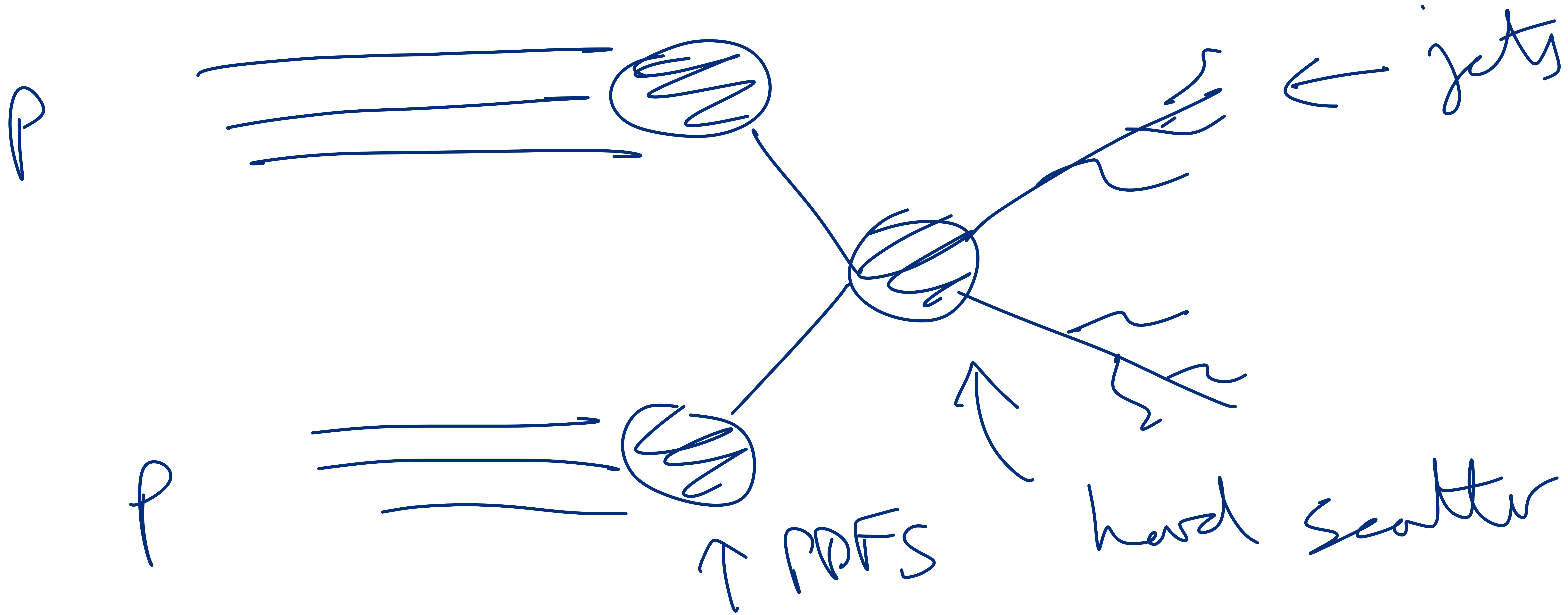
Collider Phenomenology

A typical LHC event



How can we use this to discover new physics / understand the SM

* LHC is a QCD machine!

Main focus: understand how to deal w/ that:

- $\alpha_s(M_Z) \sim 0.1$: pQCD ✓
- Need separation of short distance from long distance → Factorisation & PDFs

- Brief overview

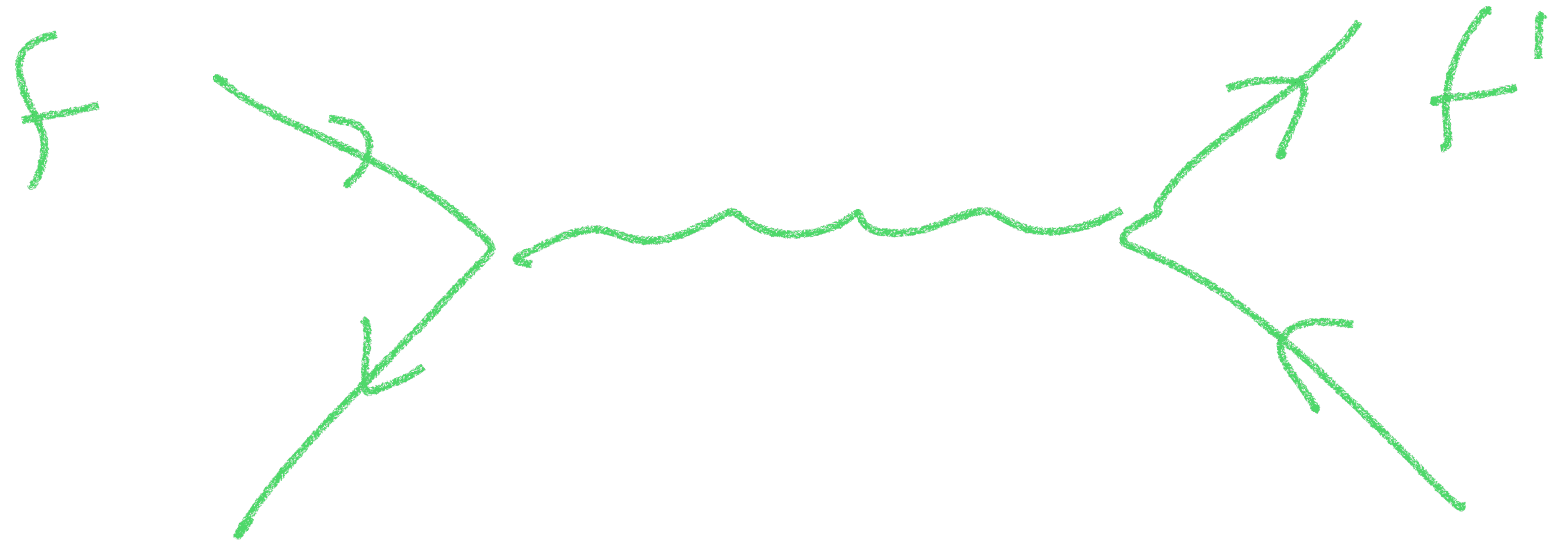
Initially deal with e^+e^- collisions.

Way to introduce key elements.

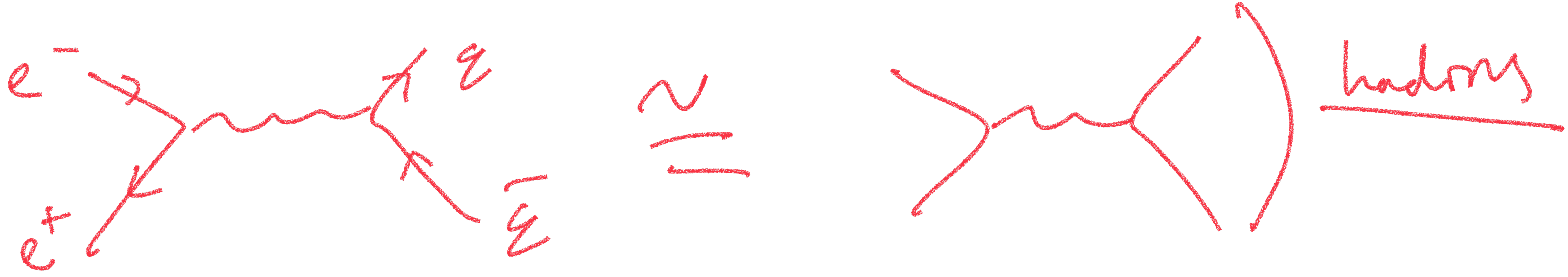
$e^+e^- \to$ hadrons

Recall QED lectures:

$$\frac{d\sigma}{d\cos\theta}(f\bar{f} \to f'\bar{f}') = f_Q^2\, f_Q'^2\, \alpha^2 \cdot \frac{\pi}{2s}\left(1+\cos^2\theta\right)$$



$e^+e^- \to$ hadrons ?



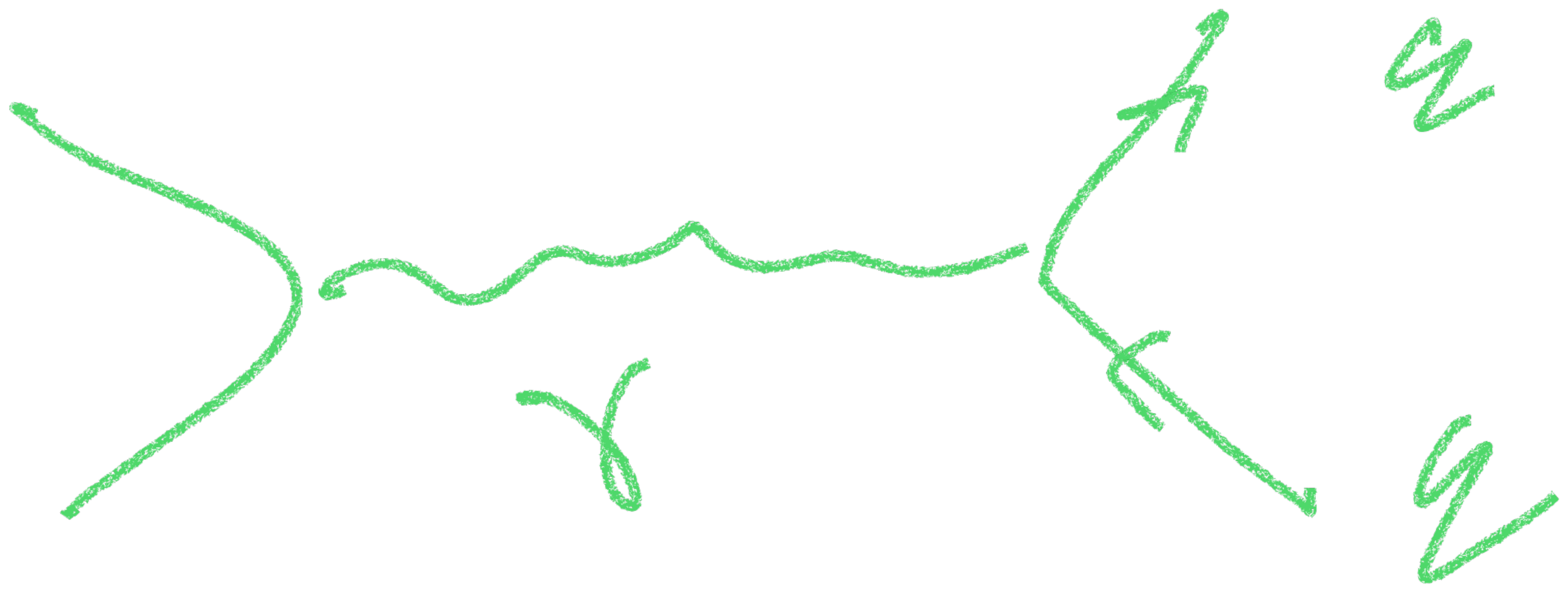
$$\frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \equiv R = N_c \sum_f f_q^2$$

