Introduction to Dark Matter

Diego Blas KCL
diego.blas@kcl.ac.uk

- See my notes for references + extra explanations

1) Evidence of dark matter (DM)
   - Properties of DM
2) Thermal production in early universe
3) Axions (ALPS) as DM candidates
4) Direct detection of WIMPS
5) Indirect detection / challenges / new ideas

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0 Why YOU want to study DM?

- Cosmological SM (ΛCDM)
- BSM : neutrino masses
  BAU

  ↓
  hierarchy problem
  quantum gravity SUSY ⟷ DM

EVIDENCE OF DM

Uranus orbit had an analogous orbit 1840s

- Leverreir

31st/08/1846
23rd/09/1846

- Neptune

~~Vulcan~~

Mercury

GR

- $\phi = \frac{GM_\odot}{r} + \delta\Phi \to \delta\Phi \propto \left(\frac{GM_\odot}{r}\right)^2$

- DM fixes several "problems" at different scales

- DM in galactic curves

$$\boxed{v^2(r) = \frac{GM(r)}{r}}$$

$$v \propto 1/r^{1/2}$$

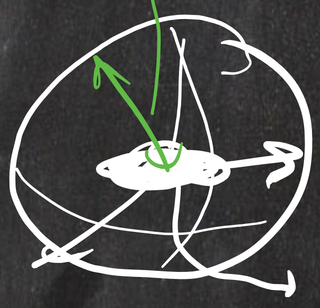
$M(r) = \rho \cdot r^3$

$\hookrightarrow \boxed{v \propto r}$

$v = \text{cst} \rightarrow \rho \propto \frac{1}{r^2}$

$\Downarrow$

spherically symmetric

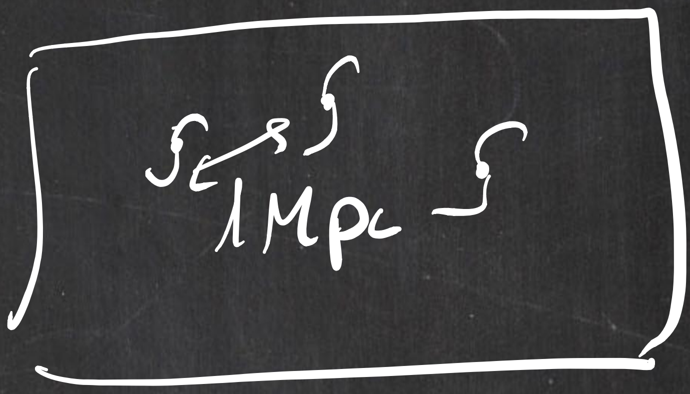


~ DM halo

$\boxed{\text{Vera Rubin}}$

kpc

Dynamics of galaxy cluster



10 Mpc

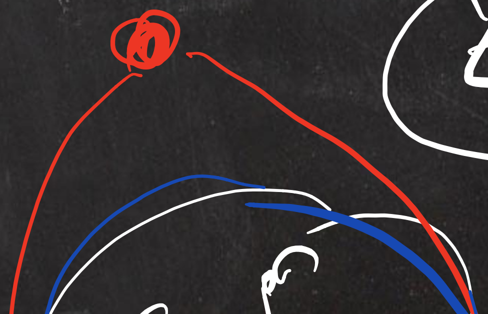
Virial theorem

$$\langle K \rangle = -\frac{1}{2}\langle V \rangle$$

$$K = \frac{1}{2}\sum m_i \frac{v_i^2}{2} \qquad V = -\sum_{i,j} \frac{G m_i m_j}{|r_i - r_j|}$$

