Leptons

$e^{\pm}$ ν_e , $\mu^{\pm}$ ν_μ , $\tau^{\pm}$ ν_τ

Arguably, Quantum Field Theory Began Here

"Who ordered that?"

?

Gauge and Higgs Interactions

Consider the first generation:

$$\mathcal{L} = i E^{c\dagger} \not{D} E^c + i L^\dagger \not{D} L$$

$\underline{E^c}$

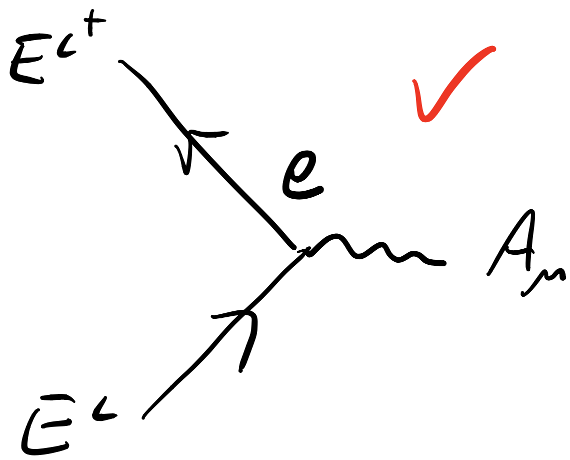
Only has Hypercharge $Y=1$. So

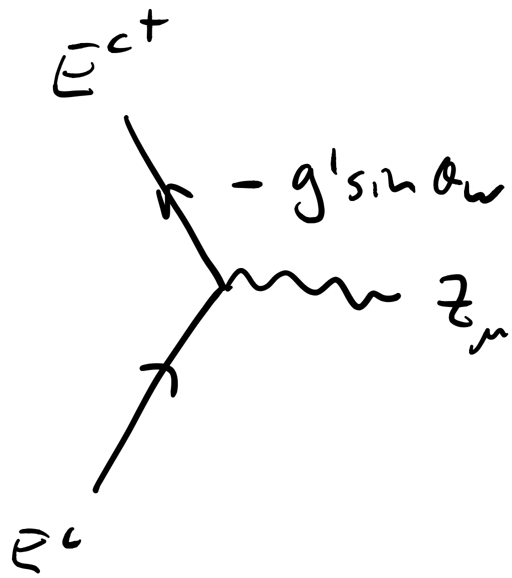
$$D_\mu = \partial_\mu + i g' B_\mu$$

but recall that

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

So we have





$\underline{L}$

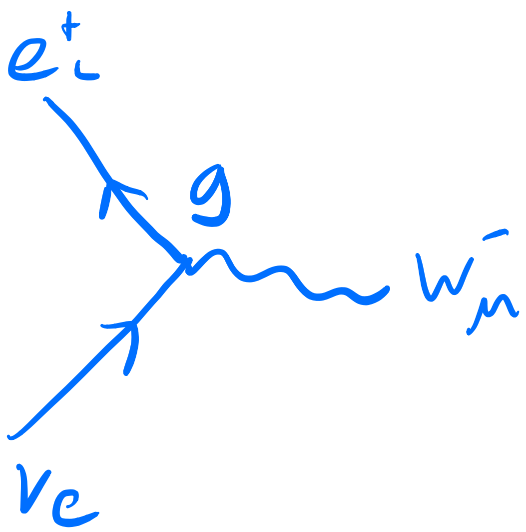
Has Hypercharge $Y = \frac{1}{2}$ but is <u>also</u> in a doublet!

$$D_\mu = \partial_\mu + i\frac{g'}{2}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} + i\frac{g}{2}\begin{pmatrix} W^3_\mu & 2W^+_\mu \\ 2W^-_\mu & -W^3_\mu \end{pmatrix}$$

Recall $W^3_\mu = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$

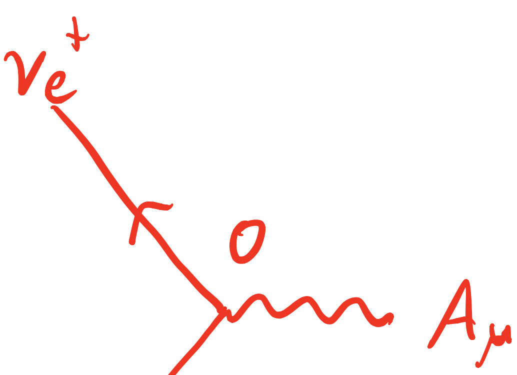
So our interactions follow from the representation: $L = \begin{pmatrix} e_L \\ \nu_e \end{pmatrix}$.

