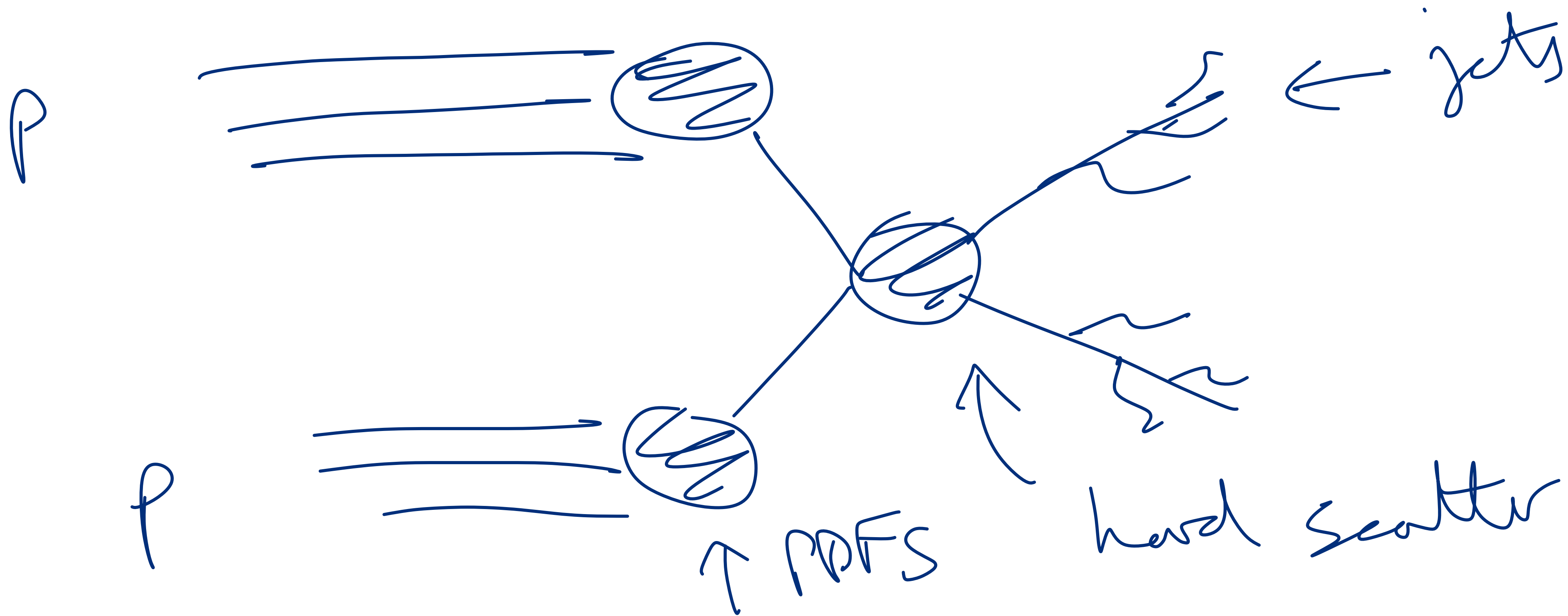


# Collider Phenomenology

A typical LHC event



How can we use this to discover  
new physics / understand the SM

\* LHC is a QCD machine!

main focus: understand how to deal

w/ that.

-  $\sqrt{s} \sim 0.1$  : p QCD ✓

- Need separation of short distance  
from long distance  $\rightarrow$  Factorisation  
Q PDFs

→ Brief overview

\* Initially deal with  $e^+e^-$  collisions.

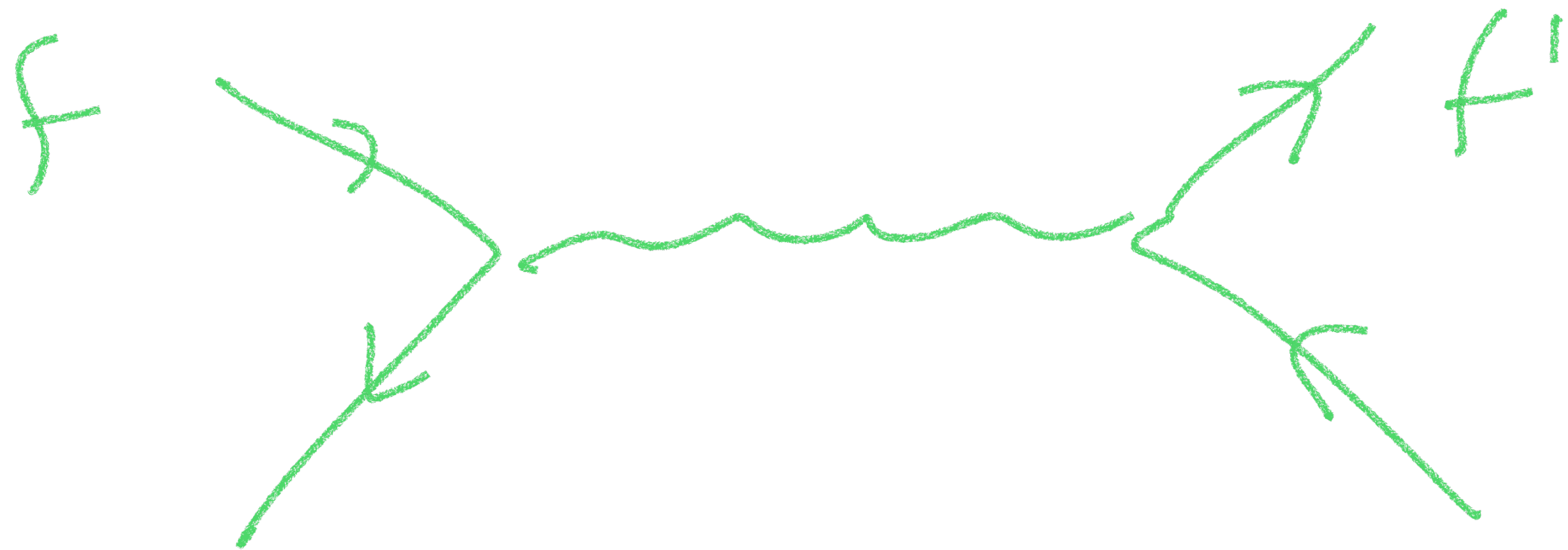
Way to introduce key elements.

$e^+ e^- \rightarrow \text{hadrons}$

Recall QED lectures:

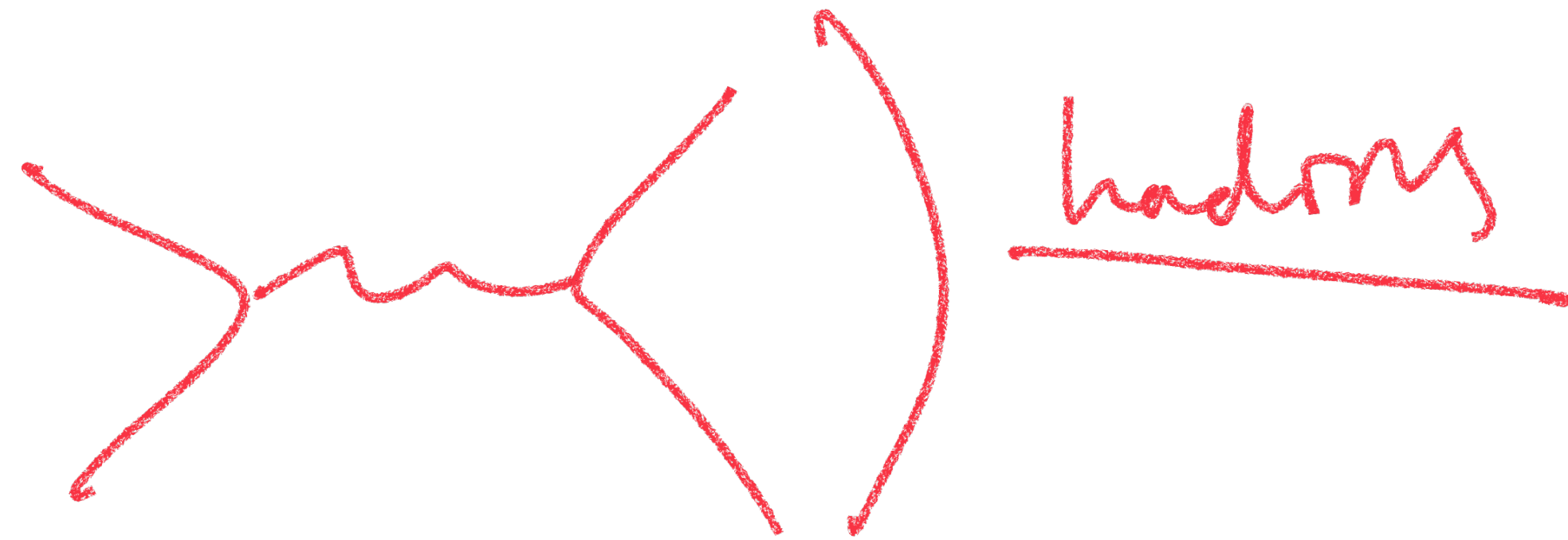
$$\frac{d\sigma}{d\cos\theta} (f\bar{f} \rightarrow f'\bar{f}') = f^2 f'^2 \alpha^2$$

$$\cdot \frac{\pi}{2s} (1 + \cos^2\theta)$$





$e^+e^- \rightarrow \text{hadrons?}$



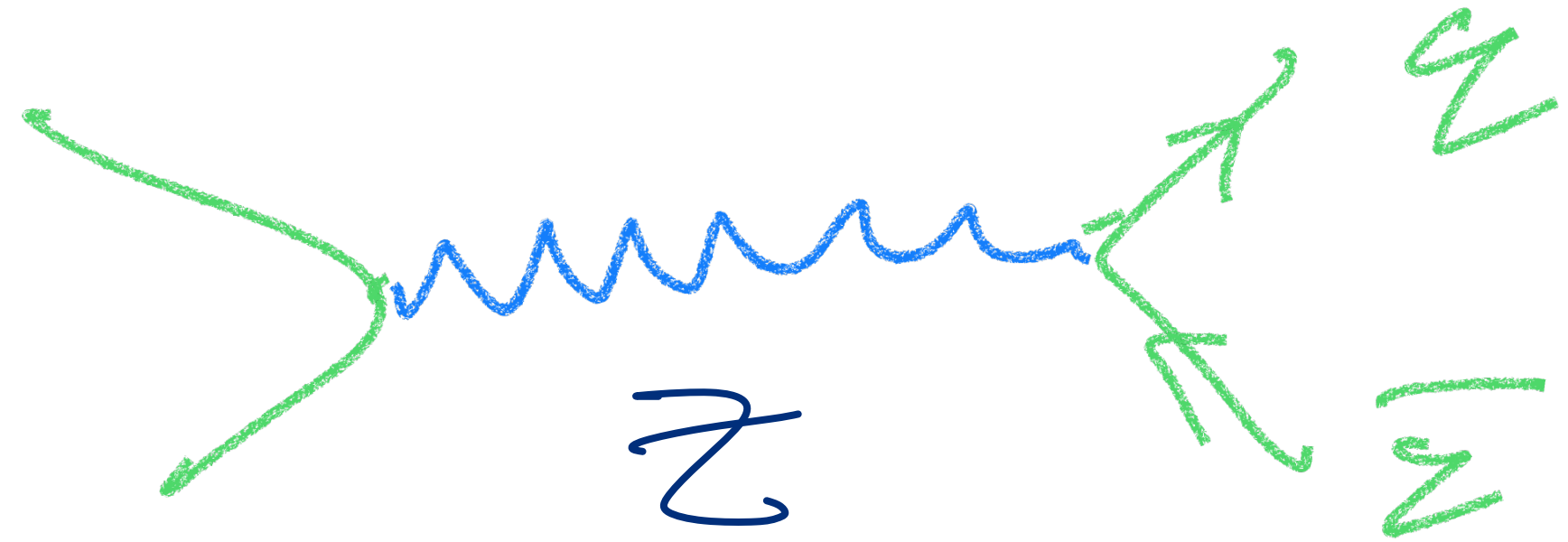
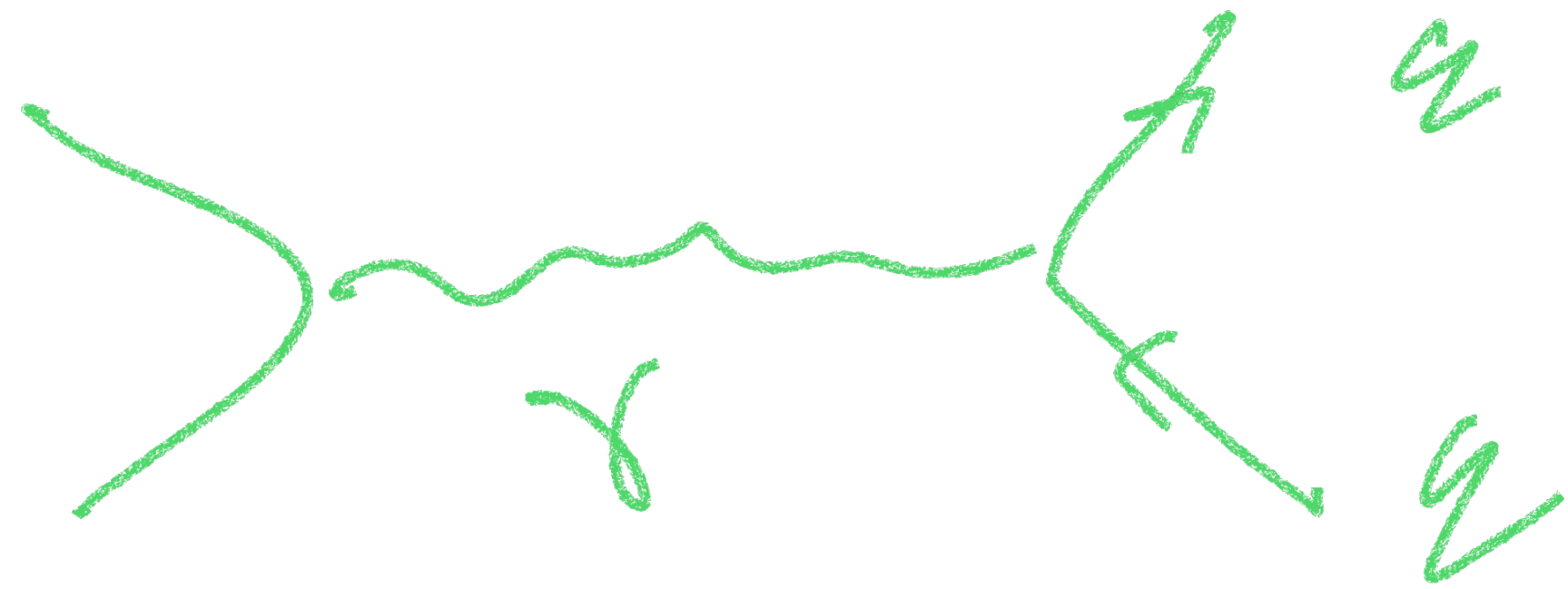
$$\frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}$$

