The Standard Model

A brief introduction to the greatest intellectual achievement of humankind...

1. What is the SM?

- Defining a quantum field theory.
- Gauge Symmetries
- Representations
- The Path Integral.
- Natural Units $\hbar$ counting. Mass scales coupling parameters.
- Effective Field Theories, Re[illegible]

Effective Field Theories, Renormalizability.

2. Approximate symmetries of the SM. Chiral B, L B-L Accidental.

3. Higgs Global Symmetry breaking Gauge Symmetry breaking

4. Leptons Weyl/Majorana/Dirac masses

5. Quarks

6. Below Λ_{QCD}
- Goldstone's Theorem
- Pions/Mesons
- Baryons

7. The Strong-CP Problem

8. Neutrino Masses and Baryogenesis

9. Microscopic Origins of Weak Scale

Lecture 1

will build it up conversely to historical dev.

What is the Standard Model?

Answer: A Quantum Field Theory.

(which happens to agree with a cornucopia of experimental measurements extremely well!)

Example: Electron anomalous magnetic moment. This deviation from classical mechanics was a major confirmation of QFT. Still is:

$$a_e = 0.00115965218643(764)$$

which agrees with the SM prediction to $\underline{\underline{10}}$ significant digits.

Quantum Field Theory

Path integral $\hbar \neq 0$

Involves fields $\phi(x)$

Makes predictions

How would one define a QFT?

- Symmetries/ Redundancies
- Representations
- Parameters

Once the first two are defined, one can measure (never predict) the remaining free parameters.

I.e. The SM cannot predict the electron mass.

Symmetries

Global

Practically a symmetry is an operation which leaves the predictions of a theory unchanged.

i.e. $\mathcal{L}_0 = (\partial_\mu \phi)^* (\partial^\mu \phi) - m^2 \phi^* \phi$

If we make a transformation

$$\phi \to e^{i\theta}$$

then $\mathcal{L}_\theta = \mathcal{L}_0$ and nothing changes. Symmetries can be continuous, like this, or discrete, like $\phi \to -\phi$.

More formally, continuous symmetry associated with a conserved current.

Noether's Theorem

Conserved current for every continuous symmetry: $\partial_\mu J^\mu = 0$.

HW: Derive this for the above example.

In a QFT this is promoted to an invariance of not just the Lagrangian, or action, but the whole path integral!

Gauge Symmetry

When we say a symmetry is "gauged" we mean it becomes a local transformation.

(This is actually not really a symmetry in the usual sense any more, but we stick with the jargon.)

For the previous example we would have $\phi \to e^{i\theta(x)}\phi$ where now θ is a function of spacetime. What happens? For small $\theta(x)$ we get

$$\mathcal{L}_0 = (\partial_\mu \phi)^*(\partial^\mu \phi) - m^2|\phi|^2$$

$$\to \mathcal{L}_0 + i\left(\phi^*\partial_\mu \phi - \phi\partial_\mu \phi^*\right)\partial^\mu \theta$$

So no longer invariant! Recovered if we promote:

$$\partial_\mu \to \partial_\mu + iA_\mu$$

Where, for $\phi \to e^{i\theta(x)}\phi$, also

$$A_\mu \to A_\mu - \partial_\mu \theta .$$

Invariance is now respected, but we have a new field A_μ. "Gauge Field". Due to antisymmetry, the term

"Field Strength" $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

is also invariant! As is $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

"Kinetic Term"

Non-Abelian Symmetries

The previous symmetry was Abelian: Two transformations gives the same result if reversed.

Consider, for example, rotations. Now,

rotations around different axes give a different result if reversed.
This is Non-Abelian.

$SU(N)$

Special Unitary Group of Degree N.
Acts on complex vectors as

$$\underline{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix} \qquad \underline{\phi} \rightarrow U \underline{\phi}$$

U: $N \times N$ matrix satisfying $U^\dagger U = 1$.

We call $\underline{\phi}$ a "representation" of $SU(N)$.
In this case the "fundamental"

representation. More generally, an object with a well-defined transformation that allows to have a group action acting in a similar manner to matrix multiplication is a <u>linear</u> representation.

How do we form an invariant Lagrangian for $\underline{\phi}$?

<u>Global</u>

$$\mathcal{L} = (\partial_\mu \phi)^\dagger \cdot (\partial^\mu \phi) - m^2 \phi^\dagger \cdot \phi$$

This works because

$$\phi^\dagger \to \phi^\dagger U^\dagger \qquad \phi \to U\phi$$

and $U^\dagger U = 1$.

Local

Now $U = e^{i\theta^a(x)T^a}$ where T^a are matrices that are the "generators" of the group. Any unitary matrix $U(x)$ can be parameterised in this way.

Now invariant if

$$\mathcal{L} = (D_\mu \phi)^\dagger \cdot D^\mu \phi - m^2 \phi^\dagger \cdot \phi$$

where $D_\mu = \partial_\mu + i A^a_\mu T^a$.

Rep $\Leftrightarrow$ Matter

$A_\mu \Leftrightarrow$ Force Carrier

HW: How does A^a_μ transform?

What is the kinetic term?