$$\langle f(t) \rangle = \frac{1}{T}\int_t^{t+T} dt'\, f(t')$$

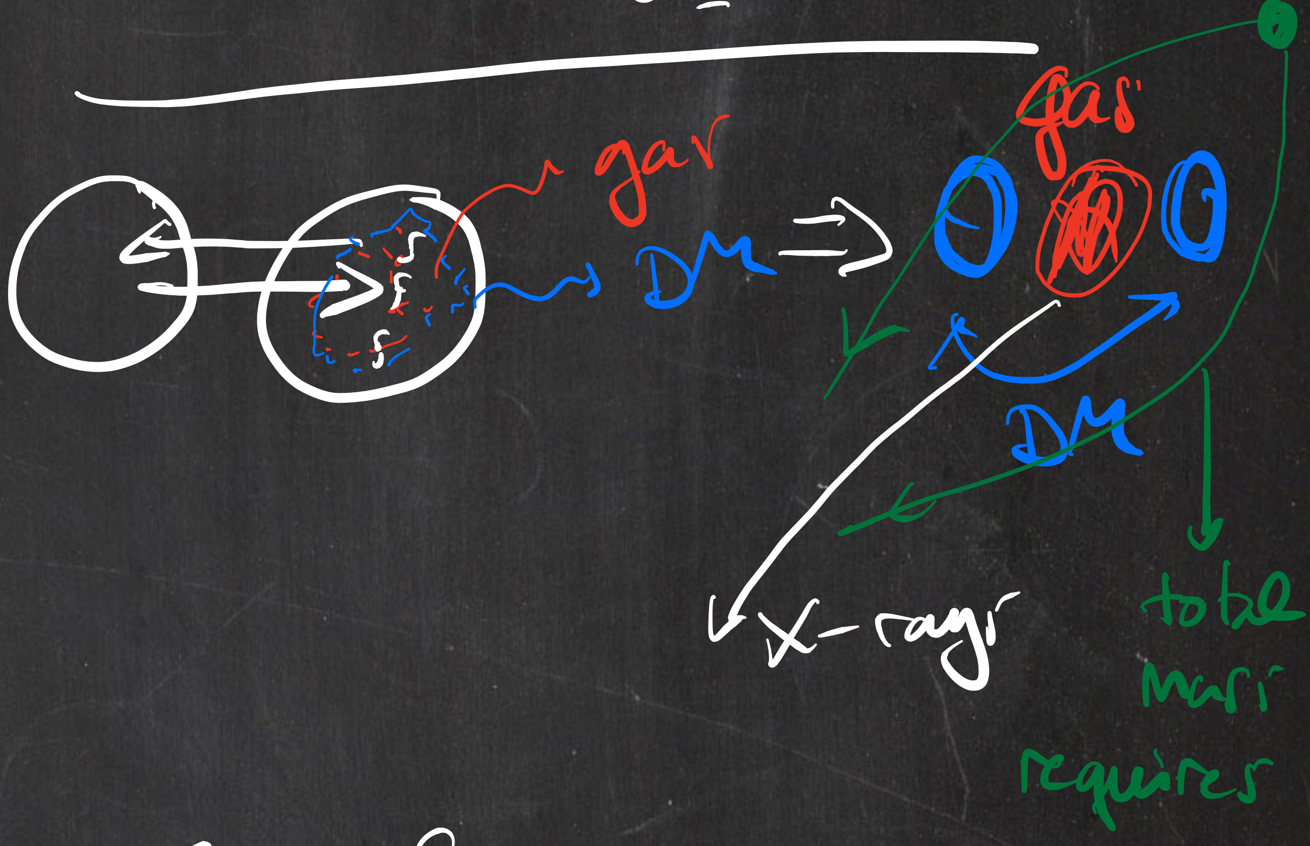
Zwicky



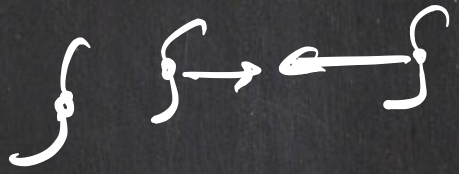
$\langle m_i\, v \rangle \leftarrow$ Total mass

· Gravitatione lensing

Collisions of galaxy cluster



Mpc

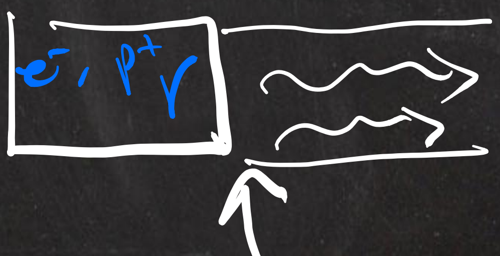


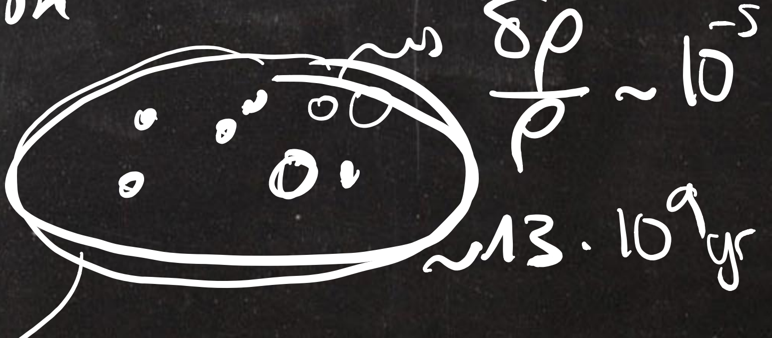
Cosmological probes

Gpc

cosmic microwave b. (CMB)

· Recombination





2.7 K

CMB

→ We need more mass!

DM

$$\Omega_{DM} h^2 = 0.1198 \pm 0.0012$$

$$\Omega_{DM} = \frac{\rho_X}{\rho_c} \; ; \; \rho_c = 10^{-6}\ \mathrm{GeV/cm^3}$$

$$\boxed{h \simeq 0.7}$$

DM properties

• **Dark**

· interaction w/ SM

· $q \lesssim 10^{-4} \left(\frac{m_{DM}}{\mathrm{TeV}}\right)^{1/2}$

·

• **Cold**

$$\boxed{T \ll m}$$

· non-relativistic → small typ. velocities $v \ll c$

$$\boxed{m_{DM}^{th} \gtrsim \mathrm{keV}}$$

axion, $m_a \gtrsim 10^{-21}\,eV$

- Stable $\boxed{\tau_d \sim 13\,Gyr}$

- Non-baryonic
  - BBNucleosynt. $\rightarrow \Omega_b = \dfrac{\rho_b}{\rho_c}$

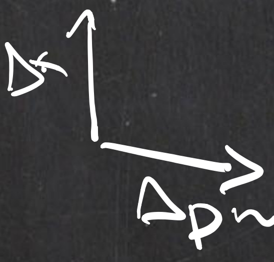
$$\Omega_b \approx \frac{1}{5}\Omega_{cdm}$$

- Collisionless $\quad \sigma/m_{DM} \lesssim 1\,cm^2/gr$

- mass, spin, charges...