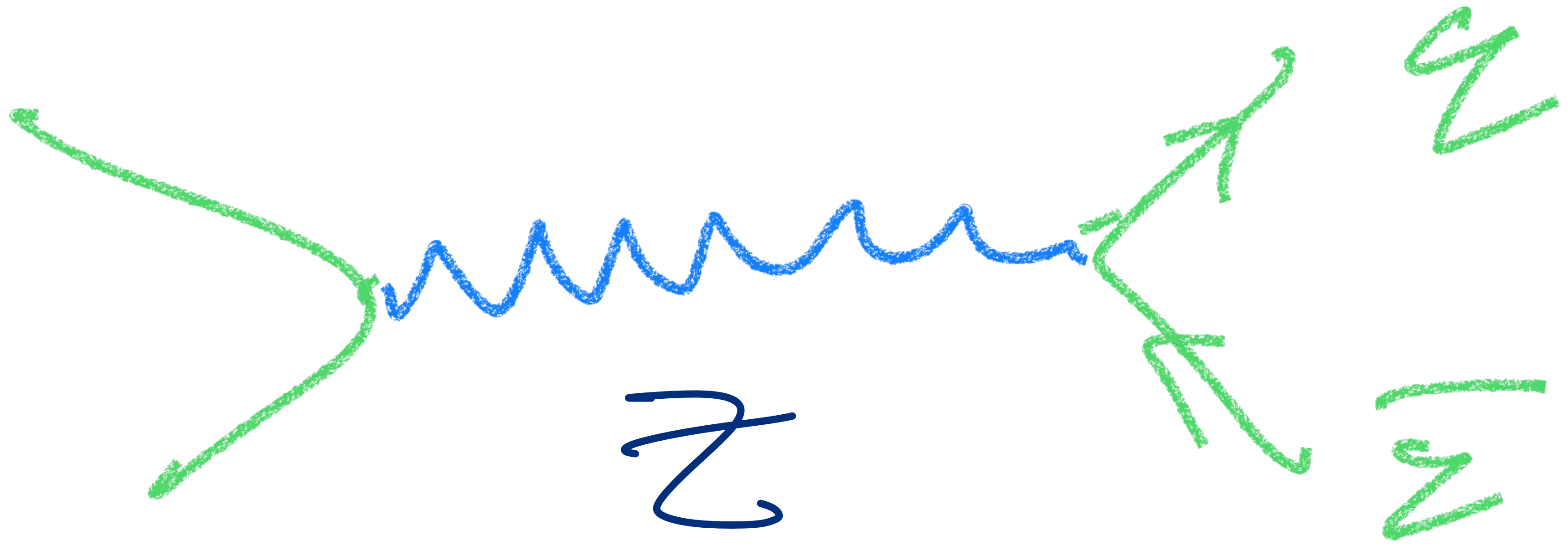
$u, d \ldots$

$s \geq 4 m_q^2$

Z resonance

If $\sqrt{s} \sim M_Z$:





F. rules: $\dfrac{g_{\mu\nu}}{p^2}$ $\qquad$ $g_{\mu\nu} - \cancel{\dfrac{p_\mu p_\nu}{M_Z^2}}$

$M_{e,Z} = 0$

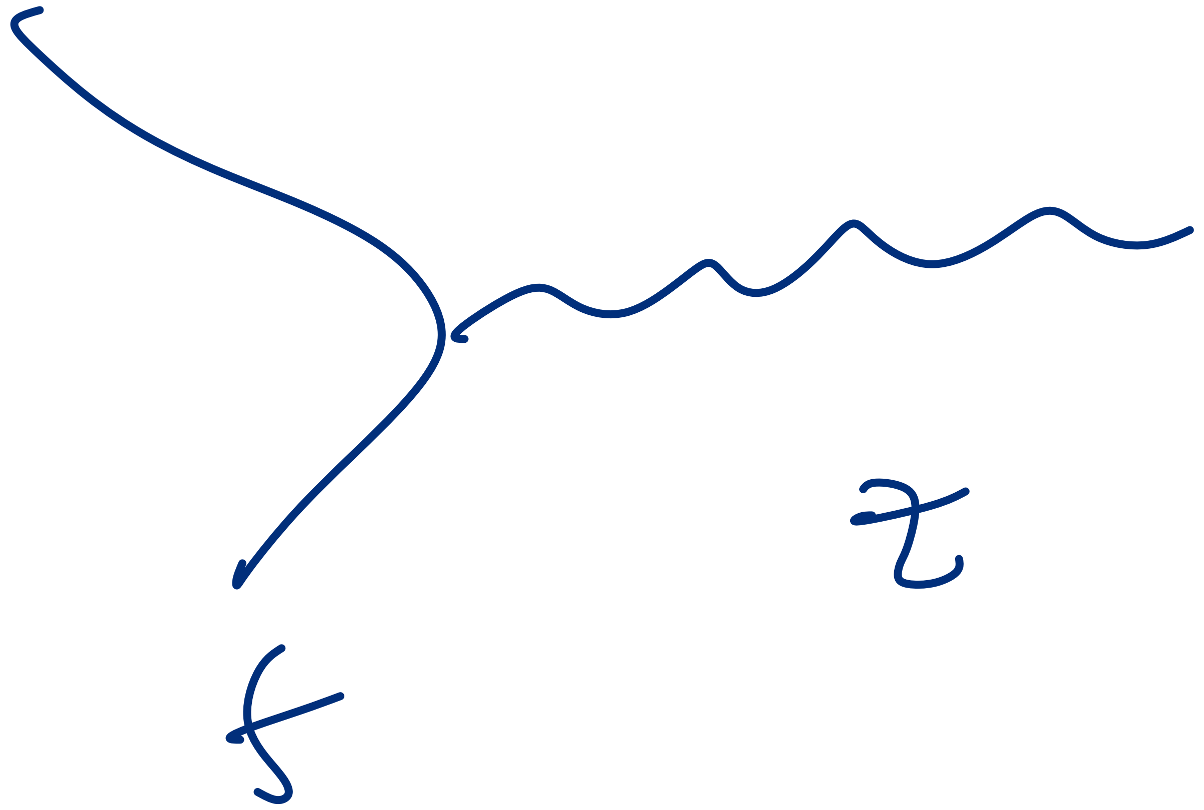
↑

Try!

Z unstable $\Rightarrow$ will decay w/ width Γ_Z $\longrightarrow$

$$\frac{g_{\mu\nu} - \cancel{\frac{p_\mu p_\nu}{M_Z^2}}}{p^2 - M_Z^2 + i\Gamma_Z M_Z}$$

In addition, more complex structure due to our V − A:



μ : $i g_Z \left(v_f \gamma^\mu + a_f \gamma^\mu \gamma^5 \right)$

(see problem & SM).

Width Γ that appears in Z propagator

precisely the usual $Z \to \ldots$ decay

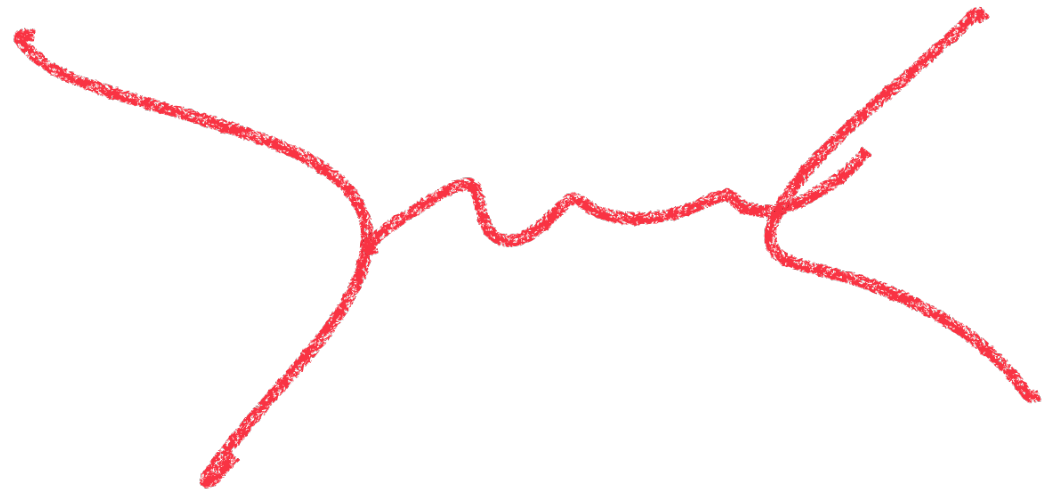
width

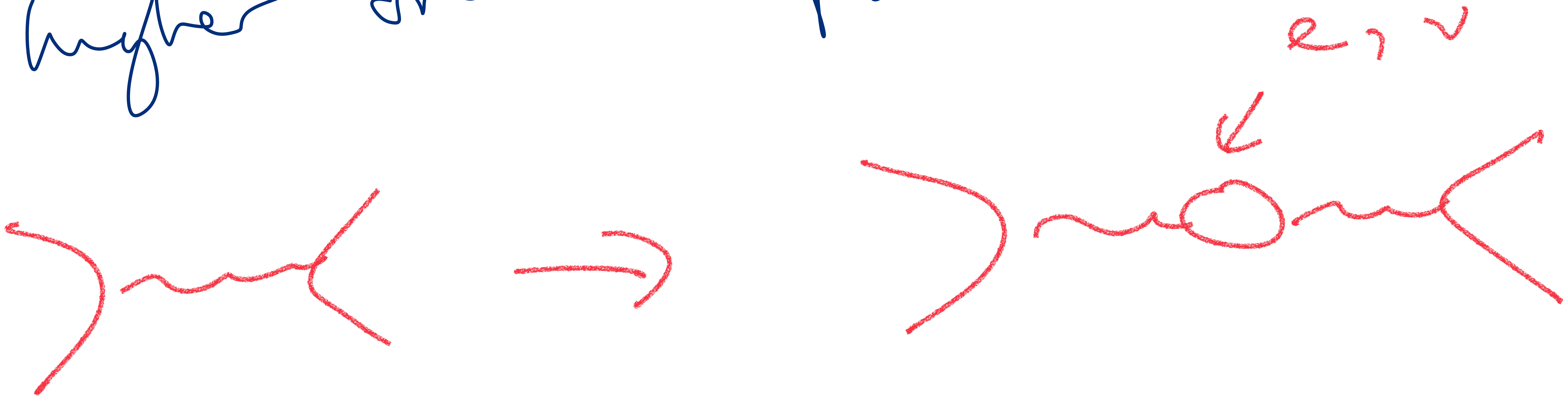
$$\rightarrow \quad \Gamma_Z \propto \int dPS \, |M(Z \to X)|^2$$

$X = e^+e^-, \nu\bar{\nu} \ldots$

What is the connection?

This is a rather subtle aspect of QFT.

Basic point:  is not the only contributing diagram. QFT $\Rightarrow$ higher order loops:



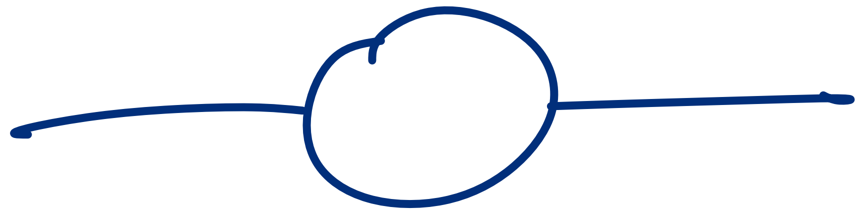
$+ \dots$

Consider e.g. a scalar propagator

$$= \frac{1}{p^2 - M_0^2}$$

$\leftarrow$ bare mass

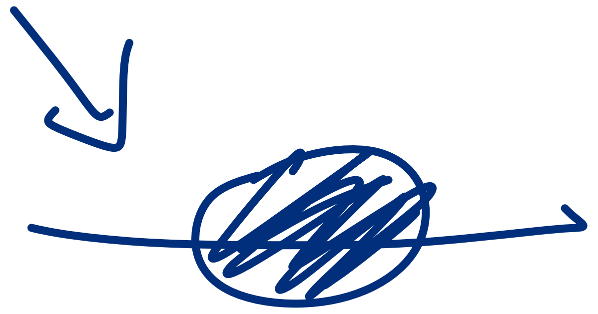
NLO:



$$= \frac{1}{p^2 - M_0^2} \; \Sigma(p)$$

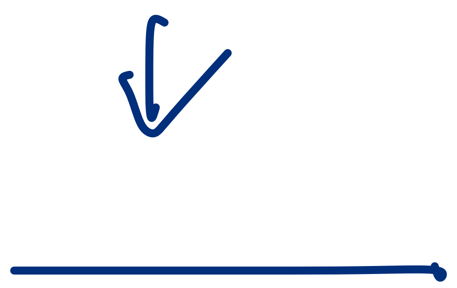
e.g. Σ

Then we write:



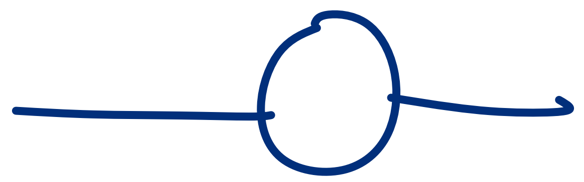
All NLO

$=$



$+$

Σ



$+$

$\Sigma \quad \Sigma$

$+ \ldots$

$$= \frac{1}{p^2 - m_0^2}\left(1 + \frac{\Sigma(p)}{p^2 - m_0^2} + \left(\frac{\Sigma(p)}{p^2 - m_0^2}\right)^2 + \dots\right)$$

geometric series

$$= \frac{1}{p^2 - m_0^2} \frac{1}{\left(1 - \frac{\Sigma}{p^2 - m_0^2}\right)}$$

$$= \frac{1}{p^2 - m_0^2 - \Sigma}$$

Input of loops: $\frac{1}{p^2 - m_0^2} \rightarrow \frac{1}{p^2 - m_0^2 - \Sigma}$

$im\Gamma$??

