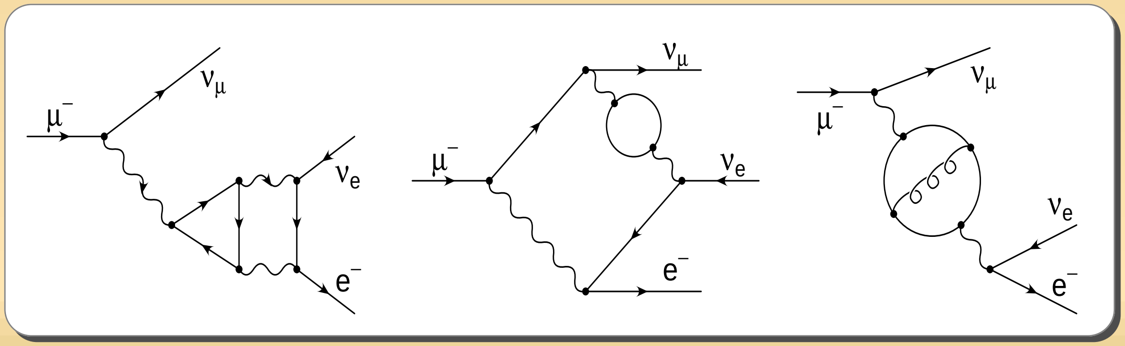
SM loop corrections

Objective: Comparison of EWPO measurements with SM theory predictions

	δM_W [MeV]	$\delta \sin\theta^{\text{lept}}_{\text{eff}}$ [10^{-5}]	
now	$\pm$ 12	$\pm$ 16	
LHC	$\pm$ 10	$\pm$ 15	
ILC	$\pm$ 2.5	$\pm$ 1.3	
1-loop	$\pm$ 450	$\pm$ 1000	Marciano, Sirlin '80
2-/3-loop QCD	$\pm$ 70	$\pm$ 45	Djouadi et al. '88 Chetyrkin, Kühn, Steinhauser '95
ferm. 2-loop EW	$\pm$ 50	$\pm$ 90	Freitas et al. '00 Awramik, Czakon '03 Awramik, Czakon, Freitas, Weiglein '04
bos. 2-loop EW	$\pm$ 2	$\pm$ 1	Awramik, Czakon, Freitas '06
leading 3-loop	$\pm$ 5	$\pm$ 25	Faisst, Kühn, Seidensticker, Veretin '03

Experimental precision sensitive to 2-/3-loop effects





Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_W, Z-pole observables

  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas, Huang '12

- Approximate 3- and 4-loop results (to ρ parameter)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06

Leading terms in SM prediction

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8(1-\frac{M_W^2}{M_Z^2})M_W^2}(1+\Delta r) \qquad \Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}$$

$$\sin^2\theta_{\text{eff}}^f = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1+\Delta\kappa) \qquad \Delta\kappa = \frac{c_W^2}{s_W^2}\Delta\rho + \Delta\kappa_{\text{rem}}$$

$$\begin{aligned} \Delta\rho &\propto \alpha_t && \text{(1 loop)} \\ &\propto \alpha_t^2,\ \alpha_t\alpha_s && \text{(2-loop)} \\ &\propto \alpha_t^3,\ \alpha_t^2\alpha_s,\ \alpha_t\alpha_s^2 && \text{(3-loop)} \end{aligned}$$

$$\alpha_t \equiv \frac{y_t^2}{4\pi} = \frac{g^2}{8\pi}\frac{m_t^2}{M_W^2}$$

$\Delta\alpha \approx 6\%$

$\frac{c_W^2}{s_W^2}\Delta\rho \approx 3\%$

$\Delta r_{\text{rem}},\ \Delta\kappa_{\text{rem}} \lesssim 1\%$

Leading terms in SM prediction

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8(1-\frac{M_W^2}{M_Z^2})M_W^2}(1+\Delta r) \qquad \Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}$$

depends on $\log M_H$

$$\sin^2\theta^f_{\text{eff}} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1+\Delta\kappa) \qquad \Delta\kappa = \frac{c_W^2}{s_W^2}\Delta\rho + \Delta\kappa_{\text{rem}}$$

$\Delta\rho \propto \alpha_t$ (1 loop)

$\propto \alpha_t^2,\ \alpha_t\alpha_s$ (2-loop)

$\propto \alpha_t^3,\ \alpha_t^2\alpha_s,\ \alpha_t\alpha_s^2$ (3-loop)

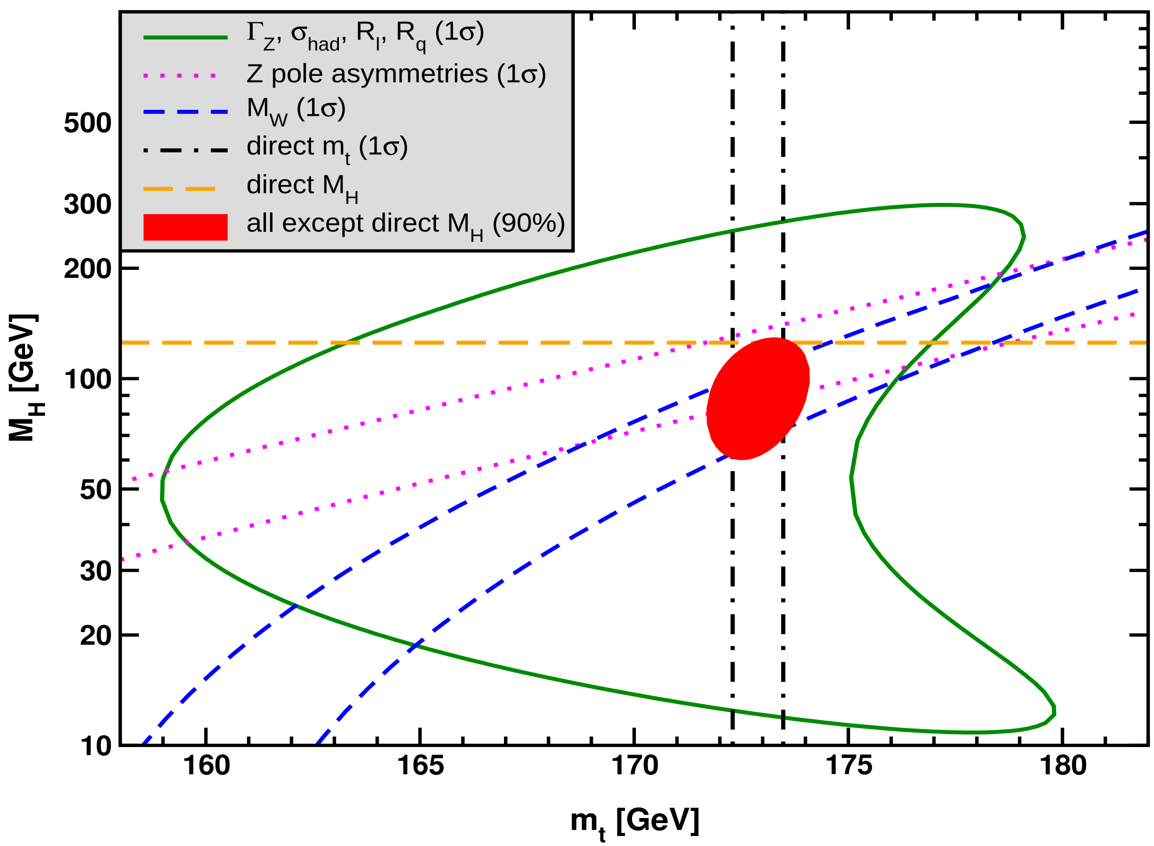
$$\alpha_t \equiv \frac{y_t^2}{4\pi} = \frac{g^2}{8\pi}\frac{m_t^2}{M_W^2}$$