KNOWNS

- Thermally produced $\quad m \geq KeV$
- Bosonic $\quad \boxed{m \gtrsim 10^{-22}\,eV}$



$$\Delta x \Delta p \gtrsim 2\pi\hbar$$

$\Delta p \sim m.v \sim m v_{esc}$

$v < v_{esc}$

- Fermionic $\quad \boxed{m \gtrsim 400\,eV}$

- $\Delta x \Delta p \gtrsim 2\pi\hbar$

$$N = \frac{(\Delta x)^3 (m v_{esc})^3}{(2\pi\hbar)^3}$$

$$\boxed{N_m \equiv M_{gal.}} \quad \text{e.g.}$$

- Black holes

Thermal production in the early universe

(WIMP miracle)

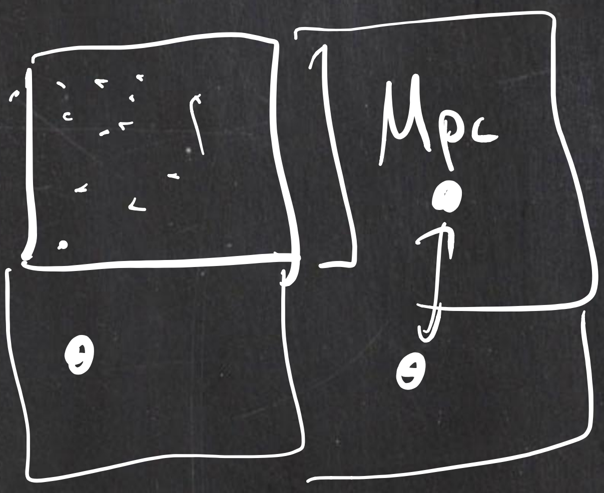
- thermodynamics
- particle physics
- general relativity

$\Rightarrow$ cosmogony

Physics in expanding universe

- isotropic & homogeneous

$\updownarrow$



$$x_i(t) = \boxed{\frac{a(t)}{a(t_0)}} x_i(t_0)$$

$$\boxed{v(t) = \left(\frac{\dot{a}}{a(t_0)}\right) x_i(t)}$$

Hubble

$$\left(\frac{\dot a}{a}\right)_{today} \simeq 100\,\mathrm{km/s/Mpc}\cdot h \quad h \sim 0.7$$

- **GR**

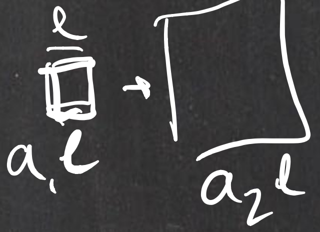
$$\boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G_N}{3}\rho}$$

$$\rho_c = \frac{3}{8\pi G_N}\left(\frac{\dot a}{a}\right)^2_{today}$$

$$\boxed{1+z = \frac{a(t_0)}{a(t)}}$$

z ↑ redshift

- CMB ~ 13 billion year, $z \sim 1000$

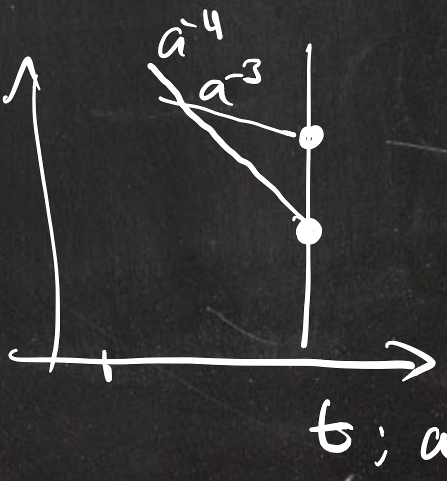


$$\rho_{NR} = \frac{E}{V} = \frac{m}{a^3\ell^3}$$ · non-rel.

- relativ (radiation) $\quad \boxed{E \approx p} \to p \propto \frac{1}{a}$

$$\rho_R \propto \frac{1/a}{a^3} \propto a^{-4}$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$$



• Thermal distribution

$$\frac{1}{e^{p/T}-1}$$

Dark energy

• $\rho = cte$

• Species → relativistic

$$\boxed{\frac{1}{e^{E/T}-1}}$$

; $E \simeq p$

; $E \simeq \frac{mv^2}{2}$

• $T \gg m \Rightarrow \rho \propto \frac{1}{a^4}$

• $T \ll m \Rightarrow \rho \propto \frac{1}{a^3}$

• $\rho \Rightarrow a(t)$

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32 Thermal equilibrium in an expanding Universe

• $f(\vec{x}, \vec{p}, t)$

$$f(\vec{x}, \vec{p}, t)\,\frac{d^3x\, d^3p}{(2\pi\hbar)^3} \equiv$$ # of particles with $\vec{p} + d\vec{p}$, $\vec{x} + d\vec{x}$

$\int dx$

$$N = \int f(\vec{x}, \vec{p}, t)\, d^3p$$

$$n = \frac{N}{V} = \frac{\int d^3x \int f(\vec{x}, \vec{p}, t)\, d^3p}{V}$$

$$\rho = \frac{\int\int E(p)\, f(\vec{x},\vec{p},t)\, d^3p\, d^3x}{V}$$

$$\bullet \quad f(\cancel{\vec{x}},\vec{p},t) = \frac{1}{e^{E/T} \pm 1}$$

$$E = p \Rightarrow \underbrace{\rho_r}_{\hookrightarrow \frac{1}{a^4}} = \frac{\pi^2}{15} \underbrace{T^4}_{1/a^4}$$

$$n_r = \frac{12\, g_{eff}\, T^3}{\pi^2}$$

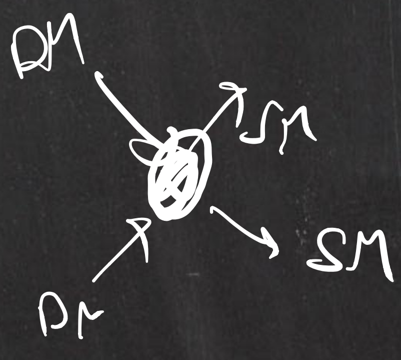
$$g_{eff} = \begin{cases} 3/4 & \text{fermions} \\ 1 & \text{bosons} \end{cases}$$

$$\underbrace{E \simeq m\left(1 + \frac{v^2}{2}\right)} \quad ; \quad \rho = m \cdot n$$