Σ : amplitude $\Rightarrow$ complex value

$$\frac{1}{p^2 - m_0^2 - \Sigma} \quad \text{vs.} \quad \frac{1}{p^2 - m^2 - i\Gamma M}$$

'physical mass'

$$\mathrm{Re}(\Sigma) + m^2 = m_0^2$$

$$\& \ \mathrm{Im}(\Sigma) = m\Gamma$$

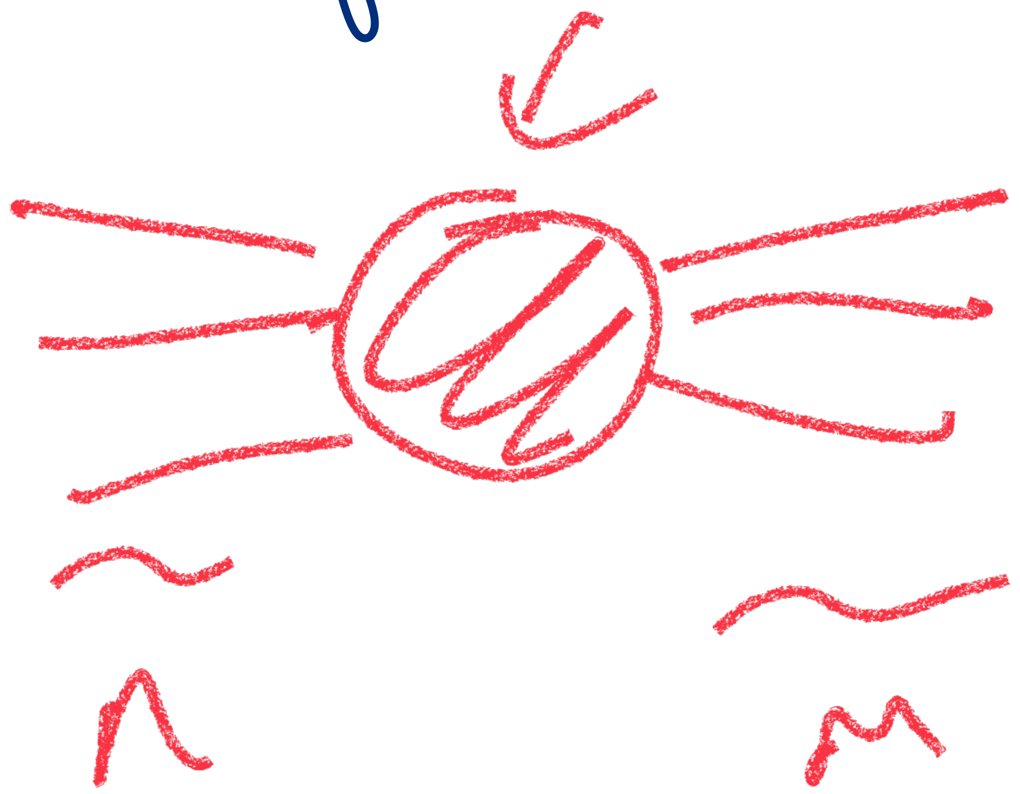
what we want

Recall 'unitarity' relation in QED / QCD.

Aka 'optical theorem'.

Let's start from a general expression for scattering amplitude A!

$$A = \langle m | S | n \rangle$$



Then $$\sum_m |\langle m | S | n \rangle|^2 = 1$$

i.e. prob. that $|a\rangle$ ends up in some state $= 1$.

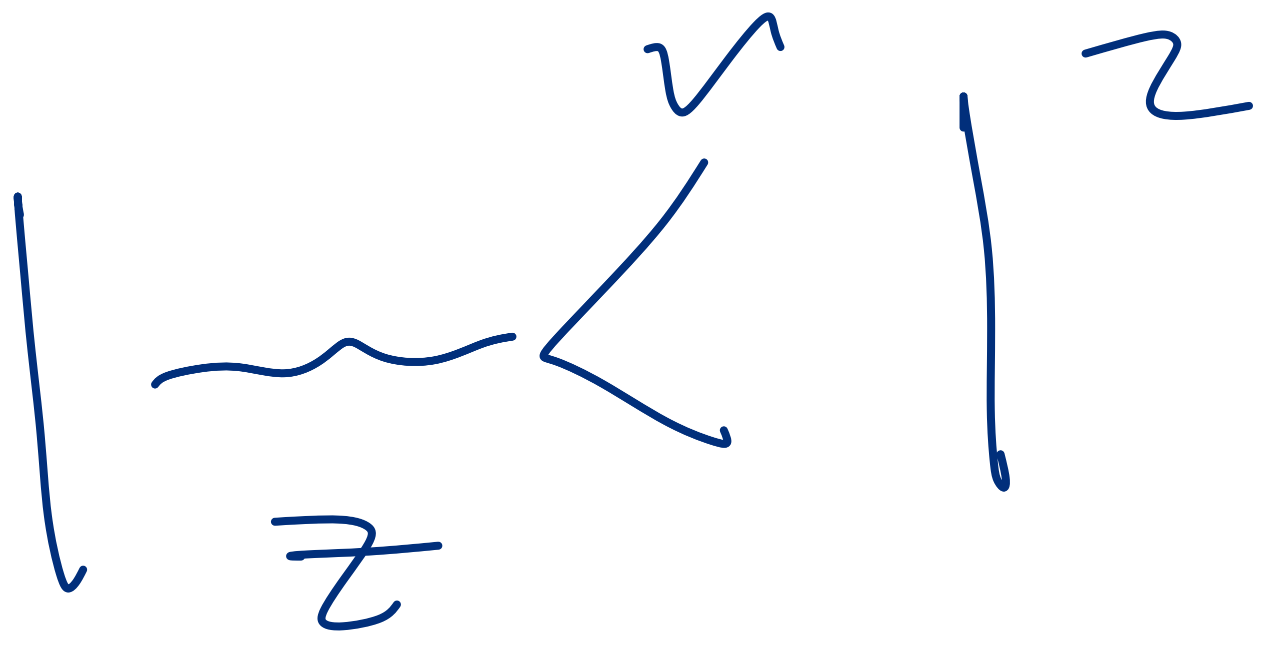
- With a bit of work (skip details), this is equivalent to:

$$2\,\mathrm{Im}\left(A(a \to a)\right) = \sum_f \int d\Pi_f \, |A(a \to f)|^2$$

$$\Rightarrow \quad 2\,\mathrm{Im}(\Sigma) = m\Gamma$$

e.g.

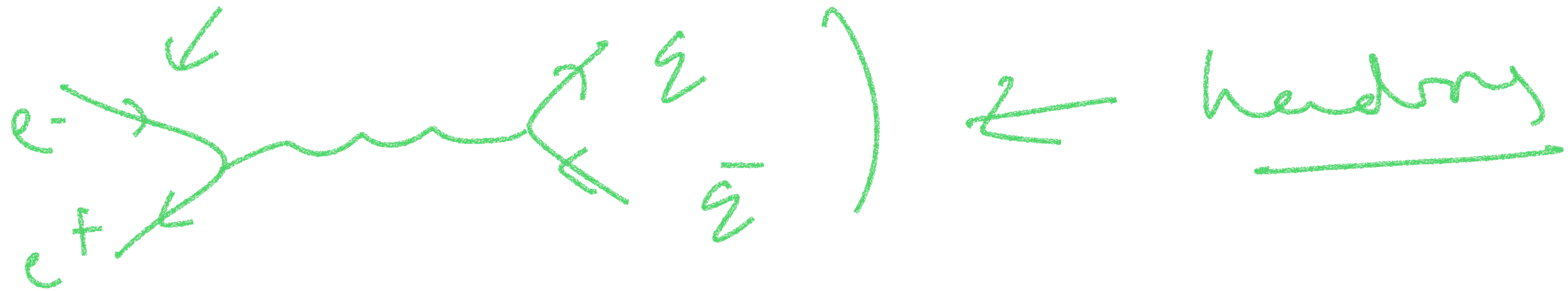
$\sim$



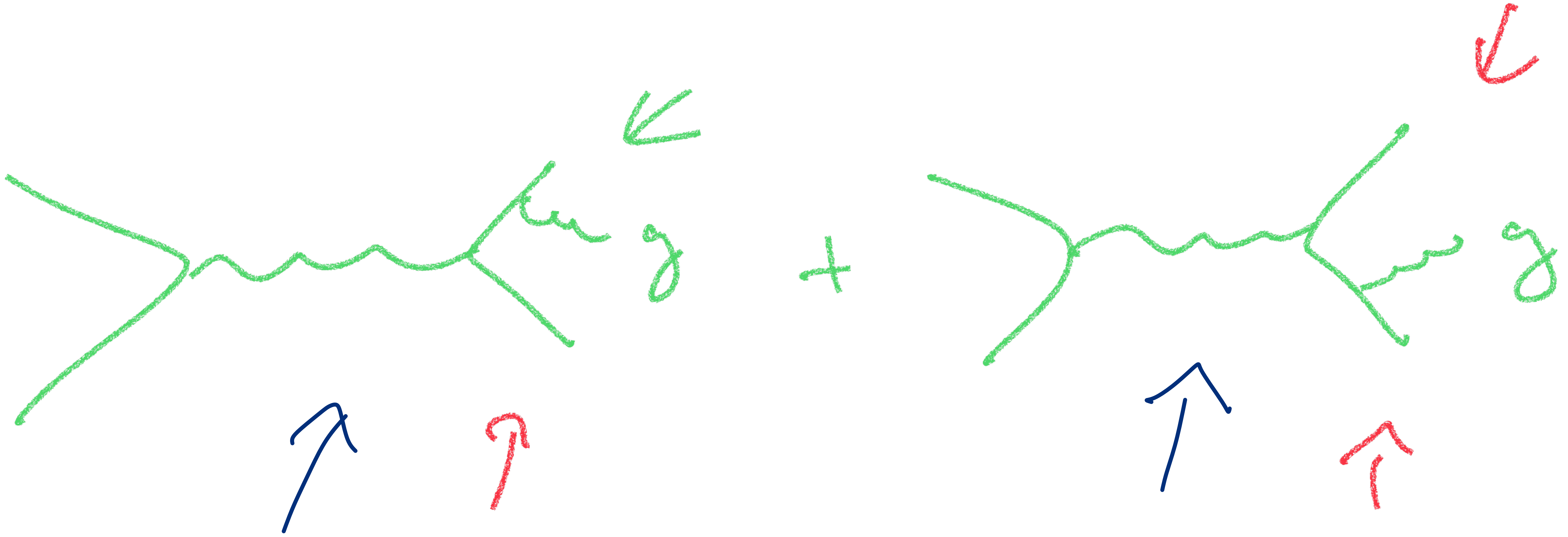
$e^+e^- \to$ hadrons : radiative corrections