Building the Standard Model

Now we have the recipe for a QFT (and the Standard Model).

Step 1: Specify the global symmetries.

SM: Lorentz Invariance
(Boosts, Rotations, etc)

In GR becomes gauged, but that's not strictly part of SM.

Other symmetries may show up, but we won't impose them.

Step 2: Specify the gauge symmetries.

SM: $U(1)_Y$ } Becomes EW

$SU(2)_W$

$SU(3)_c$ Becomes QCD

These become the forces.

Step 3: Specify the representations.

SM:

	Poincaré	$U(1)_Y$	$SU(2)_W$	$SU(3)_c$
H	1	$1/2$	2	1
Q	$1/2$	$1/6$	$\bar{2}$	3
u^c	$1/2$	$-2/3$	1	3
D^c	$1/2$	$1/3$	1	3

L	$\frac{1}{2}$	$\frac{1}{2}$	$\bar{2}$	1
E^c	$\frac{1}{2}$	-1	1	1

Like a recipe!

Step 4: Write the most general action consistent with the symmetries.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6 + \mathcal{L}_7 + \cdots$$

Each term is for each mass dimension.

Parameters are "bare", in that they can only be fixed by an experiment.

$$\mathcal{L}_0 = \Lambda^4$$ Bare cosmological constant.

$$\mathcal{L}_1 = 0$$

$$\mathcal{L}_2 = -M_H^2 |H|^2$$ Bare Higgs Mass

$$\mathcal{L}_3 = 0$$

$$\mathcal{L}_4 = -\frac{1}{4}\sum F_{\mu\nu}F^{\mu\nu}$$ Gauge kinetic terms

$$+ i \sum \bar{\psi} \not{D} \psi$$ Fermion kinetic terms

$$+ \sum y_{ij} H \psi_i \psi_j$$ Yukawas

$$-\frac{\lambda}{4}|H|^4$$ Higgs Quartic

$$\mathcal{L}_5 = \frac{(L\cdot H)^2}{\Lambda_L}$$ Ohh.. violates $U(1)_L$ generates neutrino masses!

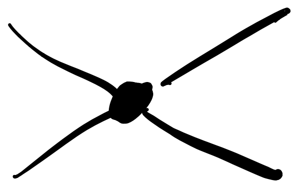
$$\mathcal{L}_6 = \frac{|H|^6}{\Lambda_H^2} + \text{Lots more ...}$$ Thousands....

$\mathcal{L}_7 =$ Can't be bothered.

All of this goes in the path integral. Some comments....

Relevance

Cross sections have the same dimension. On dimensional grounds operators of higher mass dimension must have extra kinematic factors to absorb dimension.

$\sigma\Big($  $\Big)$

$$\sim \frac{1}{m^2}\left(c_4 + c_5 \frac{E}{M} + c_6 \frac{E^2}{M^2} + \cdots \right)$$

Dimension-4 couplings have no mass scale. "Marginal"

Mass-scale

Thus at low energies $E \ll M$, lower dimension interactions are more "relevant".

Operators at dimension 4 are relevant at all energies.

In the SM there is presently no evidence for higher dimension operators suggesting mass scales could be high!

Natural Units Suck

All of this goes in a path integral.

$$Z_{SM} \sim \int \mathcal{D}\phi \, e^{\frac{i}{\hbar}\int d^4x \, \mathcal{L}}$$

The exponent must be dimensionless.

$$\therefore \left[\mathcal{L}/\hbar\right] = \left[x^{-4}\right]$$

From kinetic terms,

$$\left[H^2\right] = \left[A_\mu^2\right] = \left[\hbar/x^2\right]$$

But also, for instance,

$$\left[\lambda H^4\right] = \left[\frac{\hbar}{x^4}\right] \Rightarrow \left[\lambda\right] = \left[\frac{1}{\hbar}\right]$$

So $\hbar\lambda$ is a dimensionless quantity. Same exercise reveals $\hbar g^2$ is the same. This is why size of quantum corrections in QED is controlled by $\alpha = \frac{\hbar e^2}{4\pi}$!

What about

$$\left[\frac{H^6}{\Lambda_H^2}\right] \text{ well } \left[\Lambda_H^2\right] = \left[\frac{1}{\hbar^2 x^2}\right]$$

Can organise all this by mass and coupling dimension.

$$C = \left[\hbar^{-\frac{1}{2}}\right] \qquad M = \left[\frac{1}{x}\right]$$

This means that

$$\left[\frac{1}{\Lambda_H^2}\right] = \frac{C^4}{M^2}$$

In higher dimension operators it is very frequently an error to interpret Λ_j as a mass scale.

Often involves couplings.

HW: What is the dimension of Λ in a four-fermion interaction like the Fermi interaction?

$$\mathcal{L}_6 \sim \frac{(\psi)^4}{\Lambda^2}$$

Renormalisability

If all operators at Dim ≤ 4, and all gauge symmetries anomaly-free (more on this later) then you can fix the finite set of parameters by a finite number of experiments and everything else is uniquely predicted.

If not renormalisable, can still calculate predictions, but to go to arbitrarily high order in $\hbar$ an arbitrarily high number of experiments are needed to fix more and more parameters.