$$= m\, g_{eff} \left(\frac{mT}{2\pi}\right)^{3/2} \underbrace{e^{-m/T}}$$

$$\bullet \quad m \gg T \to \rho \to 0 \text{ very fast}$$

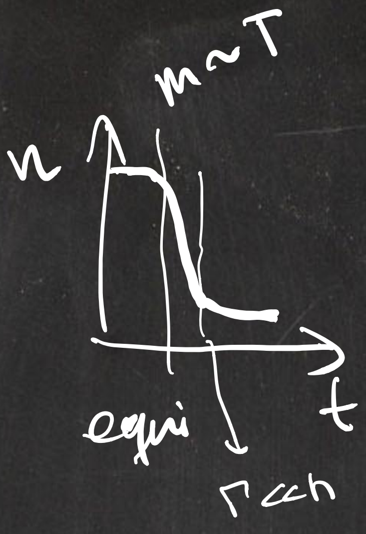
$$DM + DM \to SM + SM$$



$$\Gamma = \sigma\, v_{DM}\, n_{DM}$$

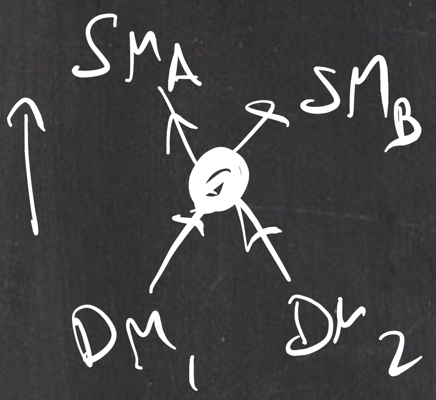
$\Gamma(t)$ decreases with time

- $\Gamma \gg H = \left(\frac{\dot a}{a}\right)$
- $\Gamma \sim H$
- $\Gamma \ll H$
- $n \propto \frac{1}{a^3}$ out of equil.



- $N = n a^3$
- $V \propto a^3$

$$\frac{d(n\cdot a^3)}{dt} = \left(\text{new particles per unit time}\right) - \left(\frac{\text{lost}}{\text{unit time}}\right)$$



$$\boxed{d\Pi_i = \frac{g_i\, d^3 p_i}{(2\pi)^3\, 2E_p}}$$

$$\int d\Pi_1 d\Pi_2 d\Pi_A d\Pi_B (2\pi)^4 \delta^{(4)}(p_A+p_B-(p_1+p_2))\{|M_{12\to AB}|^2$$

$$\cdot f_1 f_2 (1\pm f_A)(1\pm f_B)$$

$$- |M_{AB\to 12}|^2 f_A f_B (1\pm f_1)(1\pm f_2)$$

The ORIGIN OF SPECIES!

- $|M_{12\to AB}|^2 = |M_{AB\to 12}|^2$ CP inv.
- $T \to$ $f_{eq} \simeq e^{-E/T}$

$f = \frac{1}{\ldots}$

$$_{eq} \quad e^{E/T} \pm 1$$

- $A, B \rightarrow f_A f_B \approx e^{-(E_A+E_B)/T}$

- $f_{1,2} = f_{1,2}^{eq} (1 + \delta f)$

$$\frac{d(n a^3)}{dt} = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

$\downarrow$

$$\langle \sigma v \rangle = \frac{1}{n_{eq}^2} \int d\Pi_1 d\Pi_2 d\Pi_A d\Pi_B \, (2\pi)^4 \delta^{(4)}(\underline{\quad}) \, |\mathcal{M}|^2 f_1^{eq} f_2^{eq}$$

$$\frac{dn}{dt} + 3Hn$$

$$H = \frac{da}{dt}/a \equiv \frac{\dot{a}}{a}$$

$$H^2 = \frac{8\pi G}{3} \rho_r = \left(1.66\, g_*^{1/2} \frac{T^2}{M_P}\right)^2$$

$z \gg 10^3$

$$\rho = \frac{\pi^2}{15} T^4, g_*$$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

- $n = n_{eq}$ $\quad (H)$

$$\boxed{H \sim \Gamma = \langle \sigma v \rangle n}$$

$\boxed{\text{YIELD}}$

$$Y = \frac{n}{s} \quad ; \quad s = \text{entropy density} = \frac{S}{V}$$

$$TdS = dE + p\,dV$$

- $$s = \frac{\rho + P}{T}$$

- $$P = \frac{F}{A} \Rightarrow \frac{1}{(2\pi)^3}\int d^3p\, \frac{|p|^2}{3E} f(p)$$

- $s \propto \frac{1}{a^3}$; relativistic particles

$$s = \frac{2\pi^2}{45} g_* T^3$$

- $g_* = 7/8$ f
- $g_* = 1$ b

$$\frac{dY}{dt} = -s\langle\sigma v\rangle (Y^2 - Y_{eq}^2)$$

- $x = m/T$; $x \ll 1$ ~ $(T \ll M)$

$$\frac{1}{Y_{eq}}\frac{dY}{d\log x} = -\frac{\Gamma}{H}\left(\frac{Y^2}{Y_{eq}^2} - 1\right)$$

$\Gamma = n\langle\sigma v\rangle$

$$\boxed{\frac{dY}{dx} = -\frac{s\,x\langle\sigma v\rangle}{H}(Y^2 - Y_{eq}^2)}$$

$$\frac{dY}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}(Y^2 - Y_{eq}^2)$$

$M_P^2 = \frac{1}{8\pi G}$

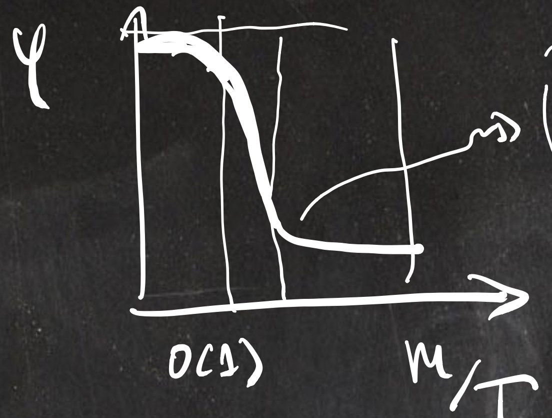
- $$\lambda = 0.26\,\frac{g_{*s}}{g_*^{1/2}} M_P M$$

$\hookrightarrow 10^{19}$ GeV

- Kinetic equil.