$$\equiv R = N_c \sum_F f_F^2$$

$\nearrow$   
 $u, d, \dots$   
 $S \geq 4m_F^2$

# Z resonance

If  $\sqrt{s} \sim M_Z$ :



F rules:

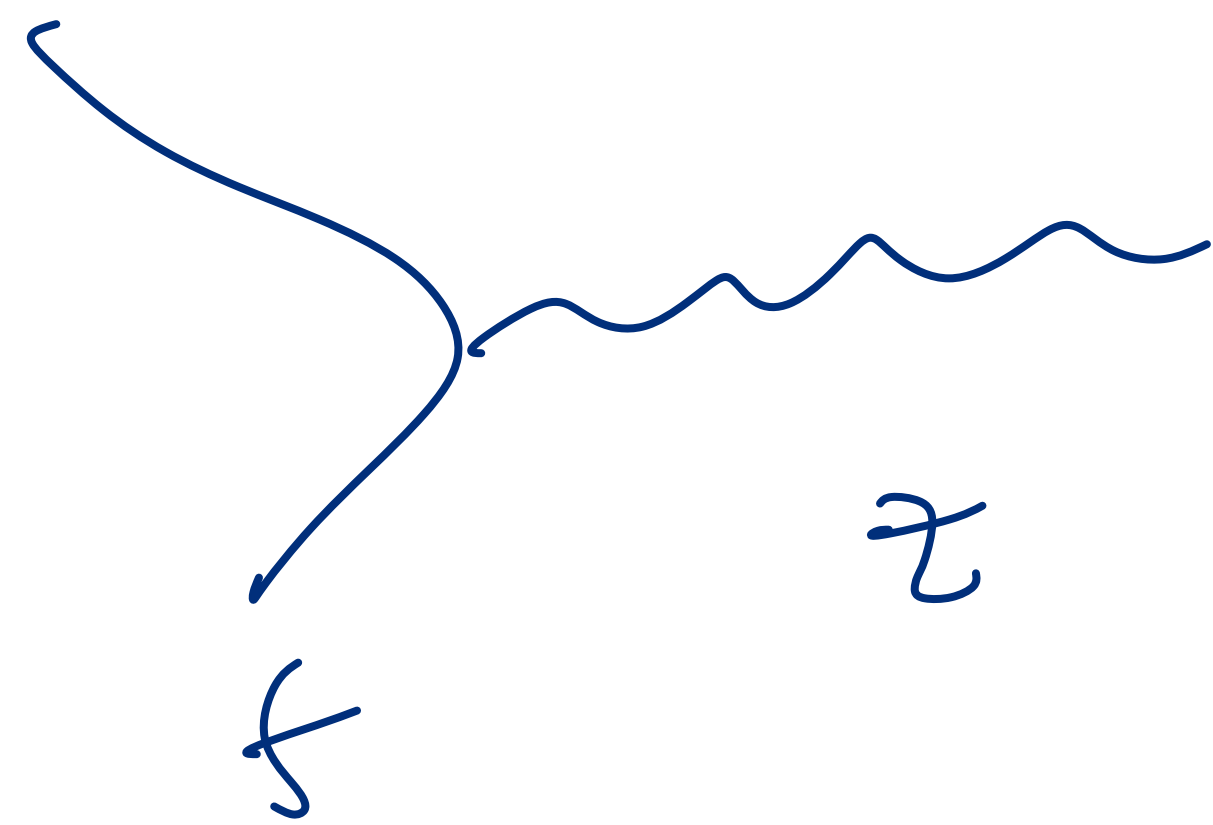
$$\frac{g_{\mu\nu}}{p^2}$$

Z unstable  $\Rightarrow$  w/ width  $\Gamma_Z$  decay  $\rightarrow$

$$\frac{g_{\mu\nu} - \cancel{\frac{p_\mu p_\nu}{M_Z^2}}}{p^2 - M_Z^2 - i\Gamma_Z M_Z} \quad \begin{matrix} \uparrow \\ \text{try!} \end{matrix} \quad \begin{matrix} \text{Re } \Gamma_Z = 0 \\ \uparrow \end{matrix}$$

In addition, more complex structure due

to our  $V-A$ :



$$\mu : i g z (v_f \gamma^\mu + a_f \gamma^\mu \gamma^5)$$

(see problem

& SM).

Width  $\Gamma$  that appears in  $Z$  propagator  
precisely the usual  $Z \rightarrow \dots$  decay  
width

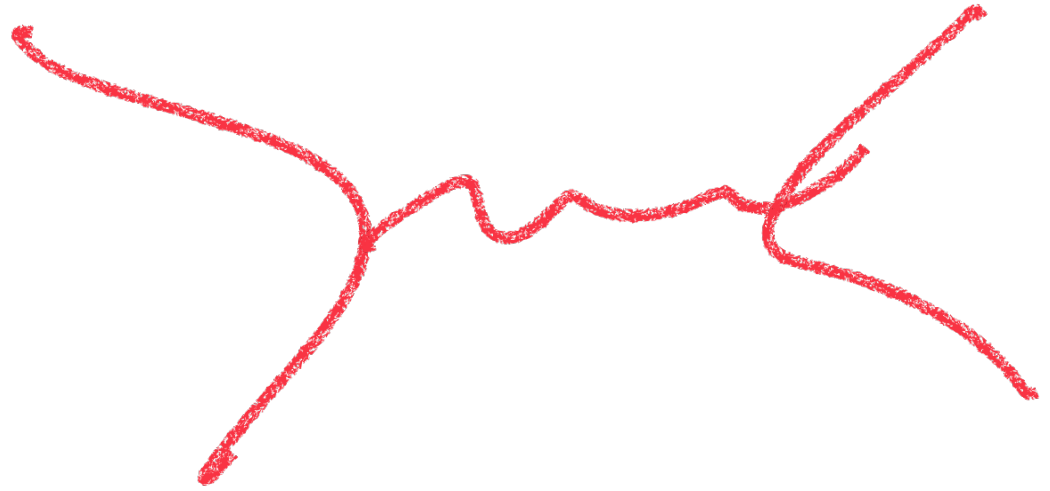
$$\Gamma_Z \propto \int dPS |M(Z \rightarrow X)|^2$$

$$X = e^+e^-, \nu\bar{\nu}, \dots$$

What is the connection?

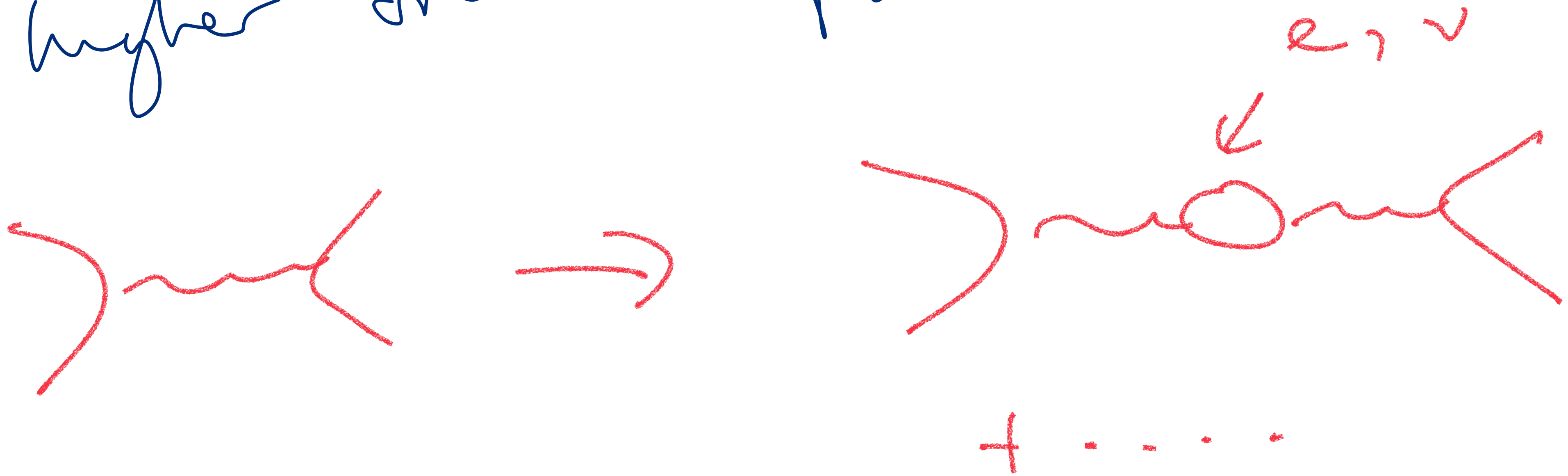
This is a rather subtle aspect of

QFT.

Basic point:  is not the

only contributing diagram. QFT  $\Rightarrow$

higher order loops:



Consider e.g. a scalar propagator



$$= \frac{1}{p^2 - M_0^2} \quad \leftarrow$$

$\hat{=}$  bare mass

NLO

$$\text{Propagator with self-energy loop} = \frac{1}{p^2 - M_0^2} + \Sigma(p)$$

e.g.

$$\text{Shaded self-energy loop} = \text{Tree-level propagator} + \text{Self-energy loop} + \text{Two-loop diagram}$$

then we write

$$= \frac{1}{p^2 - m_0^2} \left( 1 + \frac{\Sigma(p)}{p^2 - m_0^2} + \left( \frac{\Sigma(p)}{p^2 - m_0^2} \right)^2 + \dots \right)$$

↑ geometric series

$$= \frac{1}{p^2 - m_0^2} \left( \frac{1}{1 - \frac{\Sigma}{p^2 - m_0^2}} \right)$$

$$= \frac{1}{p^2 - m_0^2 - \Sigma} \leftarrow$$

Input of loops:

$$\frac{1}{p^2 - m_0^2} \rightarrow$$

$$\frac{1}{p^2 - m_0^2 - \Sigma} \leftarrow$$

int??