Stick with γ exchange for simplicity

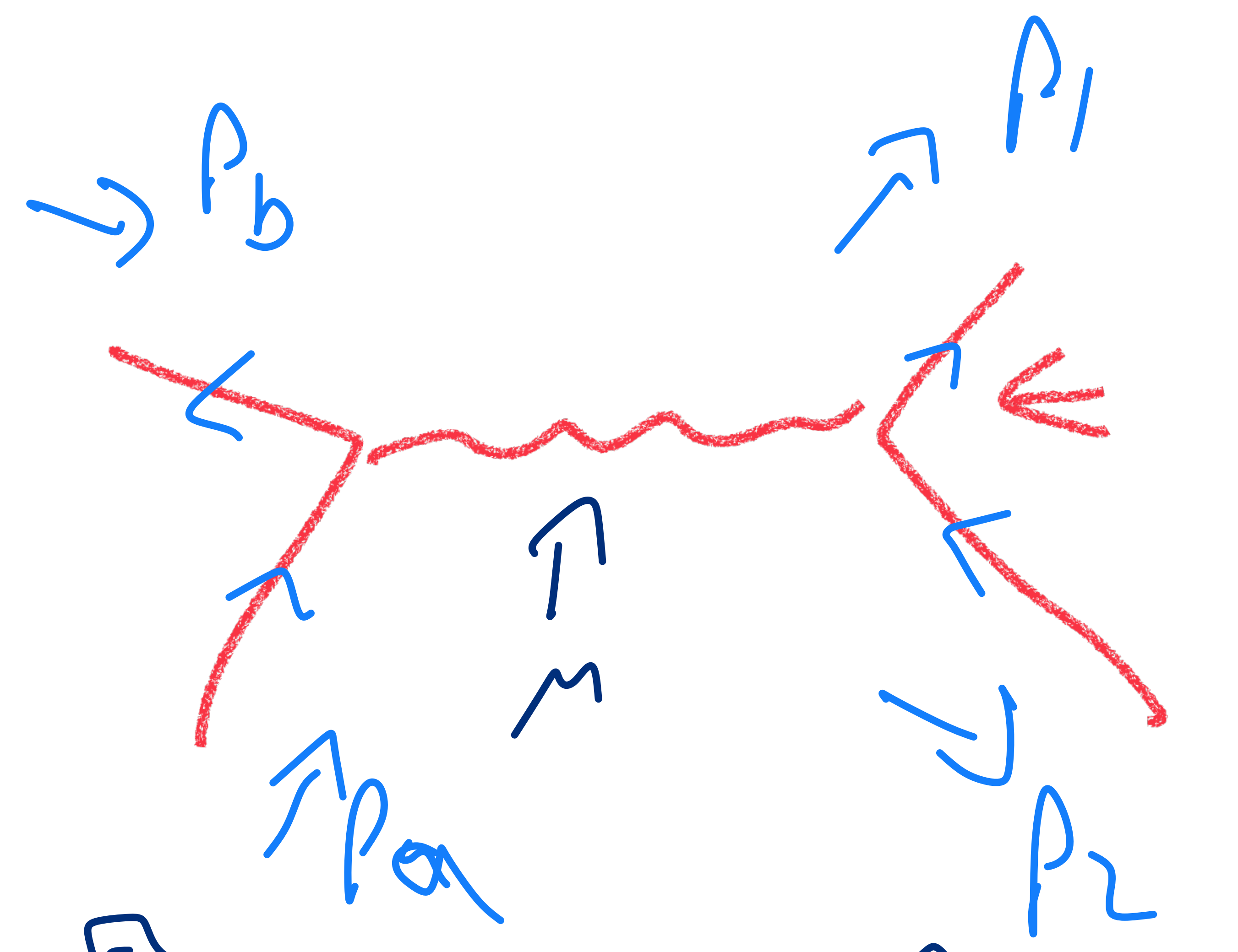
LO:



NLO:



General structure (LO)

$$\langle |M_{q\bar{q}}|^2 \rangle = \frac{1}{4} \sum_{\text{spins, colour}} \left| \vphantom{\Big|} \right.$$  $$\left. \vphantom{\Big|} \right|^2$$

$$\propto \frac{1}{s^2} e^2 \, \mathrm{Tr}\left(\gamma^\mu \not{p}_a \gamma^\nu \not{p}_b\right) \Big\} \; l^{\mu\nu}$$

$$\times f_q^2 \, e^2 \, \delta_{ii} \, \mathrm{Tr}\left(\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_1\right) \Big\} \; H_{\mu\nu}$$

$$\sum_{i,j} \delta_{ij}\delta_{ij} \qquad \equiv \underbrace{l^{\mu\nu}}_{\text{leptons}} \; \underbrace{H_{\mu\nu}}_{\text{quarks}}$$

The purely leptonic part is factored off
→ calculate once & forget about.

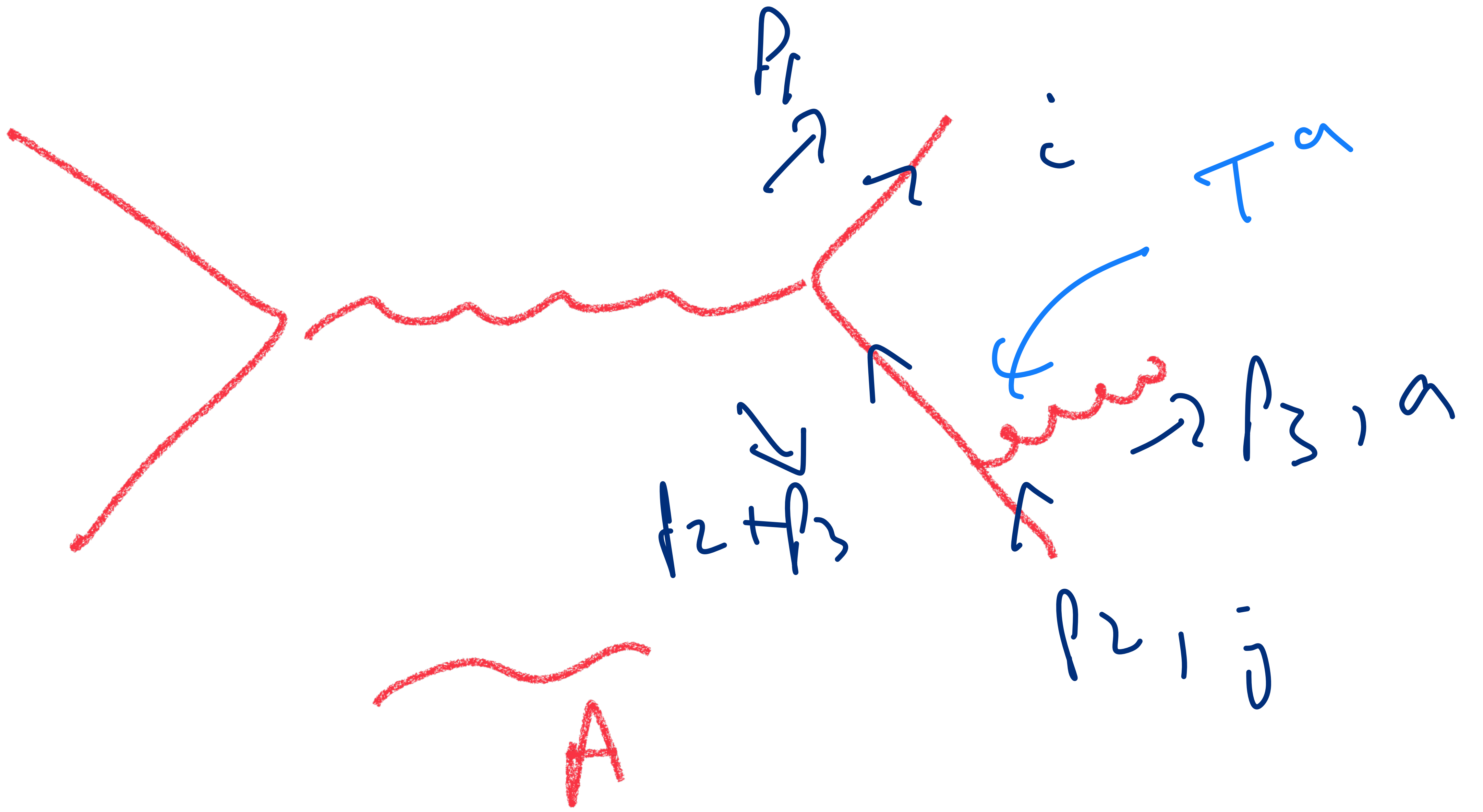
At NLO (real):

$$\langle |M_{q\bar{q}g}|^2 \rangle = L^{\mu\nu} H_{\mu\nu}^{NLO}$$

same as before

focus on that

e.g.



$\times \quad A^*$

a contribution to $H^{NLO}_{\mu\nu}$

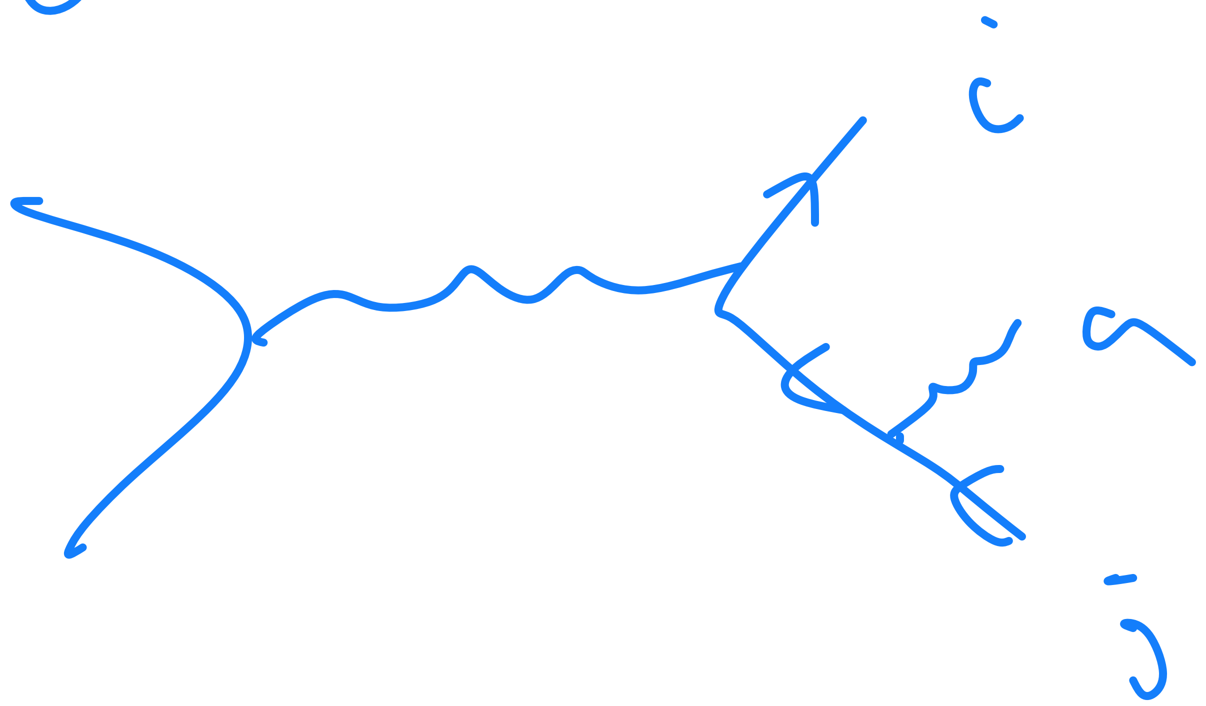
$$\propto L^{\mu\nu} \cdot e^2 g^2 \underbrace{T^a_{ij} T^a_{ji}}_{C_F} \mathrm{Tr}\left(\not{p}_1 \gamma_\mu (\not{p}_2 + \not{p}_3)\right.$$

$$\left. \cdot \gamma^\rho \not{p}_2 \gamma_\rho (\not{p}_2 + \not{p}_3) \gamma_\nu \right) \frac{1}{s_{23}^2}$$

$$\varepsilon^\rho \varepsilon^{*\sigma} \to -g^{\rho\sigma}$$

$$s_{ij} = (p_i + p_j)^2$$

Aside: Why $T^a_{ij} T^a_{ji}$

$\mathcal{M}$: 