$\Delta\alpha \approx 6\%$

$\frac{c_W^2}{s_W^2}\Delta\rho \approx 3\%$

$\Delta r_{\text{rem}},\ \Delta\kappa_{\text{rem}} \lesssim 1\%$

Quadratic dependence on m_t, but logarithmic dependence on M_H (at 1-loop)



Erler '20

Custodial symmetry

- Higgs doublet $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ has 4 indep. components, e.g. $\Omega = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$

- Higgs potential $V = -\mu^2(\phi^\dagger\phi) + \frac{\lambda}{4}(\phi^\dagger\phi)^2 = -\frac{\mu^2}{2}\,\mathrm{Tr}\{\Omega^\dagger\Omega\} + \frac{\lambda}{16}(\mathrm{Tr}\{\Omega^\dagger\Omega\})^2$
  is invariant under $\Omega \to L\Omega R^\dagger$, $L \in \mathrm{SU(2)_L}$ $R \in \mathrm{SU(2)_R}$

  [also gauge couplings, except hypercharge]

- Higgs vev $\langle\Omega\rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$ in invariant under $\Omega \to V\Omega V^\dagger$, $V \in \mathrm{SU(2)_{diag}}$

  "custodial symmetry"

- Yukawa couplings for $y_t = y_b = y$:
  $\mathcal{L}_Y = -y_t\overline{Q}_{3L}\tilde{\phi}\,t_R - y_b\overline{Q}_{3L}\phi\,b_R = -y\overline{Q}_{3L}\Omega\,Q_{3R}$ $\quad Q_{3L} = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ $\quad Q_{3R} = \begin{pmatrix} t_R \\ b_R \end{pmatrix}$
  invariant under $\mathrm{SU(2)_{diag}}$ $(Q_{L,R} \to VQ_{L,R})$

- $\mathrm{SU(2)_{diag}}$ broken for $y_t \neq y_b$ $\rightarrow$ large corrections in $\Delta\rho$

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_\mathrm{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_\mathrm{s}$, $\alpha\alpha_\mathrm{s}^2$
σ^0_had	41540 ± 37 pb	6 pb	α^3, $\alpha^2\alpha_\mathrm{s}$
$R_b \equiv \Gamma^b_\mathrm{Z}/\Gamma^\mathrm{had}_\mathrm{Z}$	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_\mathrm{s}$
$\sin^2\theta^\ell_\mathrm{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\mathrm{s}$

- Theory error estimate is not well defined, ideally $\Delta_\mathrm{th} \ll \Delta_\mathrm{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = y_t^2/(4\pi)$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.21 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

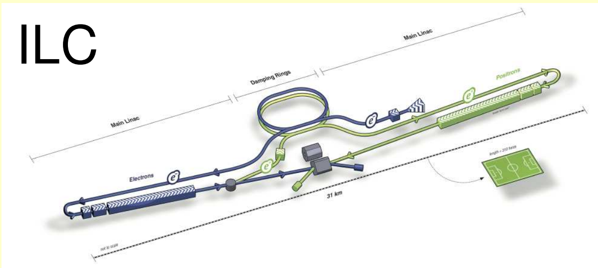
$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)}\mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

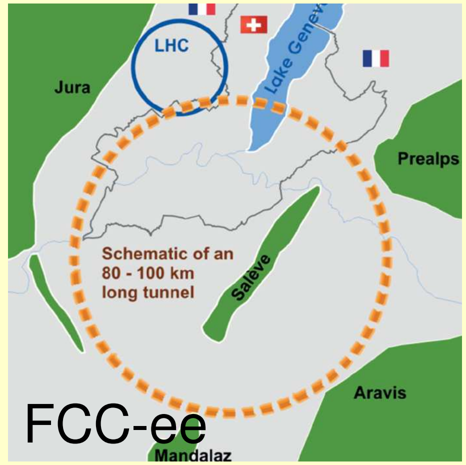
- Parametric prefactors:

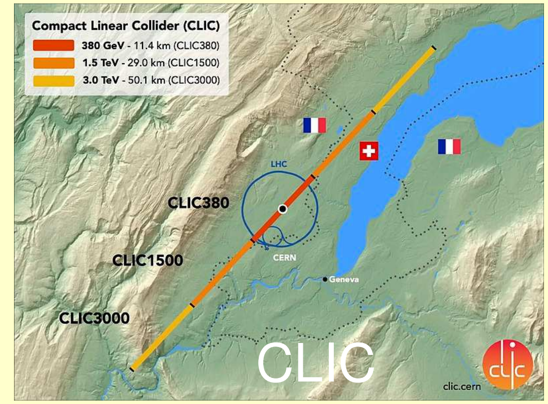
$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

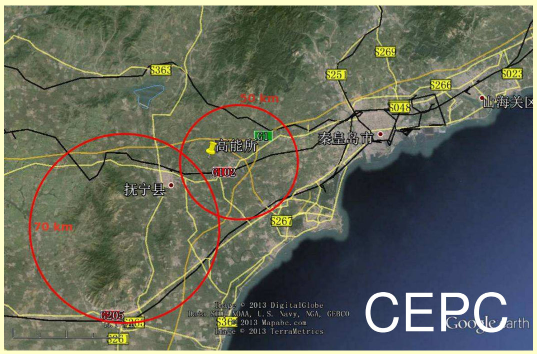
Total: $\delta\Gamma_Z \approx 0.5$ MeV

Electroweak precision physics with future e^+e^- colliders









- circular colliders: high-lumi run at $\sqrt{s} \sim M_Z$
- linear colliders: radiative return $e^+e^- \to \gamma Z$

$\sqrt{s}$	M_Z	$2M_W$	240–250 GeV	350–380 GeV
ILC	*100 fb^{-1}*	*500 fb^{-1}*	2 ab^{-1}	200 fb^{-1} (10 pts.)
CLIC	–	–	–	1 ab^{-1}
FCC-ee	230 ab^{-1}	10 ab^{-1} (2 pts.)	irrel. for EW phys.	200 fb^{-1} (8 pts.)
CEPC	45 ab^{-1}	2.6 ab^{-1} (3 pts.)	irrel. for EW phys.	–

$\longrightarrow$ lecture by P. Azzi

Electroweak precision tests at future e^+e^- colliders

Anticipated precison for EWPOs:

