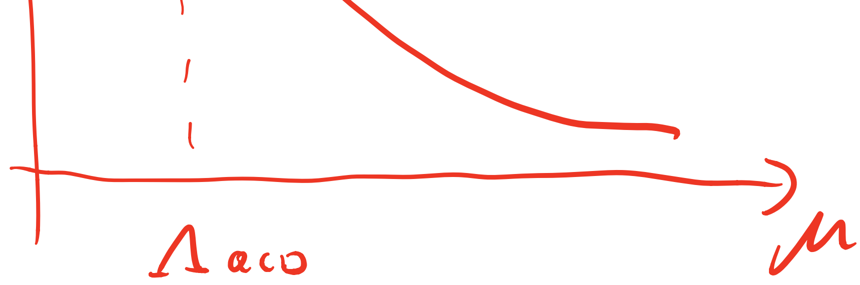
Below Λ_{QCD}

The flipside of Asymptotic Freedom is that α_s becomes large at $\mu \sim 1$ GeV. What are ~~the~~ consequences?

- Perturbation theory no longer applicable.
- Quarks and gluons bind together strongly
- Dimensional Transmutation!!

$$\alpha_s(\mu^2) \simeq \frac{1}{\beta_0 \ln\left(\mu^2/\Lambda^2_{QCD}\right)}$$



~330 MeV in $\overline{MS}$.

This is a very big deal. Because of quantum mechanics, a theory with only dimensionless parameters generates a dimensionful scale! Basically first emphasised by Coleman and Weinberg, for scalar QED. Consequences are quite profound: This is where proton mass comes from, hence the dominant source of all the mass of known matter! Discuss confinement.

Mesons

Due to strong coupling, quarks and antiquarks will bind together and condense. "Quark Condensation".

$$\langle \bar{q} q \rangle \neq 0$$

What are the consequences?

Little Known EW Breaking

Take $\langle U^c Q \rangle \neq 0$ and $\langle D^c Q \rangle \neq 0$.

Well, the gauge charges of these

combinations are

$$|Y| = \frac{1}{2}, \quad 3$$

In other words, the same gauge charges as the Higgs doublet!

Remember what happened when

$$\langle H \rangle \neq 0 \;?$$

Same here too! Quark condensation spontaneously breaks EW symmetry just like the Higgs!

$$\langle \bar{q} q \rangle \sim f_\pi^3 \sim (130\text{ MeV})^3$$

roughly...

So, even if the Higgs didn't break EW symmetry, QCD would!

$$m_W \sim g f_\pi , \quad m_Z \sim (g^2+g'^2)^{\frac{1}{2}} f_\pi$$

Totally irrelevant for us though.

Pions

Consider the 1^{st} generation, so just Up and Down quarks. Also, for now, forget masses. Under QCD we have

$\bar{3}$ $\quad U^c, D^c$

and 3 $\quad U_L, D_L$

Can package them as

$$Q^c = \begin{pmatrix} U^c \\ D^c \end{pmatrix} \qquad Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$$

and, in the limit of vanishing quark masses, the Lagrangian is invariant under

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix} \to \underbrace{V_L}_{\text{2x2 Unitary Matrix}} \begin{pmatrix} U_L \\ D_L \end{pmatrix}$$

$$(U^c \; D^c) \to (U^c D^c)\, V_R$$

So there is an $SU(2)_L \times SU(2)_R$ symmetry! When the quarks condense we have

$$\langle U^c U + D^c D \rangle \neq 0.$$

But $U^c U + D^c D = Q^c \cdot Q$. This

object is invariant under $SU(2)_V$

where $SU(2)_V$ is defined as

$$V_L = V_R^\dagger$$

But that means it isn't invariant

under $SU(2)_A$ where

$$V_L = V_R.$$

So, the quark condensate spontaneously

breaks $SU(2)_A$ symmetry.

Recall Goldstone's Theorem:

1 Massless scalar for every broken generator.

$SU(2)_A$ contains 3 generators, thus 3 Goldstone bosons!

Enter π^0, π^+, π^- !

Pion <u>Masses</u>

Pions aren't exactly massless. Why?

$SU(2)_A$ must only be an approximate symmetry. <u>Not</u> exact.