$\sim T^a_{ij}$

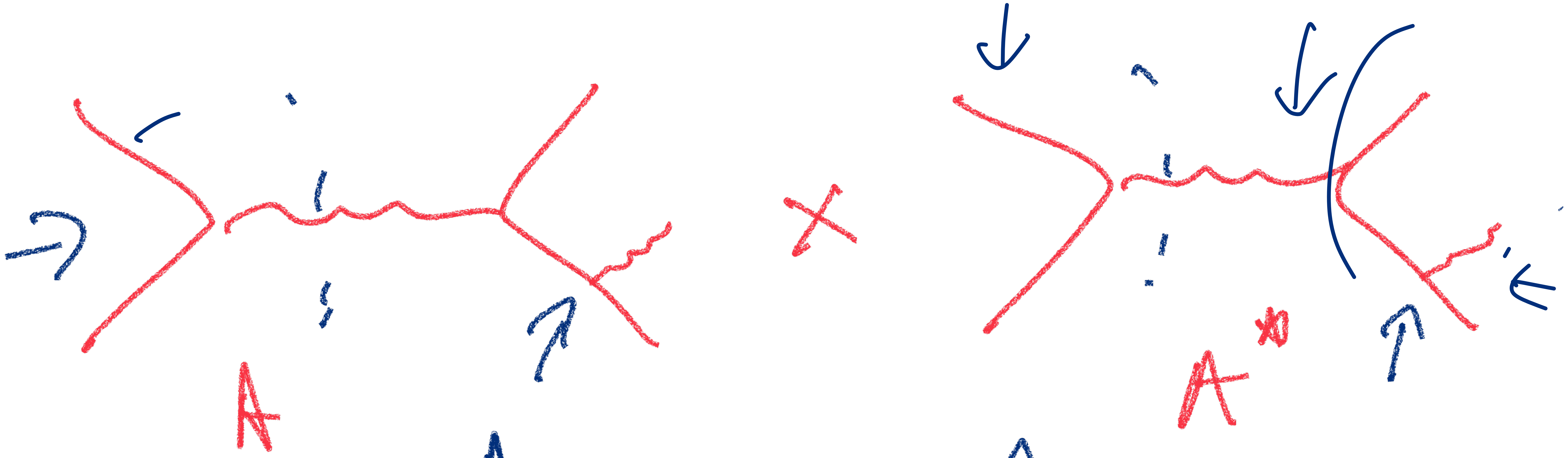
$$\mathcal{M}^* \sim \left(T^{a'}_{i'j'}\right)^*$$

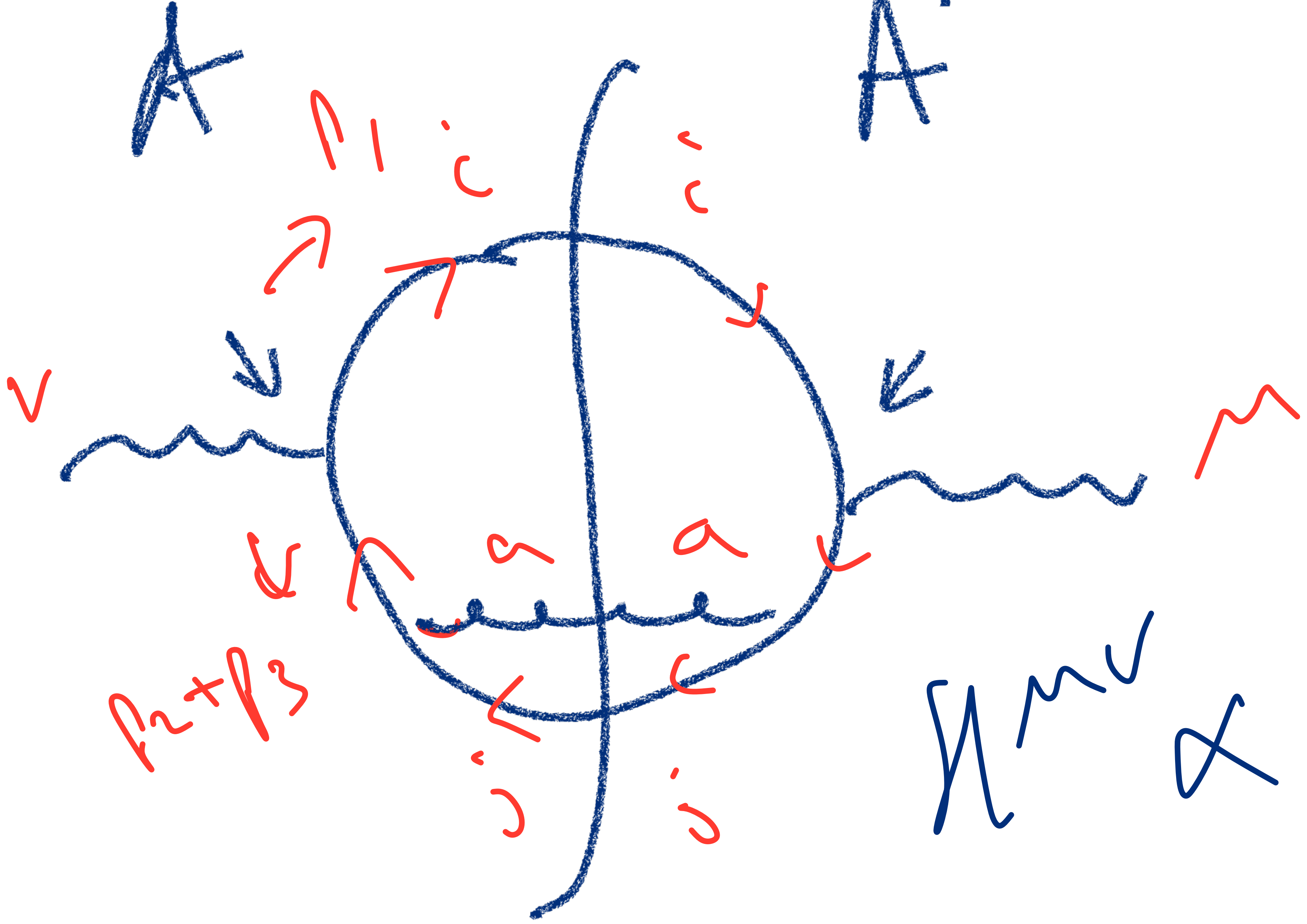
- The T^a's are hermitian $\Rightarrow T = T^\dagger$

i.e. $\left(T^{a'}_{i'j'}\right)^* = T^{a'}_{j'i'}$

$$\Rightarrow |\langle \mathcal{M}\rangle|^2 \sim \delta_{ii'}\,\delta_{jj'}\,\delta_{aa'}\,T^a_{ij}\,T^{a'}_{j'i'} \quad ✓$$

Can be useful to represent $H_{\mu\nu}$ pictorially:

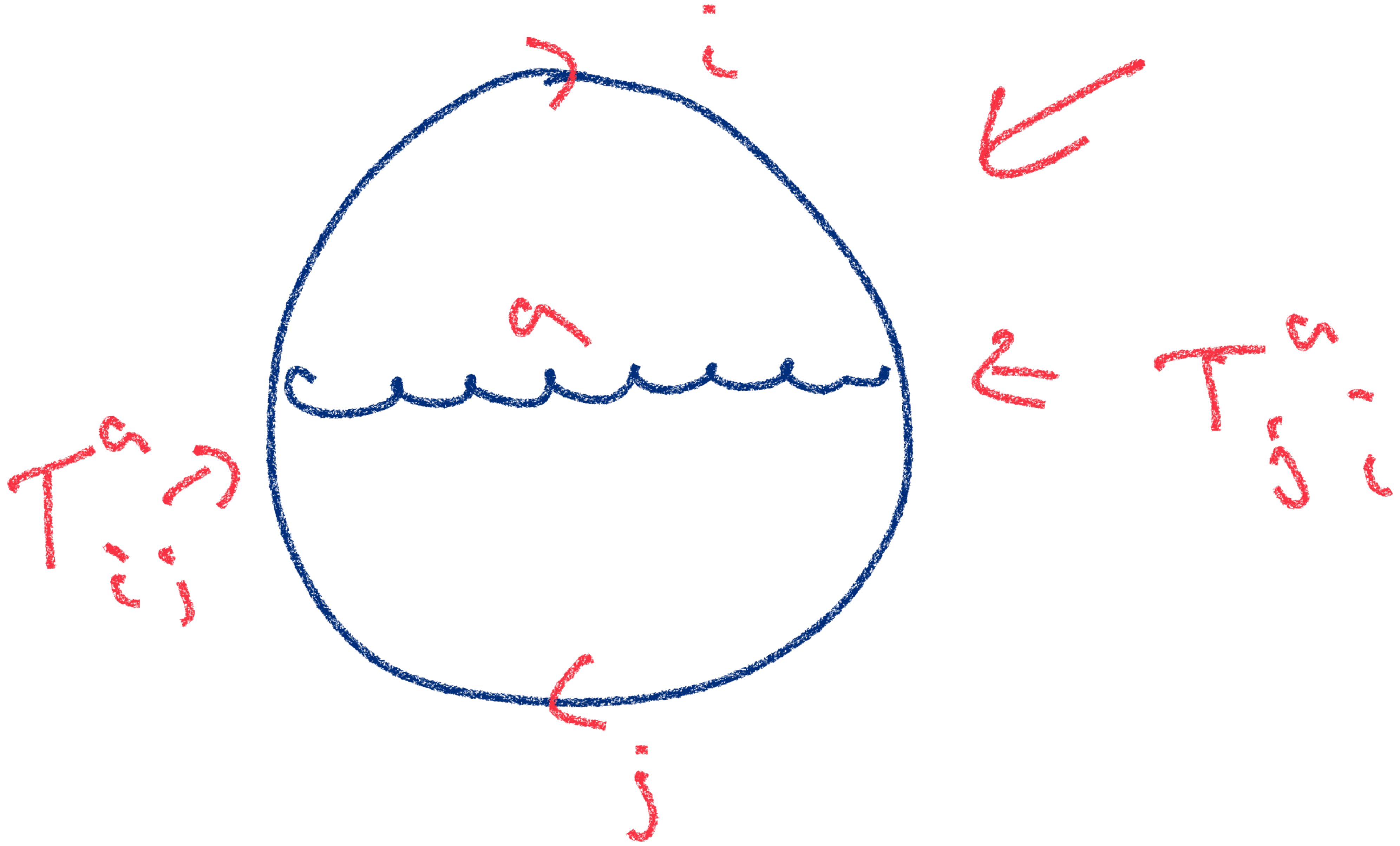




Not Feynman diagrams. Just a way of keeping track:

$$H^{\mu\nu} \propto T^a_{ij} T^a_{jc} \, \mathrm{Tr}\left(\not p_1 \gamma^\mu (\not p_2 + \not p_3) \gamma^\rho \not p_2 \gamma_\rho (\not p_2 + \not p_3) \gamma^\nu\right)$$

$$T^a_{ij}\, T^a_{ji} \equiv$$



$$\mathrm{Tr}(T^a T^a) = \frac{1}{2}\delta^{aa} = \frac{1}{2}(N_c^2 - 1)$$

Can always manipulate colour factors down to a small number of known relations:

$\rightarrow$ $\mathrm{Tr}(T^a) = 0$, $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$

Can also represent pictorially:

$$\delta_{ii} = N_c$$

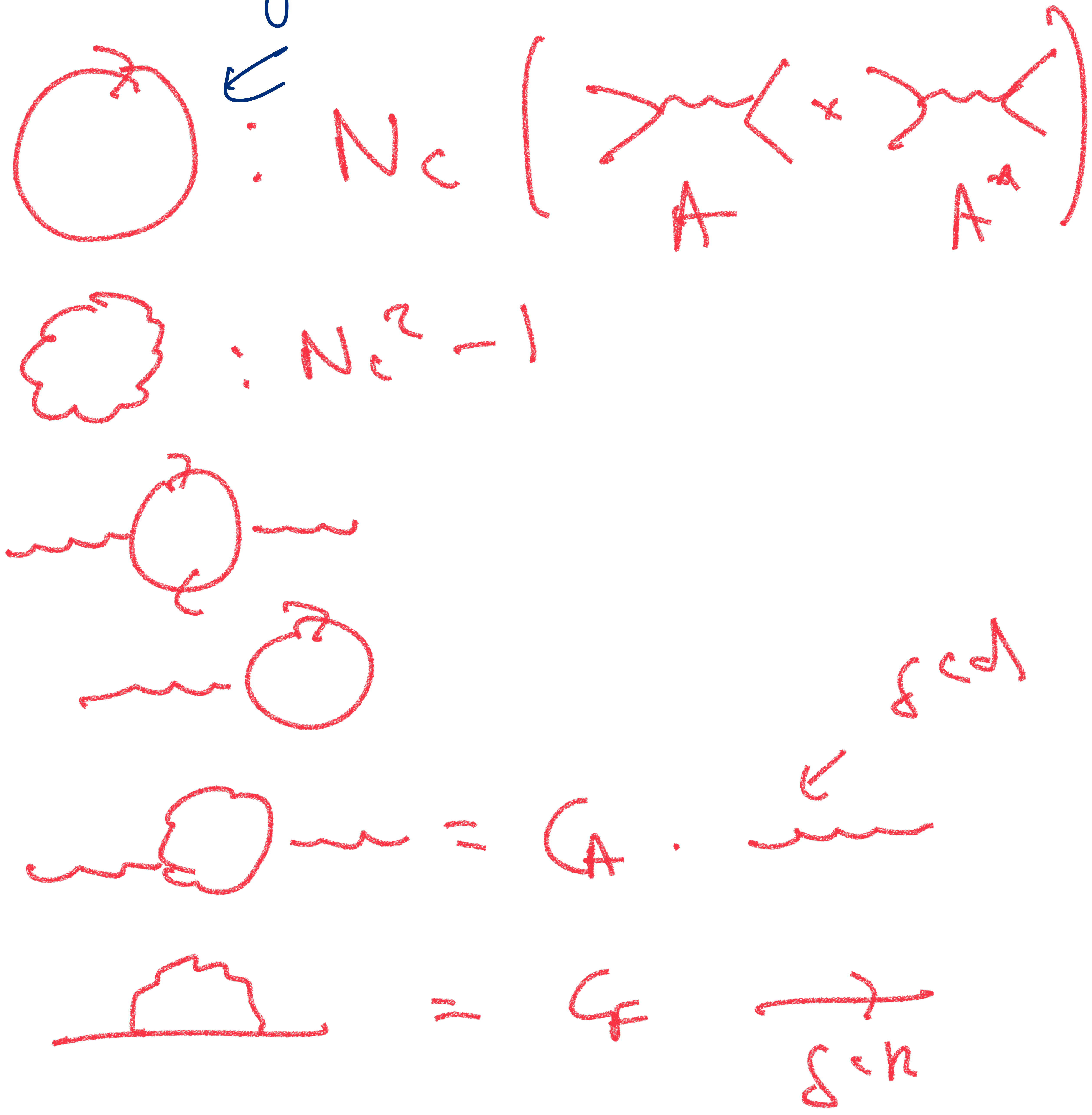
$$\delta_{aa} = N_c^2 - 1$$

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

$$\mathrm{Tr}(T^a) = 0$$

$$f^{abc} f^{abd} = C_A \delta^{cd}$$

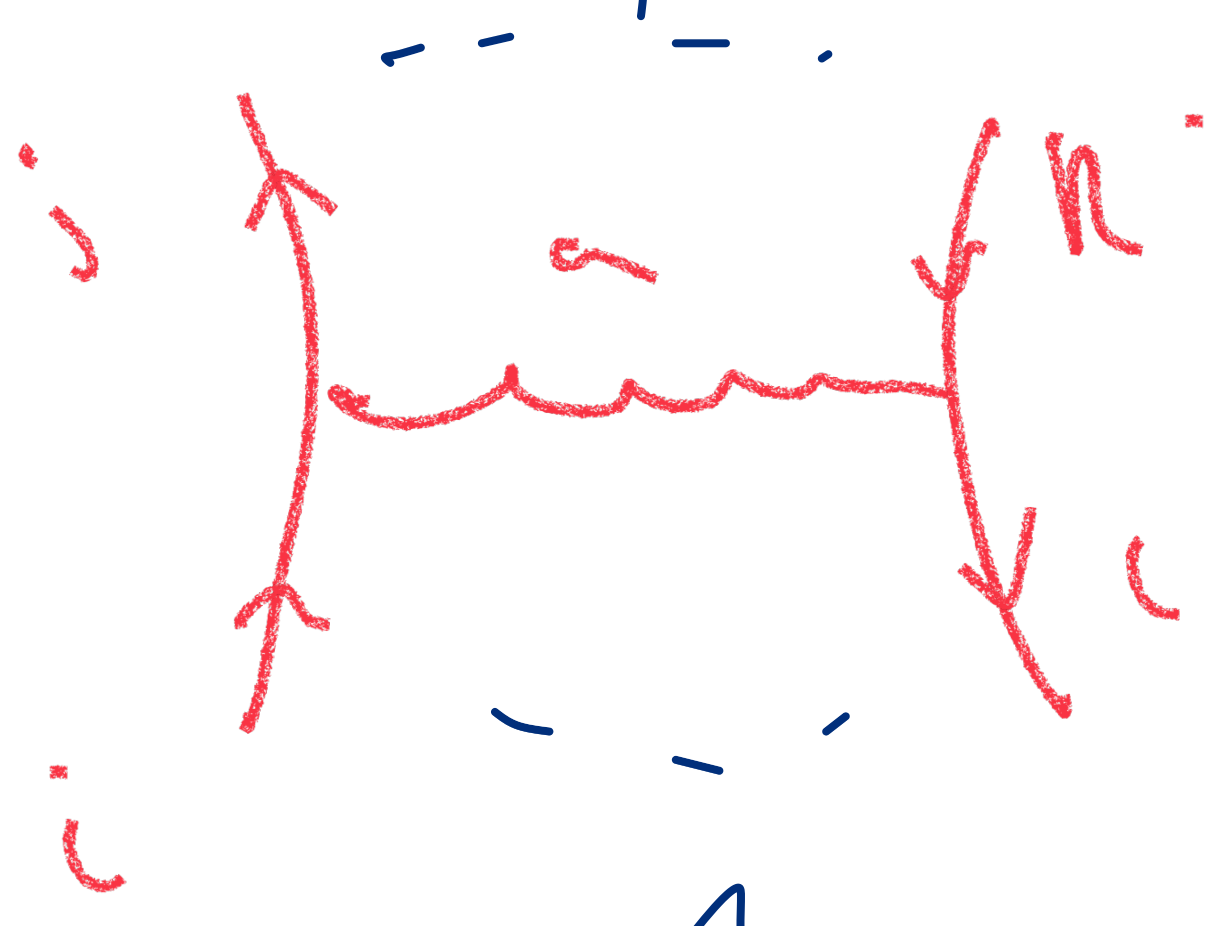
$$T^a_{ij} T^a_{jk} = C_F \delta_{ik}$$



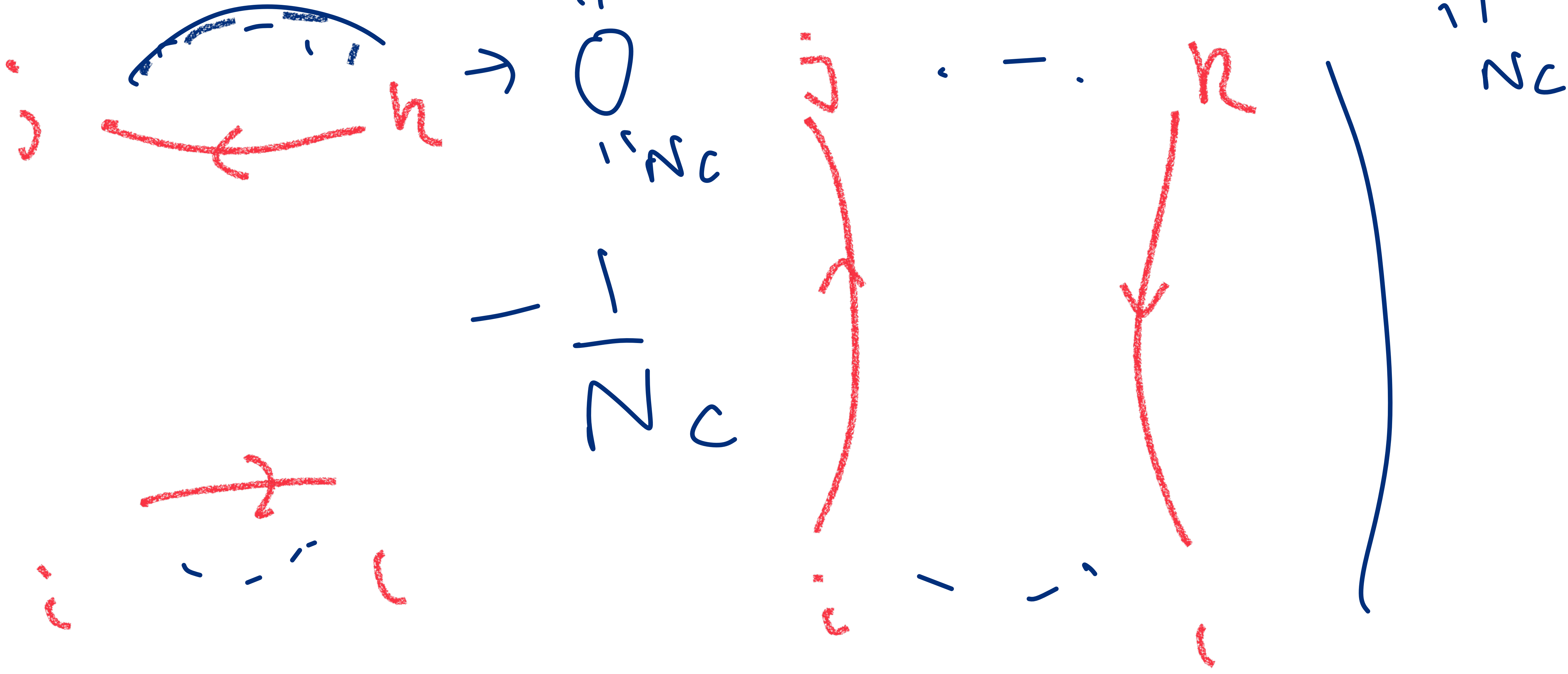
Finally we need the Fierz identity:

