The Higgs Sector

Spontaneous Symmetry Breaking

Consider a theory with a global $U(1)$ symmetry: $\phi \to e^{i\theta}\phi$.

Lagrangian is

$$\mathcal{L} = |\partial_\mu \phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - f^2/2\right)^2$$

which is just

$$\mathcal{L} = |\partial_\mu \phi|^2 + \frac{\lambda f^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4 - \frac{\lambda f^4}{16}$$

Sloppy $(\partial_\mu \phi)^*(\partial^\mu \phi)$

$m^2|\phi|^2$

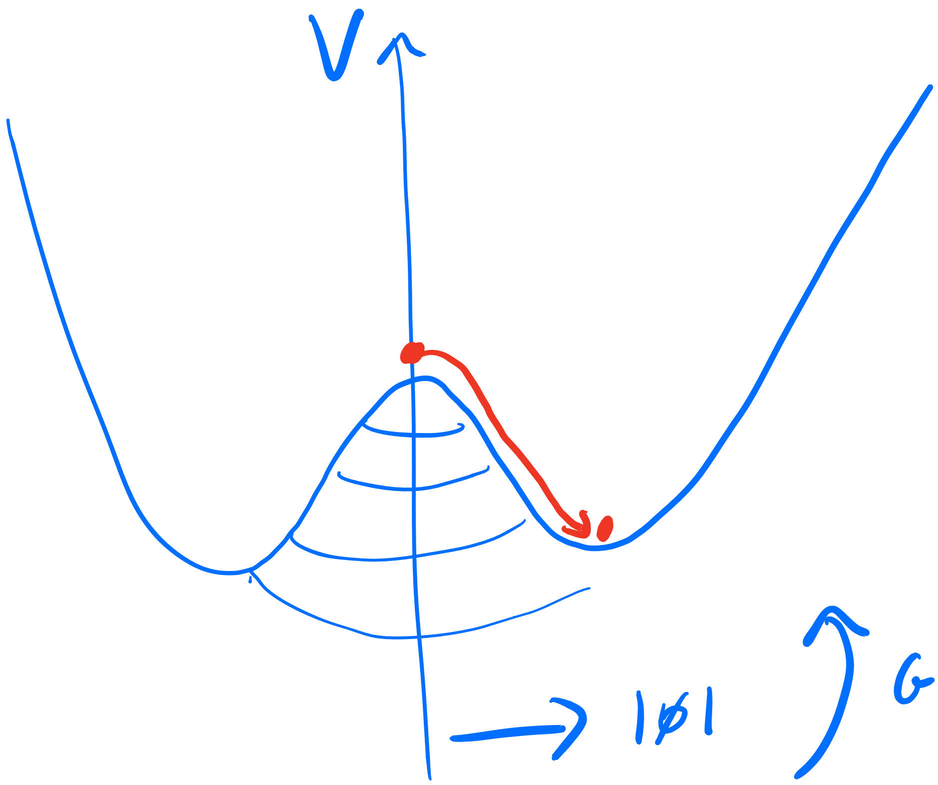
Unimportant constant.

Recall $\mathcal{L} = T - V$, so, if $f^2 < 0$

then positive mass scalar. No fun.

If $f^2 > 0$ then

$$V = \frac{\lambda}{4}\left(|\phi|^2 - \frac{f^2}{2}\right)^2$$



Locally

$\uparrow v$ Costs energy to move this way.

So out of the two degrees of freedom in $\phi = \phi_r + i\phi_i$, we expect a field that's massive, and one that's massless.

Can parameterise fields any way we wish. Writing

$$\phi(x) = \frac{f + \rho(x)}{\sqrt{2}} e^{iG(x)/f}$$

We see that

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu G)^2 - \frac{\lambda}{4}\left(\sqrt{2} f \rho + \frac{\rho^2}{2}\right)^2$$

$$= \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu G)^2 \frac{(f+\rho)^2}{f^2} - \frac{\lambda f^2}{2}\rho^2 - \frac{\lambda}{2\sqrt{2}} f \rho^3 - \frac{\lambda}{8}\rho^4$$

So we have a <u>massless</u> scalar field G, and a massive scalar field ρ. $(m^2 = \lambda f^2)$

Where did the symmetry go?...

