Muon $g-2$ theory

UK HEP Forum 2021: From Laboratories to the Universe and Back
The Cosener's House, Abington & Online
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Image: National Gallery

Overview

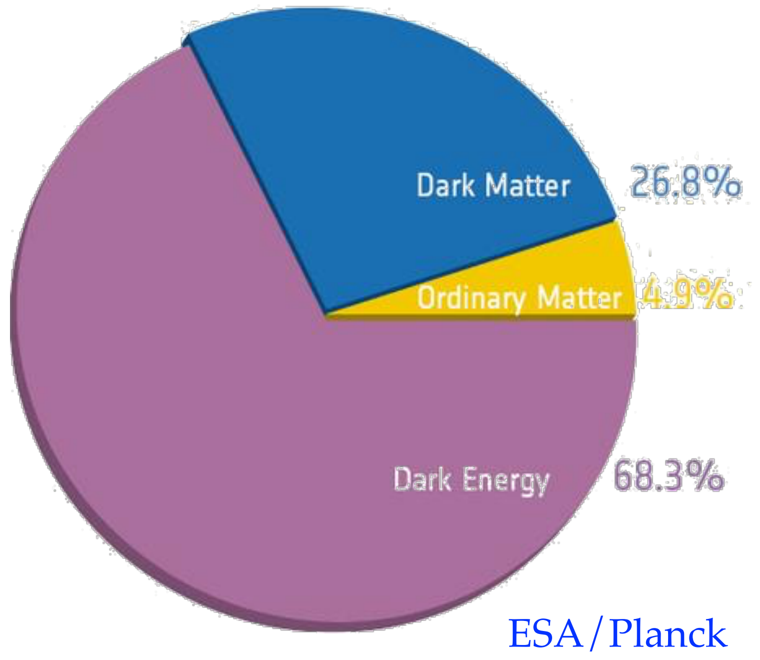
- $(g-2)_\mu$ in the Standard Model
- hadronic vacuum polarisation (HVP)
- hadronic light-by-light (HLbL)
- conclusions and outlook

Testing the Standard Model

SM describes electromagnetic, weak and strong interactions consistently in terms of renormalisable quantum field theory

While there are no ‘discovery’-level cracks in the SM there is ample phenomenological evidence of its limitations, e.g.

- dark matter and energy

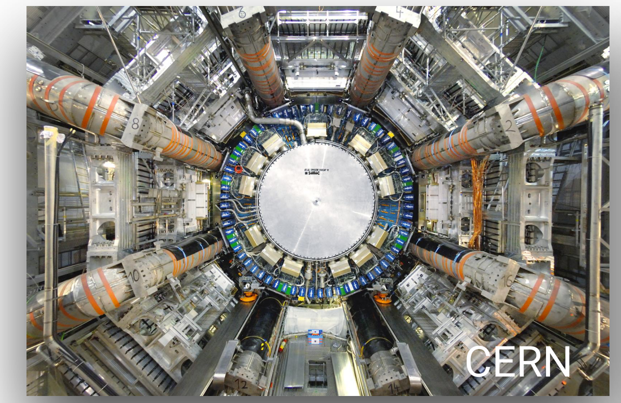


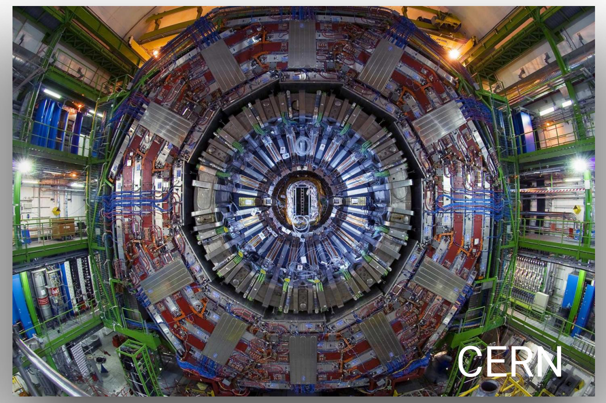
- matter-antimatter asymmetry

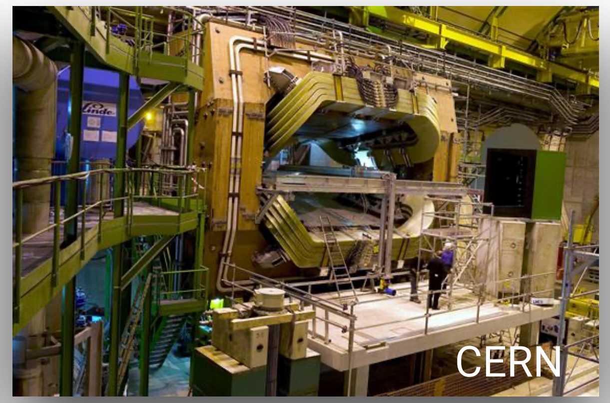
- particle hierarchy and couplings

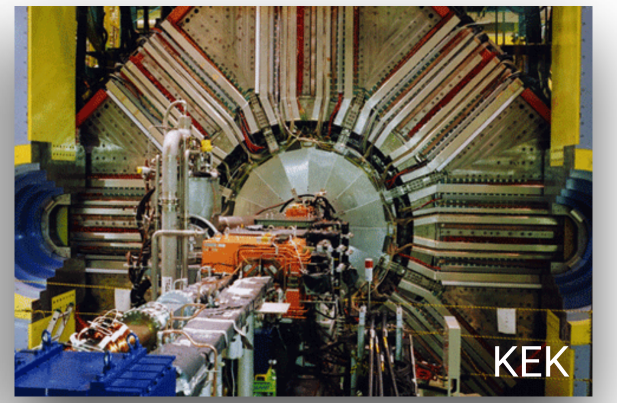
We are frantically looking for signs of what could explain these:

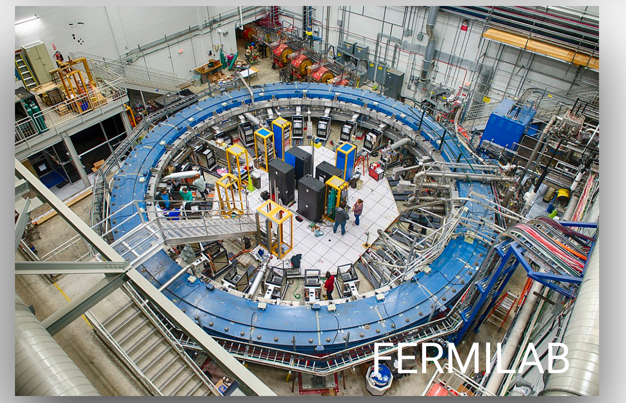
- direct searches
- indirect searches







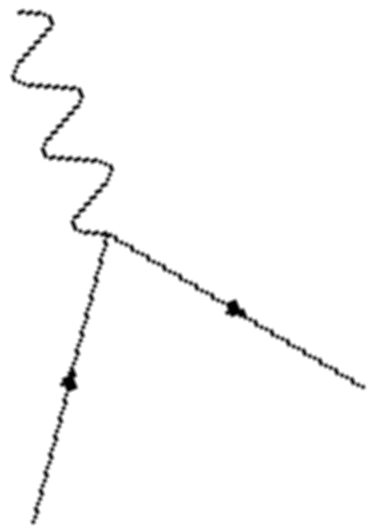




Charged lepton magnetic moment

spin-1/2 particle

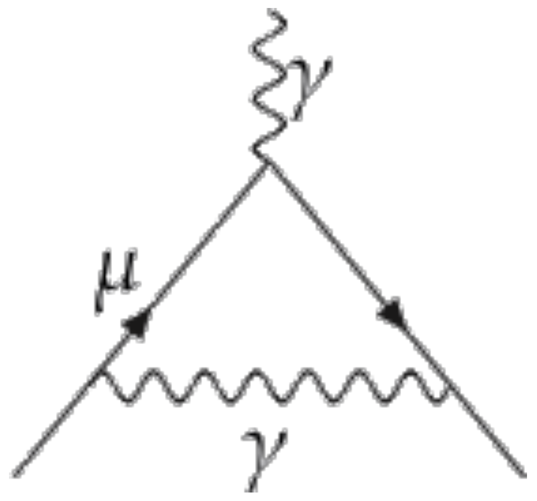
$$\vec{\mu} = g\frac{e}{2m_e}\vec{s} \qquad g \approx 2$$



$$\langle l^-(p_2)\,|\,j_\mu^{\text{em}}(0)\,|\,l^-(p_1)\rangle = (-ie)\bar{u}(p_2)\Gamma^\mu(P,q)u(p_1)$$

$$\Gamma^\mu(P,q) = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_l}$$

$$F_1(0) = 1 \qquad F_2(0) = a_l = \frac{g-2}{2}$$



$$a_l = \frac{(g-2)_l}{2} = \frac{\alpha}{2\pi} = 0.0011614\ldots$$

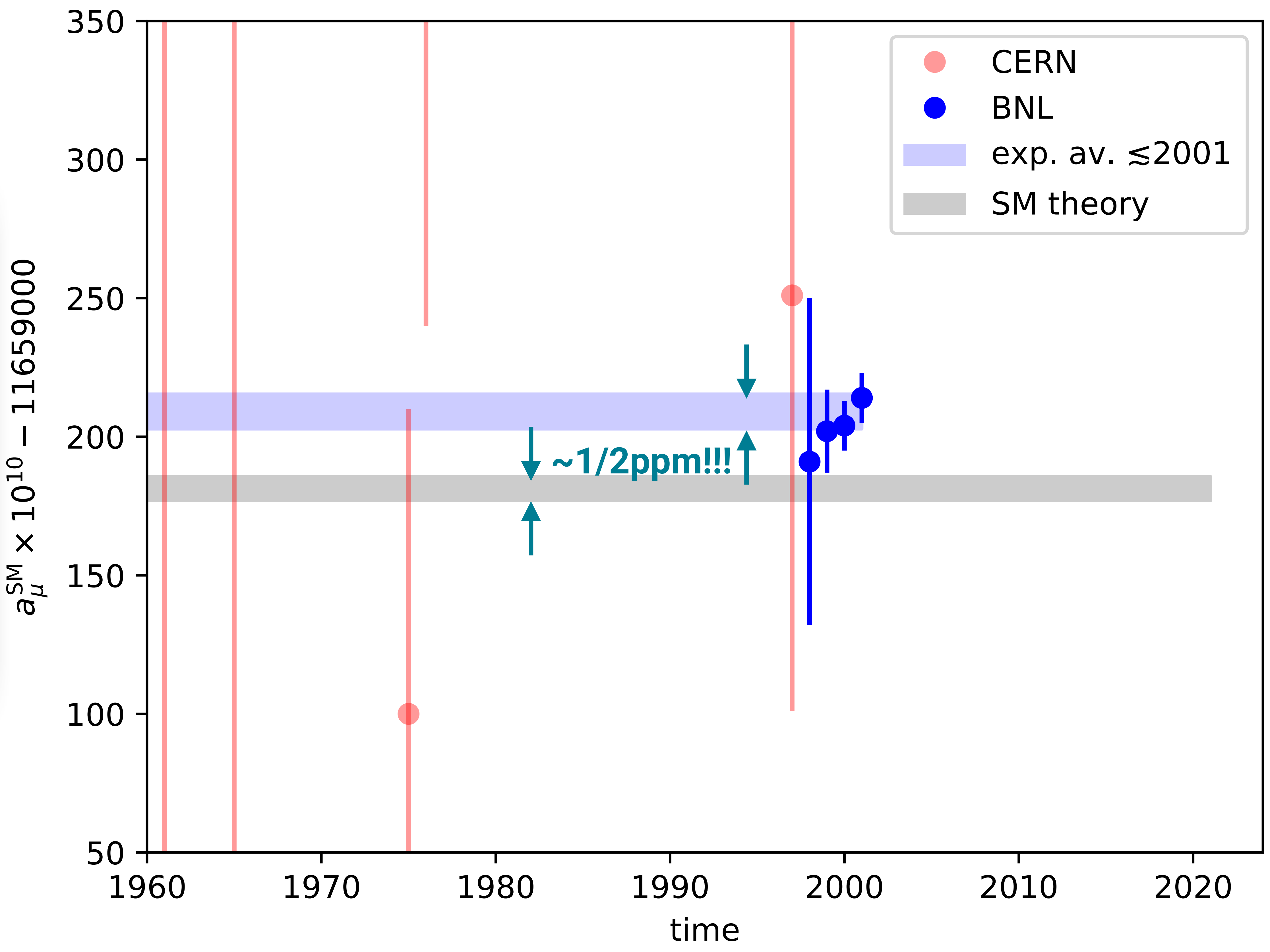
[Schwinger (1948)]

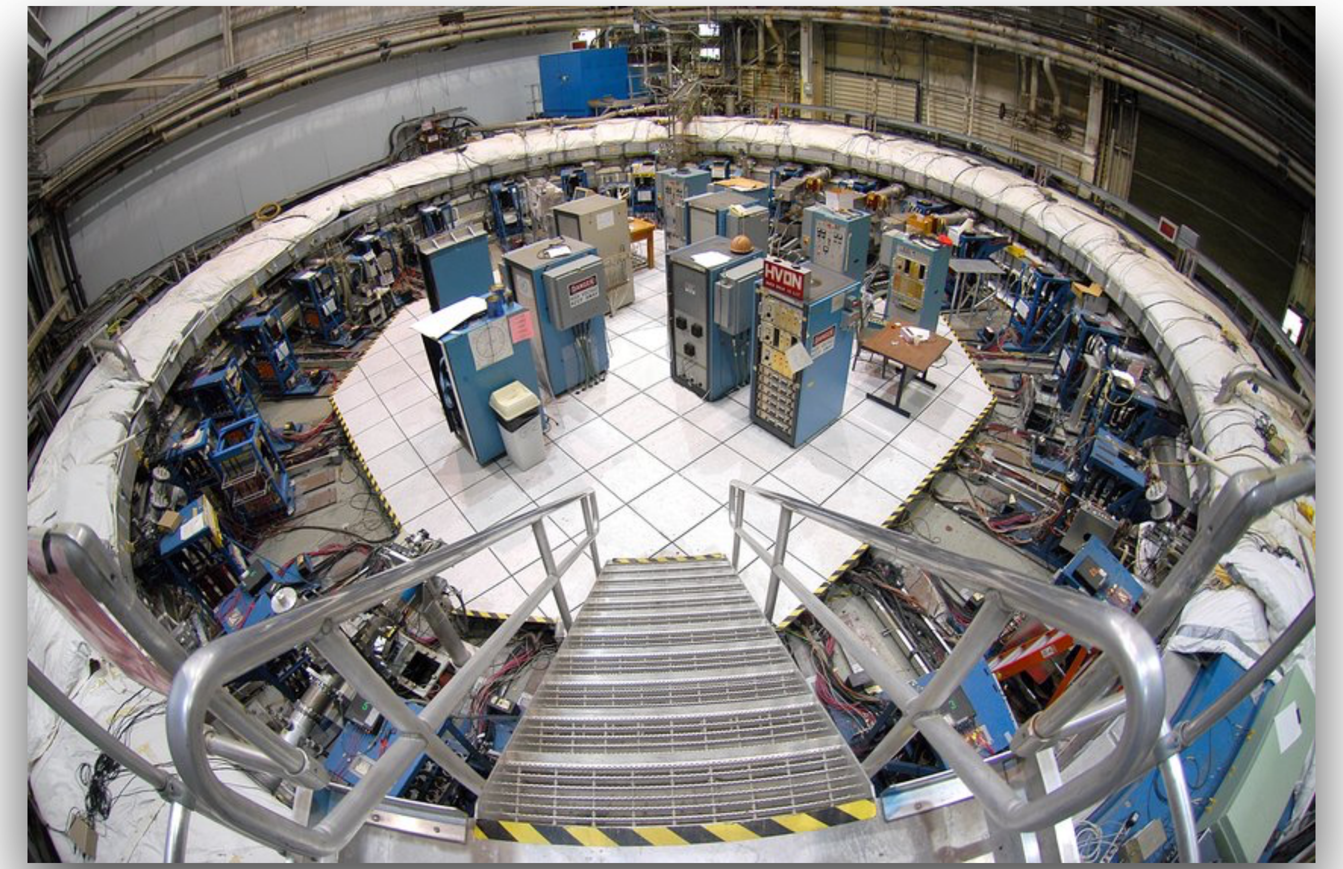
$a_l^{\text{NP}} \sim m_l^2/M_{\text{NP}}^2$ for $l = e, \mu, \tau - \mu$ is sweet spot between sensitivity and feasibility

Status — a_μ theory

	$a_\mu \times 10^{10}$	$\delta a_\mu \times 10^{10}$	
QED 5-loops	11658471.893	0.10	[Aoyama et al. 12 Aoyama et al. 19]
EW 2-loops	15.4	0.1	[Gnendinger et al. 13]
QCD HVP	684.5	4.0	[g-2 Theory Initiative 20]
QCD LbL	9.2	1.8	[g-2 Theory Initiative 20]
SM TOTAL	11659181.0	4.3	[g-2 Theory Initiative 20]