Because of the Higgs vev,

$$\mathcal{L} = m\left(u^c u + D^c D\right)$$

approximately. But this term explicitly breaks $SU(2)_A$!

UV —— Approximate $SU(2)_A$

$\downarrow$ spontaneously broken

IR —— "Pseudo-Goldstone Bosons"

So expect $M_\pi^2 \sim m\Lambda_\pi$

mass scale

Expect $\Lambda_\pi \sim g_* f_\pi$

$*$ analysis

$\sim 4\pi \times 130$

And for $m \sim 10$ MeV, we get

$$m_\pi \sim (10 \times 130 \times 12)^{\frac{1}{2}} \text{ MeV}$$

$$\sim 125 \text{ MeV}$$

Not bad! $m_\pi \sim 140$ MeV.

The Punchline

The real punchline isn't the numbers. This shows a scenario where scalars with gauge interactions are comfortably lighter than the other mass scale.

"UV-Completion"

q, q^c, g

Spontaneously broken approximate symmetry.

"IR Effective theory"

π, n, p, K etc

Comfortably light scalar.

The η' Puzzle

There's another symmetry I've ignored...

$$U(1)_A: \quad u, u^c, D, D^c \to e^{i\theta_A}(u, u^c, D, D^c)$$

This is an approximate symmetry. So where is the Goldstone boson???

This puzzle was challenging to understand for a long time. Resolution is that, under that symmetry,

$$\int \mathcal{D}\phi \, e^{\frac{i}{\hbar}\int d^4x \, \mathcal{L}} \to \int \mathcal{D}\phi \, e^{\frac{i}{\hbar}\int d^4x \left(\mathcal{L} + \frac{\theta_A g_s^2}{32\pi^2} G\tilde{G}\right)}$$

It's an anomalous symmetry!

So the symmetry that was spontaneously broken was never a symmetry in the first place! Anomalies are real-world stuff!

Pion Decays

$\pi^{\pm}$

Pions are the lightest hadronic particles. So, must decay to leptons.

But what are the rates?

- W coupling: Yukawa.

So the coupling piece must be,

$$R_{e/\mu} \propto \frac{y_e^2}{y_\mu^2} \sim \left(\frac{m_e}{m_\mu}\right)^2$$

- Phase space:

$$\Gamma \sim g^4 \frac{\;}{M_W^4} m_\pi (m_\pi^2 - m_l^2)$$

So overall

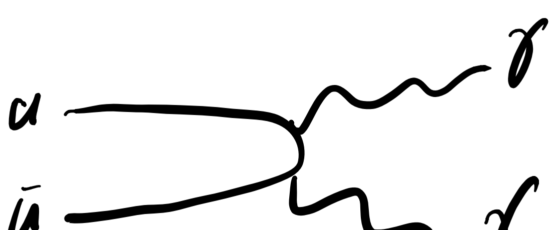
$$R_{e/\mu} \simeq \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)$$

$$\simeq 10^{-4}$$

Matches expectation.

π^0

Could decay to two electrons or, consistent with C-symmetry, two photons:



What is the coupling?

CP-odd.

$$\mathcal{L} \sim \frac{\alpha}{2\pi} c \frac{\pi^0}{f_\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

Unknown.

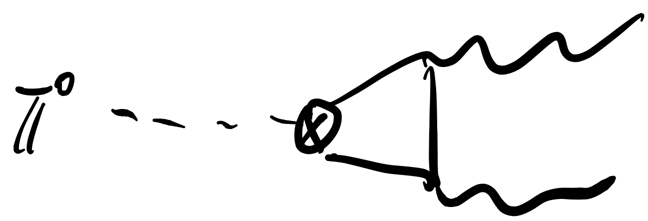
This coupling breaks shift symmetry, so expect proportional to explicit symmetry breaking: $c \sim y_u, y_d$

$$\therefore \; c \sim \frac{m_\pi^2}{M_\rho^2}$$

This scaling predicts:

$$\Gamma(\pi^0 \to 2\gamma) \approx 2\times 10^{13}\, s^{-1}$$

But observed rate is $10^{16}\, s^{-1}$. Much bigger!! Explanation: The symmetry of which the pion is a GB is anomalous. $c \sim 1$, no need for extra explicit symmetry breaking!



"Triangle Anomaly". Adler, Bell, Jackiw.

Anomalies Are Real.