$$T^a_{ij} T^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl} \right)$$

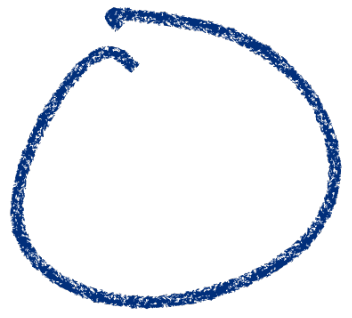
$j = k$ $\quad \delta_{jj} = N_c \quad \delta_{ij}\delta_{ji} = \delta_{ii} = N_c$



$$= \frac{1}{2}\Bigg(\quad - \frac{1}{N_c} \quad \Bigg)$$



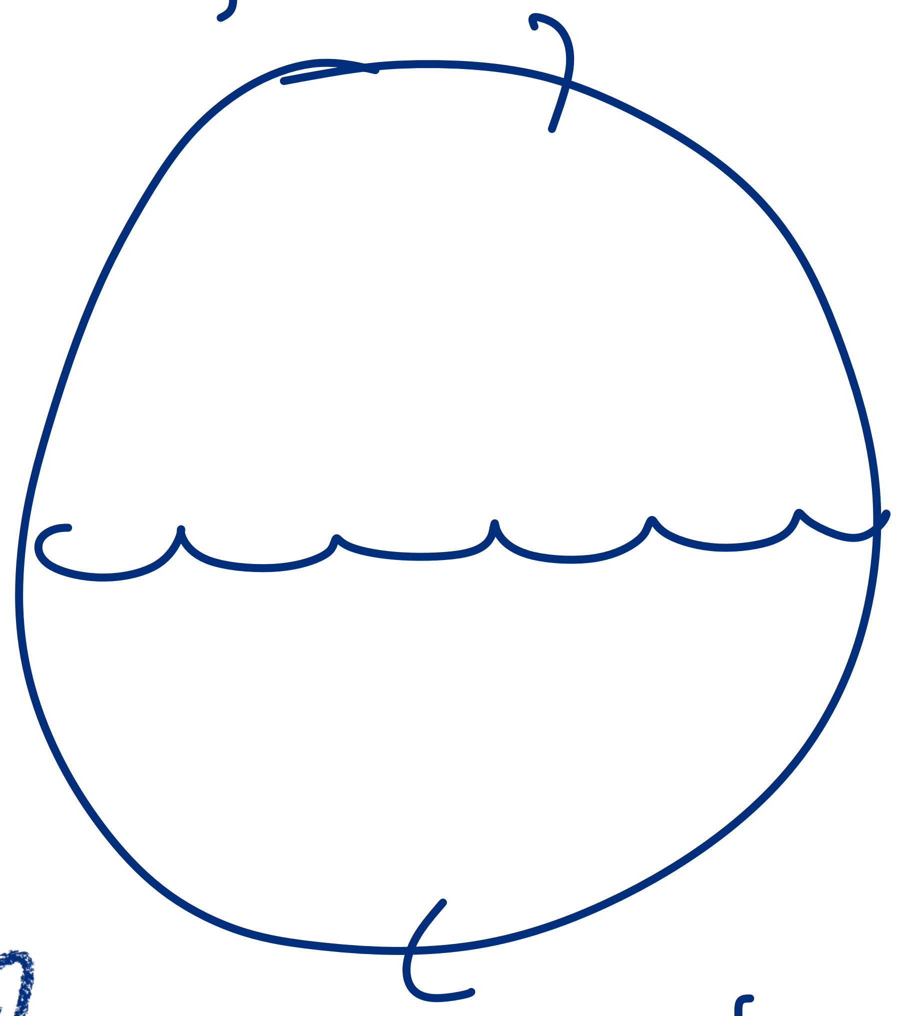
$T^a_{ij} T^a_{ji}$

$\delta_{jk} \xrightarrow{j=k} \delta_{jj} = N_c$ 

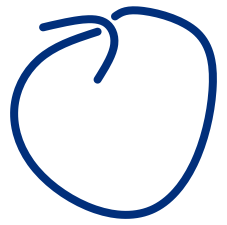
Let's see this in action:

E.g.

$j = k$



$i = l$

$$= \frac{1}{2}\left(\right.$$  $$- \frac{1}{N_c}$$  $$\left. \right)$$

$$= \frac{1}{2}\left(N_c^2 - \frac{1}{N_c} \cdot N_c \right)$$

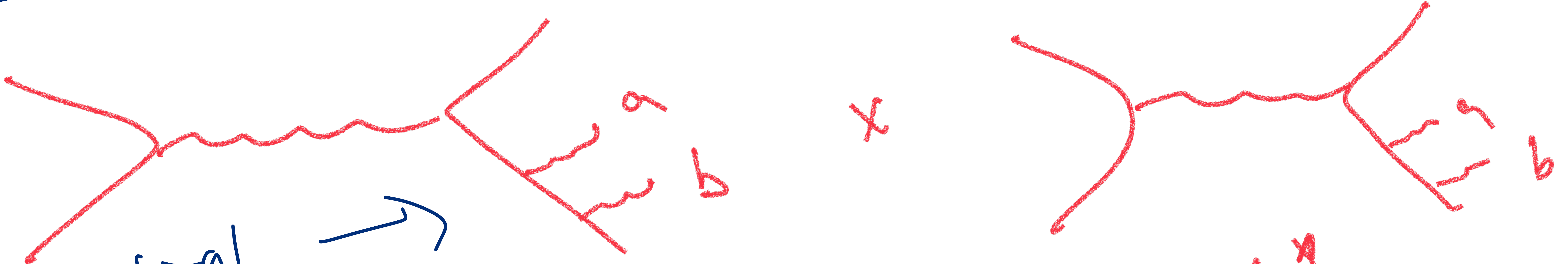
$$= \mathrm{Tr}(T^a T^a)$$

$$\cancel{\delta^{ac}}\; T^a_{ij} T^a_{ji}$$

$$= \frac{1}{2}\left(N_c^2 - 1 \right) = C_F N_c$$

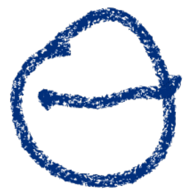
$$C_F = \frac{N_c^2 - 1}{2N_c}$$

More complex case:

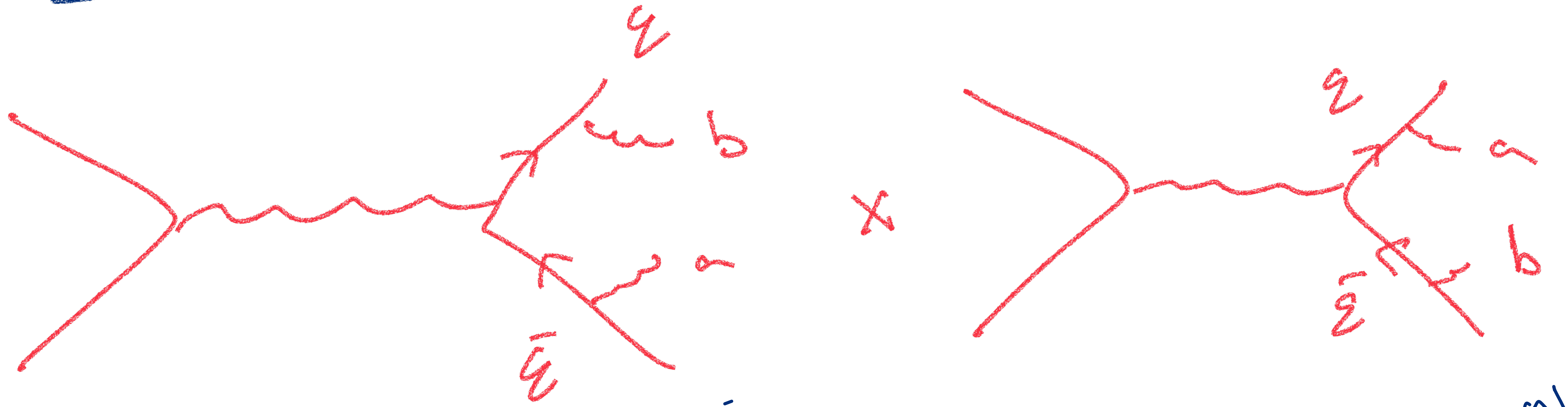


A $\qquad\qquad$ A^*

$\mathrm{Tr}[T^a T^b T^b T^a]$

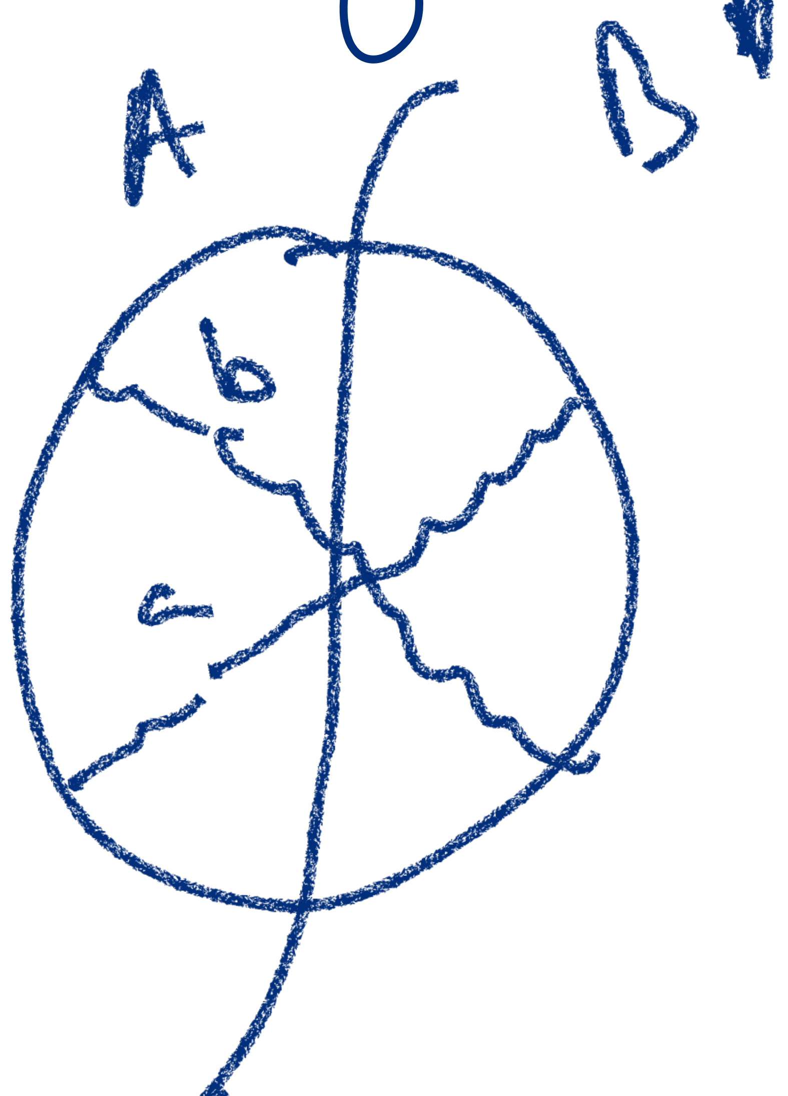
$= \frac{1}{2}\left(N_c - \frac{1}{N_c}\right)$  $= C_F$  $= C_F^2 N_c$