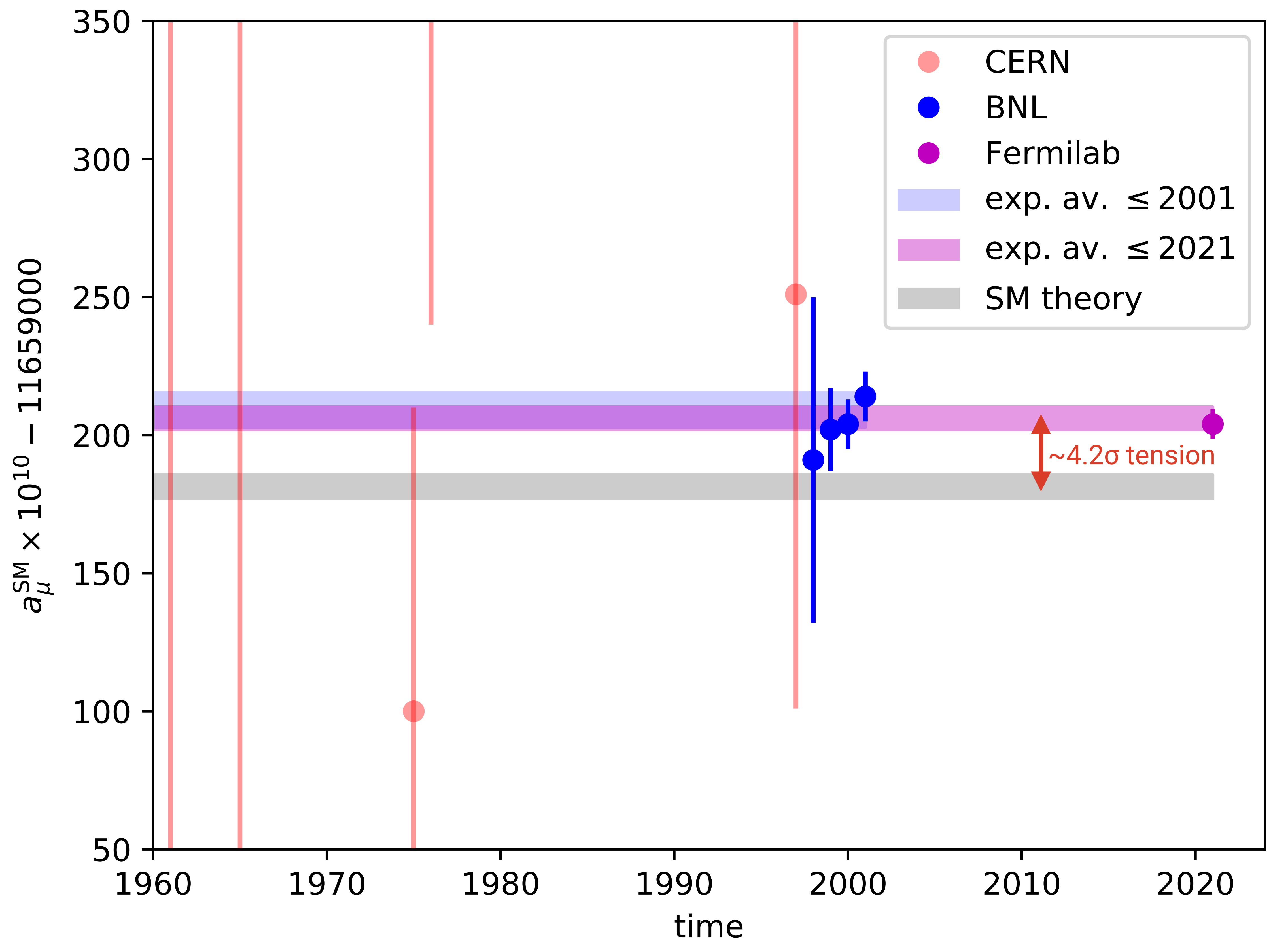
Status — a_μ experiment pre 2021





SM TOTAL	11659181.0	4.3	[g-2 Theory Initiative 20]	
Experiment	11659209.1	6.3	[Muon g-2 collab 06]	0.54ppm
th.-exp.	-28.1		~3.7σ tension	

Status — Experiment



SM TOTAL	11659181.0	4.3	[g-2 Theory Initiative 20]	
Experiment	11659206.1	4.1	[historical average 21]	0.35ppm
th.-exp.	-25.1	5.9	~4.2σ tension	

Fermilab's co-spokesperson Chris Polly:
*"So far we have analysed **less than 6%** of the data that the experiment will eventually collect.*

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The anomalous magnetic moment of the muon in the Standard Model

https://muon-gm2-theory.illinois.edu

Contribution	Section	Equation	Value x 10^{11}	Citation Refs
Experimental average (E821+E989)		[updated]	116 592 061(41)	bib, cite
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	bib, cite
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	bib, cite
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	bib, cite
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	bib, cite
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	bib, cite
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	bib, cite
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	bib, cite
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	bib, cite
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	bib, cite
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	bib, cite
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	bib, cite
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	bib, cite
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	bib, cite
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 8	[updated]	251(59)	

Theory — dominant errors

	$a_\mu \times 10^{10}$	$\delta a_\mu \times 10^{10}$	
QED 5-loops	11658471.893	0.10	[Aoyama et al. (2012) Ayoama et al. (2019)]
EW 2-loops	15.4	0.1	[Gnendinger et al. (2013)]
QCD HVP	684.5	4.0	[g-2 Theory Initiative (2020)]
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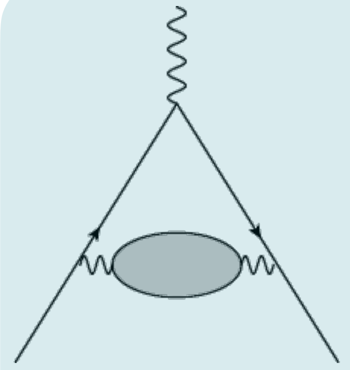
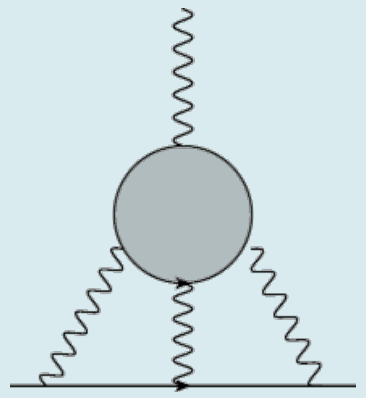
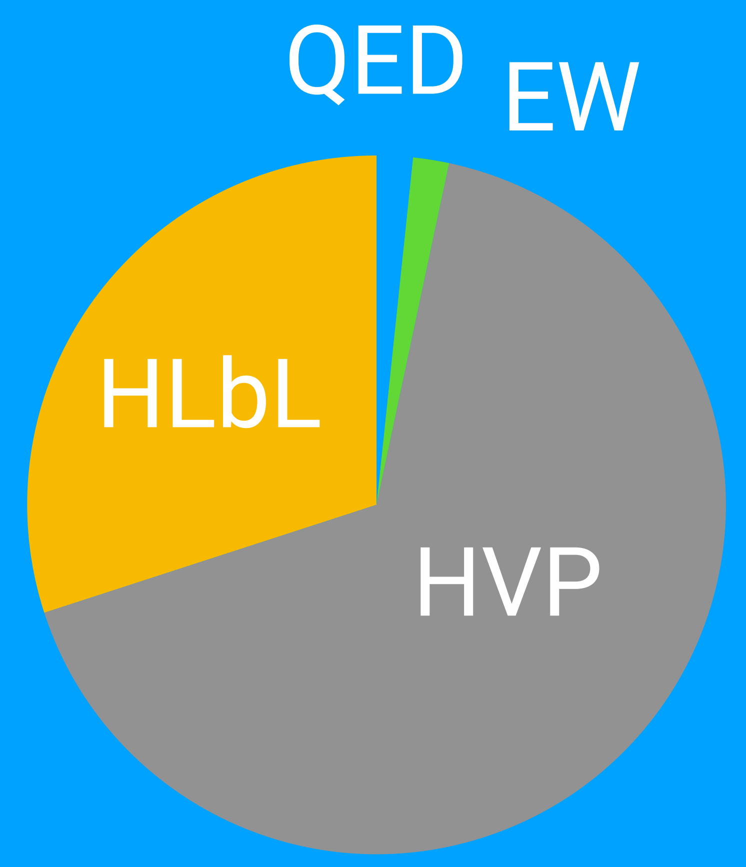


$O(\alpha^2)$, known <1%, dominates total uncertainty



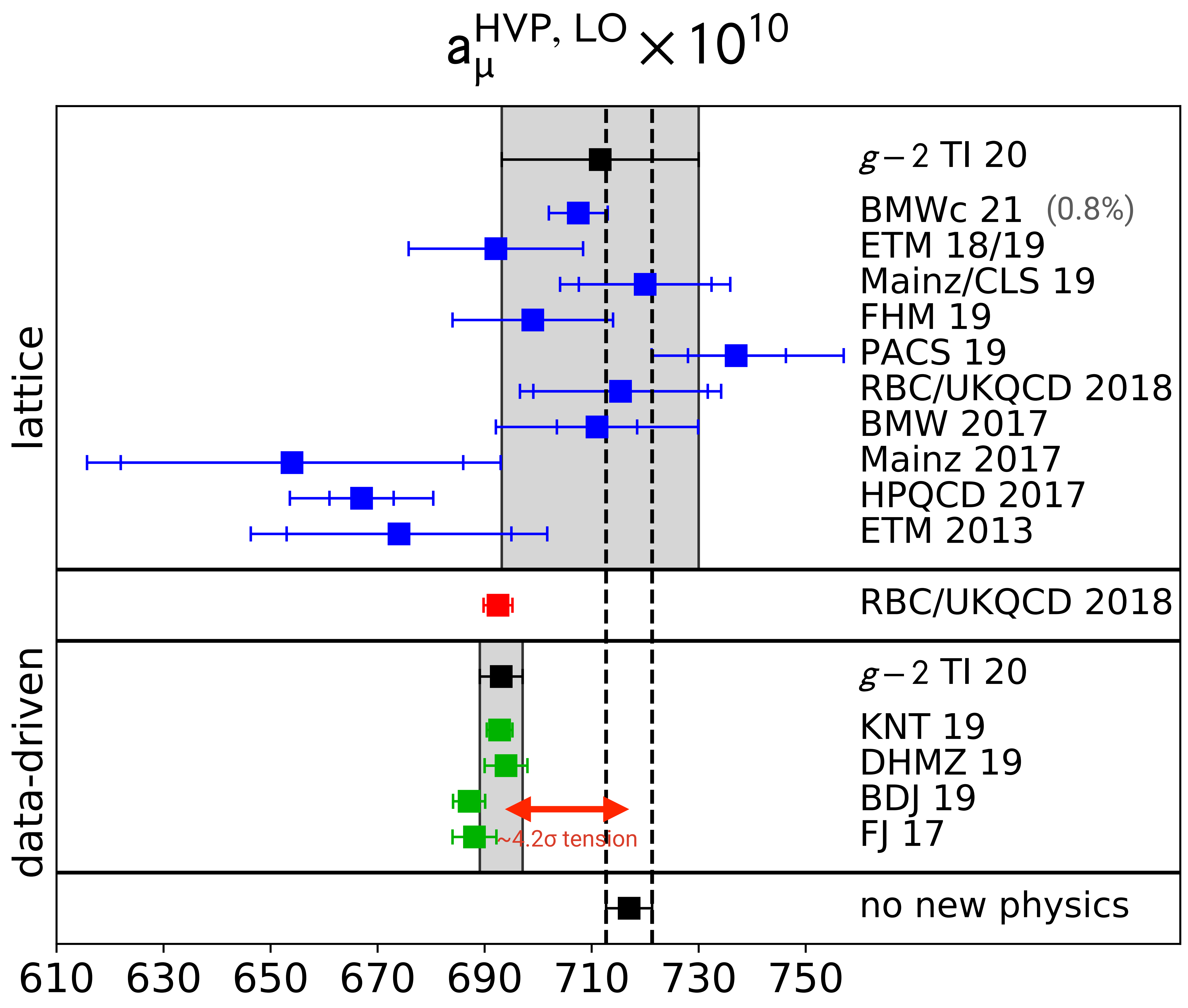
$O(\alpha^3)$, known $\sim 20\,\%$ 2nd largest uncertainty

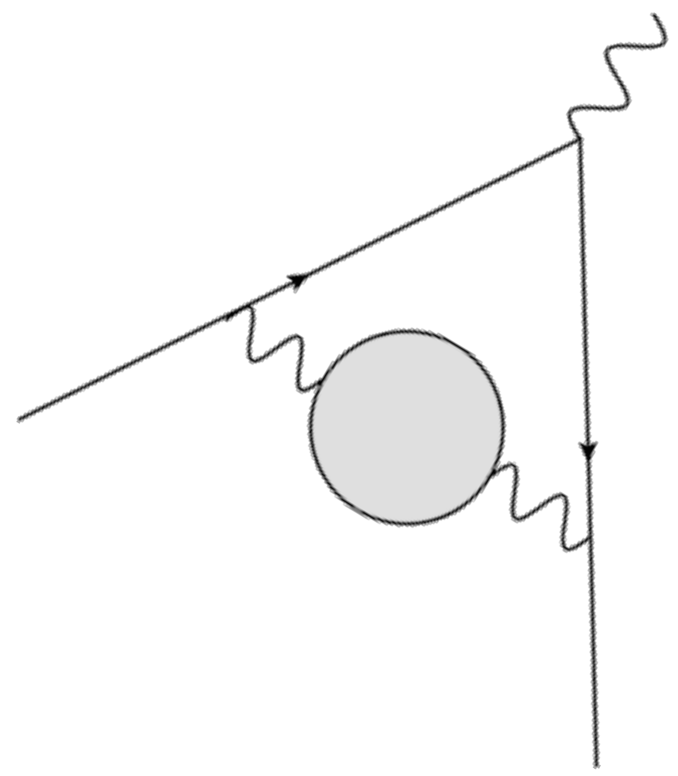
Error contributions



Theory needs to improve on hadronic contributions!

HVP status





- $a_\mu^{\text{SM}} \approx a_\mu^{\text{EW}} + a_\mu^{\text{QED}} + a_\mu^{\text{QCD}}$
  $a_\mu^{\text{QCD}} = a_\mu^{\text{HVP, LO}} + a_\mu^{\text{HVP, NLO}} + \ldots + a_\mu^{\text{HLbL}} + \ldots$

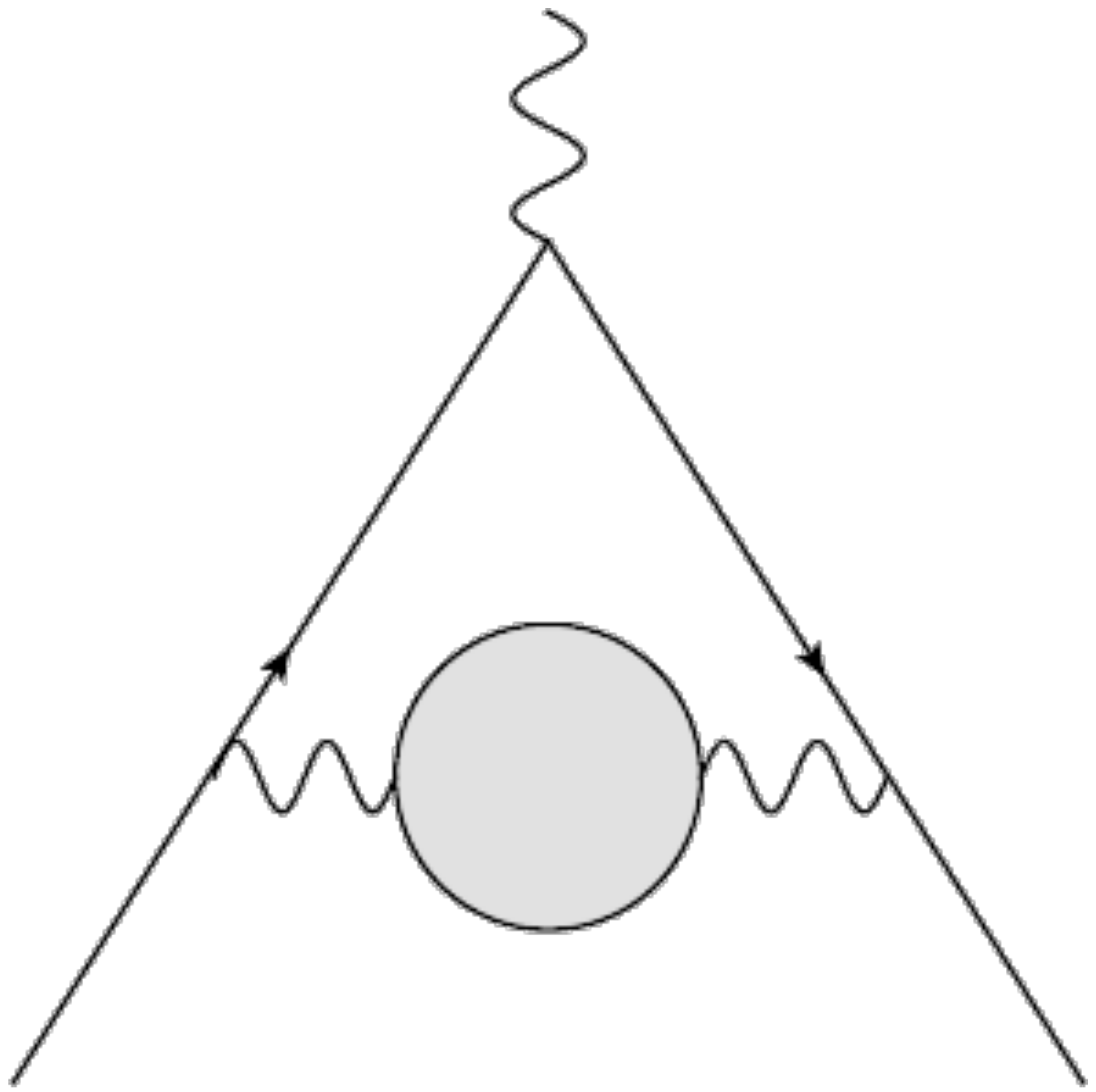
  "no new physics" — HVP value required to bring experiment an SM into agreement