Charged Current (CC)

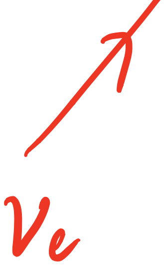


Pretty Simple!

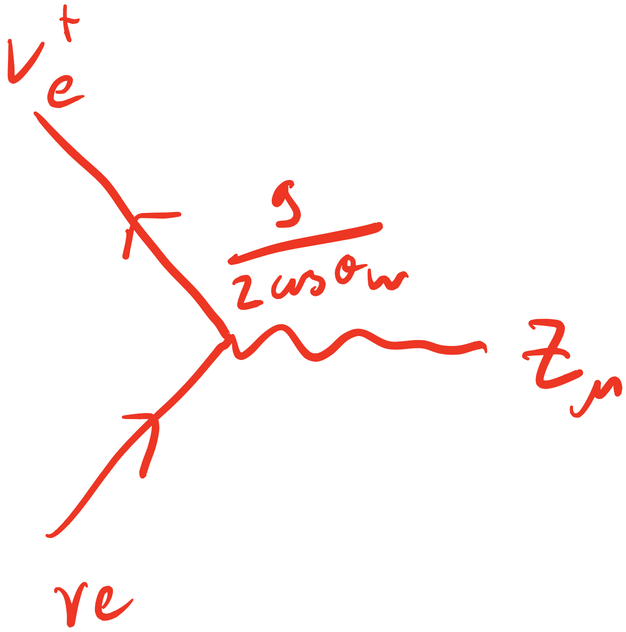
Neutral Current (NC)