Quantity	current	ILC	CLIC	FCC-ee	CEPC
M_Z [MeV]	2.1	–	–	0.1	0.5
Γ_Z [MeV]	2.3	–	–	0.1	0.5
M_W [MeV]	12	2.5	?	0.7	1.0
$\sin^2\theta^\ell_{\rm eff}$ $[10^{-5}]$	14	2	7.8	0.5	2.3
$R_b = \Gamma^b_Z/\Gamma^{\rm had}_Z$ $[10^{-5}]$	66	23	38	6	4.3

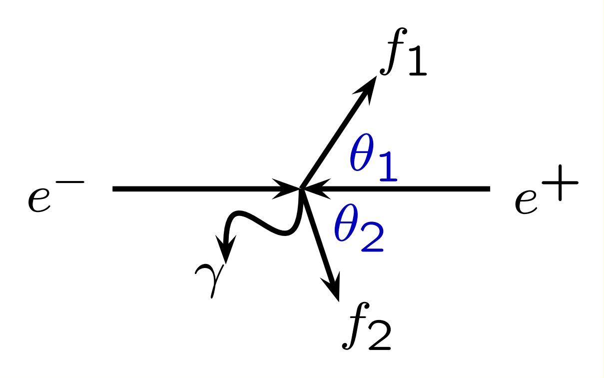
- Can be optimized with different run scenarios
- Polarized beams at ILC (P_{e^-}=0.8, P_{e^+}=0.3) and CLIC (P_{e^-}=0.8)

Electroweak precision at $\sqrt{s} = 250$ GeV

EWPOs accessible through **radiative return** $e^+e^- \to \gamma Z$

- γ mostly collinear with beam
- Reduction in cross-section by $\sim \frac{\alpha}{\pi} \ln \frac{s}{m_e^2} \sim 0.06$
- Precise det. of m_{ff} from measured angles:

$$m_{ff}^2 = s\,\frac{1-\beta}{1+\beta}, \quad \beta = \frac{|\sin(\theta_1+\theta_2)|}{\sin\theta_1 + \sin\theta_2}$$



- Additional backgrounds from $e^+e^- \to WW, ZZ$ that are not flat in m_{ff}

Ueno '19

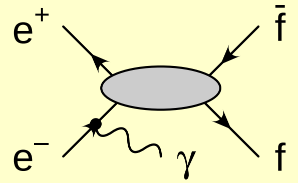
$Z\gamma$ electroweak precision: observables

Fujii et al. '19

- $A_{\mathrm{LR}} \to \sin^2\theta^{\ell}_{\mathrm{eff}}$ (limited by sys. err. on beam polarization)
- $A^{\mu,\tau,b}_{\mathrm{FB}}$ (statistics limited)
- R_ℓ, R_c, R_b (limited by sys. err. on flavor tag)
- No competitive measurements on M_Z, Γ_Z, σ^0 (need to use LEP values)

$Z\gamma$ electroweak precision: theory input

Leading effect: Soft+collinear multi-γ ISR



$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Bardin et al. '91; Skrzypek '92
Montagna, Nicrosini, Piccinini '97

$$\mathcal{R}_{\text{ini}} = \sum_n \left(\frac{\alpha}{\pi}\right)^n \sum_{m=0}^{n} h_{nm} \ln^m\left(\frac{s}{m_e^2}\right)$$

Universal ($m{=}n$) logs known to $n = 6$,
also some sub-leading terms

Ablinger, Blümlein, De Freitas, Schönwald '20

Exclusive description: MC tools

Subleading effects:

Radiative corrections to $e^+e^- \to f\bar{f}\gamma\,(+n\gamma)$

- Some corrections cancel for A_{LR}, A_{FB}, BRs
- NLO for $ee \to f\bar{f}\gamma$ + NNLO for $ee \to Z\gamma$, $Z \to f\bar{f}$ could be sufficient

W mass measurement from $e^+e^- \to WW$: Baak et al. '13

- $\ell\nu_\ell\ell'\nu_{\ell'}$: Endpoints of E_ℓ or other distributions
- $\ell\nu_\ell jj$: Kinematic reconstruction
- $jjjj$: Systematic uncertainty from color reconnection

Expected precision with $\mathcal{L}_{\text{int}} = 2\ \text{ab}^{-1}$ at $\sqrt{s} = 250$ GeV: $\Delta M_{\text{W}} \approx 2.5\ \text{MeV}$

Theory needs: Small impact of loop corrections, but accurate decription of FSR QED effects needed

Future theory projections

To test SM and probe new physics:

Compare measurements of EWPOs with SM theory predictions

Quantity	FCC-ee	CEPC	current theory*
M_W [MeV]	0.7	1.0	4
Γ_Z [MeV]	0.1	0.5	0.4
$R_b = \Gamma^b_Z/\Gamma^{\text{had}}_Z$ [10^{-5}]	6	4.3	11
$\sin^2\theta^\ell_{\text{eff}}$ [10^{-5}]	0.5	2.3	4.5

* **Current state-of-art:** full two-loop + leading 3-loop

Future theory projections

To test SM and probe new physics:

Compare measurements of EWPOs with SM theory predictions

Freitas, Heinemeyer, et al. '19

Quantity	FCC-ee	CEPC	current theory*	projected theory†
M_W [MeV]	0.7	1.0	4	1
Γ_Z [MeV]	0.1	0.5	0.4	0.15
$R_b = \Gamma_Z^b/\Gamma_Z^{\rm had}$ [10^{-5}]	6	4.3	11	5
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	0.5	2.3	4.5	1.5

* **Current state-of-art:** full two-loop + leading 3-loop

† **Future scenario:** $\mathcal{O}(\alpha\alpha_s^2)$, $\mathcal{O}(N_f\alpha^2\alpha_s)$, $\mathcal{O}(N_f^2\alpha^3)$ + leading 4-loop

(N_f^n = at least n closed fermion loops)

Theory and parametric uncertainties

	CEPC	FCC-ee	Param. error CEPC†	Param. error FCC-ee*
M_{W} [MeV]	1	0.7	2.1	0.6
Γ_Z [MeV]	0.5	0.1	0.15	0.1
R_b [10^{-5}]	4.3	6	< 1	< 1
$\sin^2\theta^{\ell}_{\mathrm{eff}}$ [10^{-5}]	2.3	0.5	2	1

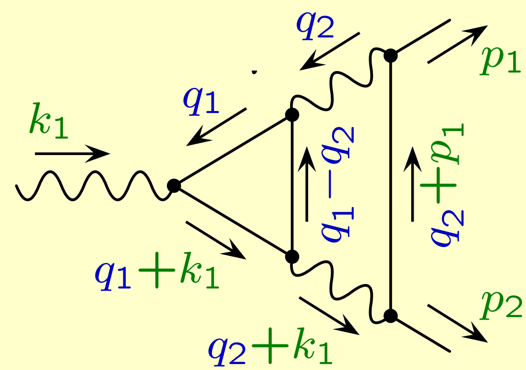
Parametric inputs from theory prediction in SM:

†**CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_{\mathrm{s}} = 0.0002$, $\delta M_{\mathrm{Z}} = 0.5$ MeV,
$\delta(\Delta\alpha) = 5 \times 10^{-5}$

