Quarks

For energies $E \gg \cancel{m_p}\,\Lambda$, can work with perturbative theory.

For $E \sim \cancel{m_p}\,\Lambda_{QCD}$, $\alpha_s(E) \sim 1$ and the theory becomes strongly coupled.

Quarks confined, separate lecture....

In the SM, the Lagrangian is

$$\mathcal{L} = \sum_j i\, Q_j^\dagger \not{D} Q_j + i\, U_j^{c\dagger} \not{D} U_j^c + i\, D_j^{c\dagger} \not{D} D_j^c$$

SU(2) invariant.

$$+ \sum_{ij} \lambda^u_{ij}\, H \cdot Q_i U_j^c + V_{CKM} \lambda^D_{ij}\, H^\dagger \cdot \varepsilon \cdot Q_i D_j^c$$

We already diagonalized this.

This is also, by definition, diagonal.

Again, after EW symmetry breaking, in the vacuum, we may perform one last gauge transformation to go to unitary gauge.

$$H \to \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

and $$Q = \begin{pmatrix} D_L \\ U_L \end{pmatrix}$$

Up-Quarks

Th...

This part is simple. We get

$$m_{ij}^{u} = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_t & 0 & 0 \\ 0 & \lambda_c & 0 \\ 0 & 0 & \lambda_u \end{pmatrix}$$

so, in this basis, Yukawa couplings and masses are diagonal. Note that, in natural units, $\lambda_t \sim 1$. This is pretty big! Especially since there are 3 colors of tops, so loop corrections scale as

$$\text{top-loop} \sim \frac{3 \times 2 \times \lambda_t^2}{(4\pi)^2} .$$

In fact, because of this, the top is quite special. For instance,

$m_t \sim 174$ GeV

thus ~~the~~ Higgs cannot decay to it!
The only SM fermion for which
~~this~~ happens!

Charm and up sort of less interesting...

Down-Quarks

$$H^{+} \cdot \mathcal{E} \cdot Q \to \frac{1}{\sqrt{2}}\begin{pmatrix} v+h & 0 \end{pmatrix} \cdot Q$$

So the mass matrix $(D_L \cdot D^c)$ is

$$m^{D}_{ij} = \frac{v}{\sqrt{2}} V_{CKM} \begin{pmatrix} \lambda_b & 0 & 0 \\ 0 & \lambda_s & 0 \\ 0 & 0 & \lambda_d \end{pmatrix}$$

from

$$\mathcal{L} \to \frac{v+h}{\sqrt{2}} \begin{pmatrix} B_L & S_L & D_L \end{pmatrix} \begin{pmatrix} V_{ckm} \end{pmatrix} \begin{pmatrix} \lambda_b & 0 & 0 \\ 0 & \lambda_s & 0 \\ 0 & 0 & \lambda_d \end{pmatrix} \begin{pmatrix} B^c \\ S^c \\ D^c \end{pmatrix}$$

To work with mass eigenstates we will need to perform one last rotation.

$$\begin{pmatrix} B_L \\ S_L \\ D_L \end{pmatrix} \to V_{ckm}^{\dagger} \begin{pmatrix} B_L \\ S_L \\ D_L \end{pmatrix}$$

This brings mass matrix to diagonal form. However, this now treats D_L differently from U_L!

Kinetic terms were:

$$i Q^\dagger \not{D} Q = i\left(D_L^\dagger \; U_L^\dagger\right) \sigma^\mu \left(\partial_\mu + \frac{i}{12} g' B_\mu + i\frac{g}{2}\begin{pmatrix} W^3_\mu & 2W^+_\mu \\ 2W^-_\mu & -W^3_\mu \end{pmatrix}\right)\begin{pmatrix} D_L \\ U_L \end{pmatrix}$$

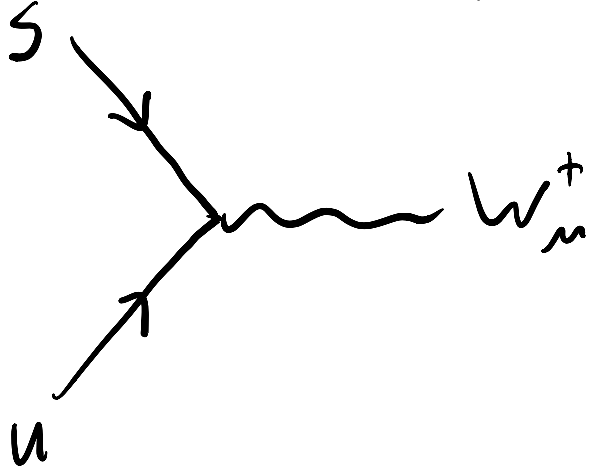
$+ h.c.$

Terms involving $D^\dagger_{L_i} (\ldots) D_{L_i}$ remain unchanged, since rotation is unitary.