$\sim n_{SM}\, n_{DM}$

n^2_{DM} DM.



$\boxed{H \sim \Gamma}$

$T \propto \frac{1}{a}$

$Y = \frac{n}{s} \propto \frac{a^{-3}}{a^{-3}} \to$

$m \gg T$

- Freeze out of relativistic species

- $\Gamma_A \sim H \Rightarrow Y_{rel} = \frac{n}{s} = \frac{0.278\, g_{eff}^{DM}}{g_{eff}^{plasma}}$

$$\boxed{\begin{array}{l} \bullet\ n = \frac{\zeta\, g_{eff}\, T^3}{\pi^2} \\ \bullet\ s = \frac{2\pi^2}{45}\, g_*^{plasma}\, T^3 \end{array}}$$

$$\Omega_0 = \frac{\rho_0^{DM}}{\rho_c} = \frac{m\, n_0^{DM}}{\rho_c} = \frac{m\, Y_0^{DM} \cdot s_0}{\rho_c}$$

$$= \frac{m\, Y_{rel}^{DM} \cdot s_{CMB}}{\rho_c} =$$

$$= \frac{g^{DM} \cdot m}{136\ \text{eV}} \quad ; \quad \Omega_0 h^2 = 0.11$$

$h \simeq 0.7$

- Freeze out of NR - particles

$-m/T$

- $m \gtrsim T \to$ $n\alpha e$

$m \sim T$; $\boxed{2 \to 2}$

- $\Gamma \sim H$; $m \sim T$

$\hookrightarrow n\langle\sigma v\rangle \sim H \sim \frac{T^2}{M_P} \sim \frac{m^2}{M_P}$

$M_P \sim 10^{19}$ GeV

- $\rho = mn \sim \frac{m^3}{\langle\sigma v\rangle M_P}$

- $\rho_t = \rho_0 \left(\frac{T_t}{T_0}\right)^3 = \rho_f \left(\frac{T_t}{T_f}\right)^3$

- $\rho_{DM} = \rho_{CMB}$; $z \sim 3000$

$\rho_f \left(\frac{T_{rme}}{T_f}\right)^3 \simeq T_{rme}^4$

- $\langle\sigma v\rangle \sim \frac{1}{T_{rme} M_P} \sim \frac{\alpha^2}{10^{-4}} \frac{1}{(10^2 \text{GeV})^2}$

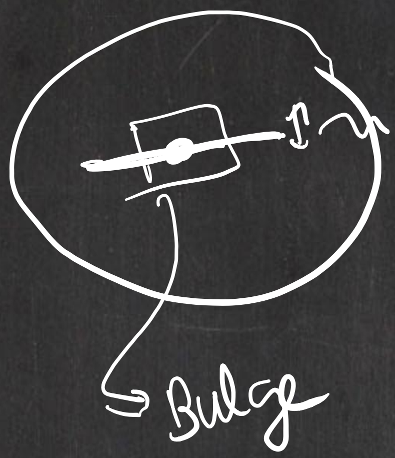
$\Omega_{DM}^0 h^2 \simeq 0.1 \left(\frac{0.01}{\alpha}\right)^2 \left(\frac{m_{DM}}{100 \text{ GeV}}\right)^2$

$\boxed{\text{WIMP MIRACLE}}$

- Local DM and direct detection of WIMPS detection of W

i) • $\underbrace{n_{DM} \cdot v}$ flux of DM on Earth

ii) • exp. sensitivities



galactic dynamics

• Milky Way

$\sim 10^{11}$ stars

$\sim 10^{11} M_\odot$

DM halo 100 kpc

DM $10^{6} M_\odot$

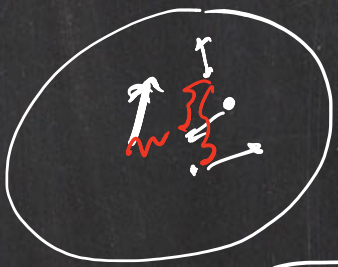
• n_{DM}

$\rightsquigarrow \rho_\odot \simeq 0.3\ \mathrm{GeV/cm^3}$

· Kinetic picture of Milky Way

· $f(v) \longrightarrow$ · typical momentum

• $v \to M(r) \propto r \to v_{(r)}^2 = \frac{GM(r)}{r}$



Jeans theorem

$f(v) \Rightarrow f(E)$

$$\boxed{f(v) = e^{-E/E_0}} \; ; \quad E = \frac{1}{2}v^2 - \psi$$

$$\rho = mn = m \int d^3v\, f(v) \propto$$

$$= m\, e^{+\psi/E_0} \int d^3v\, e^{\frac{1}{2}v^2}$$

$$\Delta\psi = 4\pi G\rho = 4\pi G\, m\, e^{\psi/\varepsilon_0}$$

- Spherical sym. ODE

$$\boxed{\rho(r) = \frac{\varepsilon_0}{2\pi G r^2}} = \frac{M}{r^3}$$

$$\rho_{NFW} = \frac{\rho_0}{r/r_s\,(1 + r/r_s)^2}$$

; $r \gg r_s$ $\sim 1/r^3$

$r \ll r_s$ $\sim 1/r$

$$f(v) = \begin{cases} \frac{1}{N_{esc}} \left(\frac{3}{2\pi\sigma_0^2 v}\right)^{3/2} e^{-\frac{3}{2}\frac{v^2}{\sigma_0^2(r)}} & ; v \ll v_{esc} \\ 0 & ; v \gg v_{esc} \end{cases}$$