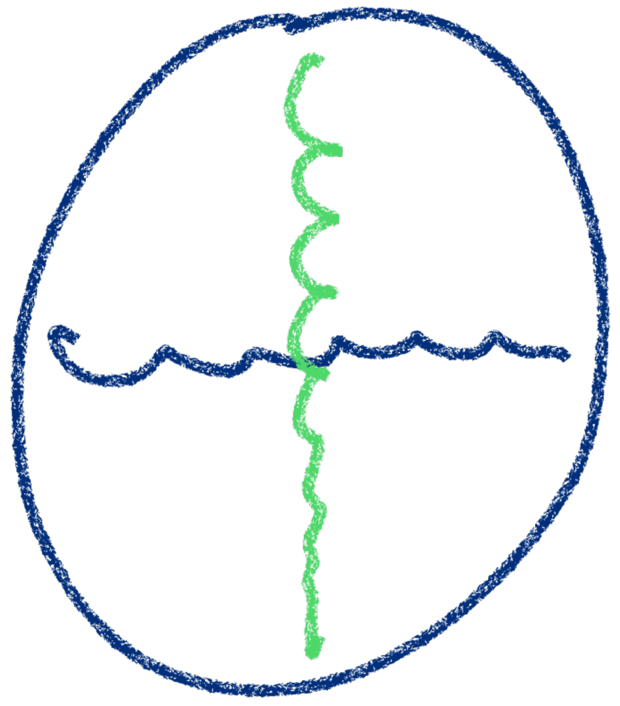
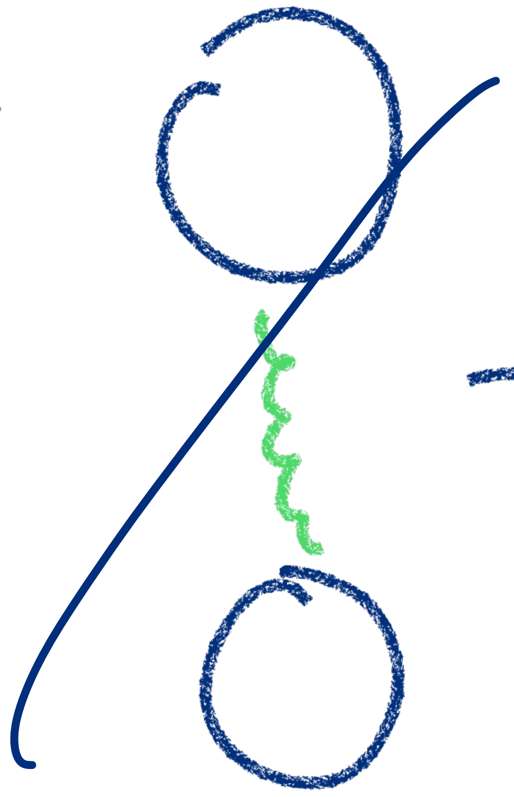
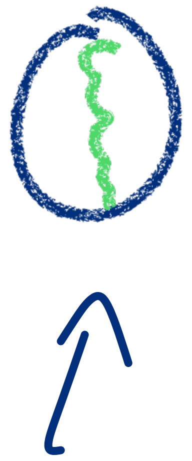
One more example :

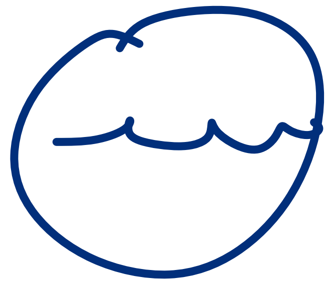


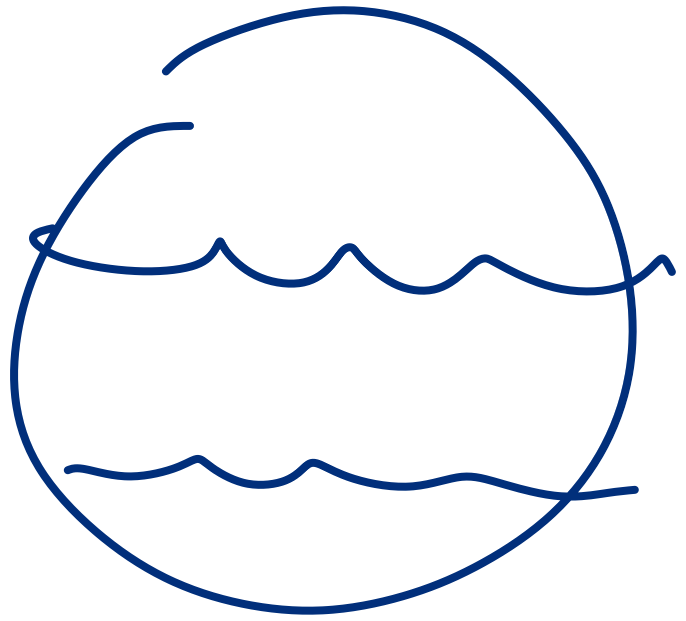
i.e an interference diagram .

$\swarrow$ $\mathrm{Tr}(T^a) = 0$

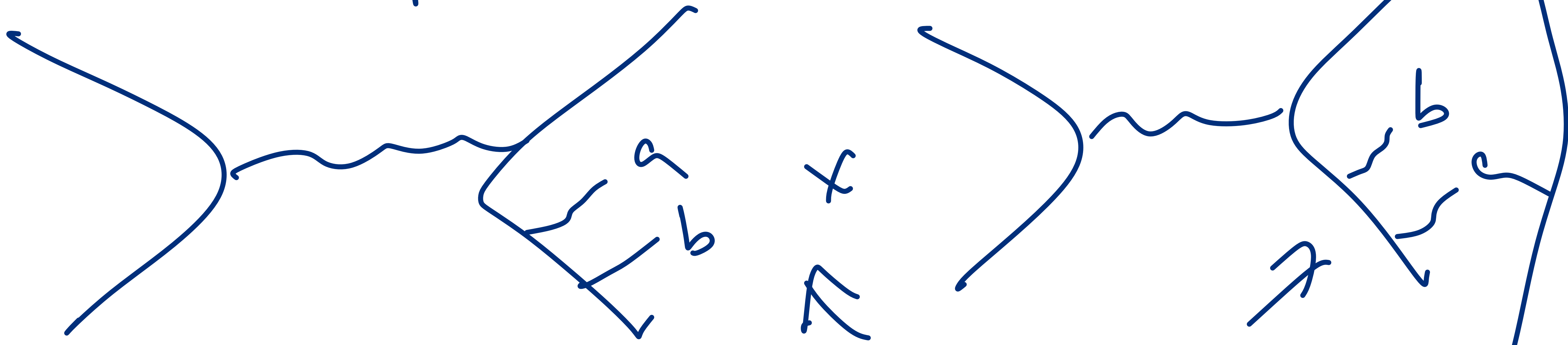
$\rightarrow$ 

$\rightarrow$  $= \frac{1}{2}\Big($  $- \frac{1}{N_c}$  $\Big)$

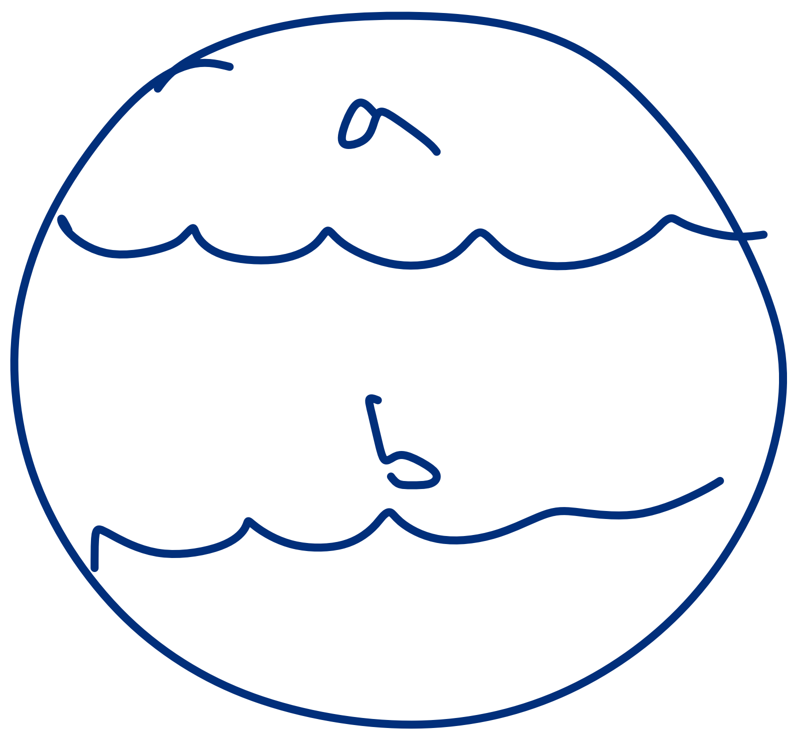
$= -\frac{1}{2N_c}$  $= \underline{\underline{-\frac{1}{2N_c} C_F N_c}}$

compare w/  : $C_F^2 N_c$

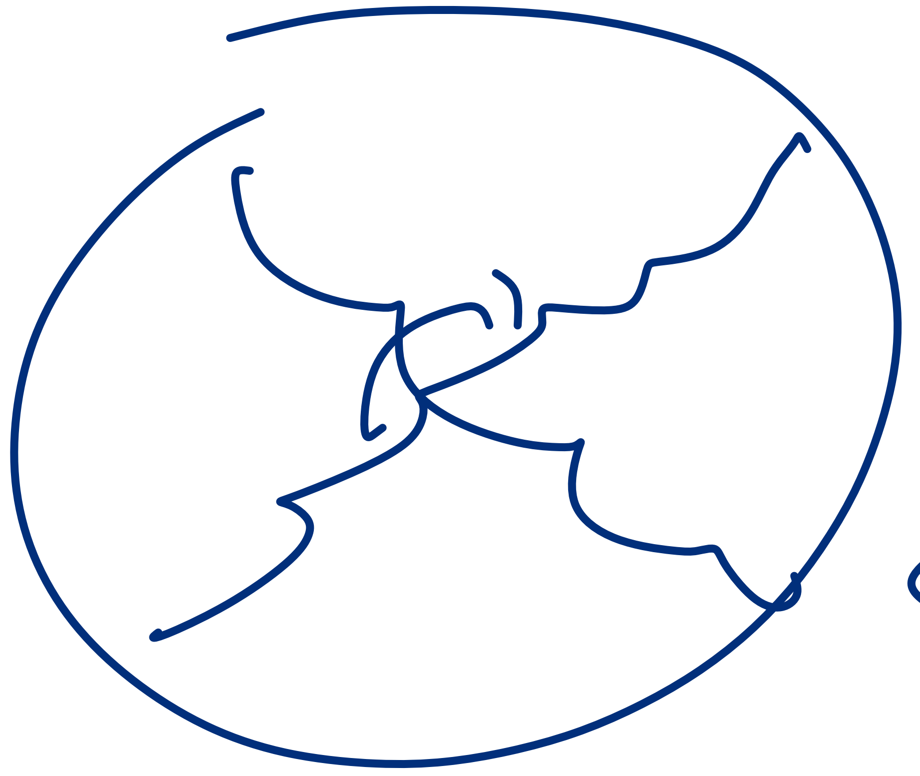
$\Rightarrow$ The interference diagram has a colour factor that is $\sim \frac{1}{N_c}$ non-interference

(same true for e.g. )

Label:



"planar"



"non-planar" $\sim \frac{1}{N_c^2}$

interference

leading in N_c

Phen. implications:

\# For higher order diagrams can get good approximation by considering "large N_c"

Though note: $N_c = 3$, which is not very large!

But:  $= C_F^2 N_c$  $= \frac{1}{2N_c} C_F N_c$

$\Rightarrow$  /  $= \frac{C_F N_c}{2N_c C_F^2 N_c} = \frac{1}{2N_c C_F} \sim \frac{1}{N_c^2}$

* Parton showers : treat showering of quarks / gluons ~ classically :

Including subleading N_c is an active area of research.