$\Sigma$ : amplitude  $\Rightarrow$  complex value

↓

$$p^2 - m_0^2 - \Sigma$$

vs.

↓

$$p^2 - m^2 - \frac{i\Gamma M}{\dots}$$

↑                      ↑

$$\text{Re}(\Sigma) + m_0^2 = m_0^2$$

'physical mass'

$$\text{Im}(\Sigma) = \Gamma$$

---

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what we want

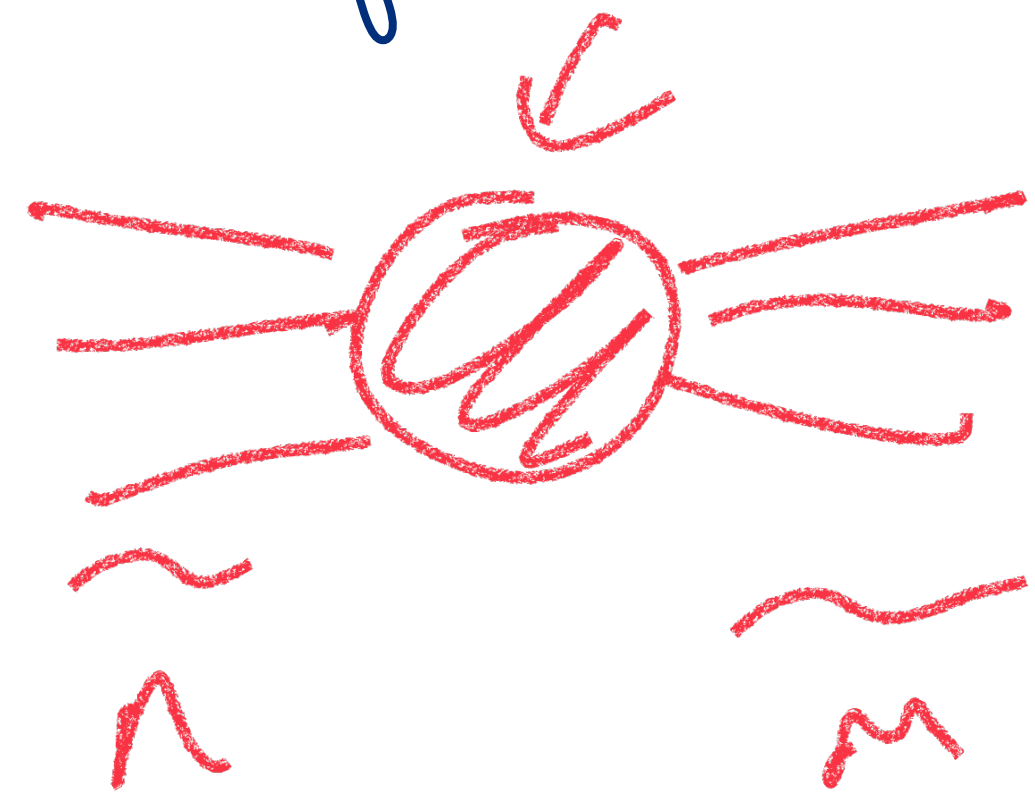


Recall 'unitarity' relation in QED / QCD.

Also 'optical theorem'.

Let's start from a general expression for scattering amplitude  $A$ !

$$A = \langle m | S | n \rangle$$

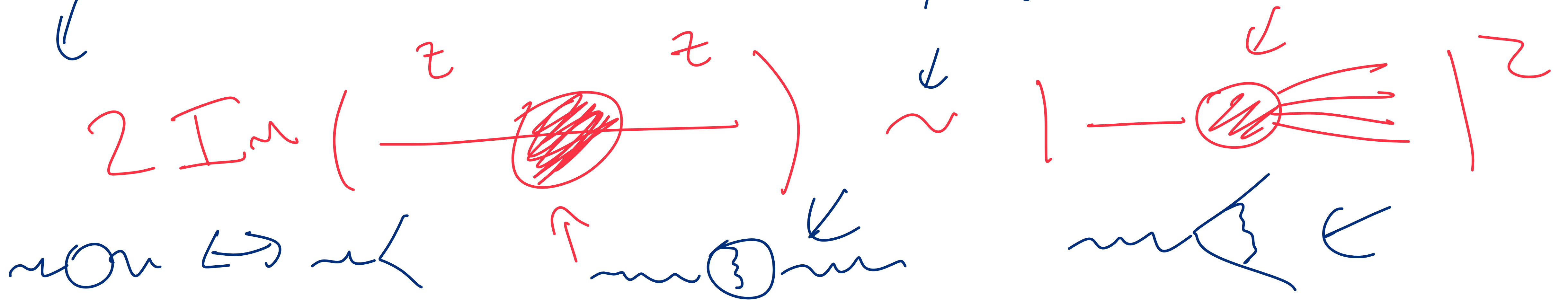


Then 
$$\sum_m |\langle m | S | n \rangle|^2 = 1$$

i.e. prob. that  $|a\rangle$  ends up in some state = 1.

- With a bit of work (ship details), this is equivalent to:

$$2 \operatorname{Im} (A(a \rightarrow a)) \approx \sum_f \int dP_f |A(a \rightarrow f)|^2$$



$$\Rightarrow \text{LIm}(\Sigma) = \text{mf}$$

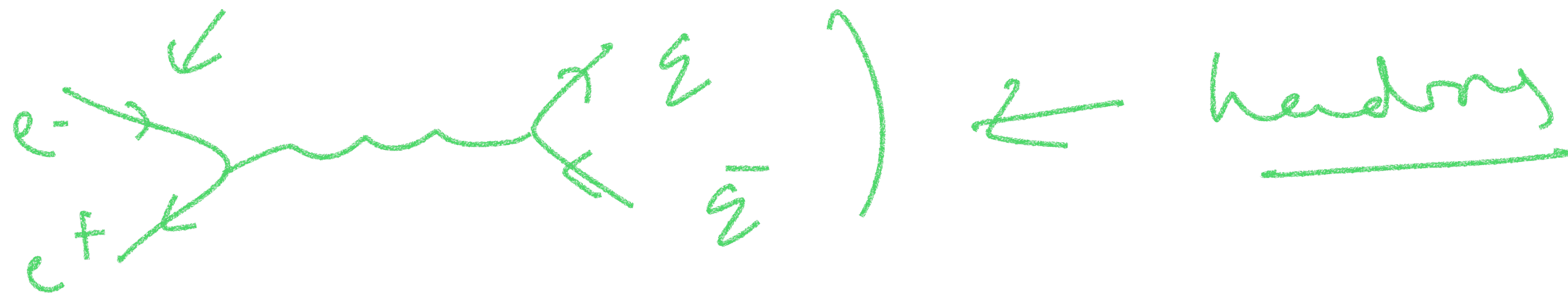
e.g



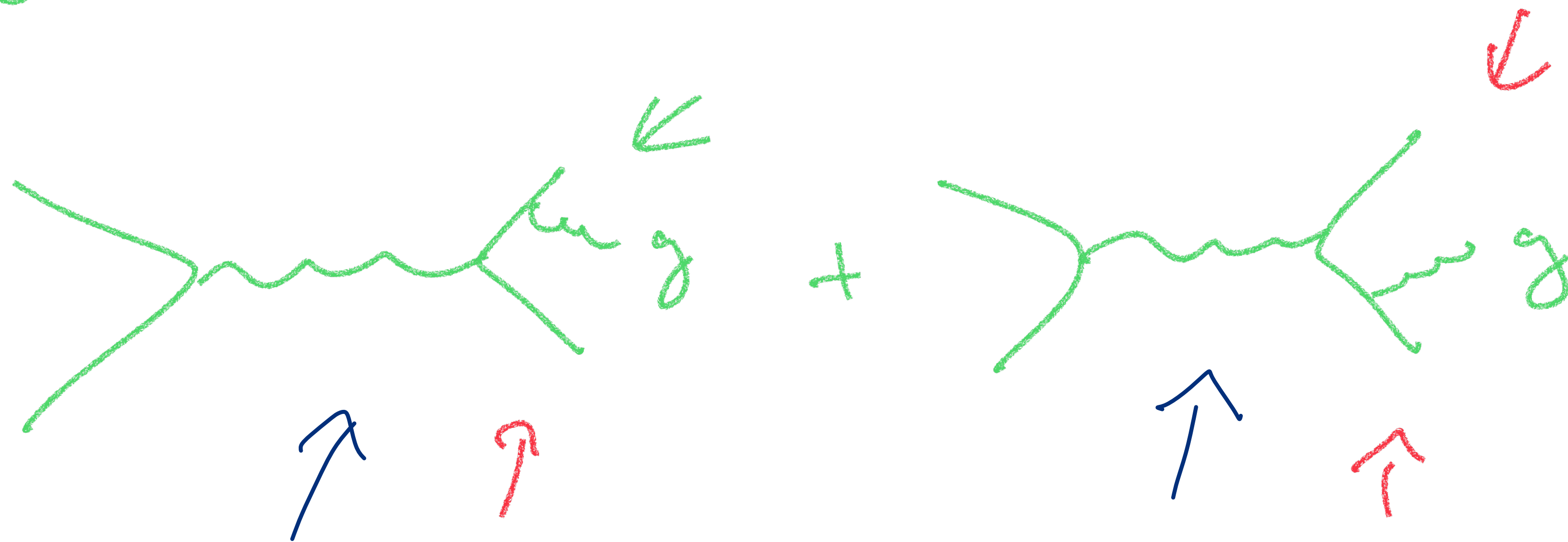
$e^+e^- \rightarrow \text{hadrons}$  : radiative corrections

Stick with  $\gamma$  exchange for simplicity

LO:



NLO:



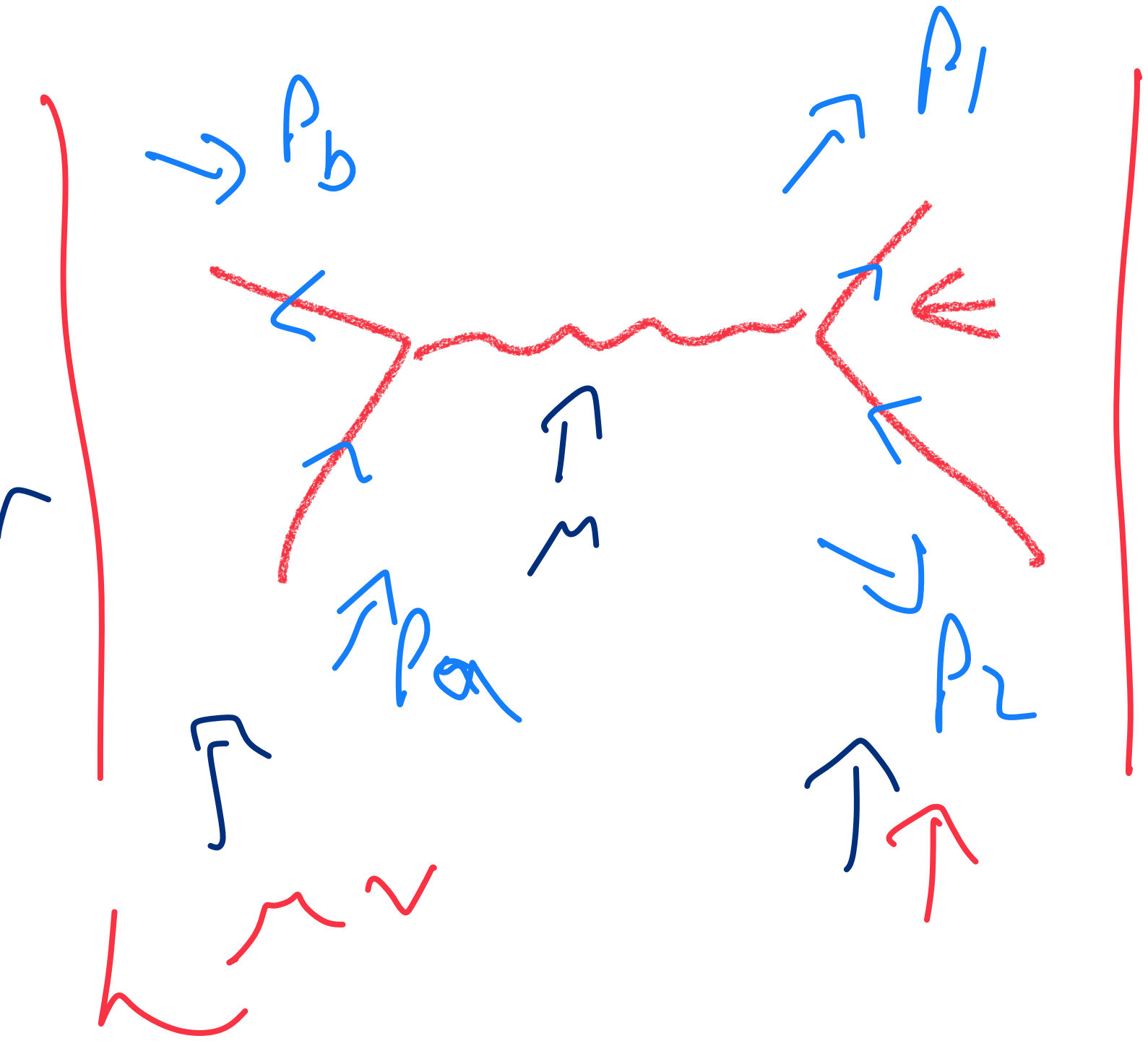
# General structure (LO)

$$\langle |M_{22}|^2 \rangle = \frac{1}{4} \sum_{\text{spins, colors}}$$

$$\propto \frac{1}{s^2} e^2 \text{Tr}(\gamma^\mu \not{p}_a \gamma^\nu \not{p}_b)$$

$$\times F_q^2 e^2 \delta_{ij} \text{Tr}(\gamma^\mu \not{p}_e \gamma^\nu \not{p}_f) \quad H_{\mu\nu}$$

$\uparrow \sum_{i,j} \delta_{ij} \delta_{ij} \equiv \underbrace{\quad}_{\text{leptons}} \underbrace{\quad}_{\text{quarks}}$



The purely leptonic part is factored off

→ calculate once & forget about.