$x' = x_0 + v_\odot t$

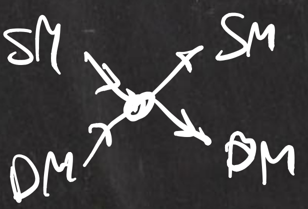
$$\langle v \rangle = \int d^3v\, v\, f(v) = 0 \; ; \quad v' = v + v_\odot$$

$$\cdot \langle v \rangle \approx 10^{-3} c \approx v_\odot$$

$$\cdot \langle v_\oplus \rangle \approx 10^{-4} c$$

$$\phi \sim n \cdot \langle v_\odot \rangle \approx 10^7 \left(\frac{\text{GeV}}{m}\right) \frac{1}{\text{cm}^2 \text{s}}$$

$$E_{DM} = \frac{1}{2} m_{DM} v_{DM}^2 \approx 10^{-6} m_{DM}$$

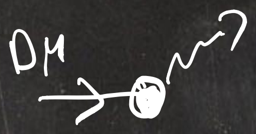


$$\hookrightarrow \quad E_R = \frac{1}{2} m_{DM} v^2 \left(\frac{4 m_D m_N}{(m_{DM}+m_N)^2}\right) \times$$

$$\left(\frac{1 + \cos\theta}{\ldots}\right)$$

$$E_R \sim E_{DM} \quad ; \quad m_N \gg m_{DM}$$

$$\Rightarrow E_R^{max} \simeq 10\,\text{keV} \left(\frac{m_{DM}}{20\,\text{GeV}}\right)^2 \left(\frac{100\,\text{GeV}}{m_N}\right)$$



- Xenon ; thr. keV $\rightarrow m_{DM} \gtrsim$ GeV

- $\lambda \sim \frac{1}{mv} \gg$ size of the atom
- E_R

$$\frac{dR}{dE_R} = N_T \cdot n_{DM} \left\langle \frac{d\sigma}{dE_R} v \right\rangle$$

$$\left\langle \frac{d\sigma}{dE_R} v \right\rangle = \int_{v_{min}}^{v_{esc}} d^3v\, f(v) \frac{d\sigma}{dE_R} v$$

$$\frac{dE_R}{d\cos\theta} = \frac{\mu^2 v^2}{m_N} \; ; \; \mu = \frac{m_N m_{DM}}{m_N + m_{DM}}$$

$$\frac{d\sigma}{d\cos\theta} = ct. + \cancel{O(x)}$$

$$\boxed{\frac{d\sigma}{d\cos\theta} = \frac{\sigma}{2}}$$

- $\displaystyle\int_{v_{min}}^{v_{esc}} d^3v\, f(v) \frac{d\sigma}{dE_R} \propto$

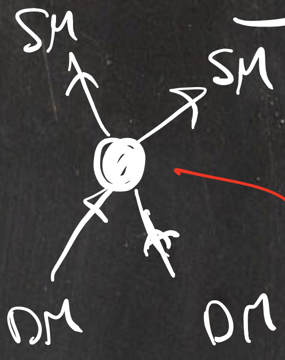
$$\propto \int_{v_{min}}^{v_{esc}} dv\, v\, e^{-v^2/\sigma^2} \sim e^{-E_R/E_0}$$

e^{-E_R/E_0} • $E_0 = \frac{2\mu^2 v_0^2}{m_N}$

$n \propto \rho / m$

$\cdot\ E_0 \sim 50\,\text{keV}$

$m_{DM} \sim 100\,\text{GeV}$



$$\mathcal{L}_{eff} = g\, \overline{DM}\, \Gamma_{DM}\, DM\, \bar{q}\, \Gamma_q\, q$$

σ :

$$\underline{\langle n |\, \bar{q}\, \Gamma_q\, q\, | n \rangle}$$

$\downarrow$

$$\rightarrow \langle At |\, \bar{q}\, \Gamma_q\, q\, | At \rangle$$

; Z protons, $(A-Z)$ neutrons

$$| At \rangle = \sum | n \rangle$$

Coherent : $\sigma_{At} \propto Z^2 \sigma_n$

Incoherent : $\sigma_{At} \propto \sigma_n$

$$\mathcal{L}_{eff} = g_{s.i.}\, \overline{DM}\, DM\, \bar{q}\, q$$

$$\mathcal{L}_{eff} = g_{s.d.}\, \overline{DM}\, \gamma^\mu \gamma_5\, DM\, \bar{q}\, \gamma_\mu \gamma^5\, q$$

$$\langle p |\, m_q\, \bar{q}\, q\, | p \rangle = m_p\, f^p_{Tq}$$

$$g_{s.i.}\, \bar{q}\, q \Longrightarrow f_p\, \bar{p}\, p$$

$$f_p = \sum_{q=u,d,s} m_q \frac{g_{s.i.}}{m_q} f^p_{Tq} + \frac{2}{27} f^p_{TG} \sum m_p \frac{g_{s.i.}}{m_q}$$

$$f^p_{TG} = 1 - \sum f^p_{Tq}$$

$$\mathcal{M} = \langle \underset{\uparrow (Z,A)\ \text{nucleus}}{f_p} \overline{DM}DM\,\bar{p}p + f_n \overline{DM}DM\bar{n}n\rangle$$