However, off-diagonal terms now have:

$$-g\, D_L^\dagger V_{CKM} \not{W}^+ U_L + h.c.$$

In the mass-basis the CC interactions are off-diagonal!!



The down-type quarks have flavour-changing CC interactions.

No NC ones though!

QCD

Let us finally get to QCD and the gluons. The interaction between quarks and gluons again comes from the kinetic terms:

$$\mathcal{L} = i Q^{\dagger} \not{D} Q + \cdots$$

Where $\not{D} = D_\mu \sigma^\mu$ and

"Generators of $SU(3)$"

$$D_\mu = \partial_\mu + \cdots EW + i g_s G^a_\mu T^a$$

So the interactions with quarks are

Gluons

For su(2) and su(3) I've glossed over some important details. Consider the "Field Strength". We may derive it more generally as

$$F_{\mu\nu} = \frac{1}{ig}\left[D_\mu, D_\nu\right]$$

Needs a good explanation.

$$= \frac{1}{ig}\left(\partial_\mu + igA^a_\mu T^a\right)\left(\partial_\nu + igA^b_\nu T^b\right) - (\mu \leftrightarrow \nu)$$

terms cancel, to give

$$= \partial_\mu A^b_\nu T^b - \partial_\nu A^b_\mu T^b + igA^a_\mu A^b_\nu \left[T^a, T^b\right]$$

$$= \left(\partial_\mu A_\nu^c - \partial_\nu A_\mu^c + i g A_\mu^a A_\nu^b f^{abc} \right) T^c$$

This is the "structure constant" which is unique to the symmetry.

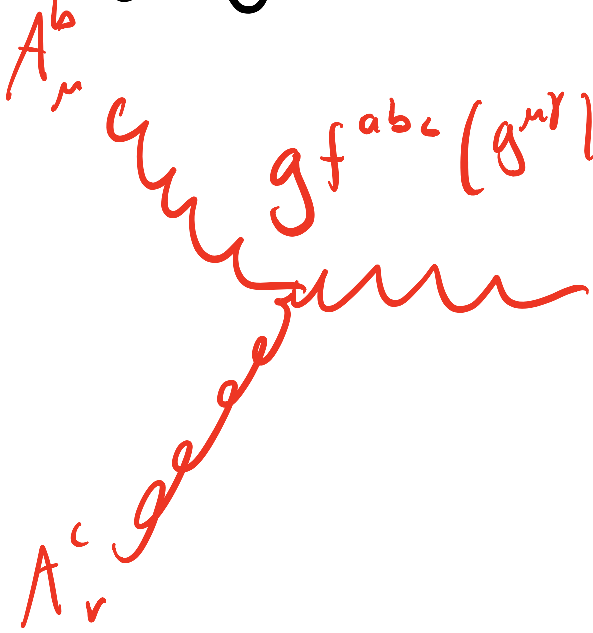
For $SU(2)$ $f^{ijk} = \varepsilon^{ijk}$, and $T^j = \frac{i\sigma^j}{2}$ the Pauli matrices. For $SU(3)$ the generators are known as the Gell-Mann matrices.

To form a gauge-invariant field strength we may then use

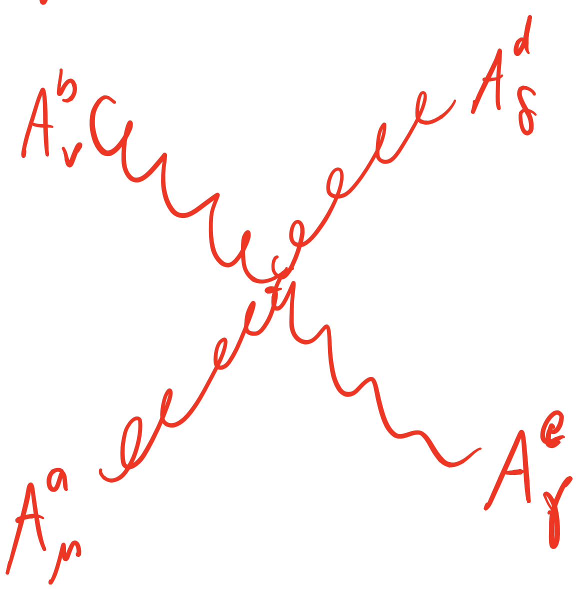
$$\mathcal{L} = -\frac{1}{4} \mathrm{Tr} \left[F^{\mu\nu} F_{\mu\nu} \right]$$

$$= -\frac{1}{4}\sum_a \left[\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i g f^{abc} A^b_\mu A^c_\nu\right]^2$$