- most precise determination data-driven (experimental data + dispersion theory)

- huge Lattice-QCD activity — catching up

- **precision goal 0.1%**

LO HVP in the SM

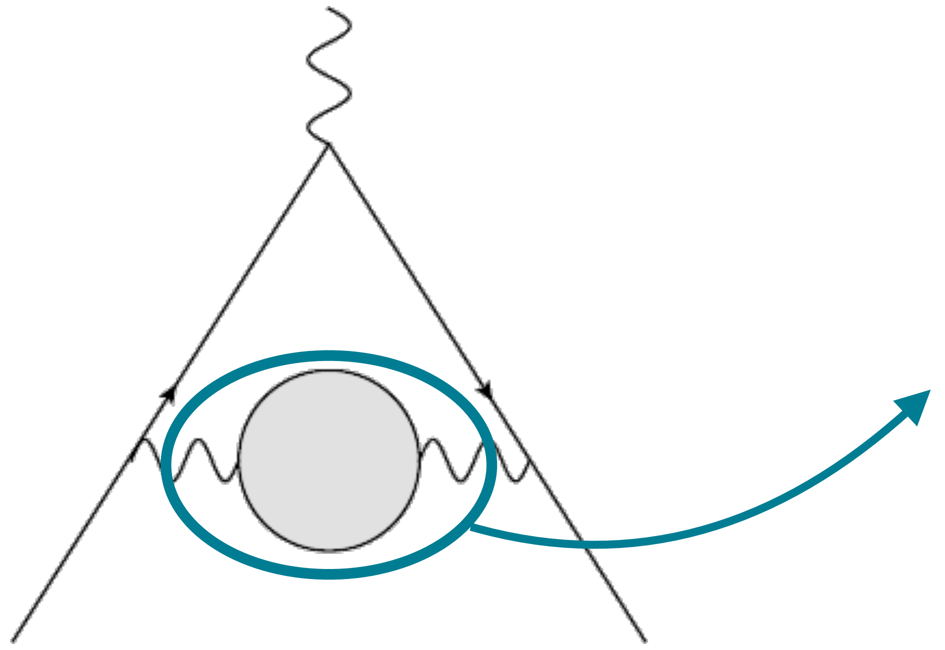


data-driven predictions

LO-HVP contribution

$$j_\mu^{\rm em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c - \frac{1}{3}\bar{b}\gamma_\mu b$$

hadronic 'blob' given by vacuum-polarisation tensor



$$\Pi_{\mu\nu}(q^2) = ie^2 \int d^4x e^{iqx} \langle 0 | T j_\mu^{\rm em}(x) j_\nu^{\rm em}(0) | 0 \rangle = \left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \Pi(q^2)$$

analyticity

$$\Pi(q^2) - \Pi(0) = \int_0^\infty ds \frac{{\rm Im}\Pi(s)}{s(s - q^2 - i\epsilon)}$$

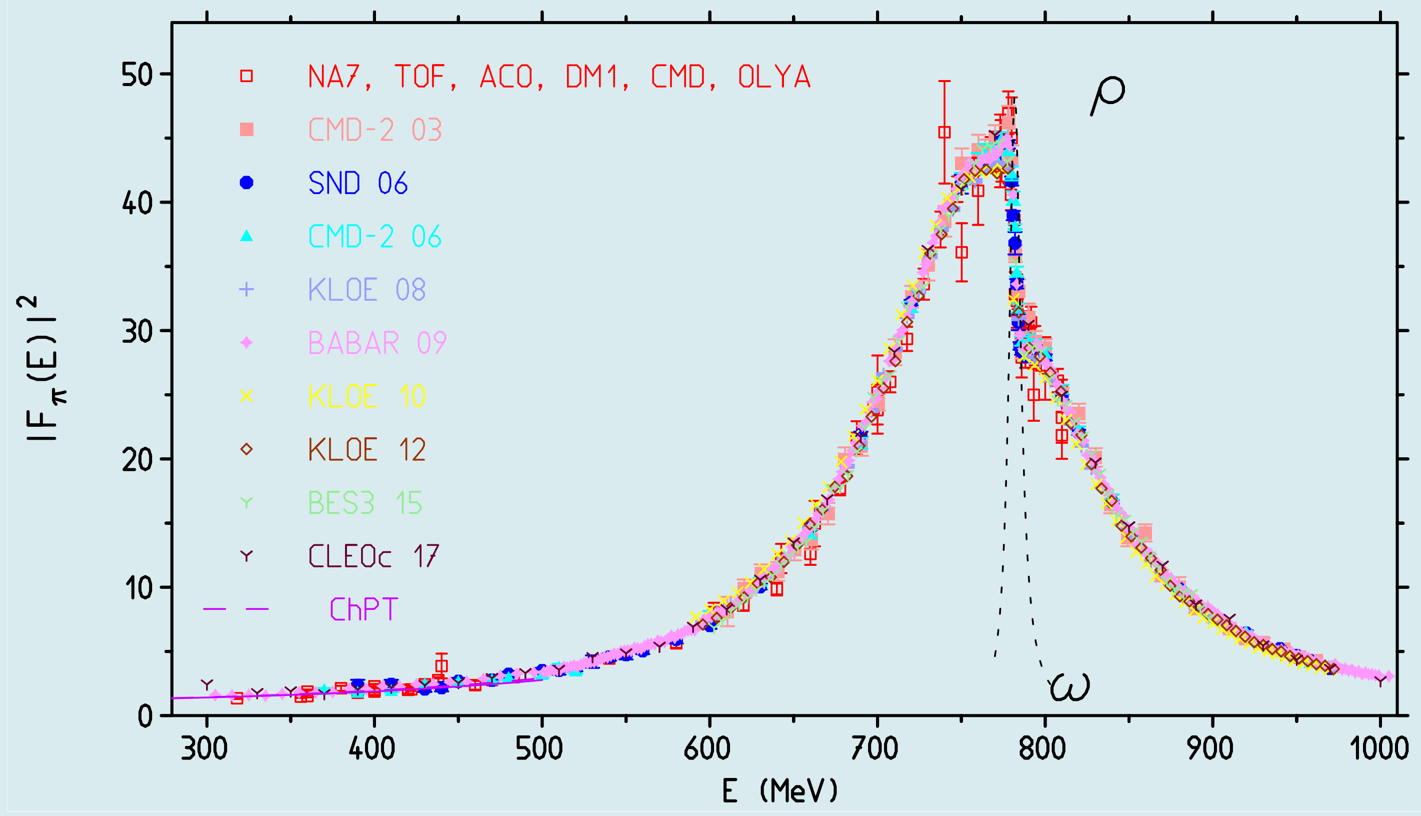
unitarity (optical theorem)

$${\rm Im}\Pi(s) = \frac{s}{4\pi\alpha(s)} \sigma_{\rm tot}(e^+e^- \to {\rm hadrons}) = \frac{\alpha(s)}{3} R_{\rm had}(s)$$

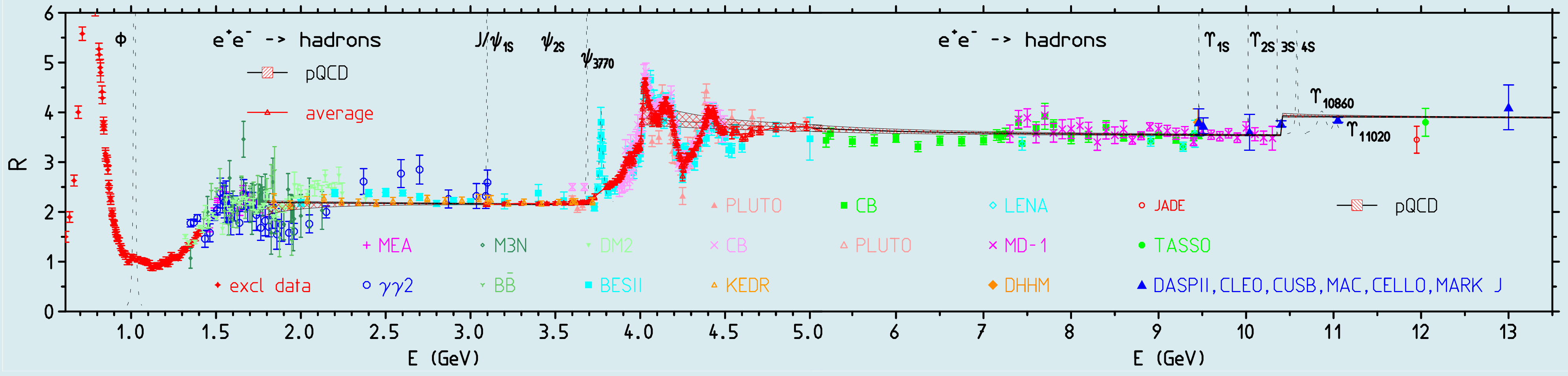
$$a_\mu^{\rm HVP,\,LO} = \left(\frac{a m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^\infty ds \frac{\hat{K}(s)\, R_{\rm had}(s)}{s^2}$$

[Bouchiat, Michel 1961]
[Durand 1962,1963]
[Gourdin, De Rafael 1969]

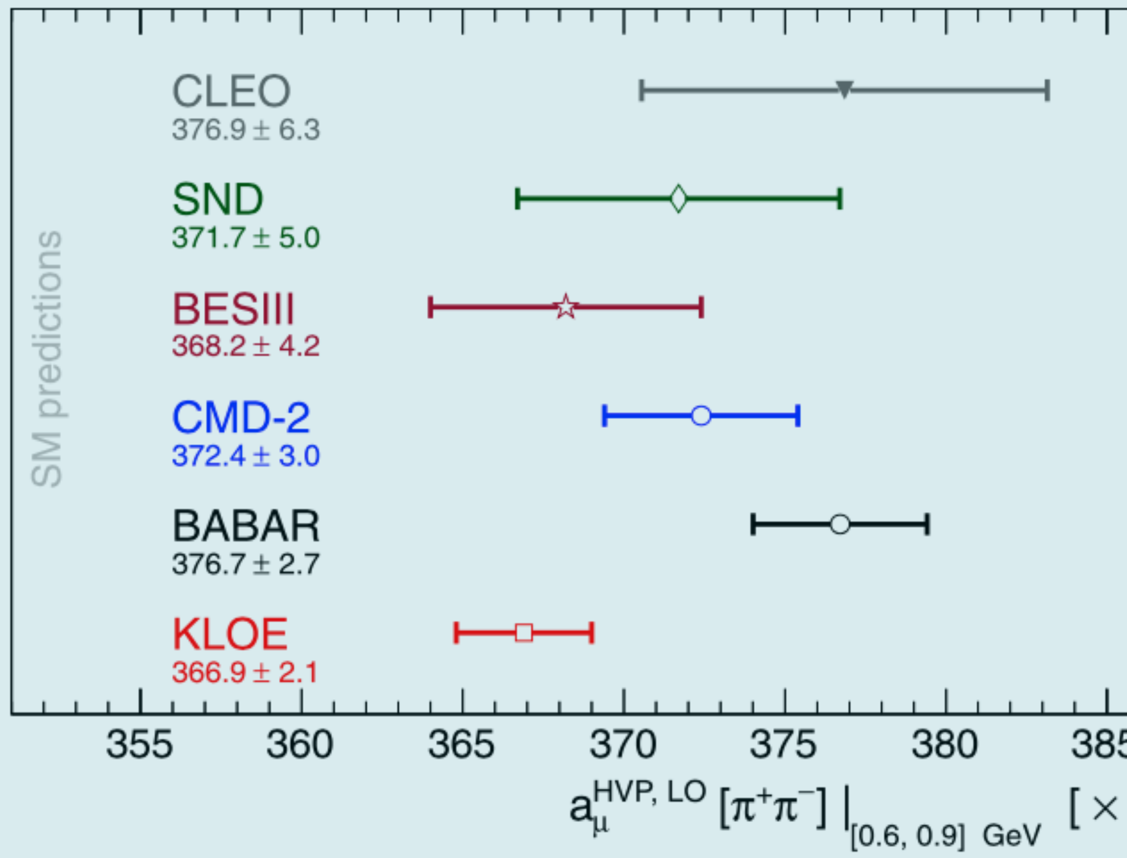
LO-HVP contribution



experimental data for R



[Jegerlehner 2017]



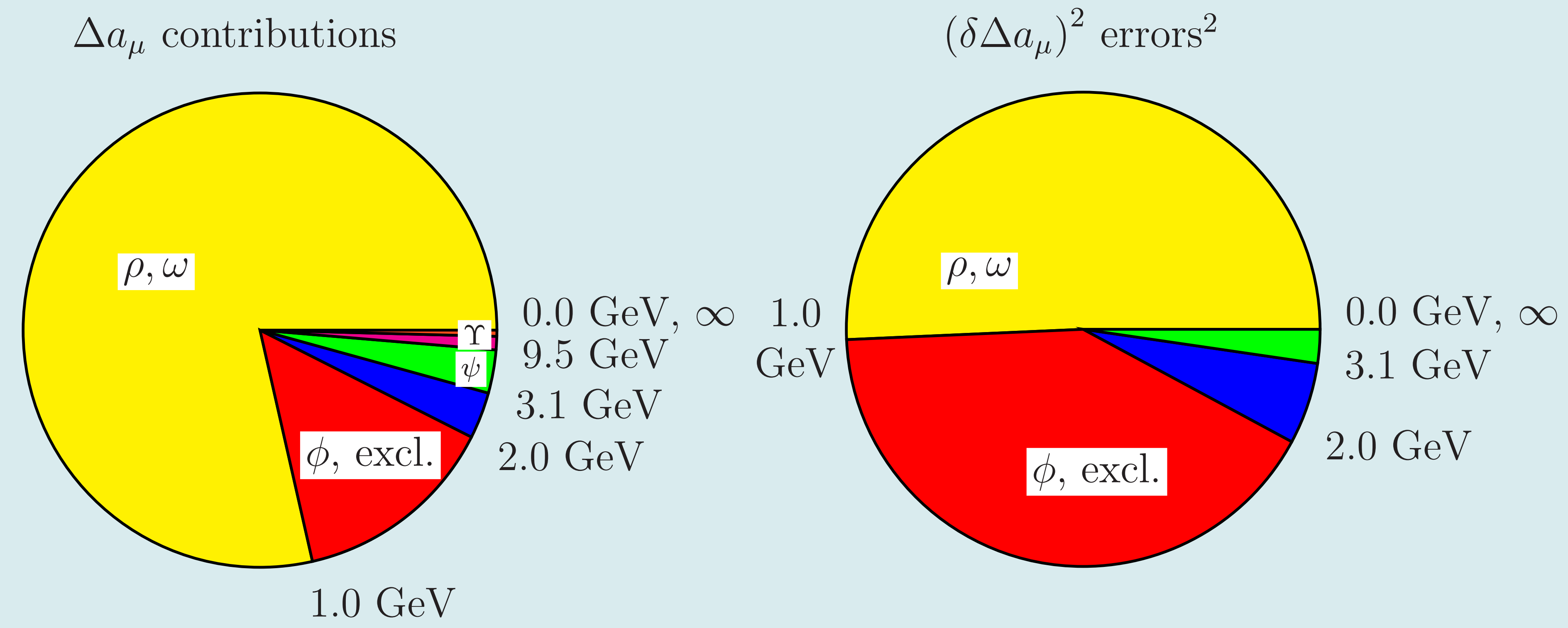
[Theory Initiative 20]

- $e^+e^- \to$ hadrons: BaBar, BESIII, CMD2/3, KLOE2, SND
- energy scan or radiative return
- loads of structure in *R*
- dominated by $\pi^+\pi^- \to \rho$ peak (~3/4, some tension in ISR data between KLOE and BABAR)
- different analysis techniques

$$a_\mu^{\mathrm{LO\,HVP}} = \left(\frac{a m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\mathrm{cut}}^2} ds \frac{\hat{K}(s)\, R_{\mathrm{had}}^{\mathrm{data}}(s)}{s^2} + \int_{E_{\mathrm{cut}}^2}^{\infty} ds \frac{\hat{K}(s)\, R_{\mathrm{had}}^{\mathrm{pQCD}}(s)}{s^2} \right)$$

$\sqrt{s} \lesssim 2\mathrm{GeV}$ exclusive channels

$\sqrt{s} > 2\mathrm{GeV}$ inclusive channels, PT



[Jegerlehner 2017]

LO-HVP contribution

$$a_\mu^{\mathrm{LO\,HVP}} = \left(\frac{a m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\mathrm{cut}}^2} ds \frac{\hat{K}(s)\, R_{\mathrm{had}}^{\mathrm{data}}(s)}{s^2} + \int_{E_{\mathrm{cut}}^2}^{\infty} ds \frac{\hat{K}(s)\, R_{\mathrm{had}}^{\mathrm{pQCD}}(s)}{s^2} \right)$$

$688.1(4.1) \cdot 10^{-10}$	[FJ 17]
$687.1(3.0) \cdot 10^{-10}$	[BDJ 19]
$694.0(4.0) \cdot 10^{-10}$	[DHMZ 19]
$692.8(2.4) \cdot 10^{-10}$	[KNT 19]
$693.1(4.0) \cdot 10^{-10}$	[TI 20]