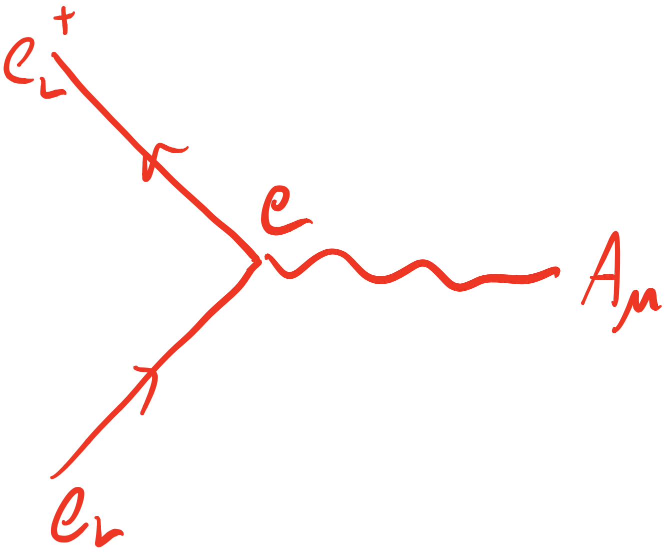


Since $g' B_\mu - g W^3_\mu$



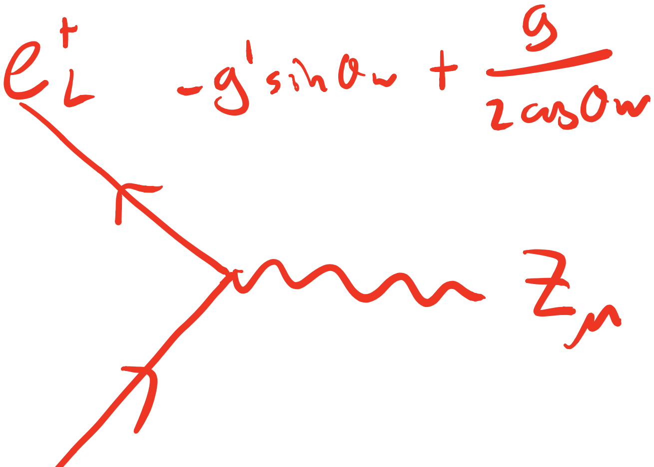
$$=(g'^2+g^2)^{\frac{1}{2}} Z_\mu$$





Since $\frac{g'}{2}\cos\theta_W + \frac{g}{2}\sin\theta_W$

$= g\sin\theta_W = e$



In Dirac Notation

We may use the projectors for Dirac spinors such that

$$P_L e = \begin{pmatrix} e_L \\ 0 \end{pmatrix}$$

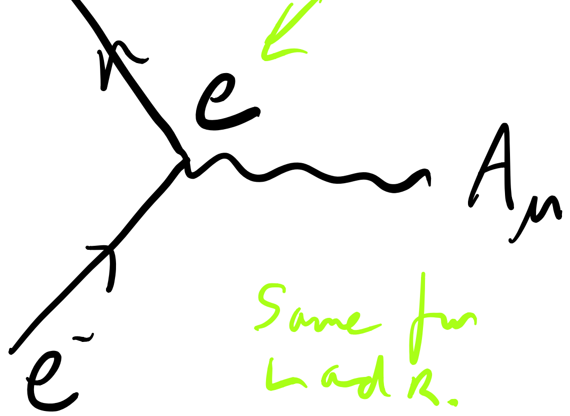
$$P_R e = \begin{pmatrix} 0 \\ e^{c\dagger} \end{pmatrix}$$

$$P_L + P_R = \mathbb{1}, \quad P_L^2 = P_L, \quad P_R^2 = P_R.$$

$$P_L P_R = 0.$$

This our Feynman rules become, for example:

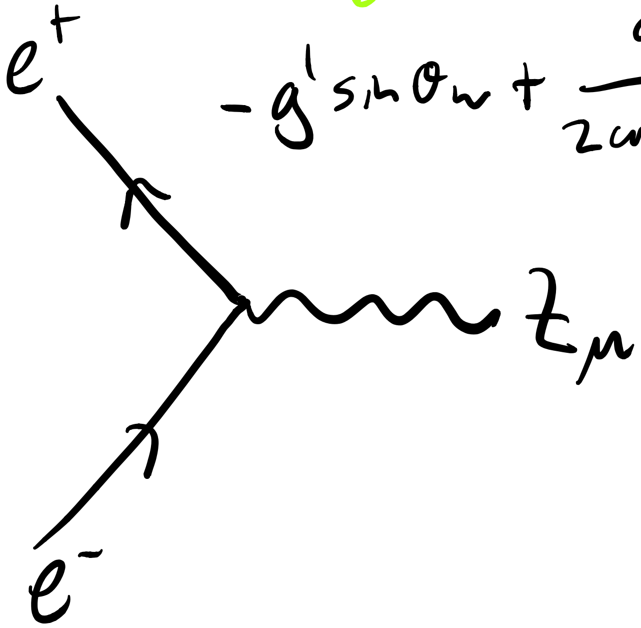
Same for L and R.

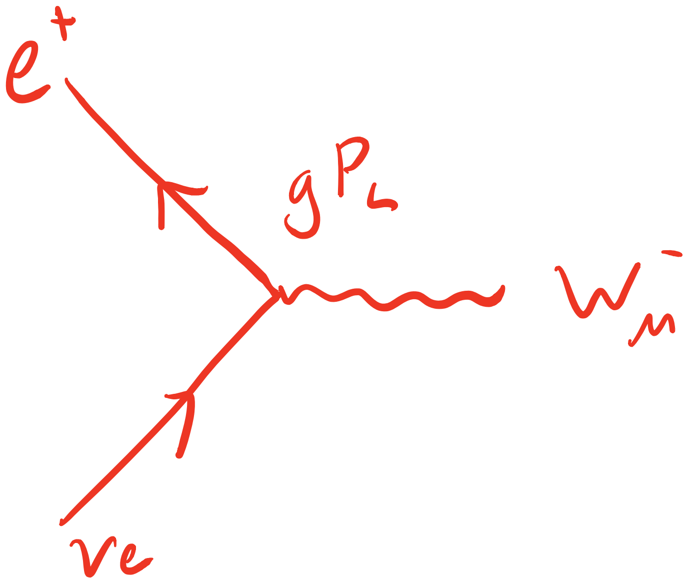


Same for L and R.