So now we also get gluon interactions fixed by gauge invariance



$g f^{abc}\left(g^{\mu\gamma} k^\nu - g^{\nu\gamma} k^\mu\right)$



$f^{abc} f^{cde} g^{\mu\gamma} g^{\nu\delta}$

$+ \cdots$

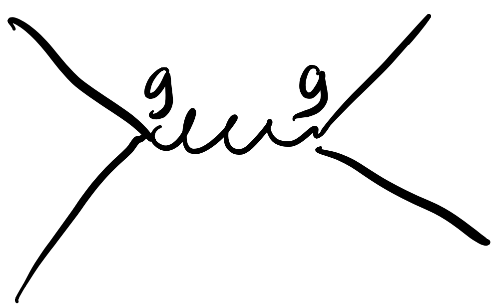
$+$ perms.

These interactions have profound

consequences!

Asymptotic Freedom

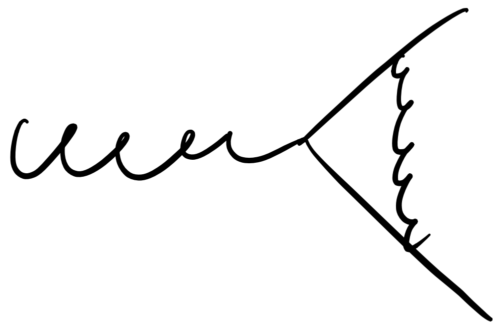
In a QFT there are some parameters that can only be measured. Gauge coupling is one of them. Suppose we measure the QCD gauge coupling at some scale μ:



$E \sim \mu$

If we measured it at some other energy what gauge coupling would we extract?

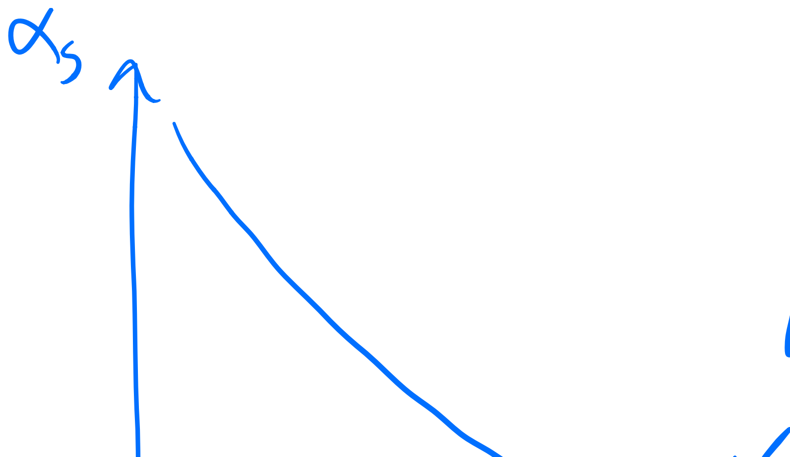
- Classically, the same!
- Quantum mechanically, not!



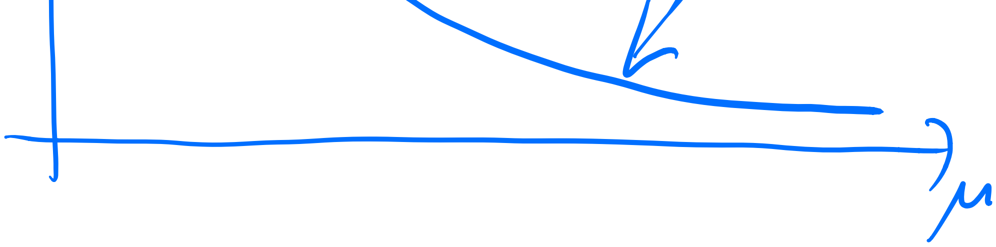
etc.

Result: $$\frac{d\alpha_s}{d\ln(\mu)} = \frac{\alpha^2}{4}\left(-\frac{11}{6}N_c + \frac{N_F}{3}\right)$$

Number of Colours. Pushes it negative!



Asymptotically becomes free!!!


μ