Approximate Symmetries of the Standard Model

Many structural aspects of the SM can be understood through the approximate symmetries.

Accidental Symmetries

Consider a symmetry where

$$q \to e^{i\theta_q} q \qquad q^c \to e^{-i\theta_q} q$$

$$l \to e^{i\theta_l} l \qquad l^c \to e^{-i\theta_l} l$$

The first is Baryon Number symmetry and the second is Lepton Number symmetry.

We may also split them up as

$$B+L: \quad \theta_l = \theta \quad \theta_q = 3\theta$$

$$B-L: \quad \theta_l = -\theta \quad \theta_q = 3\theta$$

Or take any linear combination we like. Now consider the SM at dimension 4:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4$$

It turns out that if we write all of the Lagrangian terms here, respecting only gauge symmetries, then we get $B+L$ and $B-L$ for free!

We call them accidental symmetries, since even if we don't impose them they come for free at low dimensions just because of gauge symmetries!

Going to higher dimensions operators will be allowed though.

E.g. $\mathcal{L}_5 = \frac{(L \cdot H)^2}{\Lambda}$ ← Breaks Lepton Number!

There are very strong constraints on Lepton and baryon number violation, so scales "Λ" must be very high to make rates very small.

(Remember. High Λ doesn't mean high mass, necessarily).

"Non-Perturbative" Breaking

Recall we are working with a QFT. It is not sufficient that $\mathcal{L}$ is invariant to have a symmetry. We need

$$Z = \int \mathcal{D}\phi \, e^{\frac{i}{\hbar}\int d^4x \, \mathcal{L}}$$

to be invariant....

It just so happens that for B-L everything is fine. But... for B+L we get

$$Z' = \int \mathcal{D}\phi \, e^{\frac{i}{\hbar}\int d^4x \left(\mathcal{L} + \frac{g^2 a}{32\pi^2} \tilde{W}^a_{\mu\nu} W^{a,\mu\nu}\right)}$$

Weak Field Strength (CP-odd version)

The theory is not invariant under B+L transformations!!

This is known as non-perturbative

symmetry breaking, or an "Anomaly".
The B+L-breaking effects will scale as

$$\sim e^{-\frac{4\pi}{\hbar\alpha_w}}$$

so very small when gauge interactions are perturbative. $\hbar\alpha_w \ll 1$.

So, in summary, only really 1 accidental symmetry in the SM.

There is another, more consequential, anomaly in QCD that we'll come back to later.... and other important accidental symmetries, such as "custodial" symmetry.

Approximate Symmetries

Consider the electrons. We have the Weyl spinors E^c, L, with kinetic terms and interactions

$$\mathcal{L} = i E^{c\dagger} \not{D} E^c + i L^\dagger \not{D} L + y_e H E^c L + h.c.$$

$\not{D} = D_\mu \gamma^\mu$

"Yukawa" Interaction.

Aside: You'll hear about "Weyl", "Dirac" and "Majorana" spinors.

Weyl spinors have two components: $\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$

Dirac spinors are made from two Weyl spinors

$$\psi = \begin{pmatrix} \chi \\ \phi^c \end{pmatrix} = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \phi_1^{c+} \\ \phi_2^{c+} \end{pmatrix}$$

and we sometimes switch to this form when two Weyl spinors pair up to get mass:

$$\mathcal{L} = i\chi^+ \not{\partial} \chi + i\phi^{c+} \not{\partial} \phi^c + m\chi\cdot\phi^c + h.c.$$

$$= i\bar{\psi} \not{\partial} \psi + m\bar{\psi}\psi$$

when $\bar{\psi} = (\phi^c \ \chi^+)$.

If a Weyl spinor has a mass all to itself we sometimes call it Majorana:

$$\mathcal{L} = i\chi^\dagger \not{\partial} \chi + \frac{m}{2}\chi\cdot\chi + h.c.$$

$$= i\Phi^c \not{\partial} \Phi + \frac{m}{2}\Phi^c \Phi$$

Where $\Phi = \begin{pmatrix} \chi \\ \chi^\dagger \end{pmatrix}$.

Personally I prefer to always work with Weyl!