Nowhere!

Before: $\phi \to e^{i\theta}\phi$ "Linearly Realized"

After: $\rho \to \rho$ "Nonlinearly

$\theta \to \theta + \theta_f$ Realized"

Constant shift symmetry.

We say the symmetry is "Spontaneously Broken" but that's a bit misleading, since it's still there, but hidden.

Goldstone's Theorem: For every spontaneously broken continuous global symmetry, a massless scalar!

"Nambu-Goldstone Boson"

Spontaneously Broken Gauge Symmetries

What if it were a gauge symmetry?

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D_\mu \phi|^2 + \frac{\lambda}{4}\left(|\phi|^2 - \frac{f^2}{2}\right)^2$$

Where $D_\mu = \partial_\mu + i g A_\mu$, Now

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu G)^2 \frac{(f+\rho)^2}{f^2}$$

$$+ \frac{\lambda}{4}\left(\sqrt{2} f \rho + \frac{\rho^2}{2}\right)^2$$

$$+ \frac{g^2}{2} (\rho + f)^2 A_\mu A^\mu$$

$$+ g \frac{(\rho+f)^2}{f} A_\mu \partial^\mu G$$

It's tricky to see the physical implications of these extra terms, but we may make one last gauge transformation...

$$A_\mu \to A_\mu - \frac{\partial_\mu G}{gf}$$

"G" Completely Disappears From Theory!!!

Final Lagrangian is

Gauge Boson Mass

Higgs

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{g^2}{2}(f+\rho)^2 A_\mu A^\mu$$

$$+ \frac{1}{2}(\partial_\mu \rho)^2 + \frac{\lambda}{4}\left(\sqrt{2} f \rho + \frac{\rho^2}{2}\right)^2$$

Higgs Boson

Higgs Mass

The gauge boson has become massive. This is the Higgs mechanism.

Predictions

Suppose you measure: g and m_A.

Well $m_A^2 = g^2 f^2$, $\Rightarrow f = \frac{m_A}{g}$

So $\mathcal{L} \Rightarrow g m_A \rho A_\mu A^\mu$

Interaction strength completely predicted!

Suppose you measure $m_A, m_\rho, g,$

then $\mathcal{L} \Rightarrow \frac{1}{\sqrt{2}} \frac{g m_\rho^2}{m_A} \rho^3$

$\uparrow$

Higgs
Self-Interaction
Completely Predicted!

Fermion Masses

Imagine this whole time ϕ also interacted with fermions. Example,

Weyl Fermions: ψ_a Charge "q"

ψ_b Charge "$q-1$"

Impossible to write a mass term.

$$\mathcal{L} = \cancel{\psi_a \psi_b} + h.c.$$

Breaks gauge symmetry!

We say the theory is "chiral".

(It's also anomalous, but we'll come back to that).

The gauge symmetry doesn't forbid a term like:

$$\mathcal{L} \Rightarrow y\phi\psi_a\psi_b + h.c.$$

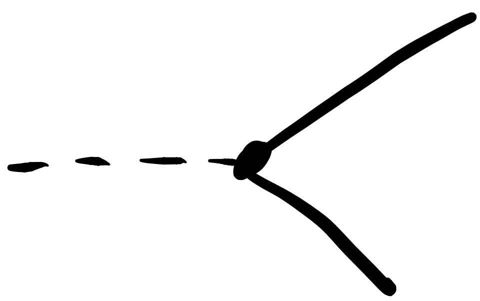
which, after that last gauge transformation, becomes

$$\mathcal{L} = \frac{y}{\sqrt{2}}(f+\rho)\psi_a\psi_b + h.c.$$

Fermion mass is $m_F = \frac{yf}{\sqrt{2}}$!