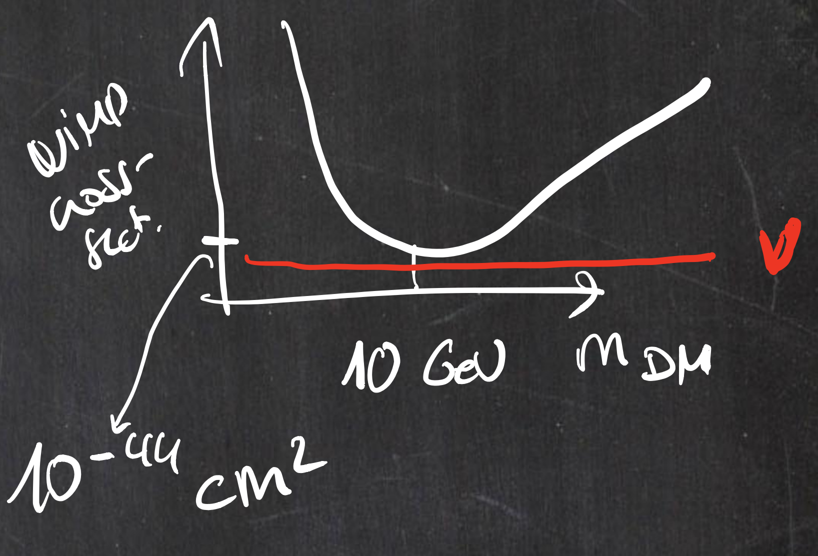
$$= [Z f_p + (A-Z) f_n]\,\overline{DM}DM\,\underbrace{\bar{N}N}_{O(1)} \cdot F(q)$$

$$\boxed{\frac{d\sigma}{dE_R} = \frac{2m_N}{\pi v^2}\left[Z f_p + (A-Z) f_n\right]^2 F^2(q)}$$

$$\frac{d\sigma}{dE_R} = \frac{2m_N}{\pi v^2}\, g^2_{s.d.}\, J(J+1)\Lambda^2 \cdot F^2(q)$$

DM- nucleon

$$\mu_p^2 \sigma_N = \mu_N^2 A^2 \sigma_p$$



Shall we build a DM detector!

$$\sigma_p \propto \frac{\mu_p^2 g^2}{\pi} \propto \frac{(GeV)^2}{\pi (300\ GeV)^2}$$

$$\cdot N = \underline{n v \sigma t}\, N_T \sim 10^{-2} \text{ events/kg/day}$$

$\sim g(100\,\mathrm{GeV})$

$\boxed{M_{DM} \sim 1\ \mathrm{GeV}}$ (WIMPS)

Axions (Axion-like Part.) ALPs

$m \gtrsim 10^{-21}\,\mathrm{eV}$ · $\boxed{m \gg T}$

↳ $T \sim 1\,\mathrm{eV}$, $m \gtrsim 1\,\mathrm{eV}$

WLDM (fuzzy DM) $\quad m \sim 10^{-5}\,\mathrm{eV}$ (QCD axion)

θ strong CP problem

· $\langle \theta \int d^4x\, \underbrace{G_{\mu\nu} G_{\alpha\beta}}_{\text{field str. } (F_{\mu\nu})} \varepsilon^{\mu\nu\alpha\beta} \rangle$

total deriv.

· exp. $\boxed{\theta < 10^{-11}}$

$\boxed{\cdot\ \text{axion}}$ $\quad \varphi \mapsto \varphi + a$

↳ $\varphi \rightsquigarrow \boxed{\varphi \mapsto \varphi + a}$

$\int d^4x\, \varphi/f_a\, G_{\mu\nu} G_{\alpha\beta}\, \varepsilon^{\mu\nu\alpha\beta}$

$\boxed{\theta_{obs} \approx 0}$

· $\int \varphi\, \partial_\mu (K^\mu)$

$(\theta + \varphi/f_a) \int d^4x\, G G \varepsilon$

$\boxed{\bar{\varphi} = -f_a \theta}$

Peccei–Quinn
$U(1)_{PQ}$

$$\varphi \mapsto \varphi + a \not\Leftrightarrow m^2\varphi^2$$

• φ acquires a mass $\boxed{m \ll f_a}$

$$\rightsquigarrow \mathcal{L}_\varphi = \frac{1}{2}\left(\partial_\mu\varphi\,\partial^\mu\varphi - m^2\varphi^2\right) + \cancel{O(\varphi^4)}$$

$$\boxed{m_\varphi = \frac{13\ \mathrm{MeV}}{f_\varphi/\mathrm{GeV}}}$$

• non-thermal production
• misalignment

$$\varphi = \bar\varphi + \delta\varphi \ ;$$

$\delta\varphi(t)$

• $t_0 \to \bar\varphi + \delta\varphi_0$

$$a(t)\,\eta^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi$$

$$\to \delta\ddot\varphi + 3\underbrace{\left(\frac{\dot a}{a}\right)}_{=H}\delta\dot\varphi + m^2\delta\varphi = 0$$

$$\rho \propto \frac{1}{a^3}$$

$$\left[\rho_{\delta\varphi} = \frac{1}{2}\left[\underset{K}{\dot\varphi^2} + \underset{\text{Pot.}}{m^2\varphi^2}\right]\right] \quad \left(\frac{\partial\varphi}{\partial x^i} = 0\right)$$

$$H^2 = \frac{8\pi G}{3}\rho$$

- $H = ct \to \delta\varphi = \delta\varphi_0 e^{\omega t}$

$$\boxed{\omega^2 + 3H\omega + m^2 = 0}$$

- $H \gg m$ · $\omega = 0$ · $\omega = -3H$

$\delta\varphi \propto ct.$

$\rho \propto ct$

- $H \ll m$; $\boxed{\omega = -\frac{3H}{2} \pm im}$

$$\boxed{\delta\varphi(t) = \delta\varphi_0 e^{-\frac{3H}{2}t} \cdot \cos(mt + \varphi_0)}$$