At NLO (real):

$$\langle |M_{q\bar{q}g}|^2 \rangle$$

$$= L^{\mu\nu}$$

↑  
same  
as before

$$H_{\mu\nu}^{\text{NLO}}$$

↑  
four or that



e.g.

a contribution

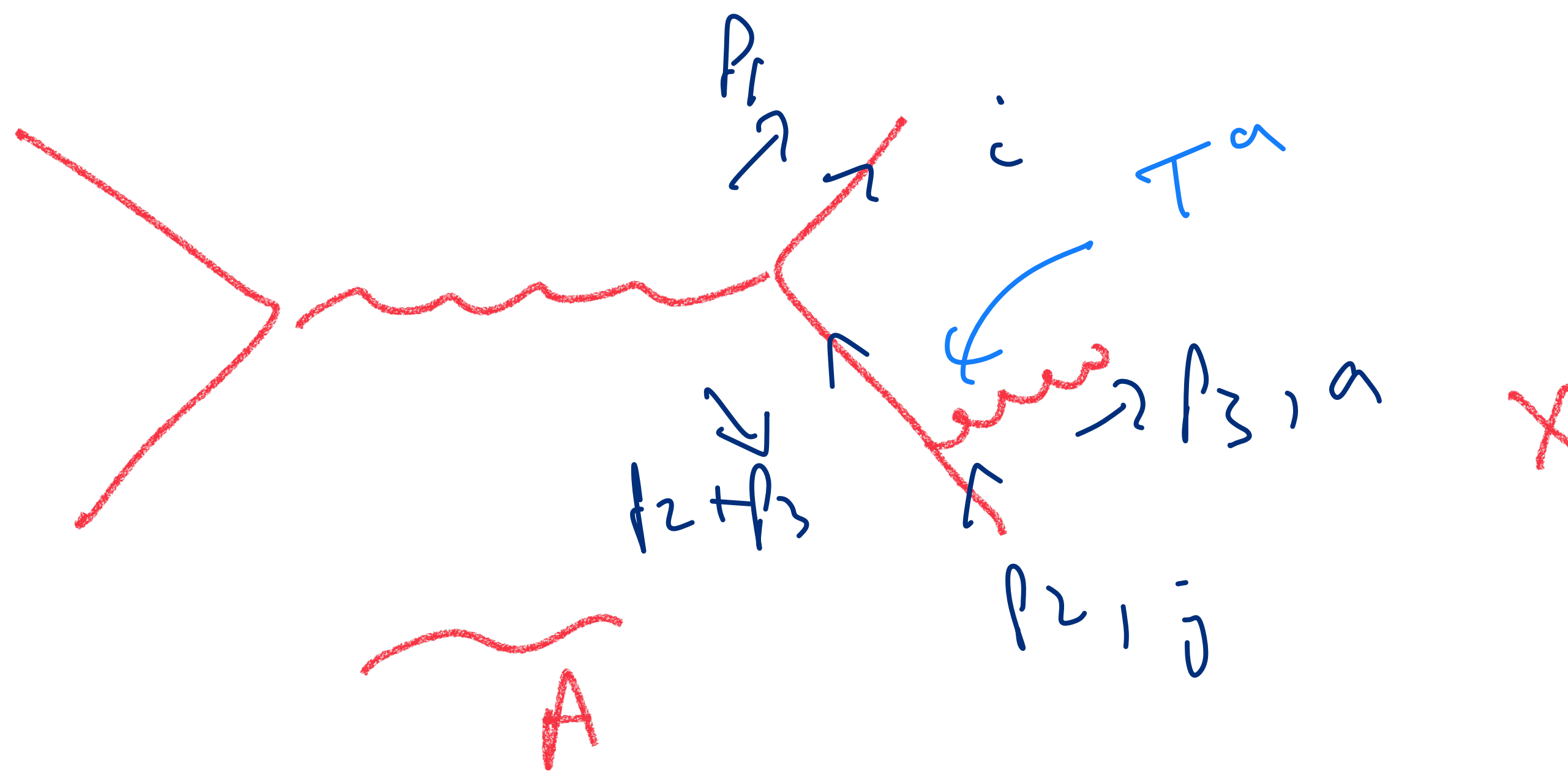
$\int H_{\mu\nu}^{\text{NLQ}}$

$$\propto L_{\mu\nu} \cdot e^2 g^2 \text{Tr} (T_{ij}^a T_{ji}^a) \text{Tr} (\not{p}_1 \gamma_\mu (\not{p}_2 + \not{p}_3))$$

$$\cdot \gamma^\rho \not{p}_2 \gamma_\rho (\not{p}_2 + \not{p}_3) \gamma_\nu \frac{1}{S_{23}}$$

$\epsilon^\rho \epsilon^\sigma \delta \rightarrow -g_{\rho\sigma}$

$\sum S_{ij} = (p_i + p_j)^2$



Aside: Why  $T_{ij}^a \rightarrow T_{ji}^a$



$$M^* \sim (T_{i'j'}^{a'})^*$$

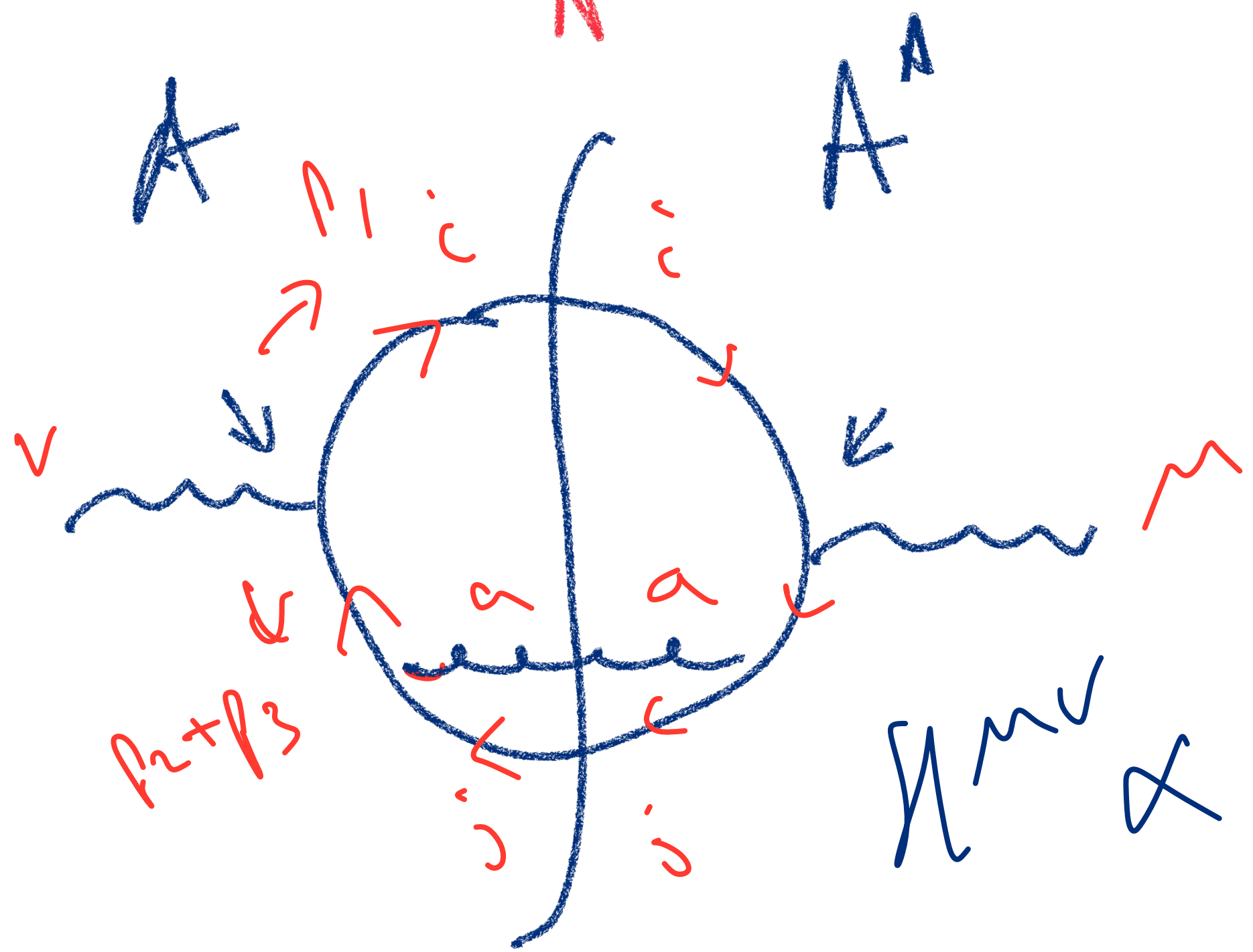
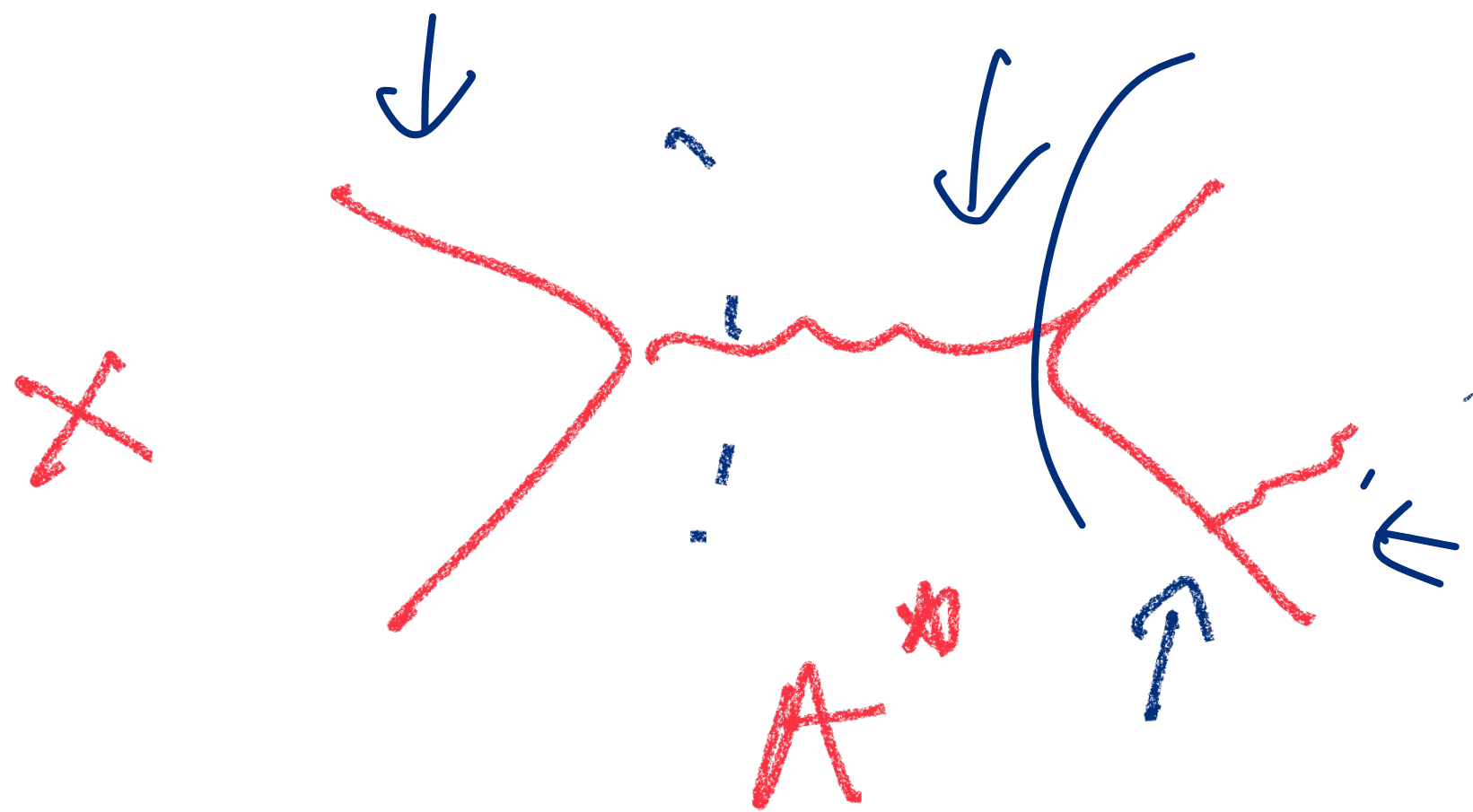
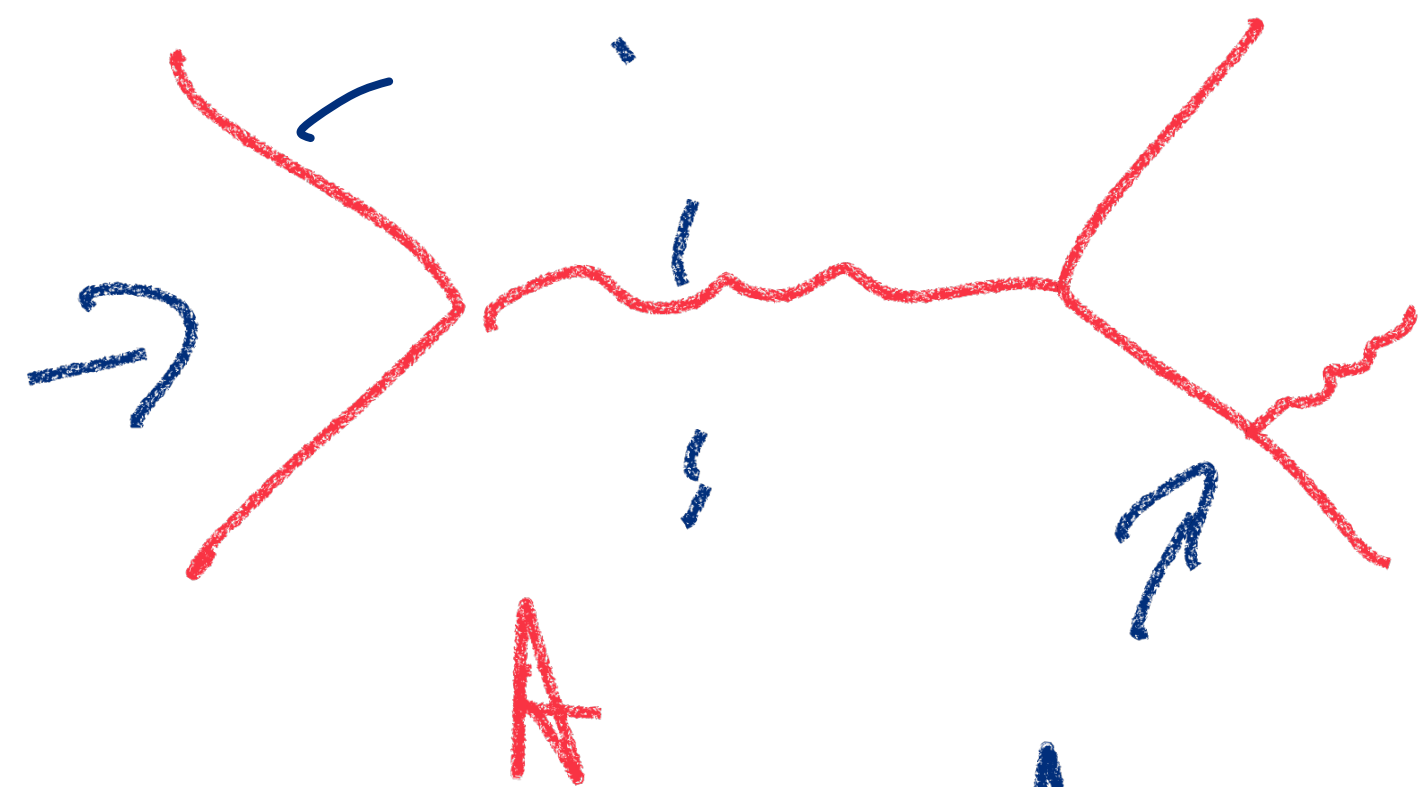
- The  $T^a$ 's are hermitian  $\Rightarrow T = T^\dagger$