Back to the electrons...

Consider again the Lagrangian:

$$\mathcal{L} = iE^{c\dagger}\not{\partial}E^c + iL^\dagger \not{\partial} L + y_e \cancel{H} L E^c$$

Forget this term...

When $y_e = 0$, we have, in addition to Hypercharge, the global $U(1)$ symmetry:

$$E^c \to e^{i\theta} E^c$$

$$L \to e^{i\theta} L$$

We call these "chiral" symmetries.

When $y_e \neq 0$ this is no longer a symmetry. But, in natural units,

$$y_e \simeq \frac{m_e}{174\,\text{GeV}} \sim 3 \times 10^{-6},$$

which is a small number. So, if

we're scattering electrons at high energies we know we have an "Approximate Chiral Symmetry". All effects involving chiral symmetry breaking will always be proportional to $y_e \lll 1$.

This is a very powerful organizational principle and allows for quick estimates of the magnitude of certain effects.

We'll also find the approximate chiral symmetries in the quark sector become very important...

CP (and T)

There are additional possible spacetime symmetries of a theory, which are discrete.

$$P: (t, \underline{x}) \to (t, -\underline{x})$$

$$T: (t, \underline{x}) \to (-t, \underline{x})$$

Charge conjugation acts similarly.

Consider a gauge field

$$A_\mu = \begin{pmatrix} A_0 \\ \underline{A} \end{pmatrix} \xrightarrow{P} \begin{pmatrix} A_0 \\ -\underline{A} \end{pmatrix} \xrightarrow{T} \begin{pmatrix} -A_0 \\ -\underline{A} \end{pmatrix} \xrightarrow{C} \begin{pmatrix} A_0 \\ \underline{A} \end{pmatrix}$$

A Lorentz-invariant theory is CPT invariant, but it needn't be CP invariant.

Is the <u>SM</u> CP-Invariant?

Consider: $\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} = \tilde{F}^{\mu\nu}$.

HW: Show this is CP-odd.

Then any terms like

$$\mathcal{L} = \frac{g_Y^2\theta_Y}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g^2\theta_W}{32\pi^2} W^a_{\mu\nu}\tilde{W}^{a\mu\nu}$$

$$+ \frac{g_s^2\theta_s}{32\pi^2} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

If $\theta \neq 0$ all terms are CP-violating.

Also, all are total derivatives, but θ_s is physical in the SM.

Experimentally, $\theta_s \sim 0$. So that part is CP-symmetric...

CP-Violation and Quarks

$$\mathcal{L} = \sum_j i Q_j^\dagger \not{\partial} Q_j + i u_j^{c\dagger} \not{\partial} u_j^c + i D_j^{c\dagger} \not{\partial} D_j^c$$

Flavors

$$+ \sum_{ij} y_{ij}^u H Q_i u_j^c + \sum_{ij} y_{ij}^D H^\dagger Q_i D_j^c$$

The kinetic terms respect the Unitary transformations:

$$\underline{Q} \to U_Q \underline{Q} \qquad \underline{D}^c \to U_D \underline{D}^c \qquad \underline{u}^c \to U_u \underline{u}^c$$

since $U_Q^\dagger \mathbb{1} U_Q = \mathbb{1}$ etc. This means we can completely diagonalize y_{ij}^u, but don't have enough freedom to diagonalize y_{ij}^D. At best, we may write

$$y^D_{ij} = U_{CKM} Y^D$$

Unitary Matrix   Diagonal

But U_{CKM} cannot be rotated away so it's physical.

A 3×3 unitary matrix can be parameterized by 3 angles and 1 phase. (Rephasing of quarks removes 5 of the 8 general parameters).

Under a CP transformation the phase changes sign $\theta \to -\theta$, essentially because $CP(e^{i\theta}) \to e^{-i\theta}$. So, if $\theta \neq 0$ then $CP(\mathcal{L}) \neq \mathcal{L}$ and CP is broken.

Indeed, this has been observed in the quark sector, where it seems the CKM phase is the only measured source of CP-violation. (Kaon oscillations, etc).

There's a similar story in the lepton sector, related to neutrino masses.