***FCC-ee:** $\delta m_t = 50$ MeV, $\delta\alpha_{\mathrm{s}} = 0.0002$, $\delta M_{\mathrm{Z}} = 0.1$ MeV,
$\delta(\Delta\alpha) = 3 \times 10^{-5}$

Theory calculations

Experimental precision requires inclusion of **multi-loop corrections** in theory



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2\ f(q_1,\, q_2,\, p_1,\, k_1,\, ...,\, m_1,\, m_2,\, ...)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(1000) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (e.g. symmetries)

} not a limiting factor

Evaluation of loop integrals:

- Analytical
- Approximate (expansions)
- Numerical

Analytical calculations

Challenge 1: reduce 1000s of integrals to a small set of *master integrals*

Simple 1-loop example:

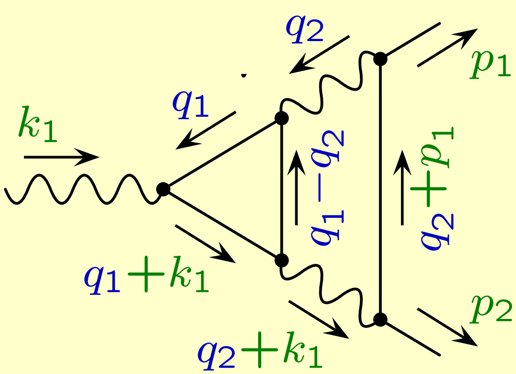
$$\int d^4q \, \frac{q\cdot p}{[q^2-m_1^2][(q+p)^2-m_2^2]}$$

$$= \int d^4q \, \frac{\frac{1}{2}(q+p)^2 - \frac{1}{2}q^2 - \frac{1}{2}p^2}{[q^2-m_1^2][(q+p)^2-m_2^2]}$$

$$= \frac{1}{2}\int \frac{d^4q}{q^2-m_1^2} - \frac{1}{2}\int \frac{d^4q}{q^2-m_2^2} + \frac{m_2^2-m_1^2-p^2}{2}\int \frac{d^4q}{[q^2-m_1^2][(q+p)^2-m_2^2]}$$

Simple cancellations do not work beyond 1-loop:

e.g. diagram to the right with $(q_1 \cdot p_1)$ in numerator



Analytical calculations

Challenge 1: reduce 1000s of integrals to a small set of *master integrals*

Integration-by-parts (IBP) relations:

1-dim. example: $$\int_{-\infty}^{\infty} dx\, \frac{d}{dx} \underbrace{\frac{1+2x}{1+x^2}}_{f(x)} = f(\infty) - f(-\infty) = 0$$

$$\frac{d}{dx} f(x) = \frac{2}{1+x^2} - \frac{2x(1+2x)}{(1+x^2)^2}$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} dx\, \frac{2x(1+2x)}{(1+x^2)^2} = \int_{-\infty}^{\infty} dx\, \frac{2}{1+x^2}$$

$\rightarrow$ Similar for $\int d^4q$ integrals

- Individual eqs. may contain integrals not in original problem
- Large enough eq. system can be fully solved — Laporta '01
- Public programs: `Reduze, FIRE, LiteRed, KIRA`
  von Manteuffel, Studerus '12; Smirnov '13,14; Lee '13; Maierhoefer, Usovitsch, Uwer '17
- Requires large computing time and memory

Analytical calculations

Challenge 2: find solutions for *master integrals*

Many methods, e.g. differential equations or Mellin-Barnes representations
Kotikov '91; Remiddi '97; Smirnov '00,01

→ Special form of diff. eq. system ("canonical basis") to solve it Henn '13

→ Complicated functions needed:
Goncharov polylogs, iterated elliptic integrals, hypergeometric functions, ...

Asymptotic expansions

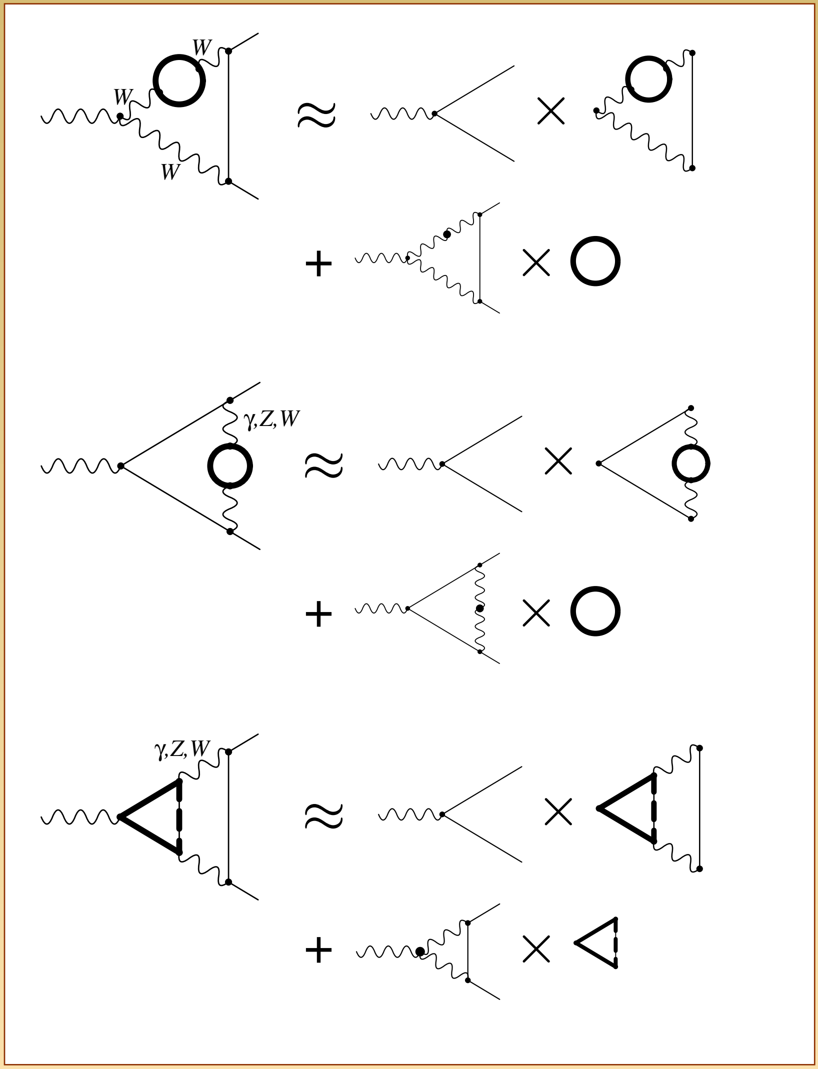
- Exploit large mass/momentum ratios, *e. g.* $M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

→ Public programs:

`exp` Harlander, Seidensticker, Steinhauser '97
`asy` Pak, Smirnov '10

→ Possible limitations:

- no appropriate mass/momentum ratios
- bad convergence
- impractical if too many mass/mom. scales



Challenge 1: presence of UV/IR divergencies

- Remove through subtraction terms

$$\underbrace{\int d^4q_1 d^4q_2\,(f - f_{\text{sub}})}_{\text{finite}} + \underbrace{\int d^4q_1 d^4q_2\, f_{\text{sub}}}_{\text{solve analytically}}$$

Cvitanovic, Kinoshita '74
Levine, Park, Roskies '82
Bauberger '97
Nagy, Soper '03
Awramik, Czakon, Freitas '06
Becker, Reuschle, Weinzierl '10
Sborlini et al. '16
...