$$\text{i.e. } (T_{ij}^a)^* = T_{ji}^a$$

$$\Rightarrow |M|^2 \sim \delta_{ii'} \delta_{jj'} \delta_{aa'} T_{ij}^a T_{i'j'}^{a'} \quad \checkmark$$



Can be useful to represent them pictorially:

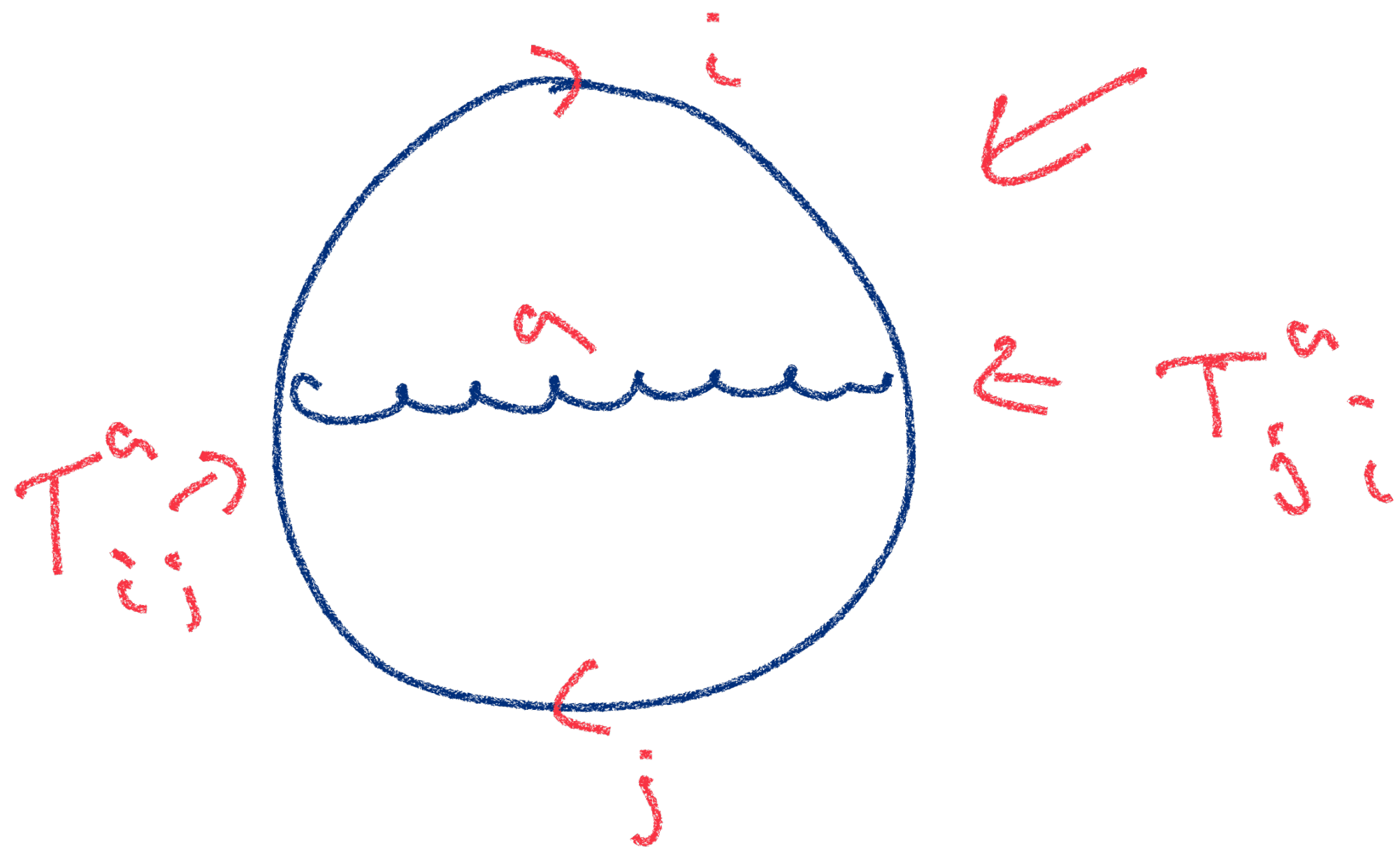


Not Feynman diagrams. Just a way of keeping track:

$$N_{\mu\nu} \propto T_{i^a} T_{j^c} T_{k^d} T_l \left( \not{p}_1 \gamma^\mu (\not{p}_2 + \not{p}_3) \gamma^\rho \not{p}_2 \not{p}_3 (\not{p}_2 + \not{p}_3) \gamma^\nu \right)$$

$$T_{ij}^a \quad T_{ji}^a$$

≡



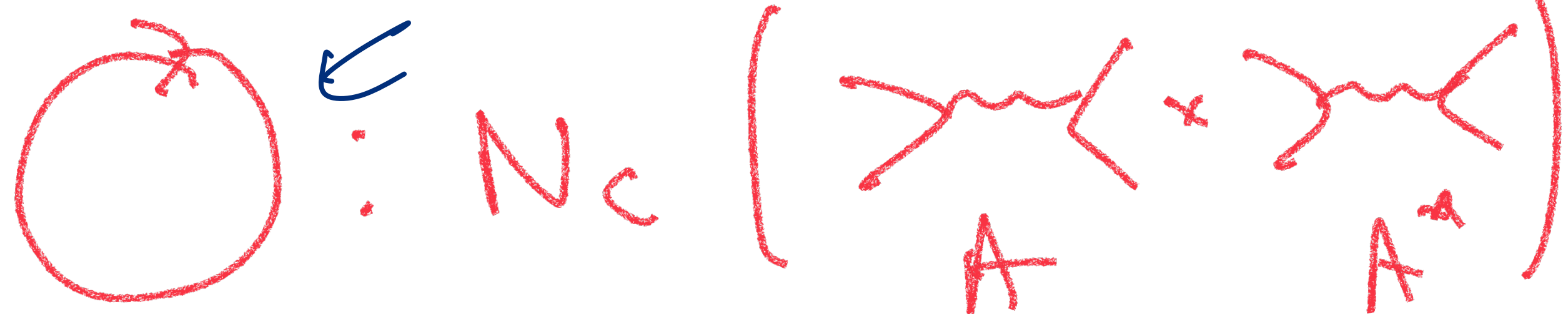
$$\text{Tr}(T^a T^a) = \frac{1}{2} \delta^{aa} = \frac{1}{2} (N_c^2 - 1)$$

Can always manipulate colour factors down to a small number of known relations:

$$\text{Tr}(T^a) = 0, \quad \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}$$

Can also represent pictorially:

$$\delta_{ii} = N_c$$



$$\delta_{aa} = N_c^2 - 1$$



$$\text{Tr}(T^a T^b) = \frac{1}{2} f^{ab}$$



$$\text{Tr}(T^a) = 0$$



$$f^{abc} f^{abd} = C_A f^{cd}$$



$$T_{ij}^a T_{jk}^a = C_F \delta_{ik}$$





Finally we need the Fierz identity:

$$T_{ij}^a T_{kl}^a = \frac{1}{2} \left( \delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right)$$