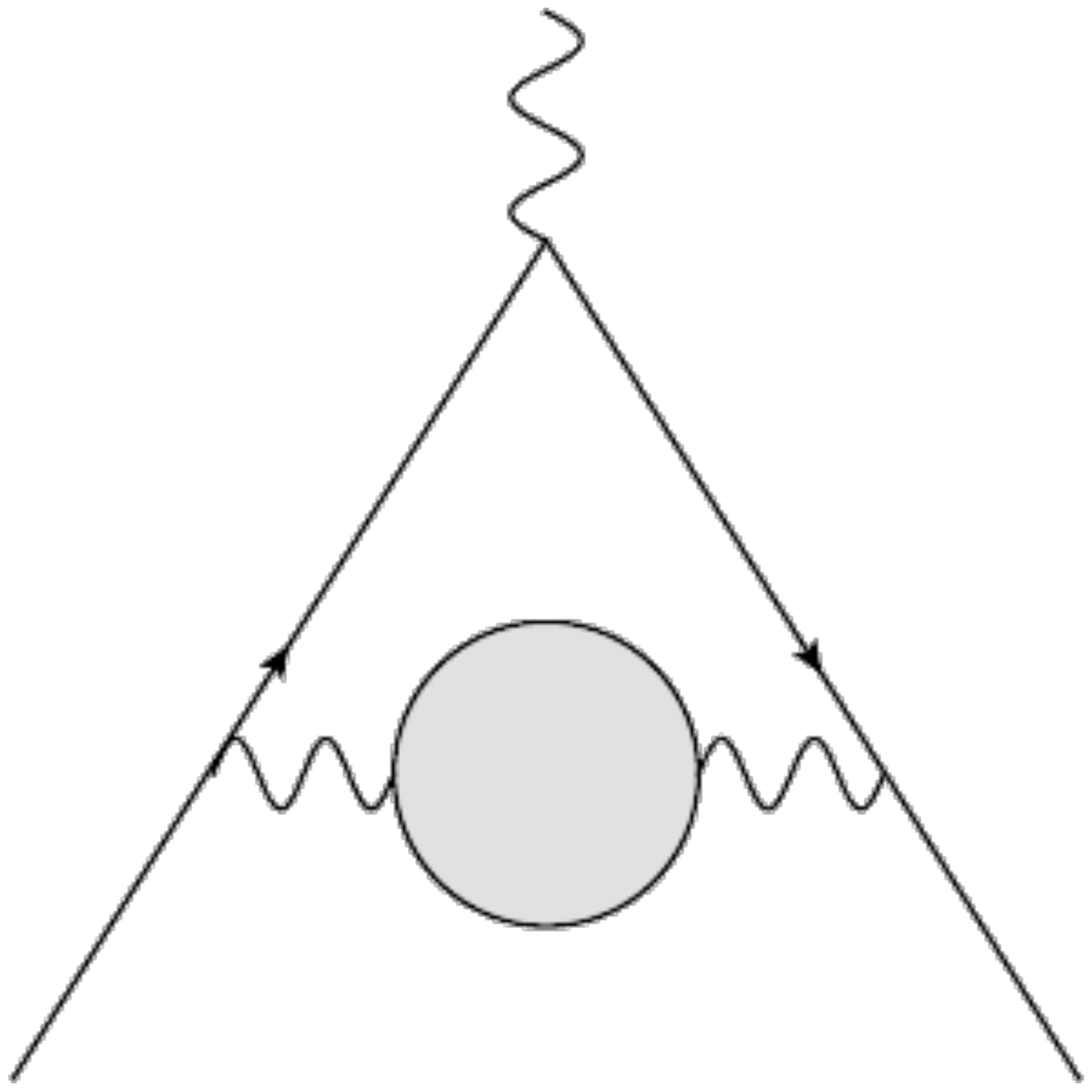
Mixture of

- fit to data/discrete integration
- MC or correlated fits for data combination
- help from EFT

(~0.6%) ← TI result is 'merged' from DHMZ 19 and KNT 19 plus analyticity and unitarity for 2π and 3π

- different data analysis leads to different systematics / error estimates
- tensions in individual channels cancel in the sum
- $e^+e^- \to$ hadrons current precision on LO HVP <1%
- Future prospects: MUonE, BABAR (new analysis), SND-3, CMD-3, BESIII, Belle II

LO HVP in the SM

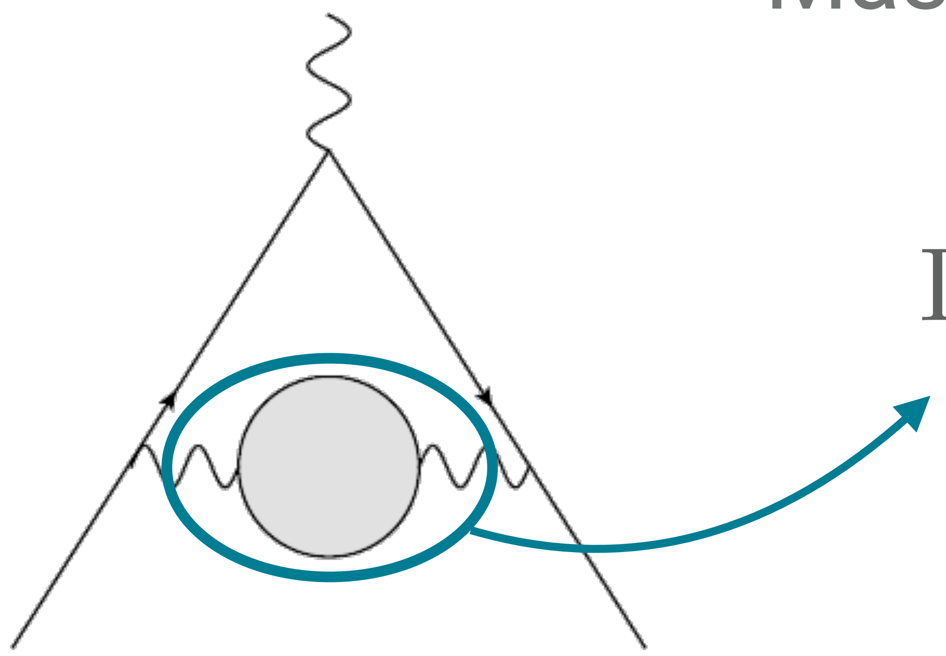


lattice-QCD predictions

LO HVP on the lattice

$$j_\mu^{\text{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c - \frac{1}{3}\bar{b}\gamma_\mu b$$

Master equation:



$$\Pi_{\mu\nu}(Q) = a^4 \sum_x e^{iQ\cdot x} \langle j_\mu^{\text{em}}(x) j_\nu^{\text{em}}(0)\rangle = (\hat{Q}_\mu \hat{Q}_\nu - \delta_{\mu\nu}\hat{Q}^2)\Pi(Q^2) + \ldots$$

$$a_\mu^{\text{LO HVP}} = 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 K_E(Q^2) \left[\Pi(Q^2) - \Pi(0)\right]$$

[Blum 2003]

Naively computing $\Pi_{\mu\nu}(Q^2)$ is a text book exercise in lattice QCD — but doing it correctly and at the **sub-%-level precision for a_μ is very hard**

Lattice QCD

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

Free parameters:

- gauge coupling $g \to \alpha_s = g^2/4\pi$
- quark masses $m_f = u,d,s,c,b,t$

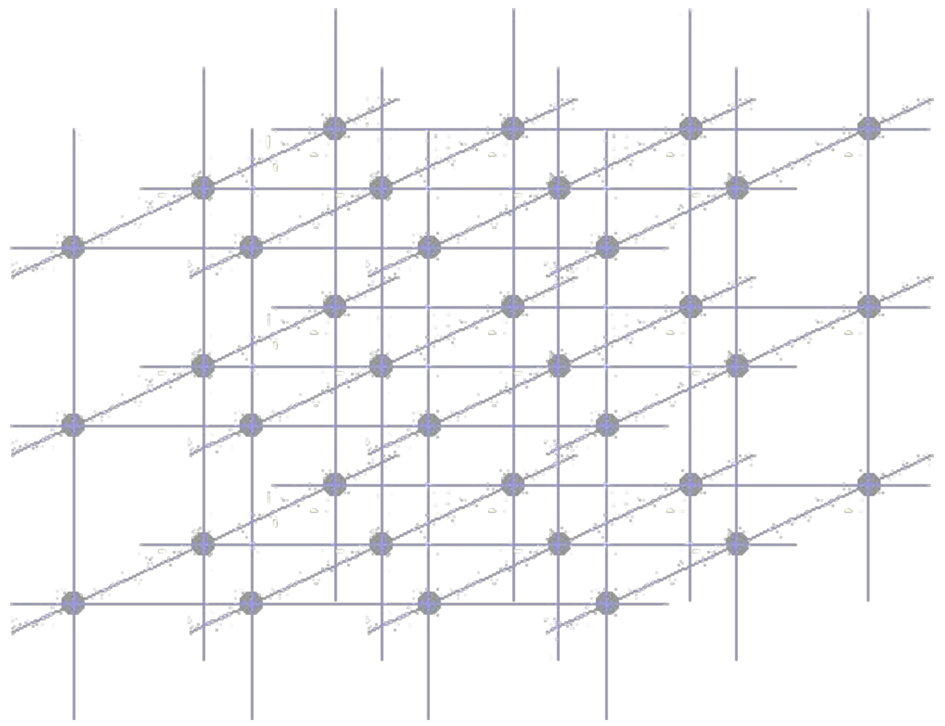
- Lagrangian of massless gluons and *almost massless quarks*
- What experiment sees are bound states, e.g. $m_\pi, m_P \gg m_{u,d}$
- Underlying physics non-perturbative

Path integral quantisation:

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-iS_{\mathrm{lat}}[U,\psi,\bar{\psi}]}$$

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-S_{\mathrm{lat}}[U,\psi,\bar{\psi}]}$$

Euclidean space-time Boltzmann factor



finite volume, space-time grid (IR and UV regulators)

$\propto L^{-1}$ $\propto a^{-1}$

→ Well defined, finite dimensional Euclidean path integral

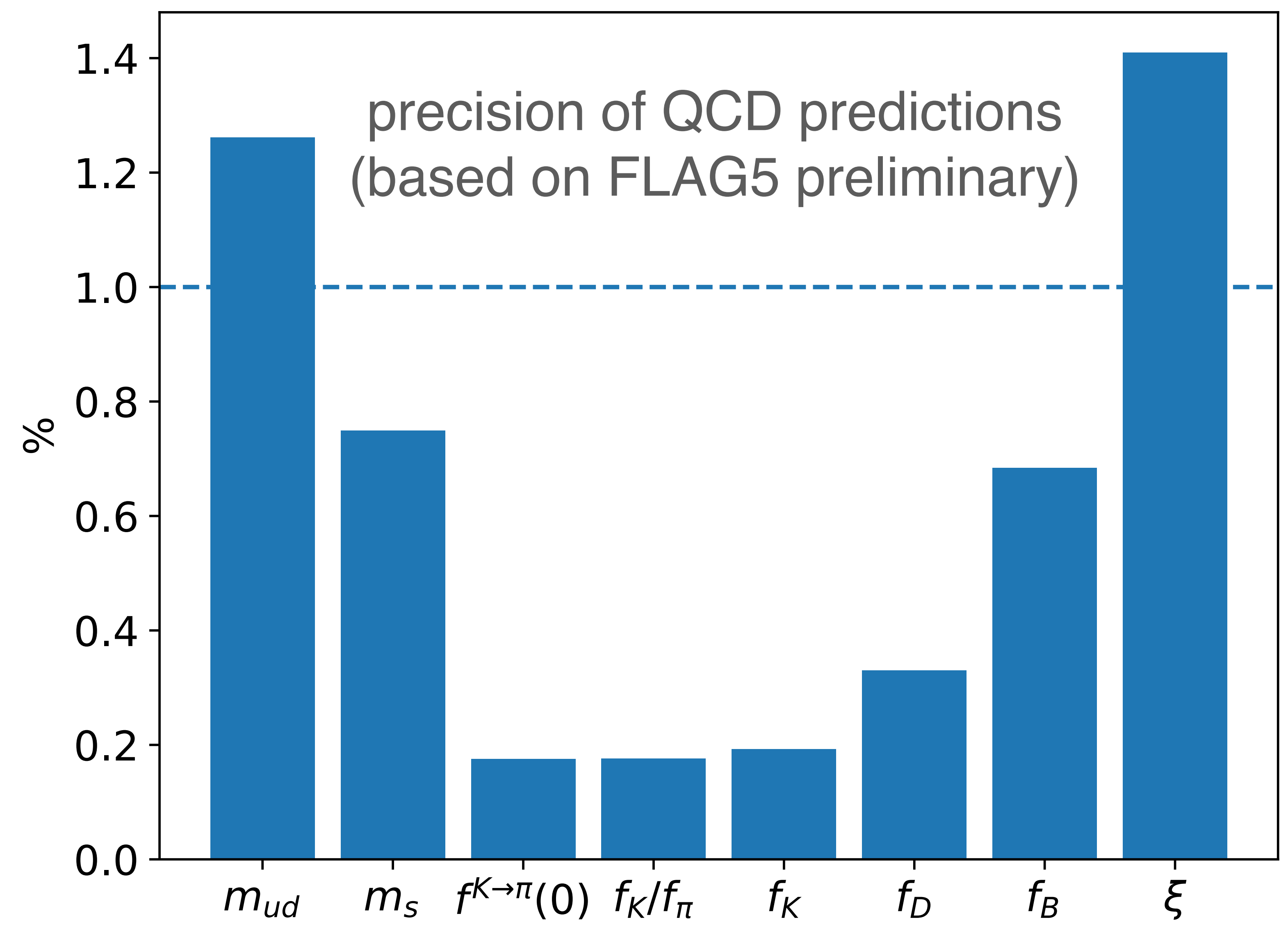
→ From first principles, solve via MCMC

Advertisement:
Flavour Lattice Averaging Group

"What's currently the best lattice value for a particular quantity?"

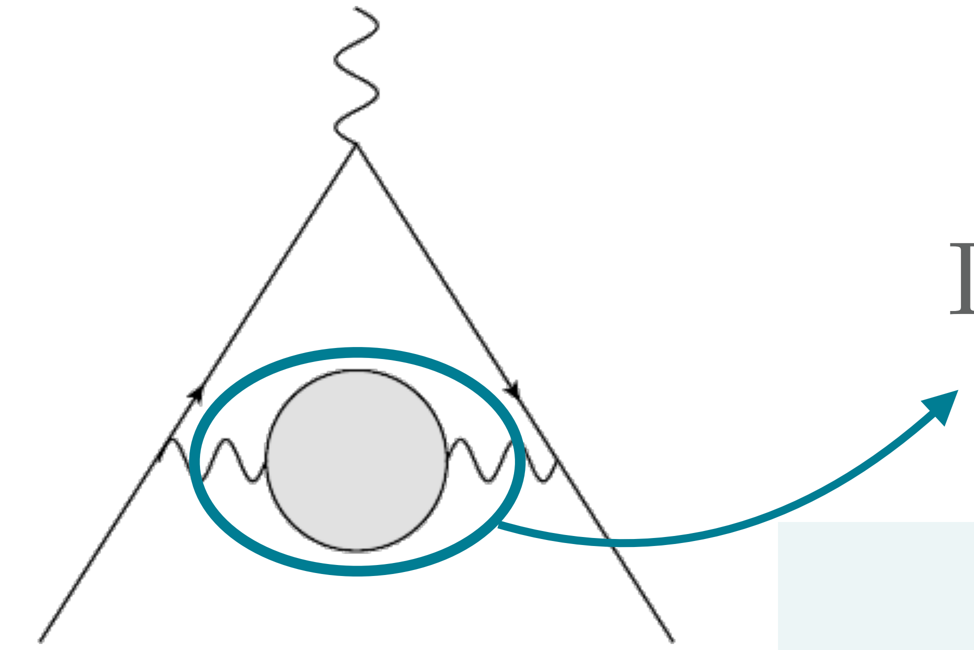
FLAG-1 (Eur. Phys. J. C71 (2011) 1695)
FLAG-2 (Eur. Phys.J. C74 (2014) 2890)
FLAG-3 (Eur.Phys.J. C77 (2017) 2, 112)
FLAG-4 (Eur.Phys.J.C 80 (2020) 2, 113)
FLAG-5 (arXiv:2111.09849 , http://itpwiki.unibe.ch/flag/)

quark masses
low-energy constants
bottom hadron decay constants
bottom hadron form factors
bottom hadron mixing
nucleon matrix elements
strong coupling constant
lattice scale setting
kaon mixing
hadronic kaon decay
charm hadron decay constants
charm hadron form factors
pion-pion scattering
pion-kaon scattering
kaon-kaon scattering



- summary of results
  - evaluation according to FLAG quality criteria (colour coding)
  - averages of best values where possible
  - detailed summary of properties of individual simulations

LO HVP on the lattice



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$$\Pi_{\mu\nu}(Q) = a^4 \sum_x e^{iQ\cdot x} \langle j_\mu^{\mathrm{em}}(x) j_\nu^{\mathrm{em}}(0)\rangle = (\hat{Q}_\mu \hat{Q}_\nu - \delta_{\mu\nu}\hat{Q}^2)\Pi(Q^2) + \ldots$$

$$a_\mu^{\mathrm{LO\ HVP}} = 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 K_E(Q^2)\left[\Pi(Q^2) - \Pi(0)\right]$$

[Blum 2003]

$$a_\mu^{\mathrm{HVP,LO}}(\alpha^2) = a_{\mu,\mathrm{conn}}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