- Remove through variable transformations:

a) Sector decomposition

Public programs: `(py)SecDec` Carter, Heinrich '10; Borowka et al. '12,15,17
`FIESTA` Smirnov, Tentyukov '08; Smirnov '13,15

b) Mellin-Barnes representations

Public programs: `MB/MBresolve` Czakon '06; Smirnov, Smirnov '09
`AMBRE/MBnumerics` Gluza, Kajda, Riemann '07
Dubovyk, Gluza, Riemann '15
Usovitsch, Dubovyk, Riemann '18

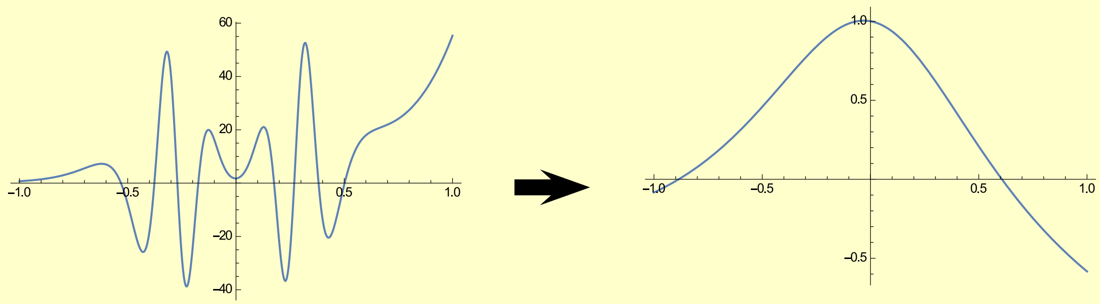
Numerical integration

Challenge 2: stability and convergence

- Integration in momentum space: $4L$ dimensions ($L =$ # of loops)
- Integration in Feynman parameters: $P-1$ dimensions ($P =$ # of propagators)

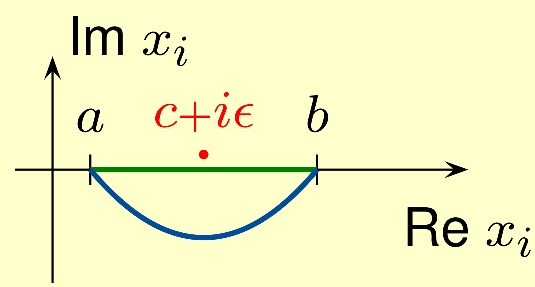
$\rightarrow$ Multi-dim. integrals need large computing resources and converge slowly

- Variable transformations to avoid singularities and peaks



- Integrals blow up when props. become on-shell:

$$\int \frac{d^4q}{[q^2 - m_1^2 + i\epsilon][(q+p)^2 - m_2^2 + i\epsilon]}$$



$\rightarrow$ complex contour integration can help

v.d. Bij, Ghinculov '94; Nagy, Soper '06

Loop calculations: Summary

Analytical techniques and expansions:

Complexity increases with ...

... more loops;

... more external particles;

... **more different masses**

Numerical techniques:

Complexity increases with ...

... more loops;

... more external particles;

... **fewer masses**

Monte-Carlo tools

Rad. corr. produce complex final states, $e^+e^- \to f\bar{f} + n\gamma \, (+f'\bar{f}')$

- Account for detector acceptance + selection cuts $\longrightarrow$ MC tools

- For EWPOs:

  a) Fixed-order (loop corrected) **matrix elements** for $e^+e^- \to f\bar{f} + n\gamma$

  b) **Parton shower** for extra γ (+ $f'\bar{f}'$)

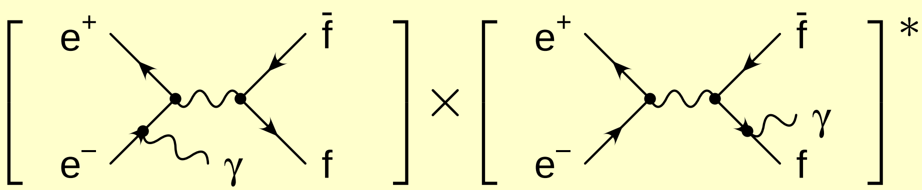
  $\longrightarrow$ **Matching procedure** to avoid double counting

- QCD parton showers: leading log + leading color $\longrightarrow$ lecture by P. Skands

  $\uparrow$ neglect terms suppressed by $1/N_c = 1/3$

- QED parton showers: no “leading color” $\longrightarrow$ full coherent treatment desirable

  (in particular large initial-final interference)



Jadach, Ward, Was ’13; Jadach, Yost ’18
Hamilton, Richardson ’06
Schoenherr, Krauss ’08
Kleiss, Verheyen ’17; Frixione, Webber ’21

New physics reach

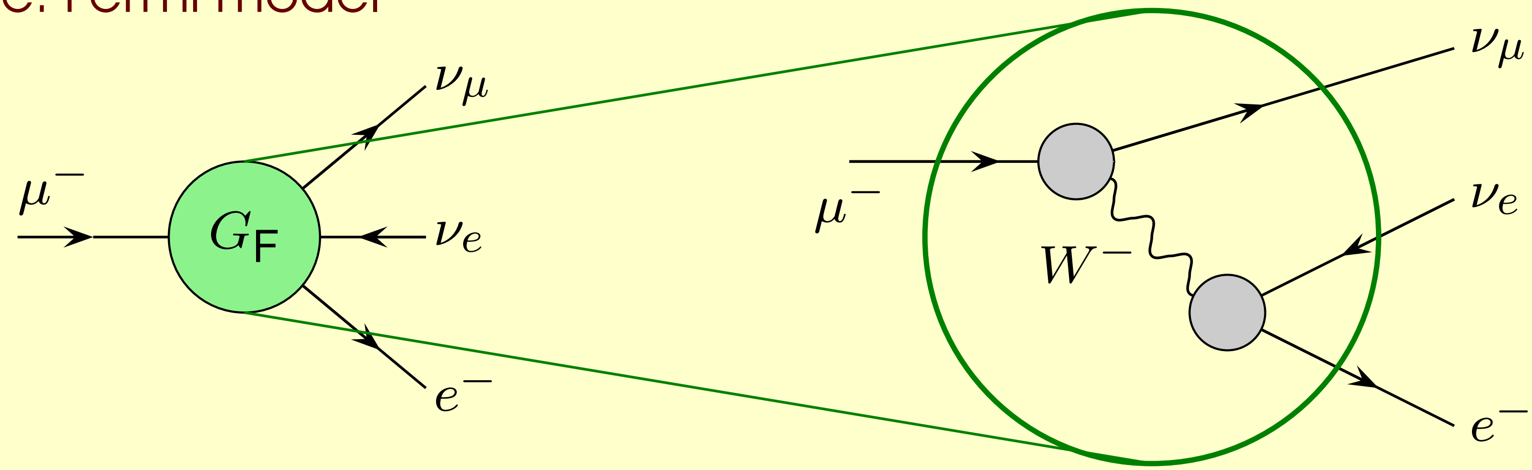
Open questions of the Standard Model:

- Is the Higgs boson part of a more complex sector?
- Is there an extended/unified symmetry group?
- What is dark matter?
- Why is there more matter than anti-matter in the universe?
- ...