Diagrammatic representation of the Fierz identity:

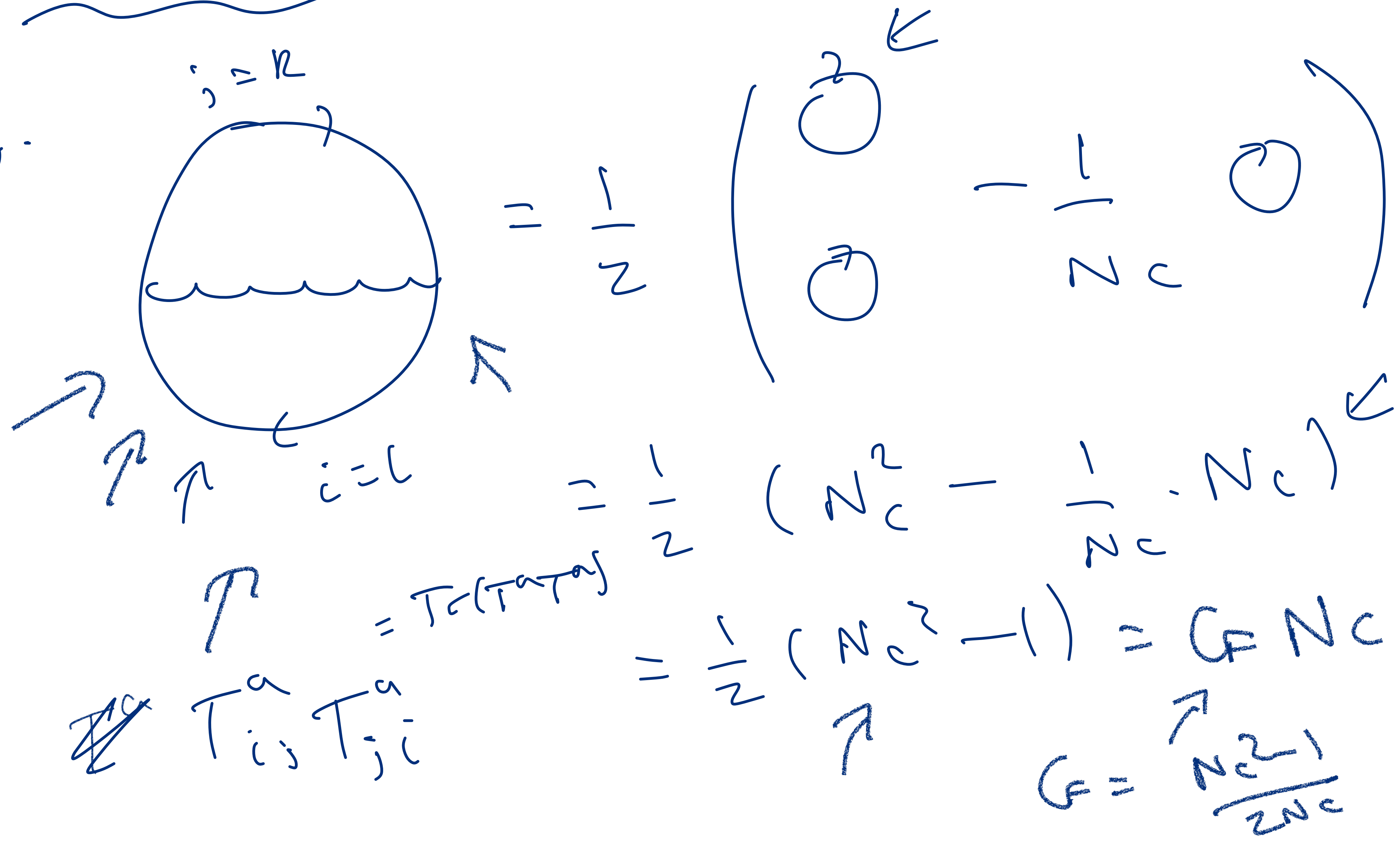
Left side:  $T_{ij}^a T_{kl}^a$  (Feynman diagram with external legs  $i, j, k, l$  and internal lines  $a, a$ )

Right side:  $\frac{1}{2} \left( \delta_{il} \delta_{kj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right)$  (Feynman diagrams with external legs  $i, j, k, l$  and internal lines  $j=k$  and  $i=l$ )

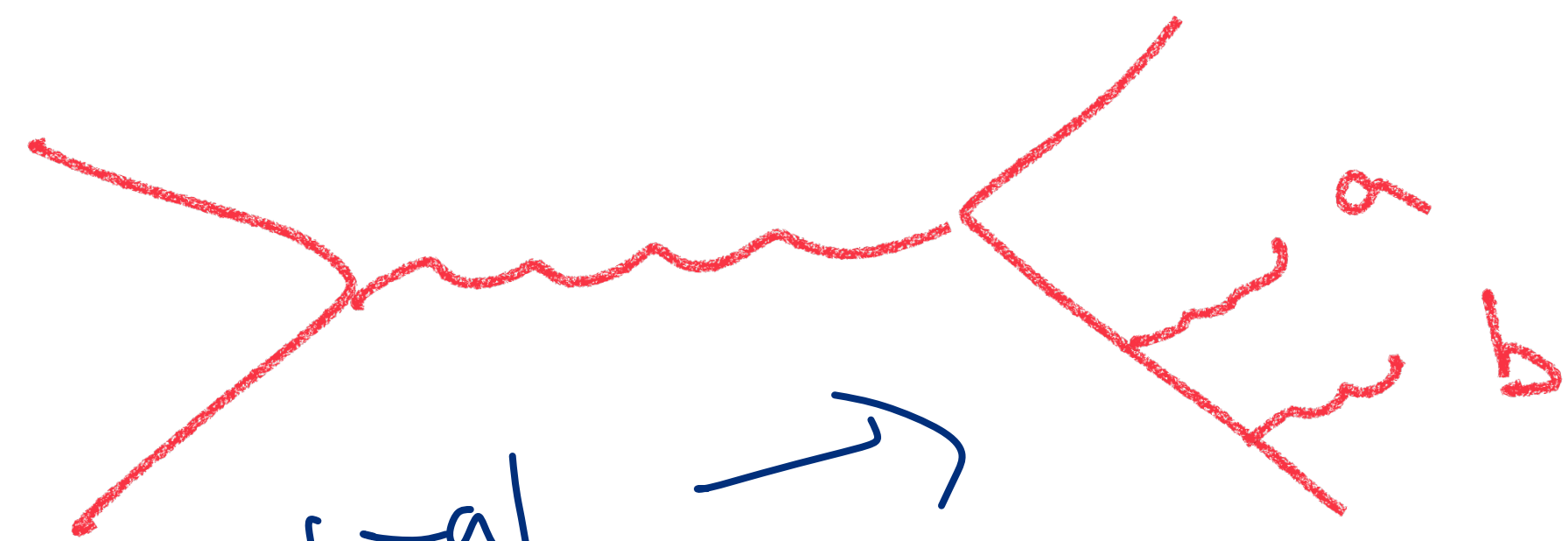
Note:  $\delta_{jk} \rightarrow \delta_{ij} = N_c$

Let's see this in action:

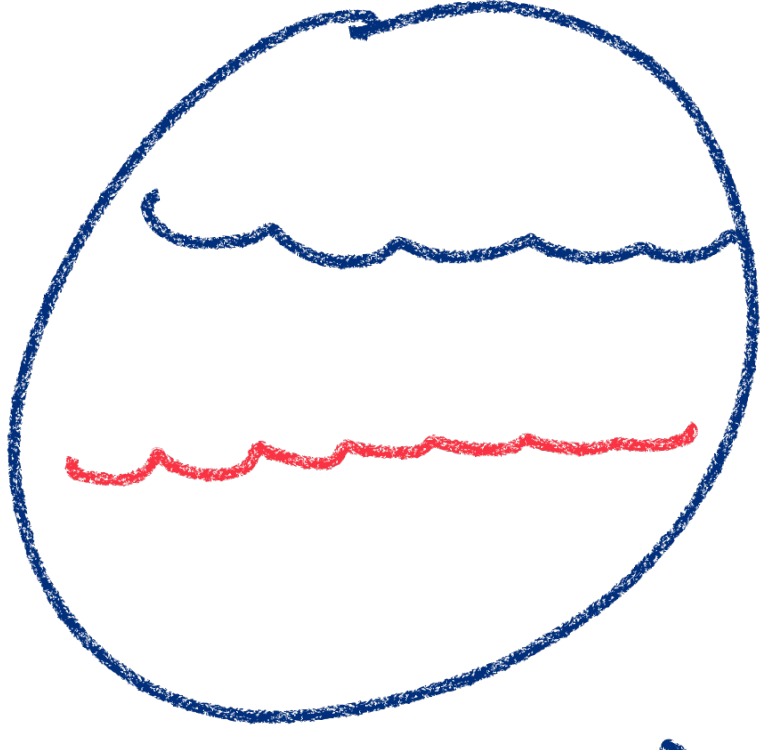
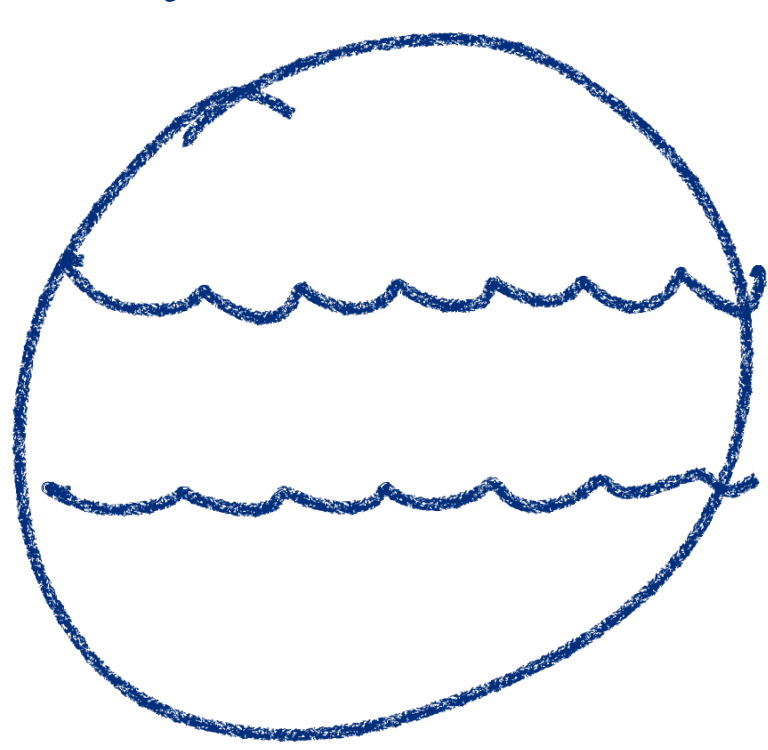
F.g.