QCD

$$a_{\mu,\mathrm{conn}}^{\mathrm{HVP,LO}} = a_{\mu,\mathrm{conn}}^{\mathrm{HVP,LO}}(ud) + a_{\mu,\mathrm{conn}}^{\mathrm{HVP,LO}}(s) + a_{\mu,\mathrm{conn}}^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{conn}}^{\mathrm{HVP,LO}}(b)$$

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(\alpha^2) + \delta a_\mu^{\mathrm{HVP,LO}}$$

QCD+QED+SIB

Modular calculation:

- different flavours — different systematics
- each component has individual continuum and infinite-volume limit → can be computed individually
- by far **dominant contribution: ud connected**

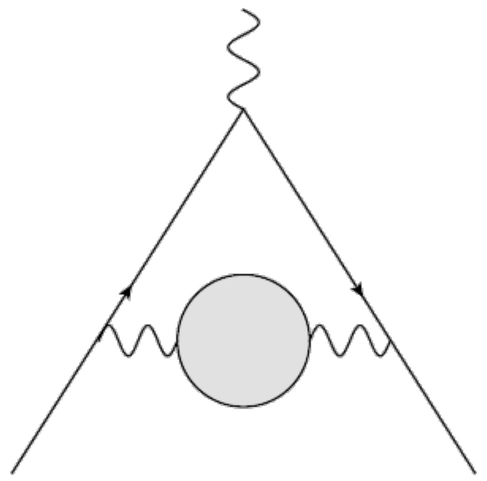
HVP on the Lattice: Challenges

- noise reduction
- scale determination
- QED and strong IB effects
- infinite-volume & continuum extrapolation

Signal-to-noise

Statistical noise from Markov-Chain Monte Carlo:

$$C(t) \equiv \Pi_{ii}(t,\vec{0}) = \frac{1}{3}\sum_{\vec{x}}\langle j_i^{\text{em}}(\vec{x},t) j_i^{\text{em}}(0)\rangle$$



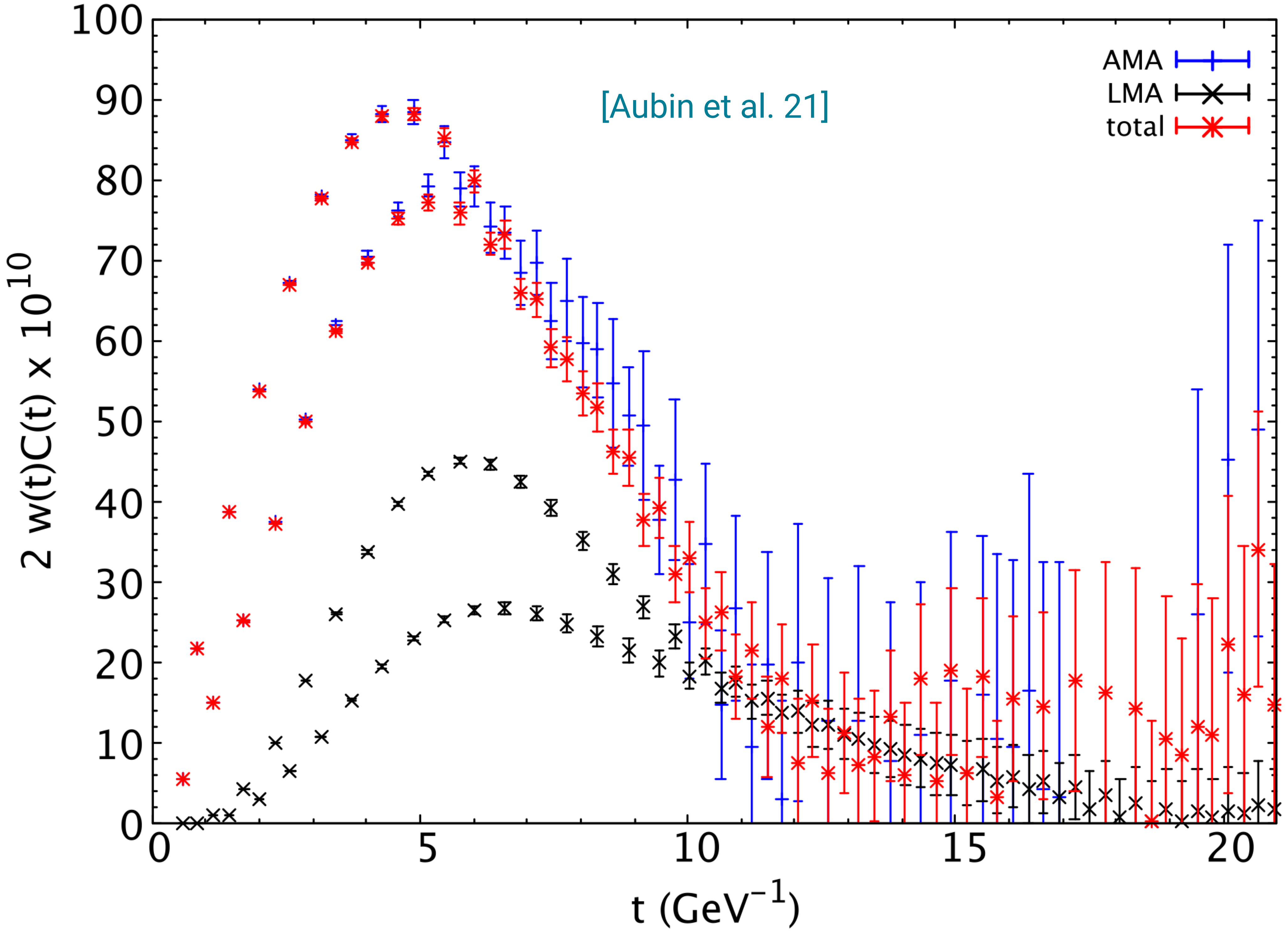
$$a_\mu^{\text{LO HVP}} = 4\pi^2\left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 K_E(Q^2)\left[\Pi(Q^2)-\Pi(0)\right]$$

$$= \sum_t w_t\, C(t)$$ [Bernecker, Meyer 11]

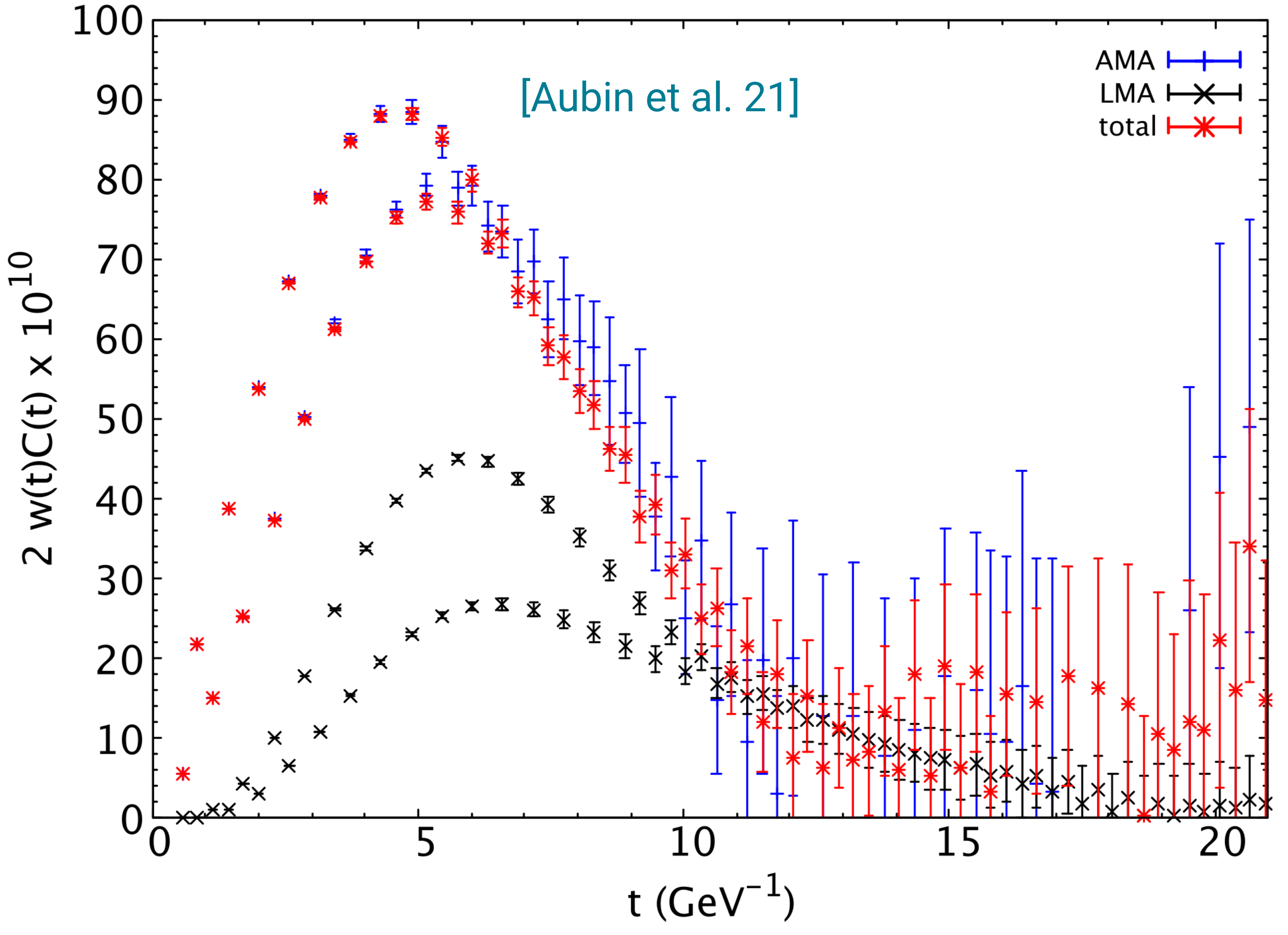
Signal-to-noise

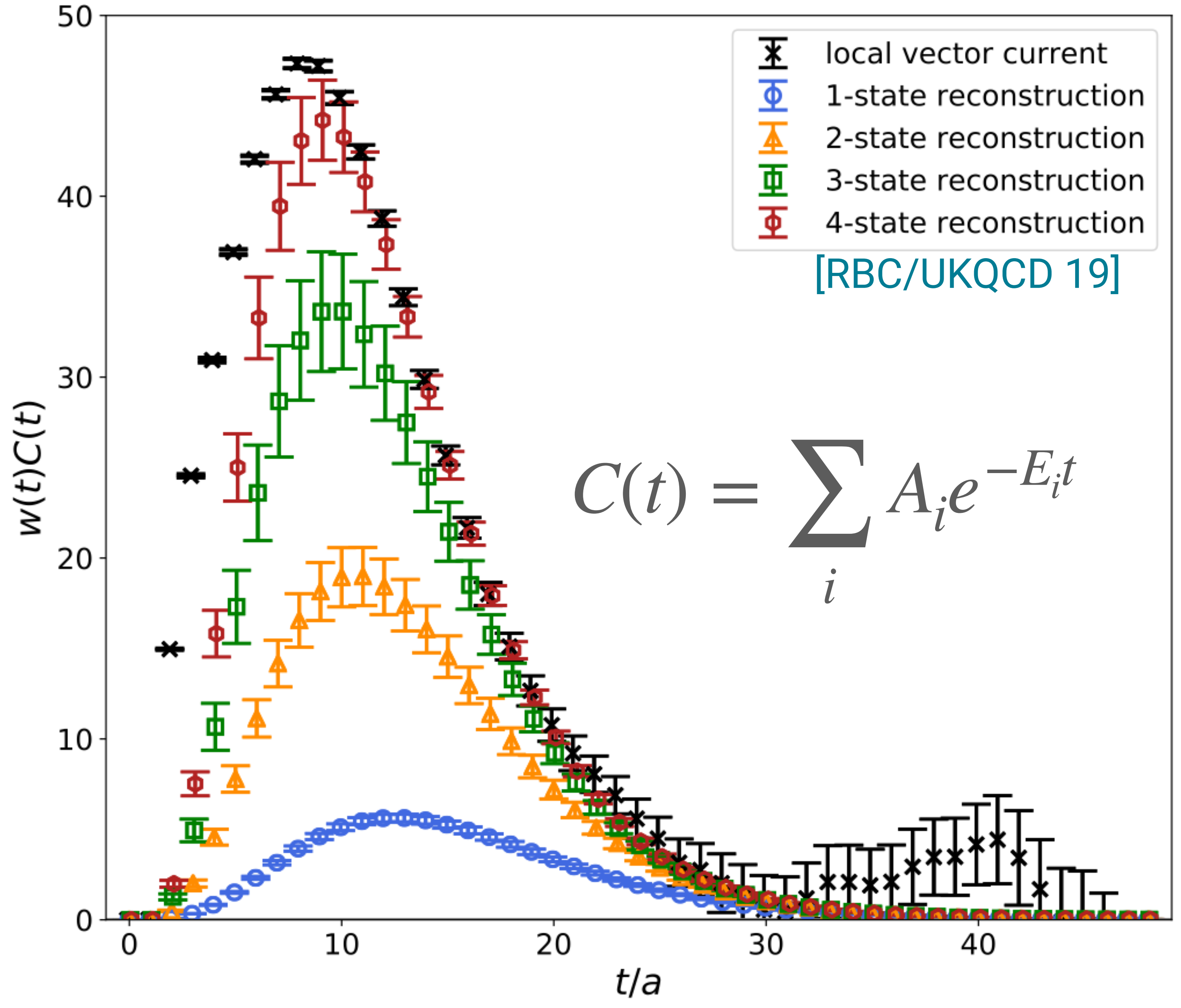
$$\frac{C^2(t)}{\sigma^2(t)} = \frac{C^2(t)}{\langle (j_i^{\text{em}}(t)\, j_i^{\text{em}}(0))^2\rangle - C(t)^2} \overset{\text{large } t}{\sim} e^{-2(m_\rho - m_\pi)t}$$

[Parisi 84, Lepage 89]



Signal-to-noise ratio





- Statistical error $a_\mu^{\mathrm{HVP,\,LO}}$ is mostly from light quarks at $t \gtrsim 1.5\ \mathrm{fm}$
- More statistics
- Noise reduction techniques (low-mode averaging, multi-level, all-to-all, …)
- Spectral reconstruction for $t > t_c$

$$C(t) = \sum_i A_i e^{-E_i t}$$

Scale determination

$a_\mu^{\mathrm{HVP,\,LO}}$ itself dimensionless but lattice scale enters through m_μ in kernel K_E

$$a_\mu^{\mathrm{LO\ HVP}} = 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 K_E(Q^2)\,\left[\Pi(Q^2) - \Pi(0)\right]$$

$$\frac{\delta a_\mu^{\mathrm{HVP,\,LO}}}{a_\mu^{\mathrm{HVP,\,LO}}} = \underbrace{\frac{1}{a_\mu^{\mathrm{HVP,\,LO}}}\left| a \frac{d a_\mu^{\mathrm{HVP,\,LO}}}{da} \right|}_{\approx 1.8} \frac{\delta a}{a}$$

[H. Wittig, TI
Della Morte et al, Lattice 2017]

→ **need lattice scale determination at per mil level**

Lattice scale determined in terms of some dimensionful experimental input: $a = \dfrac{a O_{\mathrm{lat}}}{O_{\mathrm{exp}}}$

e.g. $O = m_\Omega = 1672.45(29)$ ($\delta m_\Omega = 0.2‰$)

[FLAG 2021]

Quantity	Sec.	$N_f = 1+1+1+1$	Refs.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f > 2+1$	Refs.
$\sqrt{t_0}$[fm]	11.5.2			0.14186(88)	[37, 111, 112]	0.14464(87)	[8, 113, 114]		
w_0[fm]	11.5.2	0.17236(70)	[115]	0.17128(107)	[37, 111, 112]	0.17355(92)	[8, 114, 116]	0.17177(67)	[37, 111, 112, 115]
r_0[fm]	11.5.2			0.474(14)	[7]	0.4701(36)	[24, 116–119]		
r_1[fm]	11.5.2			0.3112(30)	[37]	0.3127(30)	[41, 117–120]		

FLAG's scale-setting summary uncertainty ~0.6%

QED & SIB

With a sub-percent precision goal
strong IB and QED effects can't be ignored

$$\frac{m_u - m_d}{\Lambda_{\text{QCD}}} \sim O(1\%) \qquad \alpha \approx \frac{1}{137} \sim O(1\%)$$

$$\langle O \rangle = \frac{1}{Z_0} \int \mathcal{D}[U]\mathcal{D}[A]\mathcal{D}[\Psi, \bar{\Psi}, U]\, O[\Psi, \bar{\Psi}, A, U]\, e^{-S_{F,0}[\Psi,\bar{\Psi},U]} e^{-S_\gamma[A]} e^{-S_G[U]}$$