- **Many models**, introducing few (minimal DM, 2HDM, ...) or many (SUSY, extra dim., clockwork, ...) new particles
- Model independent approaches:
  - **Simplified models:** only consider particles contributing to a particular observable or phenonemon
  - **Effective field theories:** low-energy description of heavy new particles

Low-energy effective theory

Example: Fermi model



$$\frac{4G_F}{\sqrt{2}}(\overline{\psi}_L^{\nu_\mu}\gamma^\alpha\psi_L^\mu)(\overline{\psi}_L^e\gamma_\alpha\psi_L^{\nu_e})$$

$$(\overline{\psi}_L^{\nu_\mu}\frac{g}{\sqrt{2}}\gamma^\alpha\psi_L^\mu)\underbrace{\frac{-g_{\alpha\beta}}{q^2-M_W^2}}_{\approx\frac{g_{\alpha\beta}}{M_W^2}}(\overline{\psi}_L^e\frac{g}{\sqrt{2}}\gamma^\beta\psi_L^{\nu_e}) \qquad (|q|<m_\mu\ll M_W)$$

$\rightarrow$ $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ ("matching")

$\rightarrow$ At low energies, G_F can be taken as independent parameter

"SM Effective Field Theory" (SMEFT)

Extension of SM by **higher-dimensional operators**:

Wilson '69
Weinberg '79

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{d-4}} \sum_i c_i \, \mathcal{O}_i^{(d)}$$

- $\mathcal{O}_i^{(d)}$ depend on all SM fields (including Higgs doublet)
- Operators must satisfy SM gauge invariance
- Valid description for energies $E \ll \Lambda$ ($\Lambda \sim$ mass of heavy particles)
- Operators ranked by suppression power Λ^{4-d}

Examples:

- $(\partial_\mu \phi)^\dagger \phi \, \phi^\dagger (\partial^\mu \phi)$ not allowed, but $(D_\mu \phi)^\dagger \phi \, \phi^\dagger (D^\mu \phi)$ is
  $[D_\mu = \partial_\mu - i g' Y B_\mu - i g \frac{\sigma^a}{2} W_\mu^a]$
- $(\phi^\dagger \phi)(\overline{\psi} \not{D} \psi)$ and $(\phi^\dagger \phi)(\overline{\psi} \phi \psi)$ are related by e.o.m.:
  $\not{D} \psi_f = \underbrace{y_f \phi}_{} \, \psi_f$ (Dirac eq.)
  $\hookrightarrow y_f v / \sqrt{2} = m_f$

“Higgs Effective Field Theory” (HEFT)

Review: 1706.08945

HEFT: similar to SMEFT, but Higgs and Goldstone bosons treated independently

Higgs ↑ gauge singlet; Goldstone ↑ SU(2) triplet

Example:

HEFT: $\frac{c_1}{\Lambda} h(\overline{\psi} \not{D} \psi), \quad \frac{c_2}{\Lambda^2} h^2(\overline{\psi} \not{D} \psi), \quad \ldots$

SMEFT: $\frac{c_{\phi\psi}}{\Lambda^2}(\phi^\dagger \phi)(\overline{\psi} \not{D} \psi) \quad \to \quad \frac{c_{\phi\psi}}{2\Lambda^2}(v+h)^2(\overline{\psi} \not{D} \psi)$

$$\Rightarrow \quad c_1 = c_{\phi\psi} v, \quad c_2 = \tfrac{1}{2} c_{\phi\psi}$$

→ Fewer operators, less freedom in SMEFT

→ HEFT needed to describe some composite-Higgs theories

SMEFT operators for electroweak precision

Leading dim-6 contribution: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3})$ $(\Lambda \gg M_Z)$

$\mathcal{O}_{\phi 1} = \frac{1}{4}|\phi^\dagger \overleftrightarrow{D}_\mu \phi|^2$ $\alpha\Delta T = -\frac{v^2}{2}\frac{c_{\phi 1}}{\Lambda^2}$

$\mathcal{O}_{BW} = \phi^\dagger B_{\mu\nu} W^{\mu\nu}\phi$ $\alpha\Delta S = -e^2 v^2 \frac{c_{BW}}{\Lambda^2}$

$O_{LL}^{(3)\mu e} = (\bar{L}_L^\mu \sigma^a \gamma_\mu L_L^\mu)(\bar{L}_L^e \sigma^a \gamma^\mu L_L^e)$ $\Delta G_F = -\sqrt{2}\frac{c_{LL}^{(3)\mu e}}{\Lambda^2}$

$O_R^f = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{f}_R \gamma^\mu f_R)$ $f = e, \mu\, \tau, b, lq$

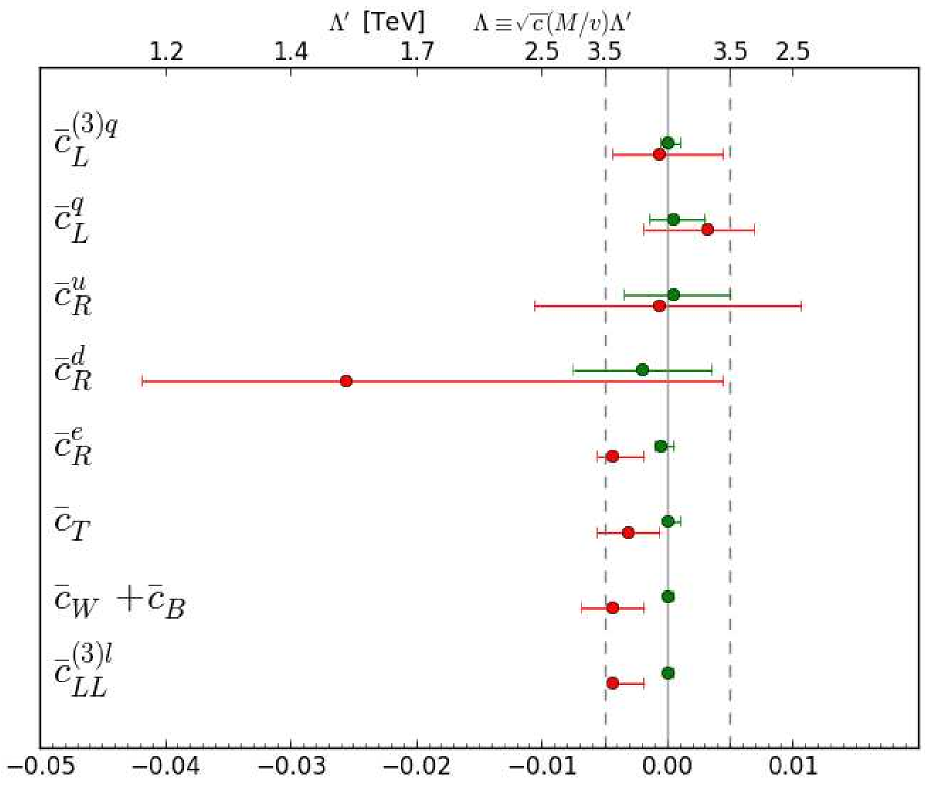
$O_L^F = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{F}_L \gamma^\mu F_L)$ $F = \begin{pmatrix}\nu_e \\ e\end{pmatrix}, \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}, \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}, \begin{pmatrix}u, c \\ d, s\end{pmatrix}, \begin{pmatrix}t \\ b\end{pmatrix}$