More complex case:



$T^a$   $T^a$   $T^b$   $T^b$   $T^a$   
 $\nearrow$   $\searrow$   $\nearrow$   $\searrow$   
**A**



$$= \frac{1}{2} (N_c - \frac{1}{2} N_c)$$

x



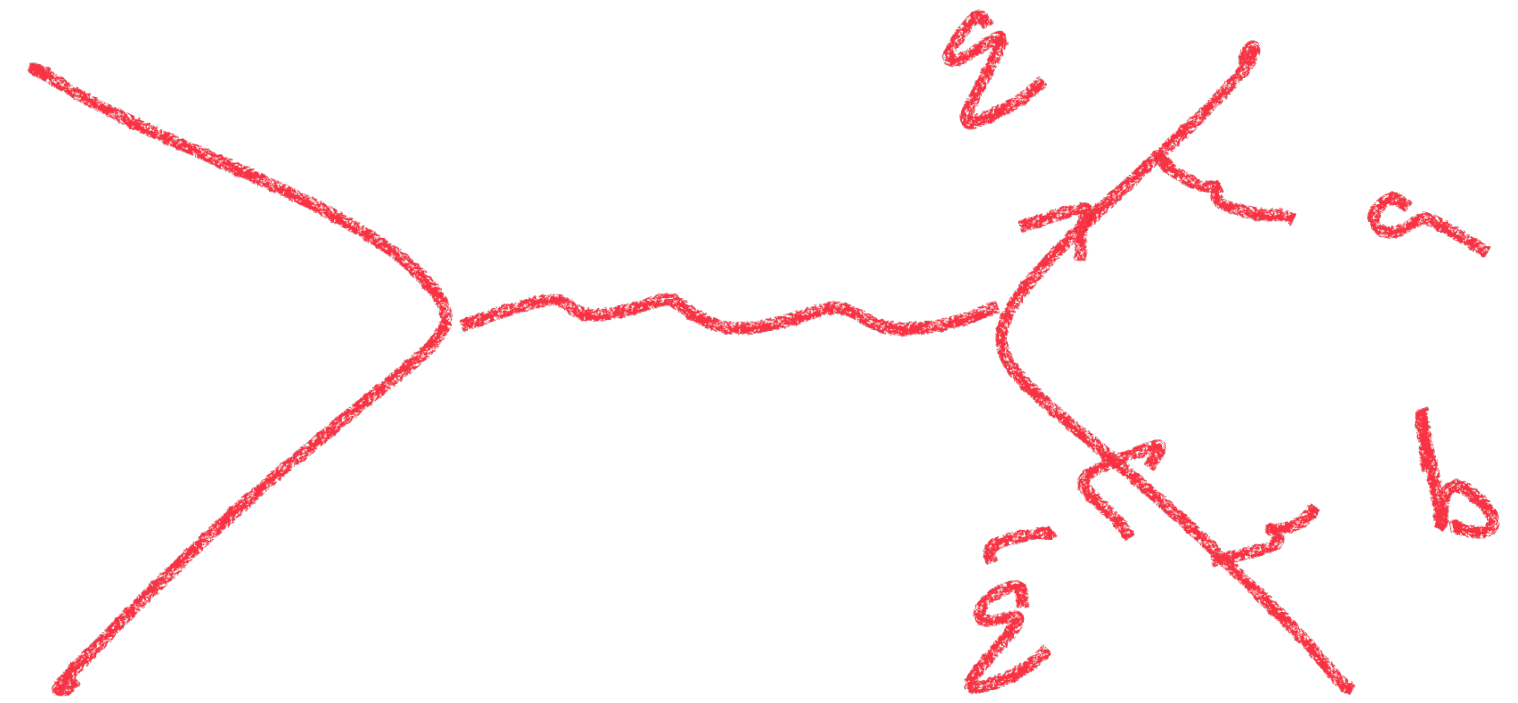
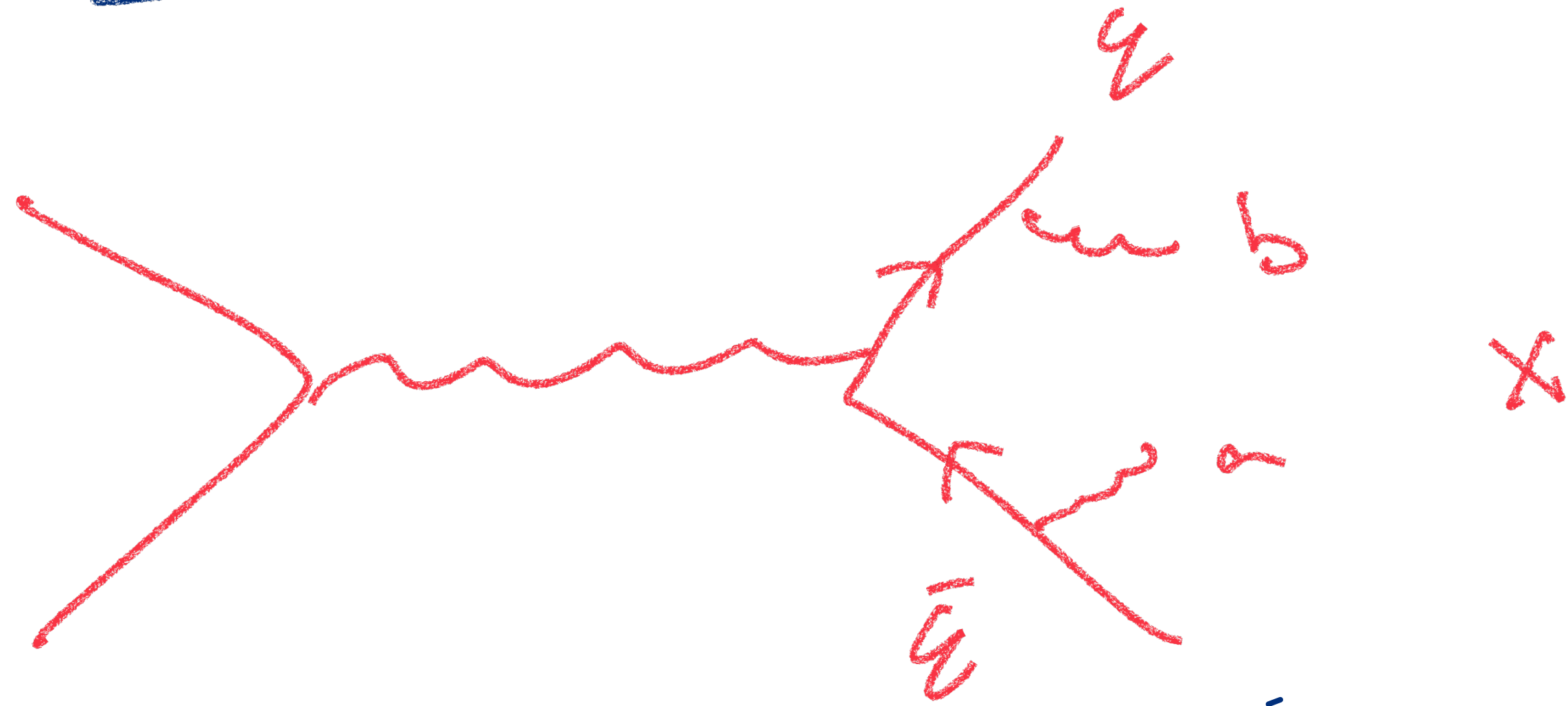
**A<sup>x</sup>**

$$= \frac{1}{2} \left( \begin{array}{c} \text{O} \\ \text{O} \\ \text{O} \end{array} \right) = G_F^2 N_c$$

The diagram shows a vertical stack of three circles. The top circle is empty. The middle circle contains a red wavy line. The bottom circle contains a red wavy line and has two arrows pointing down towards it. The entire stack is enclosed in large parentheses.

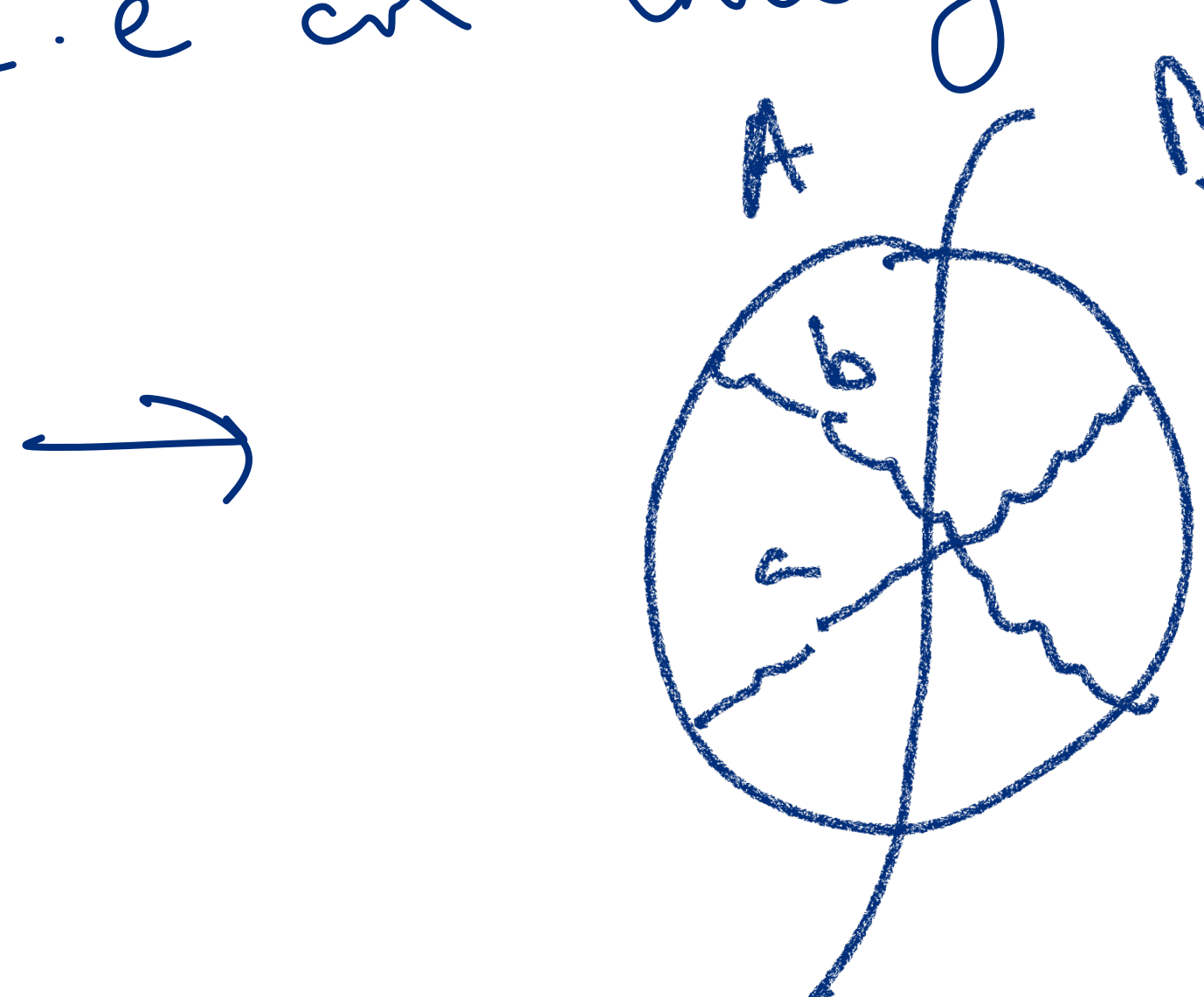


One more example :