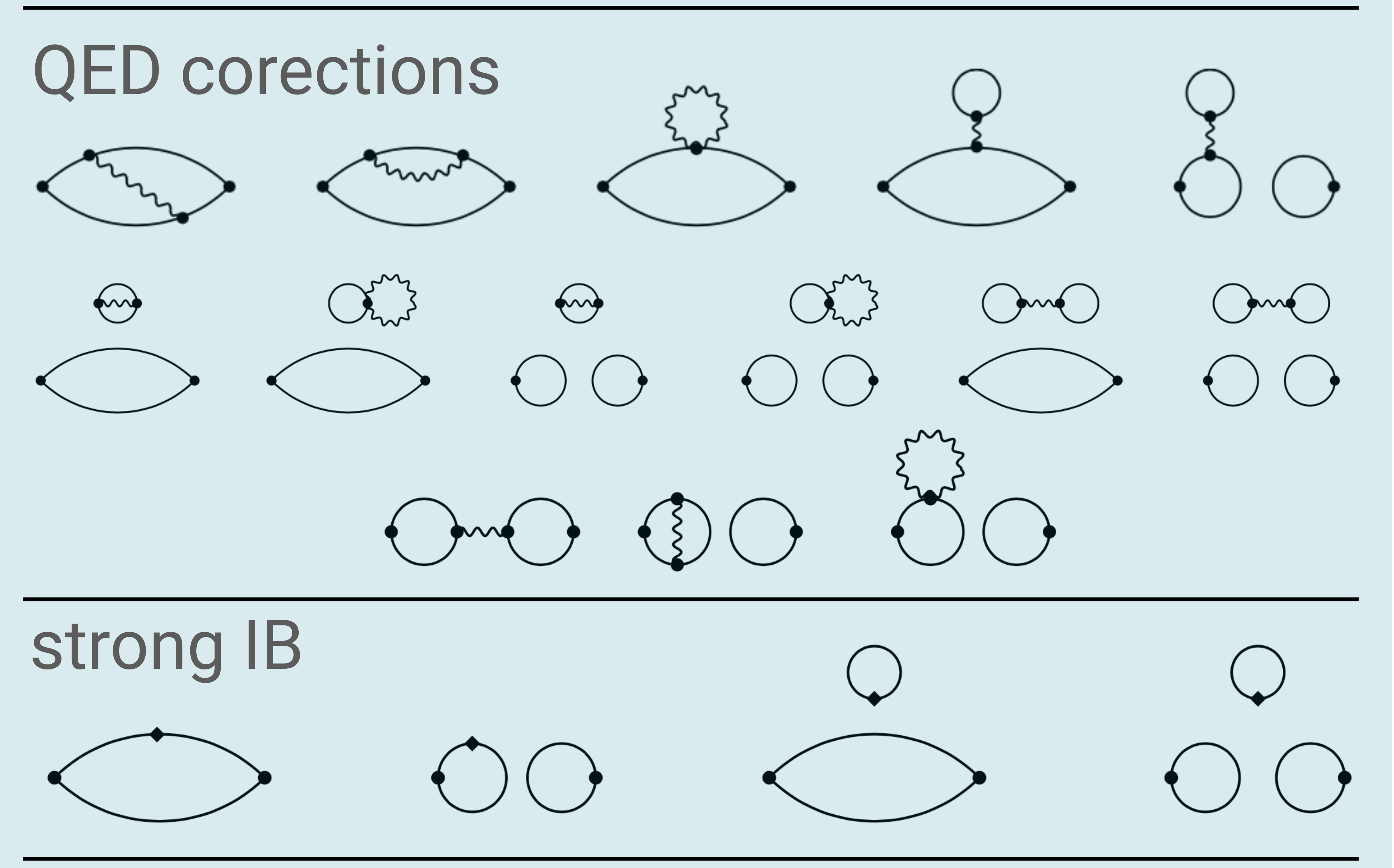
Perturbative expansions: [RM123 Collaboration (2013)]

QED $$\langle O \rangle = \langle O \rangle_0 + \frac{1}{2} e^2 \frac{\partial^2}{\partial e^2} \langle O \rangle \,|_{e=0} + \mathcal{O}(\alpha^2)$$

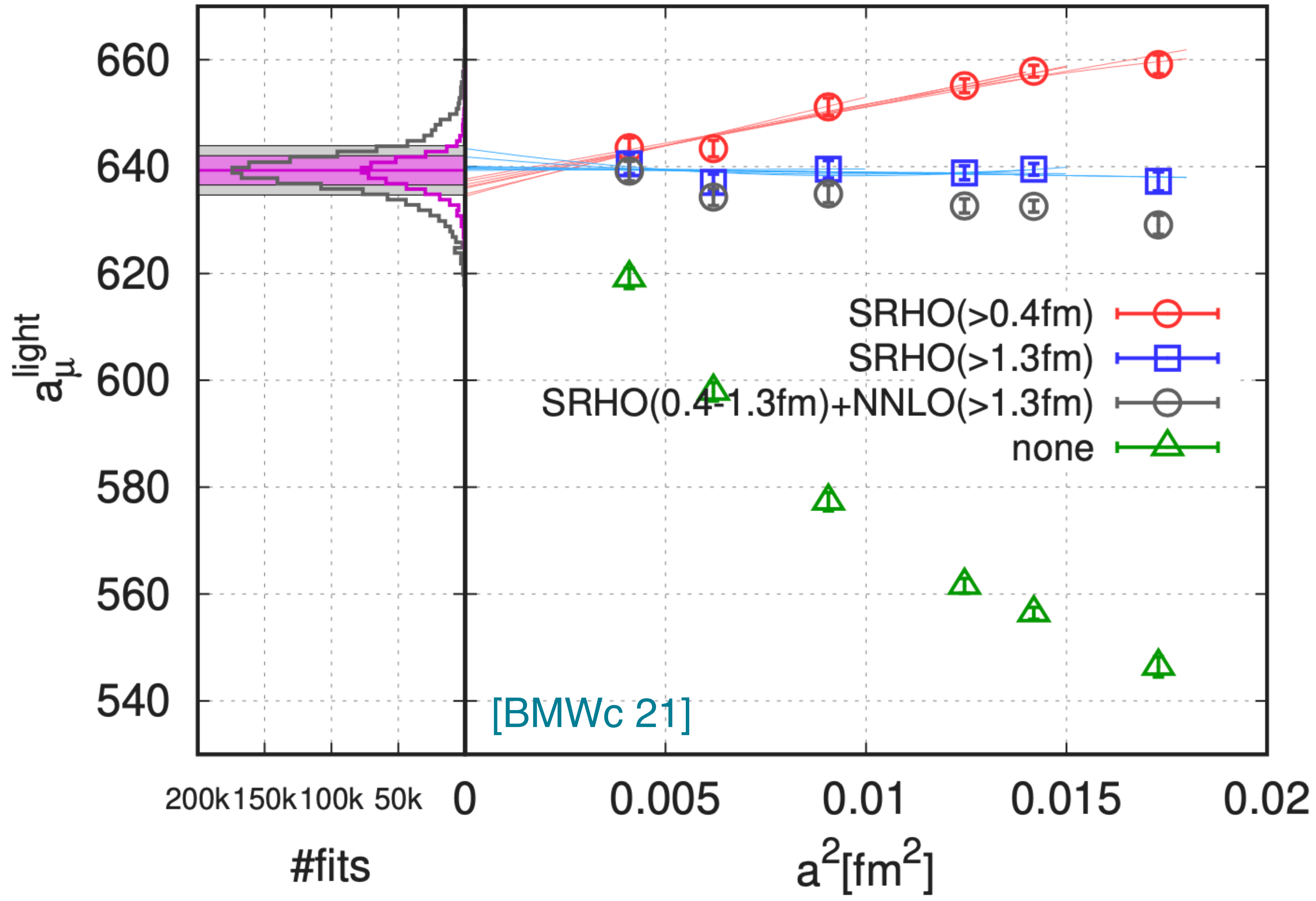
SIB $$\langle O \rangle_{m_f \neq \hat{m}_f} = \langle O \rangle_{m_f = \hat{m}_f} + \Delta m_f \frac{\partial}{\partial m_f} \langle O \rangle \,|_{m_f = \hat{m}} + O(\Delta m_f^2)$$

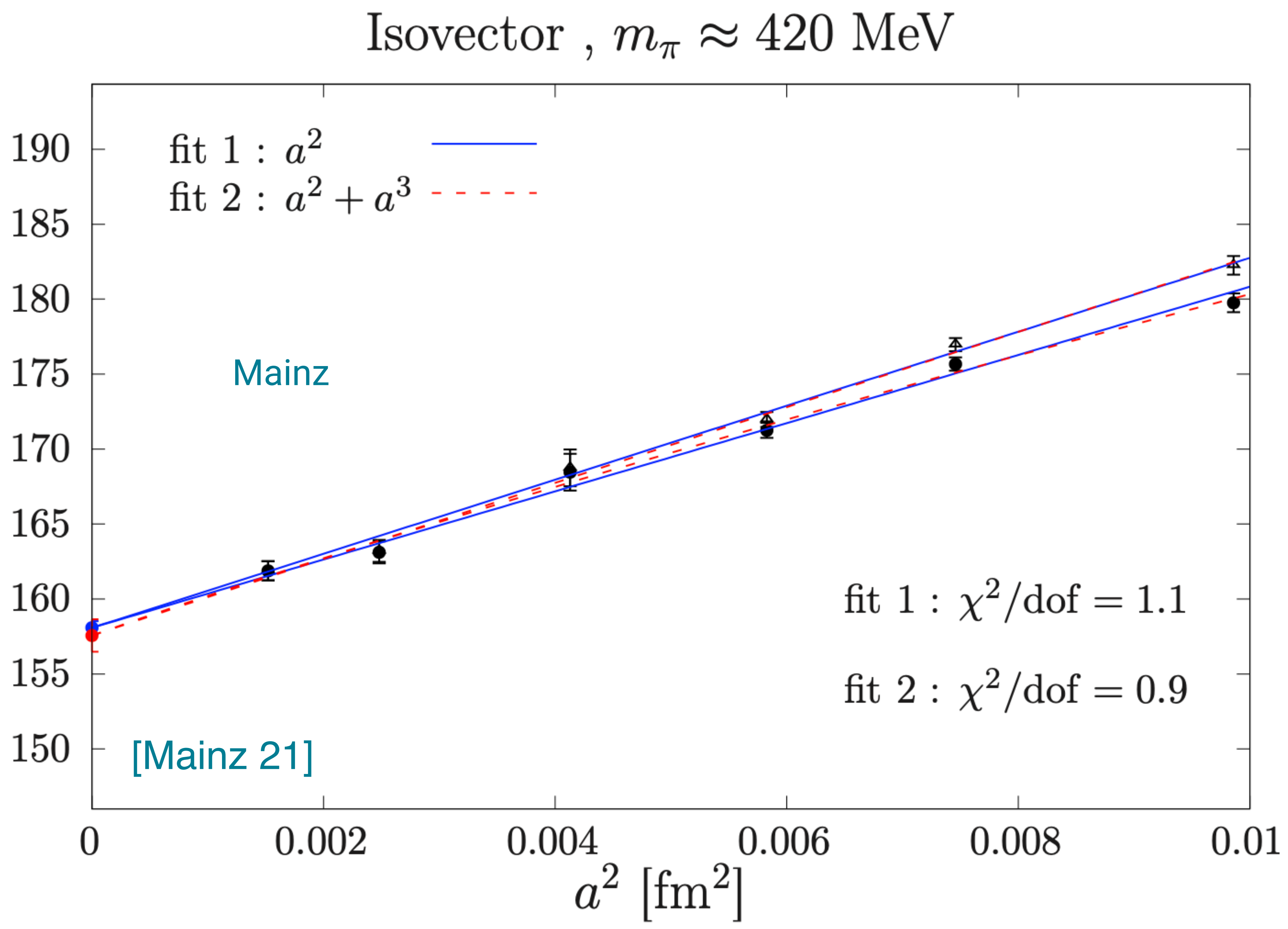
alternative: include QED in ensembles with SIB