Prediction:

$$\mathcal{L} \Rightarrow g\frac{m_F}{m_A}\rho\,\psi_a\psi_b$$



Yukawa Interaction Predicted once m_A, g, m_F are measured.

The anomaly was because

$$\sum q_i \neq 0 \text{ and } \sum q_i^3 \neq 0.$$

Easily rectified if extra "chiral"

fermions added.

Finally...

The Standard Model
Higgs Sector

For the Standard Model we need to graduate to non-Abelian gauge symmetry breaking.

Gauge Sector

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \quad \text{with } Y = \tfrac{1}{2} ,$$

thus the gauge interactions are:

$$\mathcal{L} = |D_\mu H|^2$$

Where

$$D_\mu H = \left(\partial_\mu + ig'\frac{B_\mu}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + i\frac{g}{2}\begin{pmatrix}W_\mu^3 & 2W_\mu^+\\ 2W_\mu^- & -W_\mu^3\end{pmatrix}\right)\begin{pmatrix}H_1\\ H_2\end{pmatrix}$$

As before, we can always perform one final SU(2) transformation of H to put all of the breaking in one component. Also, can go to "Unitary" gauge by performing a gauge transformation to "eat" the Goldstone bosons.

Writing $A_\mu = \dfrac{g B_\mu + g' W_\mu^3}{\sqrt{}}$

$$\sqrt{g'^2+g^2}$$

$$Z_\mu = \frac{g' B_\mu - g W^3_\mu}{\sqrt{g'^2+g^2}}$$

And $\quad H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$, we get

$$D_\mu H = \left[\partial_\mu + i \frac{\sqrt{g'^2+g^2}}{2} \begin{pmatrix} & 0 \\ 0 & Z_\mu \end{pmatrix} + ig \begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} \right] \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} + \frac{i(v+h)}{2\sqrt{2}} \begin{pmatrix} 2g W^+_\mu \\ \sqrt{g'^2+g^2}\, Z_\mu \end{pmatrix} \right]$$

So

$$|D_\mu H|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{(v+h)^2}{8} \left(2g^2 |W^+_\mu|^2 + (g'^2+g^2) Z_\mu^2 \right)$$

And we have:

$$M_W = \frac{gv}{2} \qquad M_Z = \frac{\sqrt{g'^2+g^2}\, v}{2}$$

But also:

$$g' B_\mu = \frac{g'}{\sqrt{g^2+g'^2}}\left(g A_\mu - g' Z_\mu\right)$$

$$g W^3_\mu = \frac{g}{\sqrt{g^2+g'^2}}\left(g Z_\mu + g' A_\mu\right)$$

$$\therefore \; g' B_\mu + g W^3_\mu$$

$$= \frac{1}{(g^2+g'^2)^{\frac{1}{2}}}\left(2 g' g A_\mu + (g^2 - g'^2) Z_\mu\right)$$

Recall interaction had a factor $\frac{1}{2}$,

thus

$$e = \frac{g'g}{(g^2+g'^2)^{\frac{1}{2}}}$$

Also calling $\sin\theta_w = \frac{g'}{(g^2+g'^2)^{\frac{1}{2}}}$

$$e = g\sin\theta_w \qquad M_Z\cos\theta_w = M_W$$

Finally, the gauge interactions with the lepton doublet are:

$$D_\mu H = \partial_\mu H + ig\begin{pmatrix} \sin\theta_w A_\mu + \frac{\cos 2\theta_w Z_\mu}{2\cos\theta_w} & W_\mu^+ \\ W_\mu^- & \frac{Z_\mu}{2\cos\theta_w} \end{pmatrix} H$$

Summary: The SM EW sector is extremely predictive. Only essentially 4 free parameters:

$v, g, g', m_H.$

Multiple consistency checks!