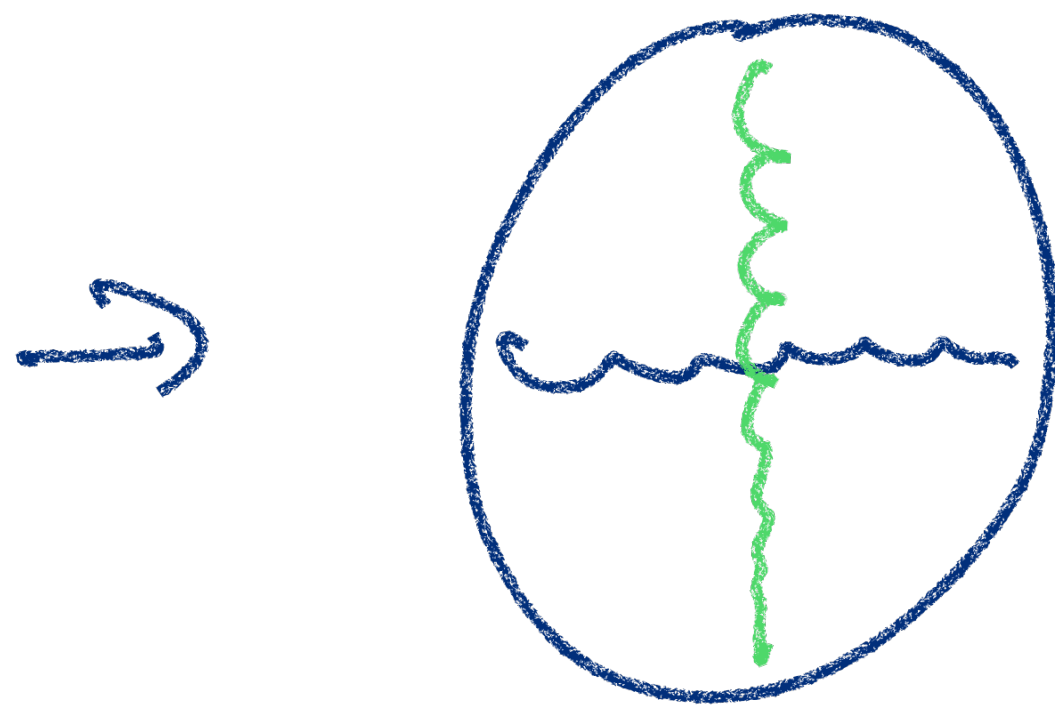


i.e an interference diagram

$\text{Tr}(T^a) = 0$



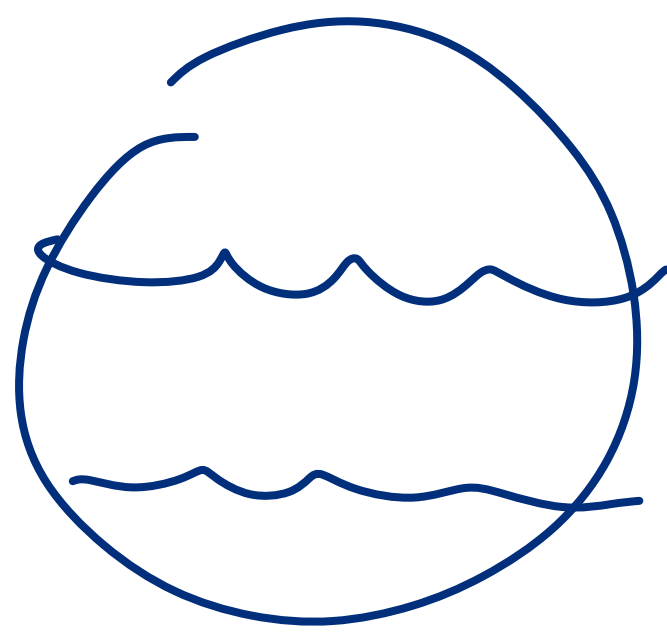
B



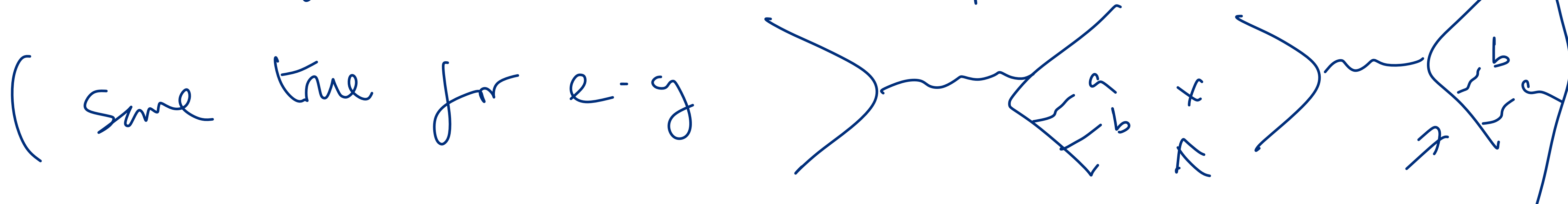
$= \frac{1}{2} \left( \begin{array}{c} \text{circle} \\ \text{---} \\ \text{circle} \end{array} - \frac{1}{2} \begin{array}{c} \text{circle} \\ \text{---} \\ \text{circle} \end{array} \right)$

The diagram shows a subtraction of two terms in large parentheses. The first term is a vertical line with two circles at the ends and a green wavy line in the middle. The second term is a circle with a green wavy line inside and an arrow pointing up.

$$= -\frac{1}{2Nc} \text{ (circle with wavy line) } = -\frac{1}{2Nc} G^2 Nc$$

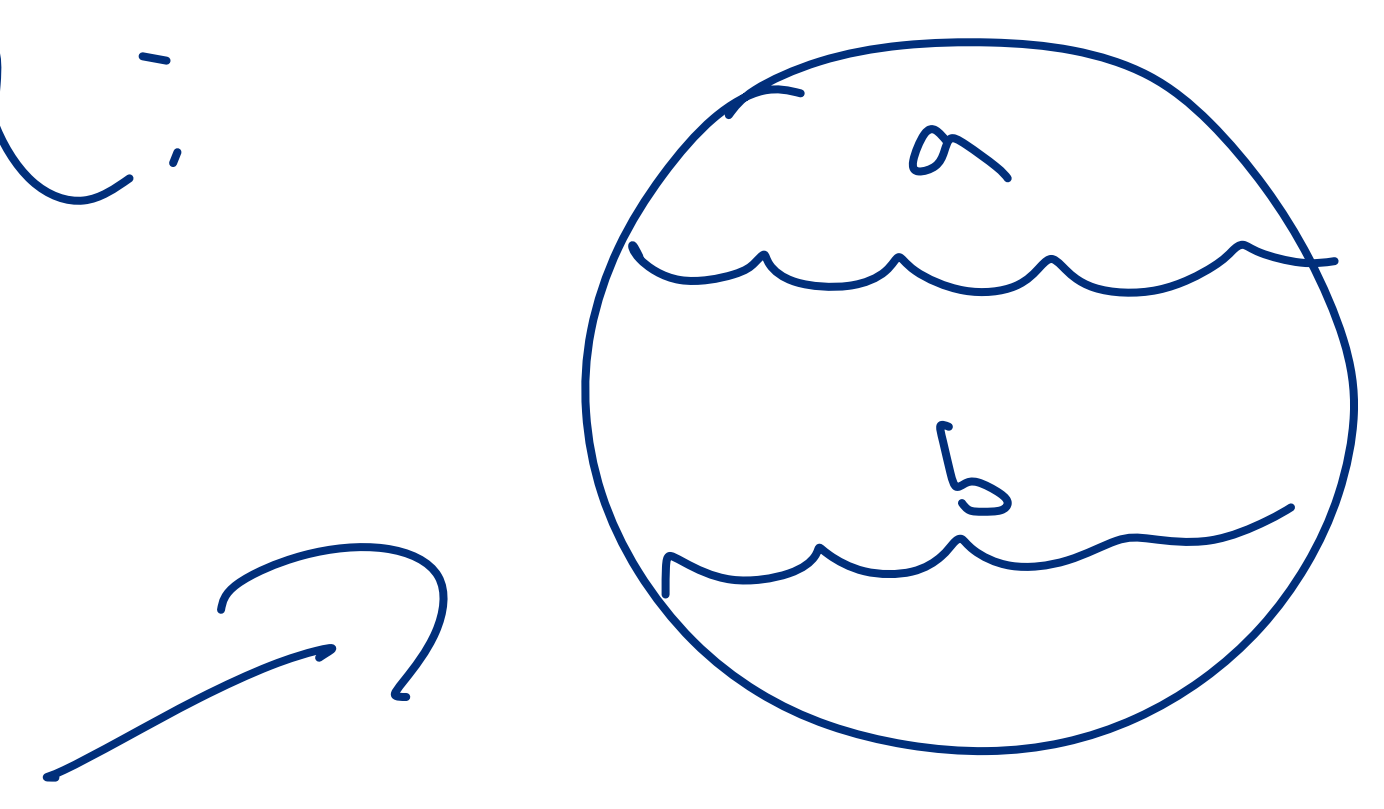
compare w/  ;  $G^2 Nc$

$\Rightarrow$  The interference diagram has a about factor that is  $\sim \frac{1}{Nc}$  non-interference



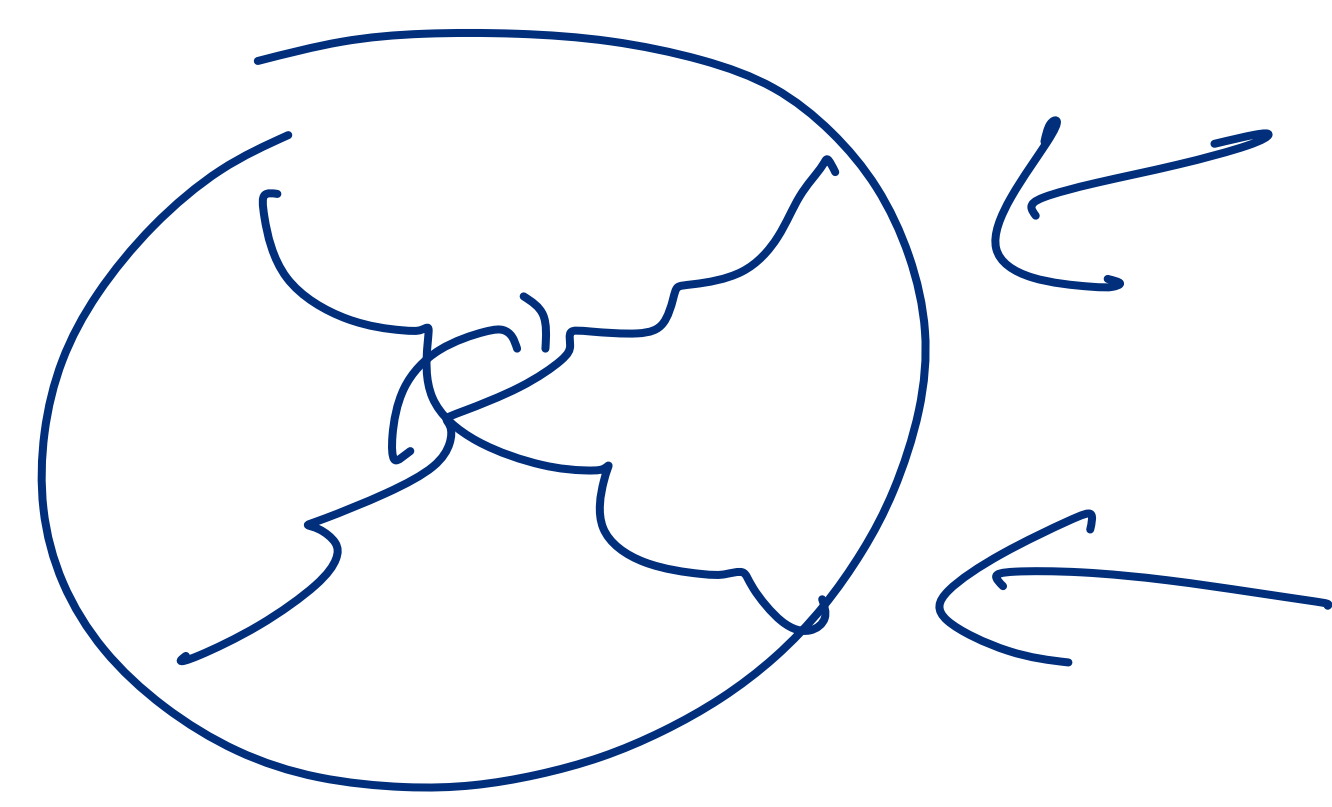


Label:



"plurals"

leading in  $N \rightarrow$



"non-plurals"

interference

$$\sim \frac{1}{N_c^2}$$

# Phenomenological implications:

\* For higher order diagrams can get good approximation by considering "large  $N_c$ "

Though note:  $N_c = 3$ , which is not very large!

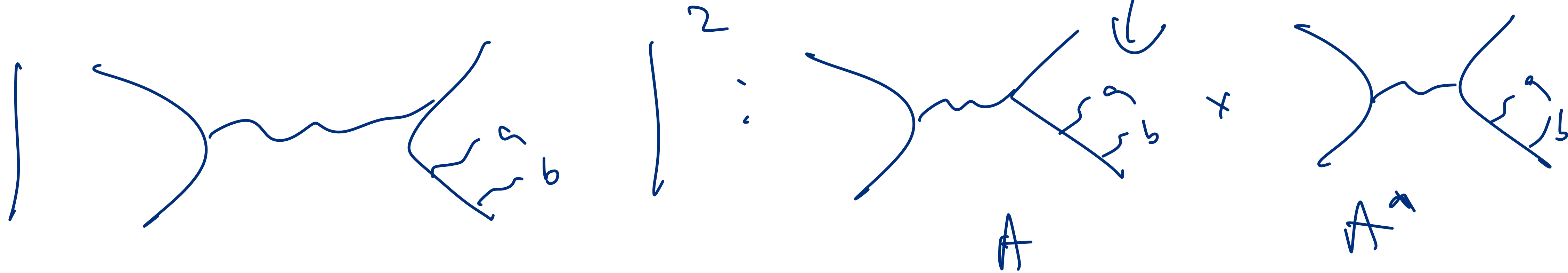
But:

$$\text{Diagram 1} = G^2 N_c \quad \text{Diagram 2} = \frac{1}{2 N_c} G N_c$$

$$\Rightarrow \frac{\text{Diagram 2}}{\text{Diagram 1}} = \frac{G N_c}{2 N_c G^2 N_c} = \frac{1}{2 N_c G} \sim \frac{1}{N_c^2}$$

\* Parton shower : Treat showering of quarks

(gluons  $\sim$  classically :



Including subleading  $N_c$  is an active area of research.