QED & SIB $\sim 1\,\%$ $\rightarrow$ require less precision



Continuum extrapolation

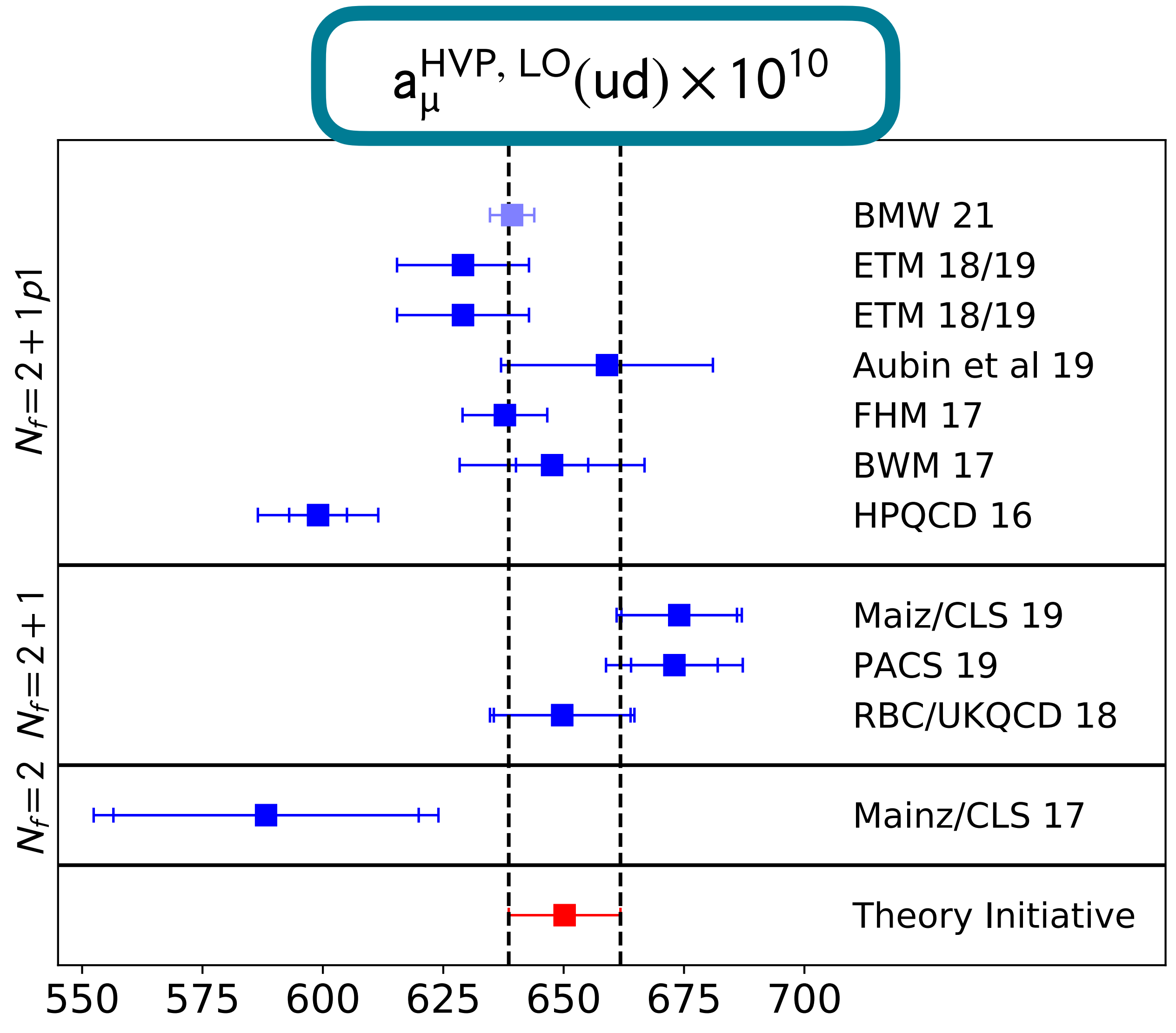


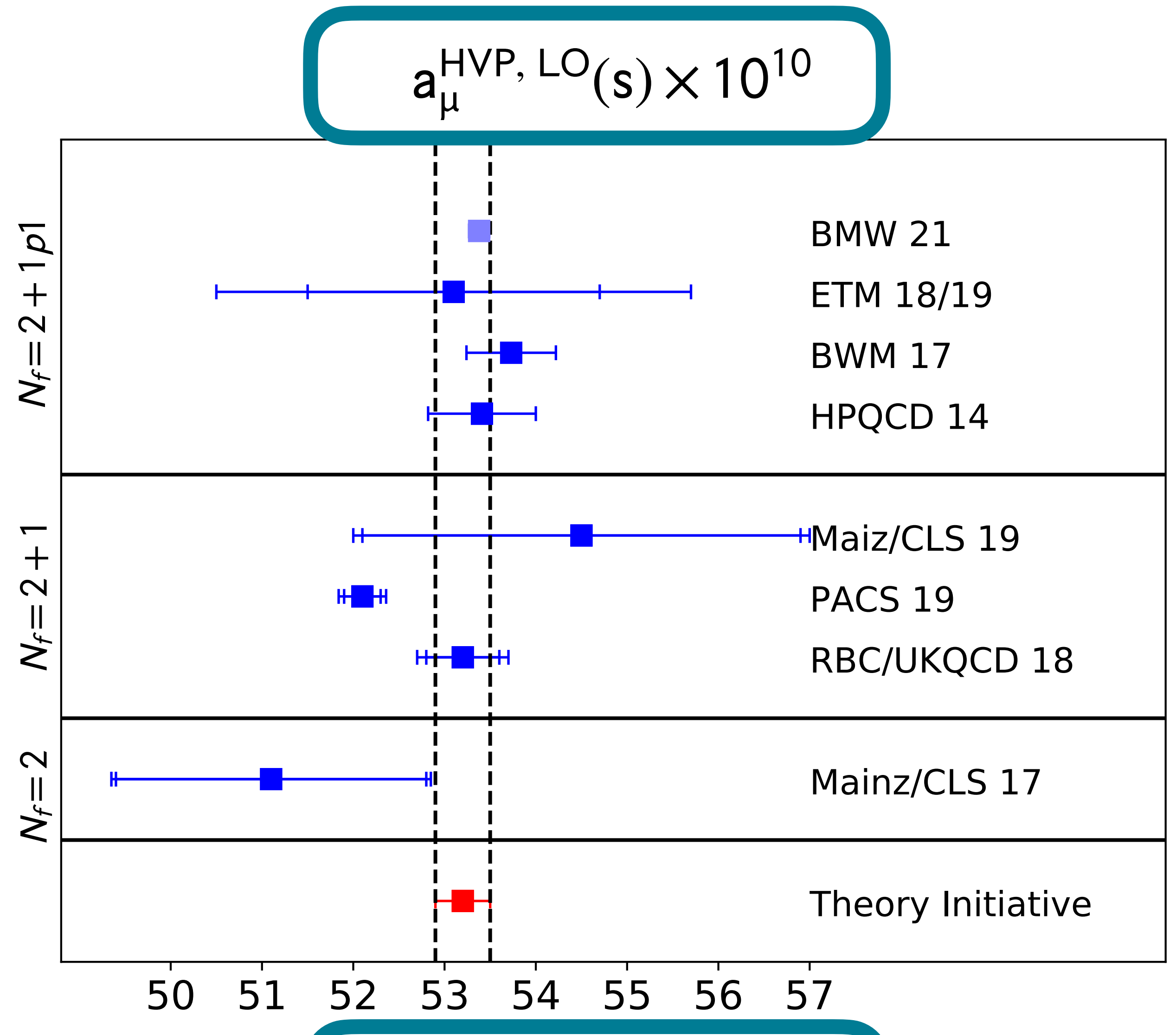


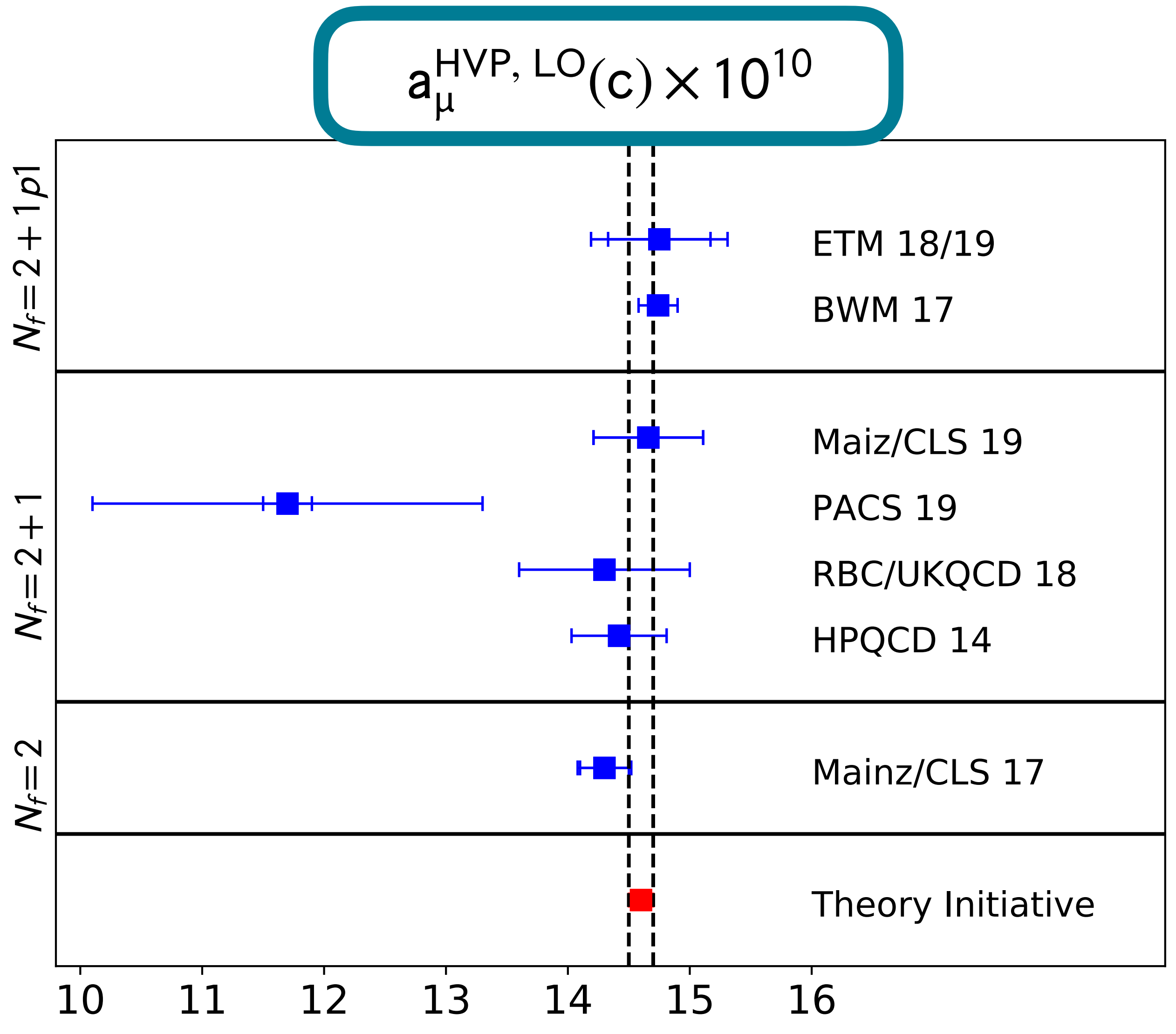
Size of correction towards continuum limit depends on discretisation, quark mass, …

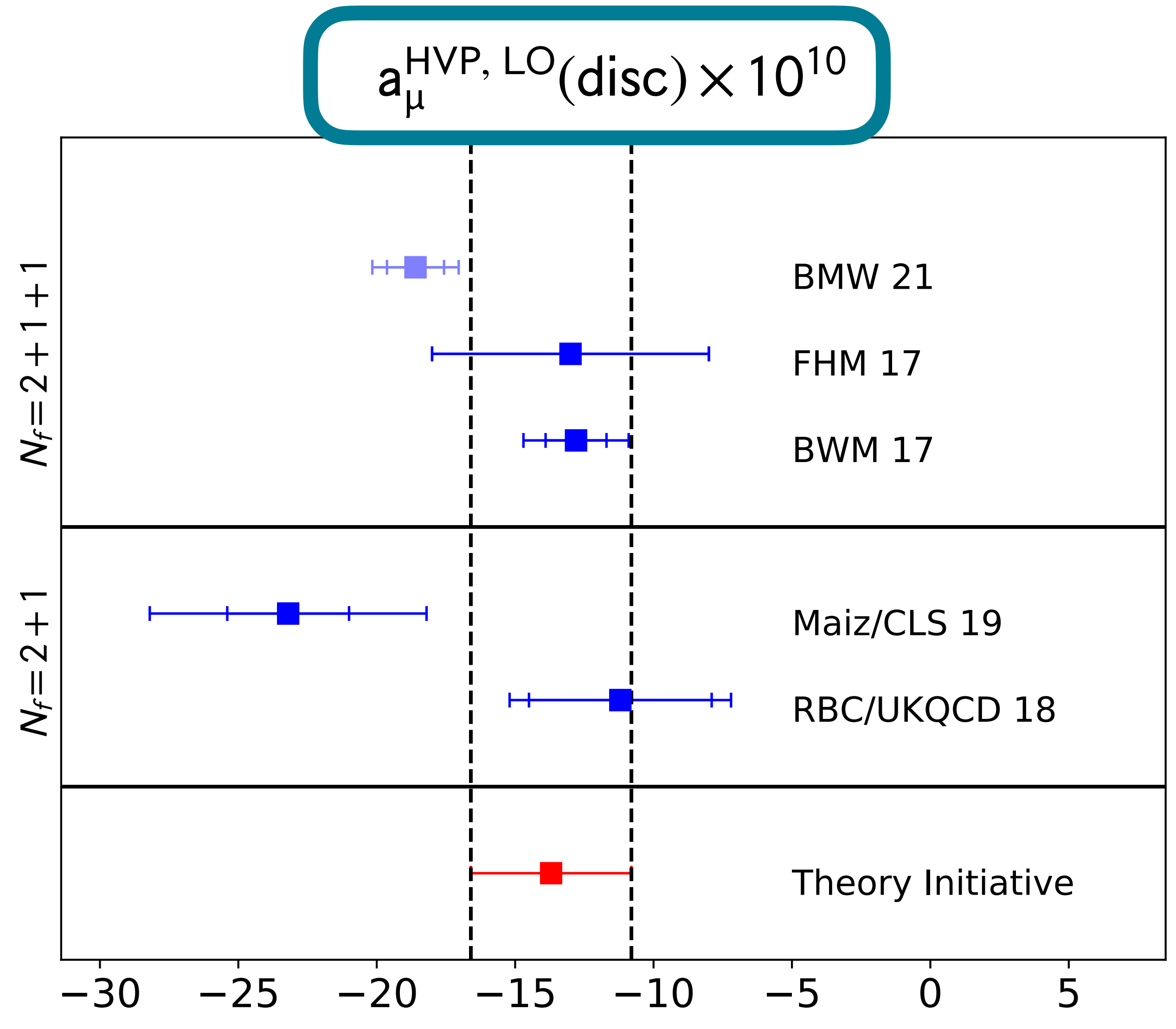
EFT used to *improve* continuum limit [BMWc 21]
EFT used to correct for finite volume effects or estimate from simulations of various volumes

Individual contributions

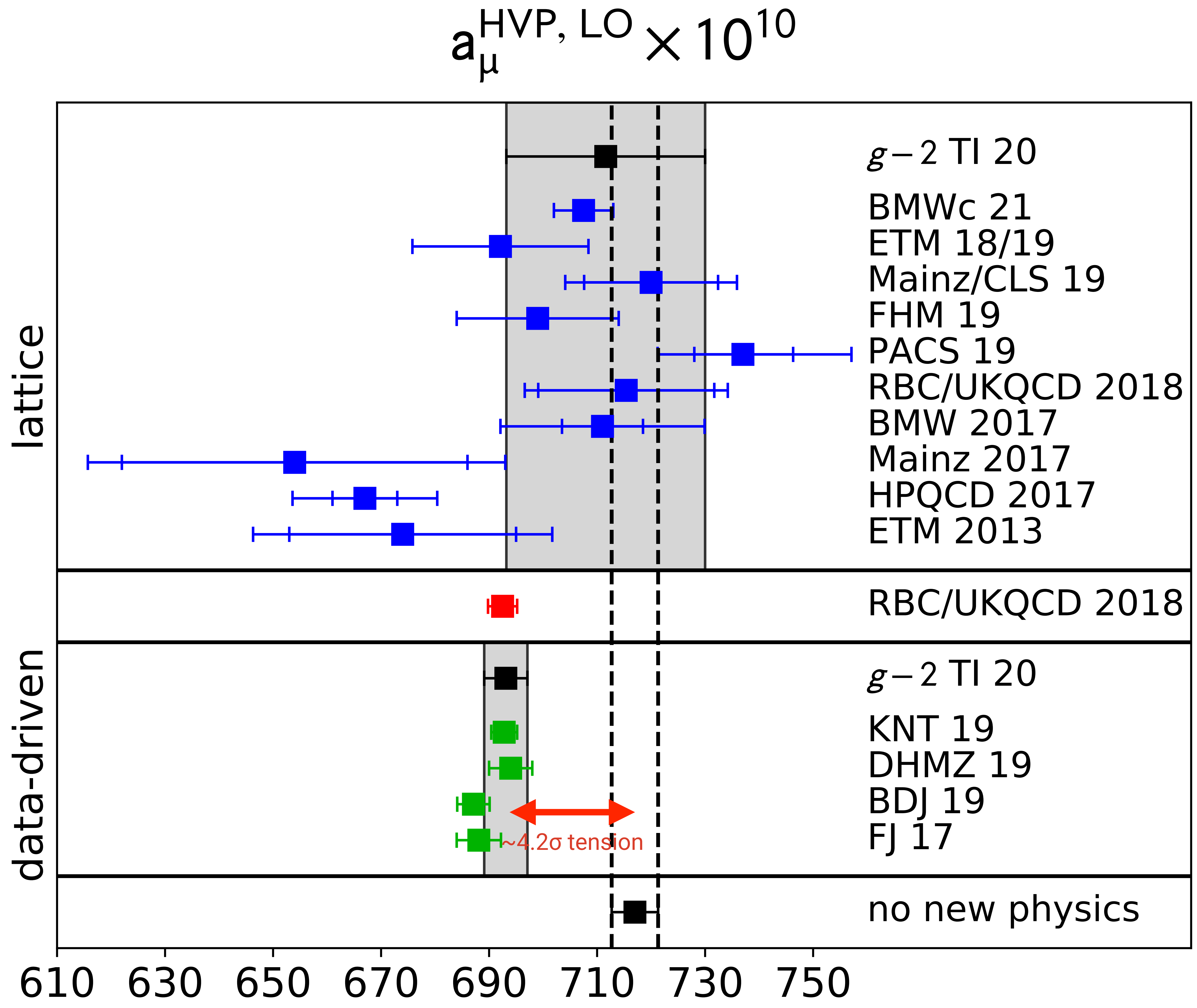








What next?

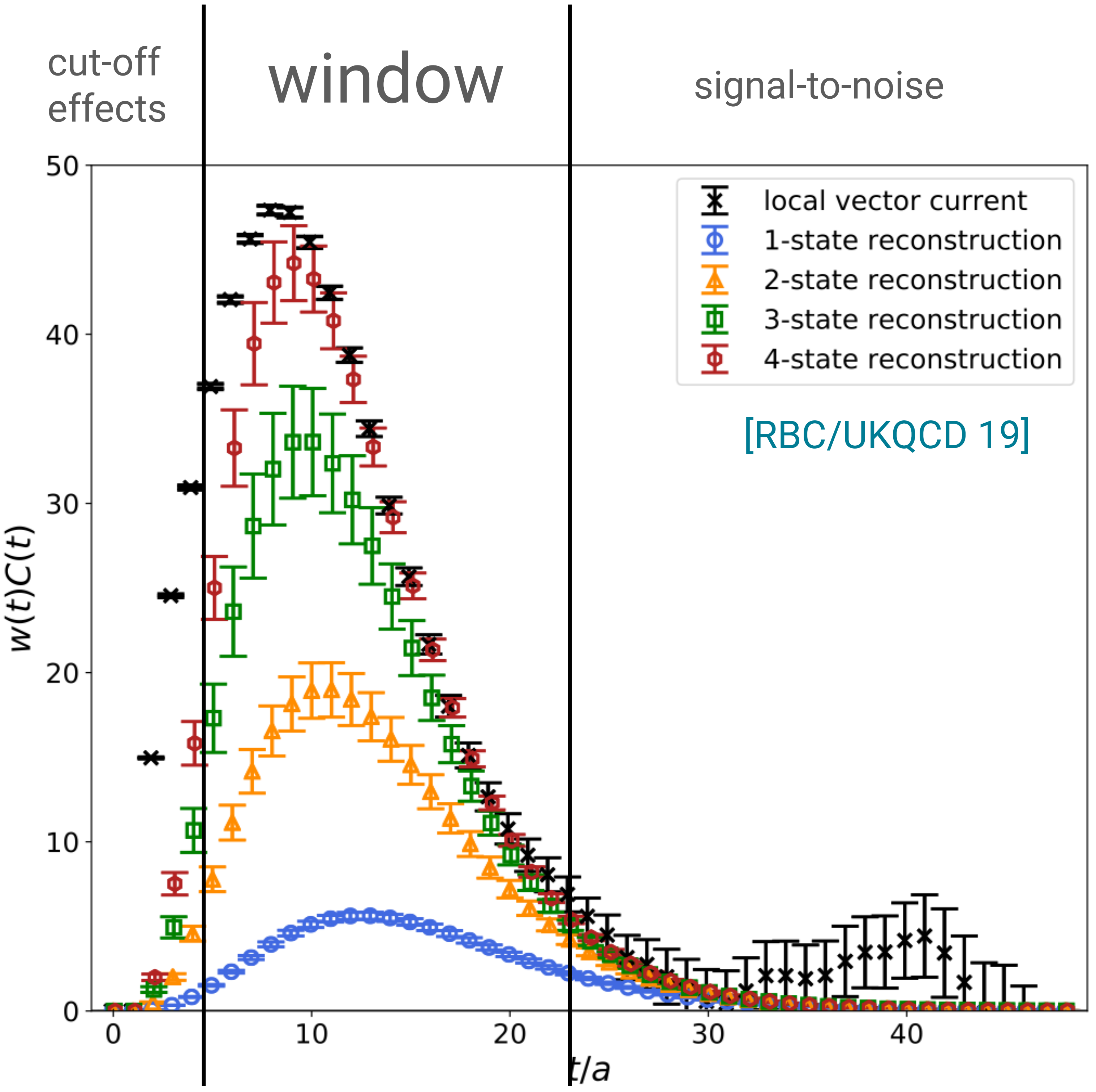


- Data-driven methods still slightly ahead of lattice QCD
- but tensions in data-driven analysis
- BMW result closer to ‘no-new-physics scenario

- while huge efforts under way to improve on lattice result for full $a_\mu^{\text{HVP, LO}}$ consider alternative observables to further scrutinise HVP results

Window method

there are regions in phase space where either lattice/experiment do better/worse
$\rightarrow$ choose range where discretisation effects, finite-volume effects and signal-to-noise issue are small



$\rightarrow$ allows for high-precision tests and comparisons

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{W} + a_\mu^{\mathrm{LD}}$$

$$a_\mu^{\mathrm{SD}} = \sum_t C(t)\, w_t\, [1 - \Theta(t, t_0, \Delta)]$$

$$a_\mu^{\mathrm{W}} = \sum_t C(t)\, w_t\, [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]$$