- $\frac{\dot a}{a} = H = ct \to \boxed{a = a_0 e^{Ht}}$

$\delta\varphi = \delta\varphi_0\, a^{-3/2} \cos(mt + \varphi_0)$

- $\rho = \frac{1}{2}\left[\delta\dot\varphi^2 + m^2 \delta\varphi^2\right] =$

$\simeq (\delta\varphi_0)^2 \cdot \cos(mt + \varphi_0)^2 \frac{1}{a^3}$

$\langle \quad \rangle = \frac{1}{2}$

$\frac{1}{T}\int_0^T \cos^2(mt)\,dt$; $T \gg m$

$\langle \rho \rangle \simeq (\delta\varphi_0)^2 \frac{1}{a^3} m^2$

$\cdot\ H^2 = \frac{8\pi G}{3}\rho$

$\rho_0 = (\delta\varphi_0)^2 m^2$

$\cdot$ PQ

$$\Omega_\varphi h^2 \sim \left(\frac{m_\varphi}{10^{-5}\,eV}\right)^{-3/2}$$

$$\Omega_\varphi = \frac{\rho_\varphi}{\rho_c}$$

- How to detect axions?

- $\rho_0 \simeq 0.3\ \mathrm{GeV/cm^3}$; mass so low ↓ ~~recoil~~

DM

- axions as a field

classical field ⟷ [· φ : a field with high oc. number
. degree incoh.

- $\varphi \, \boxed{G_{\mu\nu} G_{\alpha\beta} \varepsilon^{\mu\nu\alpha\beta}}$

↓

$\boxed{\langle\varphi\rangle}$

$$\varphi = \varphi_0 \cos(mt + \phi_0)$$

$$\rho_\varphi = \frac{1}{2}(\dot\varphi^2 + m^2\varphi^2) = \frac{m^2\varphi_0^2}{2}$$

$$\varphi_0 = \frac{\sqrt{2\rho}}{m}$$

- nEDM $\rightsquigarrow$ $\underline{\theta} + \underline{\Phi}/f_a + \varphi/f_a$

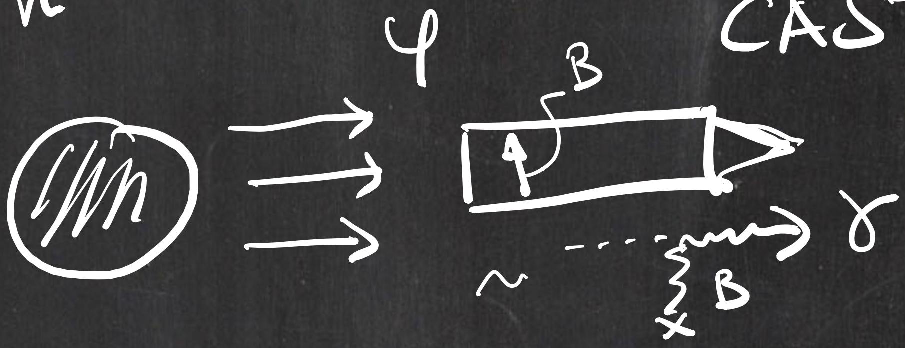
- $\varphi \mapsto \varphi + a$; $\cancel{CP}$

$$\mathcal{L}_{\gamma\gamma\varphi} = -\frac{g_{\gamma\gamma\varphi}}{4}\varphi F_{\mu\nu}F_{\alpha\beta}\varepsilon^{\mu\nu\alpha\beta}$$

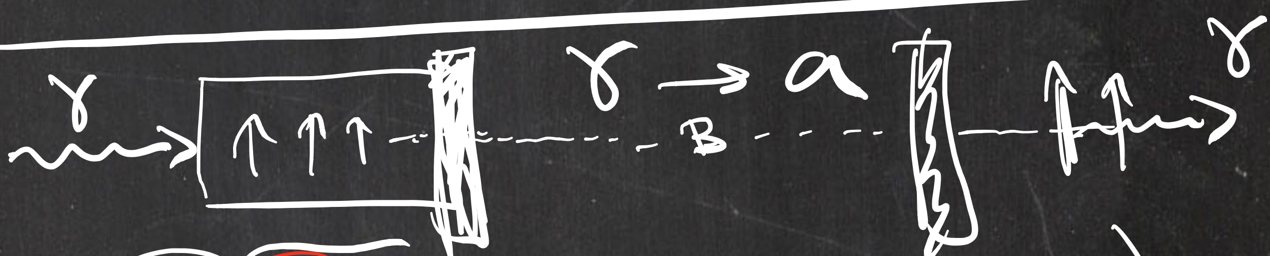
$$= -\frac{g_{\gamma\gamma\varphi}}{4}\underline{\vec{E}}\cdot\underline{\vec{B}}\,\underline{\varphi}$$

Primakoff

CAST helioscope



X-rays



$$P = \frac{4\Delta_M^2}{\Delta_m^2 + 4\Delta_M^2}\sin^2\left(\frac{1}{2}\underline{L}\Delta_{osc}\right)$$

$$\Delta_{osc}^2 = (\Delta_m^2) + 4\underline{\Delta_M^2}$$

$$\Delta_m = \frac{m^2}{2\omega} \simeq 7\times10^{-11}\left(\frac{m}{10^{-7}\,eV}\right)^2 \cdot \left(\frac{10^{19}\,eV}{\omega}\right) pc^{-1}$$

$$\Delta_M = 540\left(\frac{B}{1G}\right)\left(\frac{10^{10}\,GeV}{g_{\gamma\gamma\varphi}^{-1}}\right) pc^{-1}$$

Plasmas in star

$\downarrow$

cooling of stars

$$g_{\gamma\gamma\varphi} \lesssim \frac{10^{-10}}{\text{GeV}}$$

$m < \text{MeV}$