$O_L^{(3)F} = i(\phi^\dagger \overleftrightarrow{D}_\mu^a \phi)(\bar{F}_L \sigma_a \gamma^\mu F_L)$

More operators than EWPOs $\rightarrow$ need assumptions:

- Family universality, e.g. $c_R^e = c_R^\mu = c_R^\tau$
- U(2)×U(1) flavor symmetry, e.g. $c_R^e = c_R^\mu \neq c_R^\tau$

Assuming family universality:

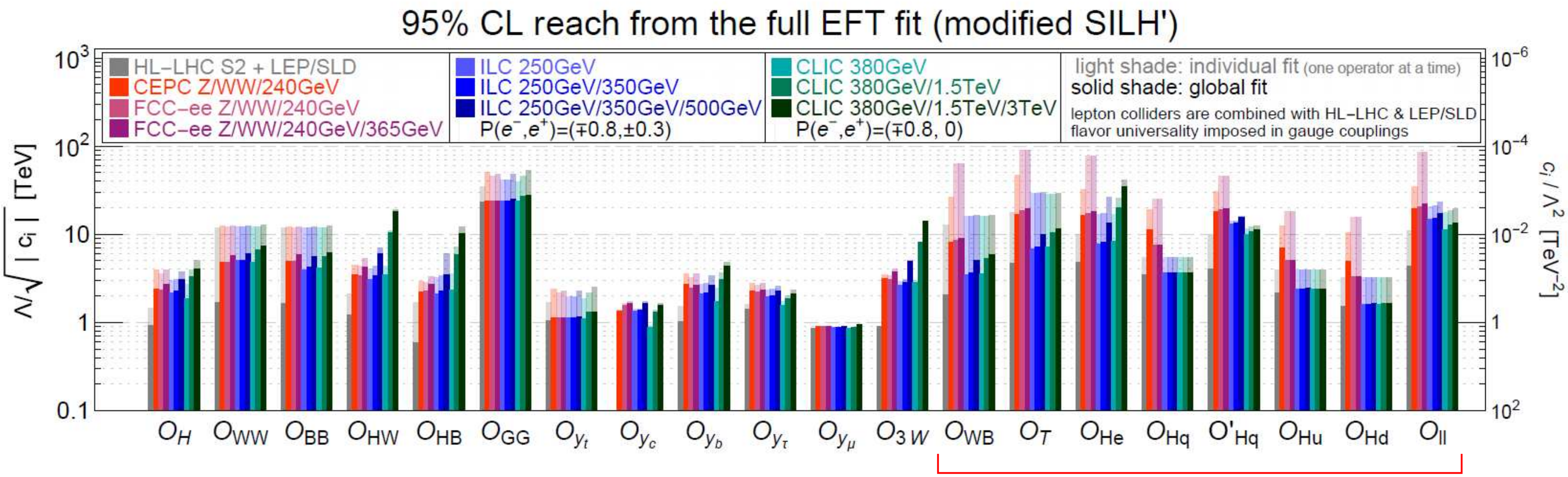


- Electroweak precision tests put constraints on new physics at **TeV scale** $\rightarrow$ Complementary to LHC
- Significant correlation/ degeneracy between different operators

Pomaral, Riva '13
Ellis, Sanz, You '14

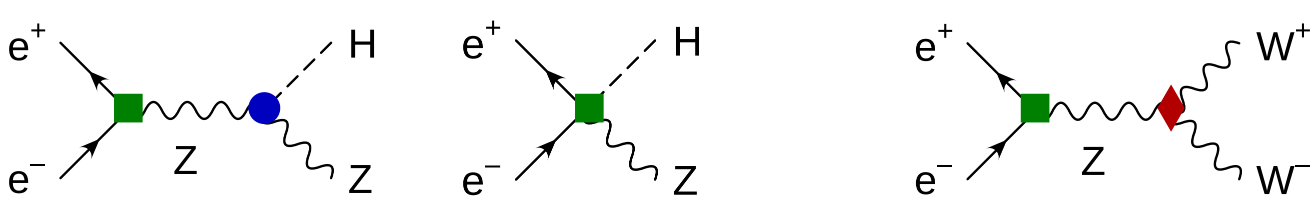
SMEFT analysis: future colliders

- SMEFT dim-6 operators provide framework for comparing experiments



de Blas, Durieux, Grojean, Gu, Paul '19

- Correlations between sectors (e.g. Higgs and EW):
  $\longrightarrow$ Improved Z-pole measurements needed for Higgs physics and aGC



SMEFT and custodial symmetry

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

Many SMEFT appearing in EWPOs violate custodial symmetry:

$$\phi^\dagger \overleftrightarrow{D}_\mu \phi \equiv \phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi = -\,\mathrm{Tr}\{\Omega^\dagger D_\mu \Omega\, \sigma_3\}$$

$\to$ not invariant under $\Omega \to V \Omega V^\dagger$, because $V \sigma_3 \neq \sigma_3 V$

Kribs, Lu, Martin, Tong '20

New physics with custodial symmetry violation generate large coefficients:

Example: 4th quark generation T, B with $y_\mathrm{T} \neq y_\mathrm{B}$

$$\mathcal{L}_4 = -y_\mathrm{T} \overline{Q}_{4\mathrm{L}} \tilde{\phi}\, T_\mathrm{R} - y_\mathrm{R} \overline{Q}_{4\mathrm{L}} \phi\, B_\mathrm{R}$$

$$\to \frac{c_{\phi 1}}{\Lambda} = -\frac{3(y_\mathrm{T}^2 - y_\mathrm{B}^2)}{4\pi^2 v^2} = -\frac{3(m_T^2 - m_B^2)}{2\pi^2 v^4}$$

not suppressed even for $m_{\mathrm{T,B}} \gg v$

New physics reach: Models

Neutrino counting and mixing

Total Z width from line-shape: $\Gamma_Z = 3\Gamma_\ell + \underbrace{\Gamma_{Z\to\text{inv}}}_{N_\nu \Gamma_\nu} + \Gamma_{\text{had}}$

$$N_\nu = \left[\left(\frac{12\pi}{M_Z^2}\frac{R_\ell}{\sigma^0_{\text{had}}}\right)^2 - R_\ell - 3\right]\frac{\Gamma_\ell}{\Gamma_\nu}$$