$$-g' \sin\theta_w + \frac{g}{2\cos\theta_w} P_L$$

Special, just for left-handed





Leptons and the Higgs

The lepton Yukawa interactions are from:

$$\mathcal{L}_L = \sum_{ij} y_{ij} E^c_i L_j \cdot H + h.c.$$

We may perform two unitary transformations on $\underline{E}^c$ and $\underline{L}$ to diagonalize y_{ij}.

$$\therefore \mathcal{L}_L = y_e E^c_e L_e^{\;H} + y_\mu E^c_\mu L_\mu^{\;H} + y_\tau E^c_\tau L_\tau^{\;H}$$

where $L_e = \begin{pmatrix} \nu_e \\ E_e \end{pmatrix}$ etc.

Recall that in Unitary gauge

we may set

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

So we get:

$$\mathcal{L}_L = \frac{(v+h)}{\sqrt{2}} \left[y_e E_e^c E_e + y_\mu E_\mu^c E_\mu + y_\tau E_\tau^c E_\tau \right]$$

So no neutrino masses! More on this later.....

The charged lepton masses are

$$m_e = \lambda_e \frac{v}{\sqrt{2}} \simeq \lambda_e \times 174 \text{ GeV (natural units)}$$

$$\therefore \quad \lambda_e = \frac{511 \text{ keV}}{174 \text{ GeV}}$$

$m_\mu = \lambda_\mu \frac{v}{\sqrt{2}}$ $\qquad$ $m_\tau = \lambda_\tau \frac{v}{\sqrt{2}}$

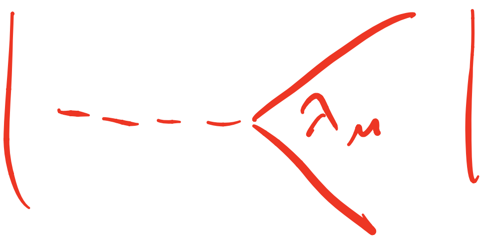
$= 105\,\text{MeV}$ $\qquad$ $= 1.8\,\text{GeV}$

So all the Yukawa interactions are very small.

Leptons in Higgs Physics

The Higgs may decay via these interactions. Exercise: How much integrated luminosity does the LHC need to see one $h \to \mu^+\mu^-$ decay given that $\sigma_h(13\,\text{TeV}) \sim 50\,\text{pb}$ and the branching ratio to τ is:

$BR(h \to \tau^+\tau^-) = 0.066$?



$^2 \propto \lambda_\mu^2$

$\therefore\ N_{\mu^+\mu^-} = L \times 50\,\text{pb} \times \frac{\lambda_\mu^2}{\lambda_\tau^2} \times 0.066$

$\therefore\ L_1 \simeq \frac{1}{50}\,\frac{1}{0.066}\left(\frac{1.8}{0.105}\right)^2\ \text{pb}^{-1}$

$\simeq 100\ \text{pb}^{-1}$

But with the HL-LHC we'll have $\sim 3000\ \text{fb}^{-1}$. So we'll get

$N_{\mu^+\mu^-} \simeq 30 \times 10^3$ events!

However, there is background too...

Check!!

What about the neutrinos?

Consider again the "Weinberg" operator:

$$\mathcal{L} = \frac{(L \cdot H^{+})^2}{\Lambda}$$

Where the SU(2) invariance is formed with the $\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ tensor:

$$L \cdot H^{+} = L \cdot \varepsilon \cdot H^{+}$$

$$= \begin{pmatrix} \nu & E^{L} \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

Thus $\frac{(L \cdot H^+)^2}{\Lambda} \to \frac{(\nu + \nu)^2}{2\Lambda} v^2$

So this operator generates a Majorana mass for the neutrinos.

$$m_\nu \approx \frac{v^2}{\Lambda}$$

So, for $\Lambda \sim 10^{13}$ GeV

$$m_\nu \simeq \frac{3 \times 10^4}{10^{13}} \text{ GeV}$$

$$\simeq \underline{0.3 \text{ eV}}$$

Our old $\hbar$ analysis shows that

$[\Lambda] = [M/c]$

So, if some microscopic physics generates this operator then it can explain neutrino masses! The mass scale is

$$M \sim \lambda \times 10^{13}\,\mathrm{GeV}$$

Some coupling in the underlying theory.