from measurement

computed in SM

Current data (LEP): $N_\nu = 2.996 \pm 0.007$

FCC-ee: ± 0.001

Janot, Jadach '20

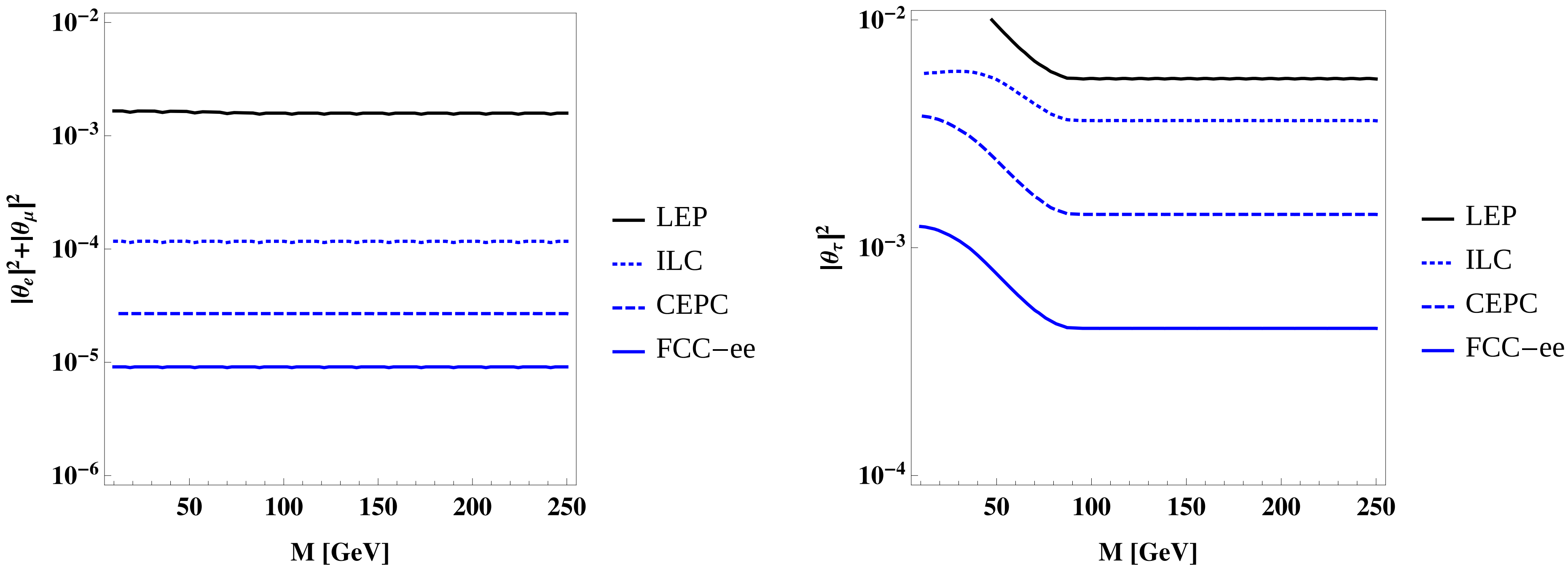
Mixing with sterile neutrino:

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1+\Delta r)(1-\theta_e^2)(1-\theta_\mu^2)$$

$$\Gamma_{Z\to\text{inv}} = \Gamma_\nu^{\text{SM}}\left(N_\nu - \sum_{\alpha,\beta}\theta_\alpha\theta_\beta\right)$$

θ_α: mixing of sterile neutrino with ν_α ($\theta_\alpha \ll 1$)

Estimated sensitivity from electroweak precision tests:

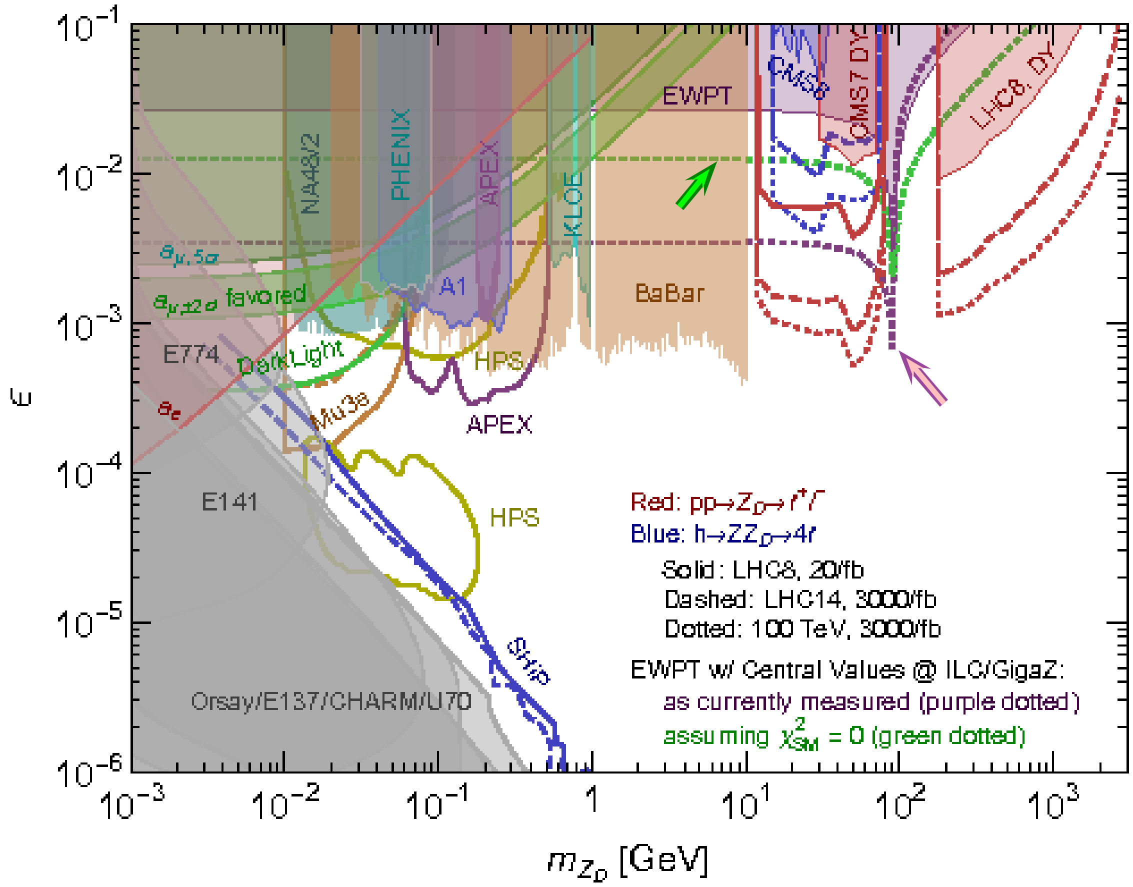


Antusch, Fischer ’15

Dark photon

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} Z'_{\mu\nu} Z'^{\mu\nu} + \frac{M_{Z'}^2}{2} Z'_\mu Z'^\mu + \overline{\chi}(i\partial\!\!\!/ + g_D Z\!\!\!/' - m_\chi)\chi + \frac{\epsilon}{2c_{\text{W}}} Z'_{\mu\nu} B^{\mu\nu}$$

Mass mixing between B_0^μ, W_0^μ and $Z_{D,0}^\mu$ modifies M_{W}–M_{Z} relation and Zff couplings (compared to SM)



Curtin, Essig, Gori, Shelton '14