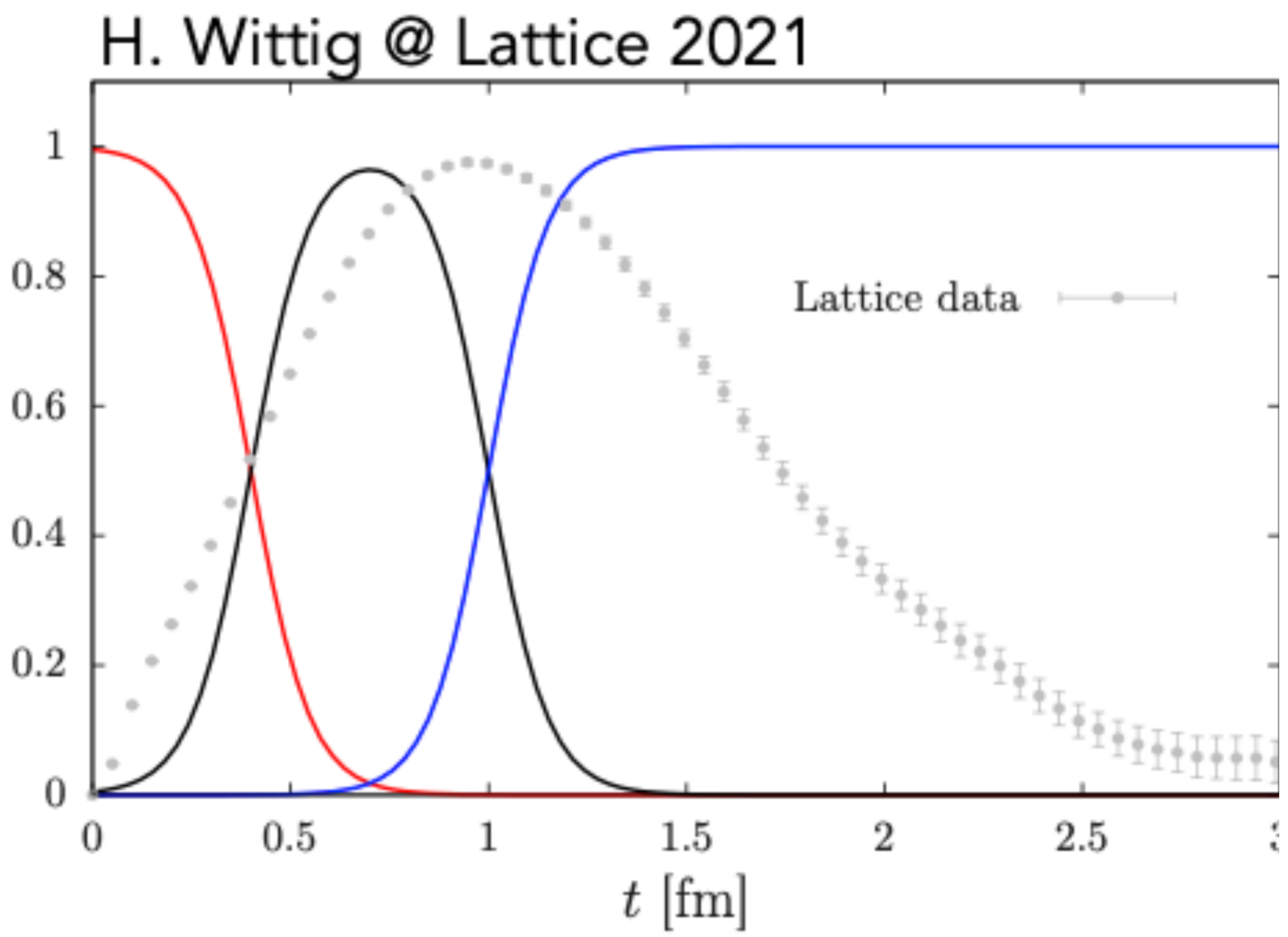
$$a_\mu^{\mathrm{LD}} = \sum_t C(t)\, w_t\, \Theta(t, t_1, \Delta)$$

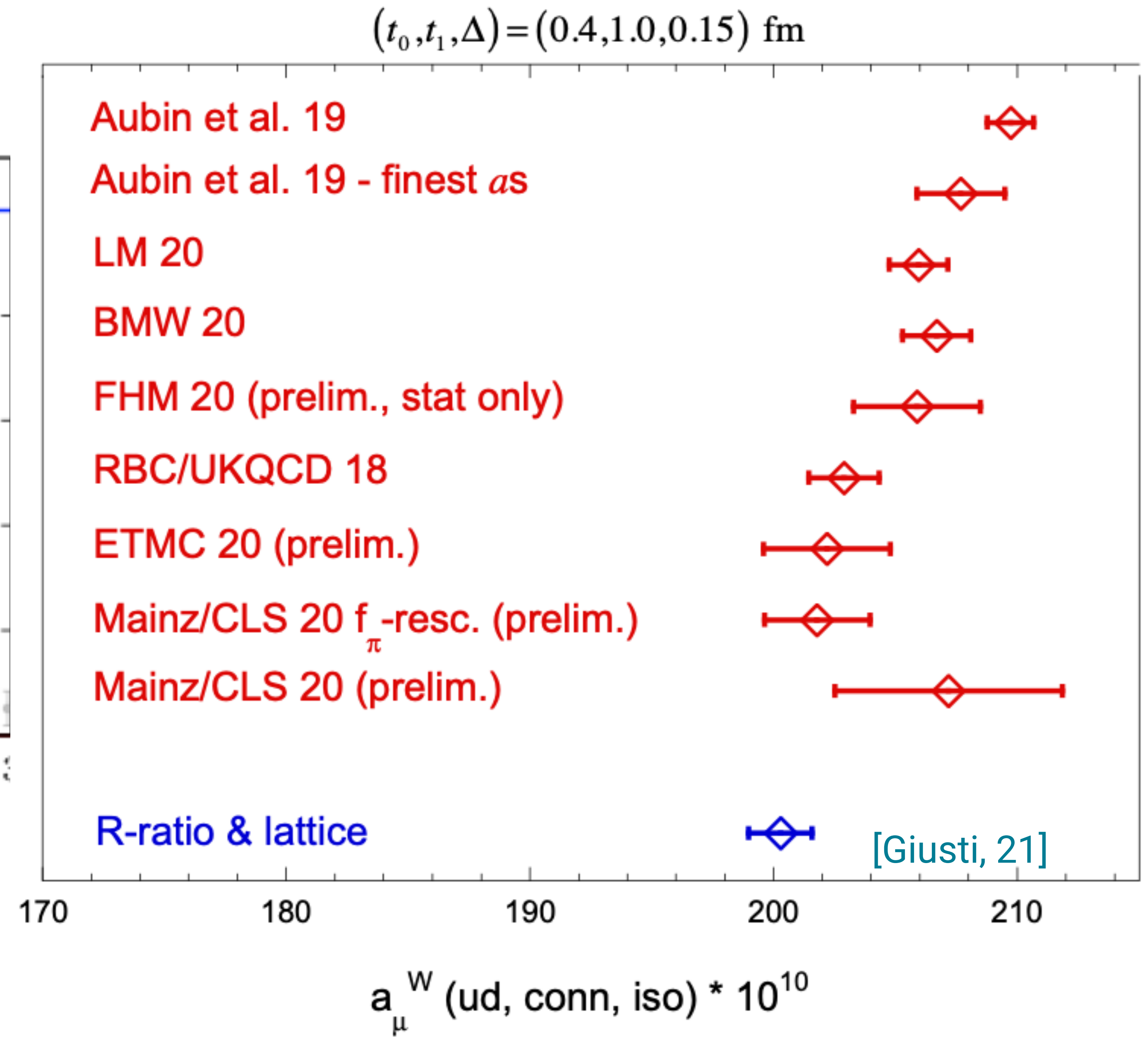
$$t_0 = 0.4\,\mathrm{fm}$$
$$t_1 = 1.0\,\mathrm{fm}$$

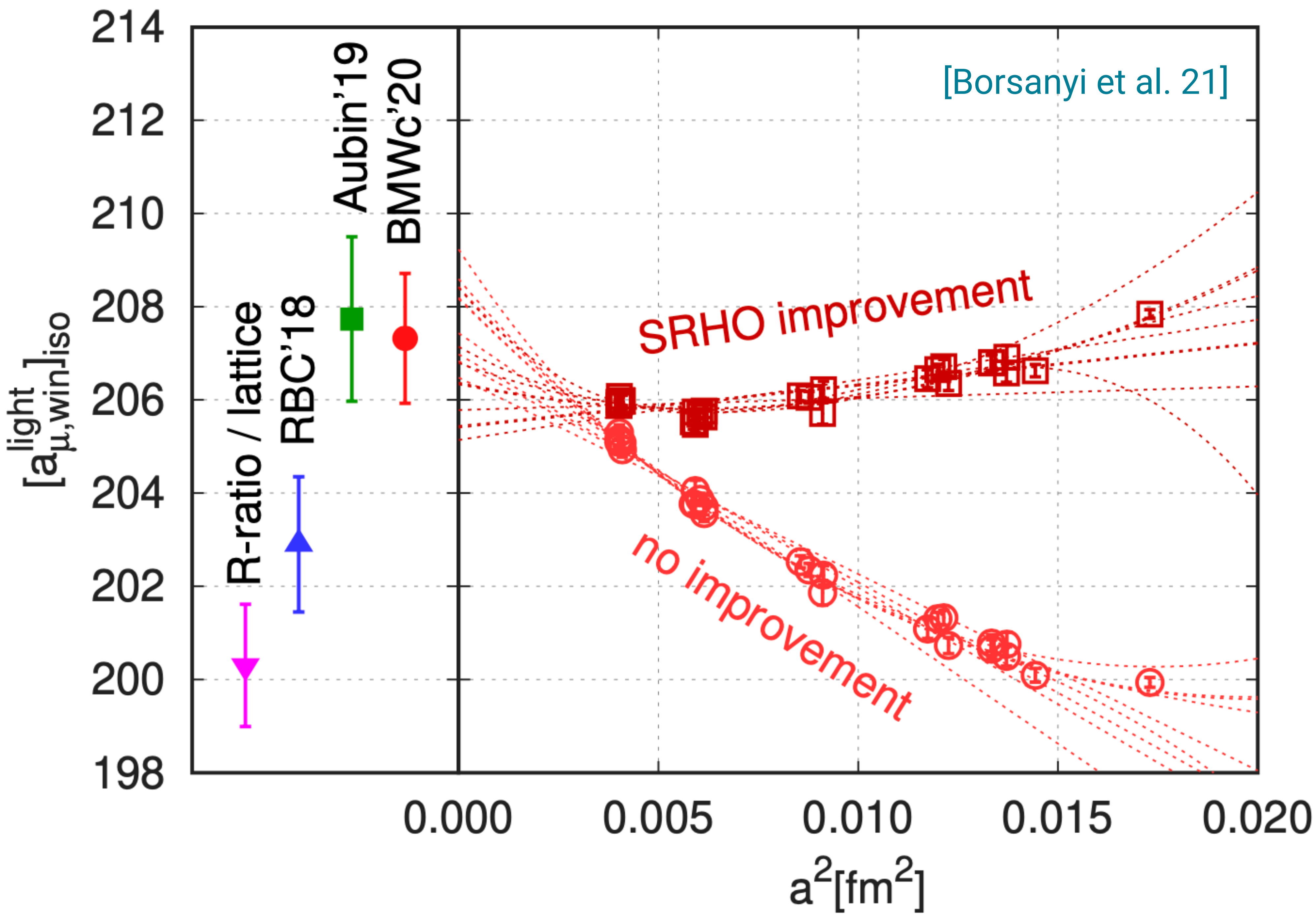
$$\Theta(t, t', \Delta) = (1 + \tanh[(t - t')/\Delta])/2$$

- apply window to $R(e^+e^-)$ as well as lattice data
- compare windows rather than results for $a_\mu^{\mathrm{HVP,LO}}$

Window method

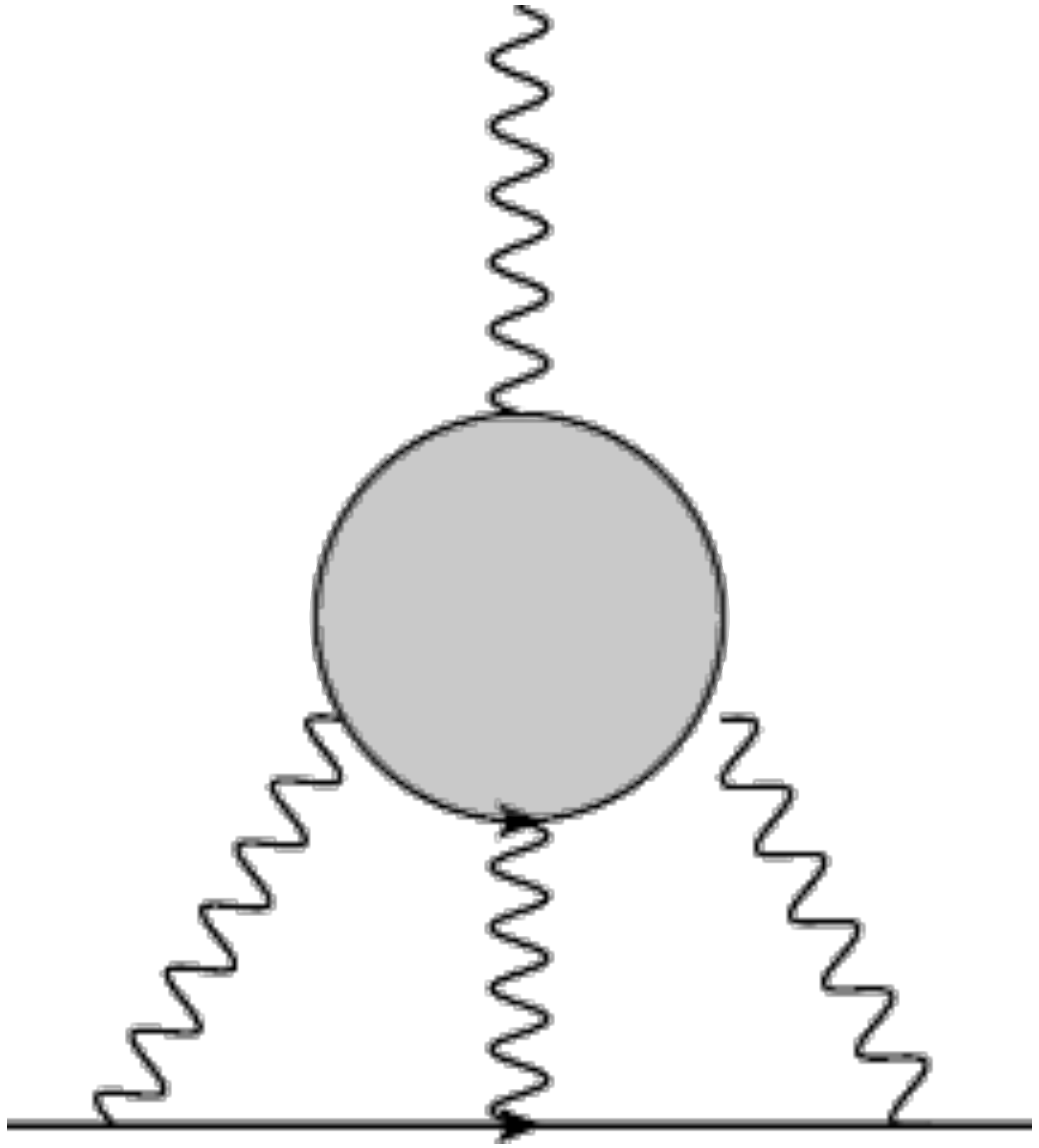






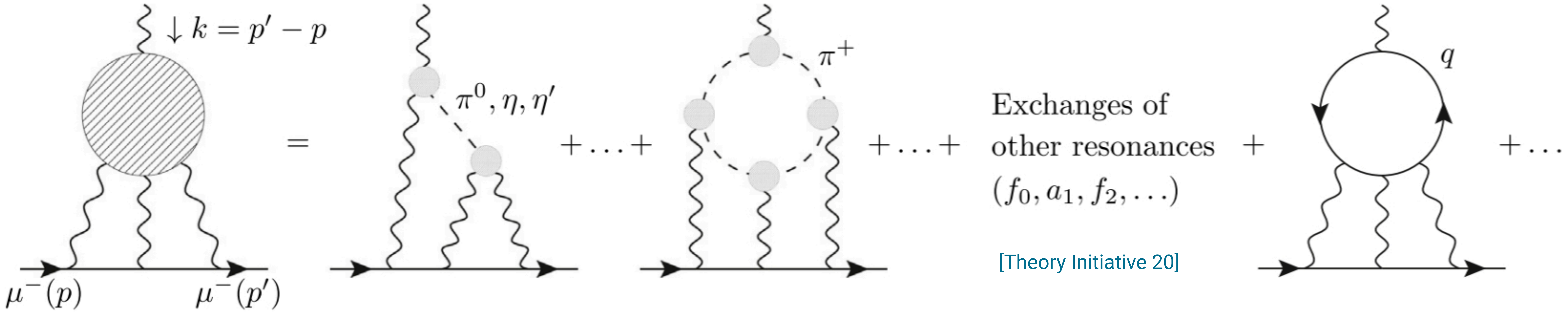
- reduced continuum-limit and finite-volume systematics, better signal-to-noise
- tension ($\sim 3.7\sigma$?) between lattice and $R(e^+e^-)$?
- collaborations are using the *window* as a high-precision probe

HLbL in the SM



data-driven

HLbL —data driven



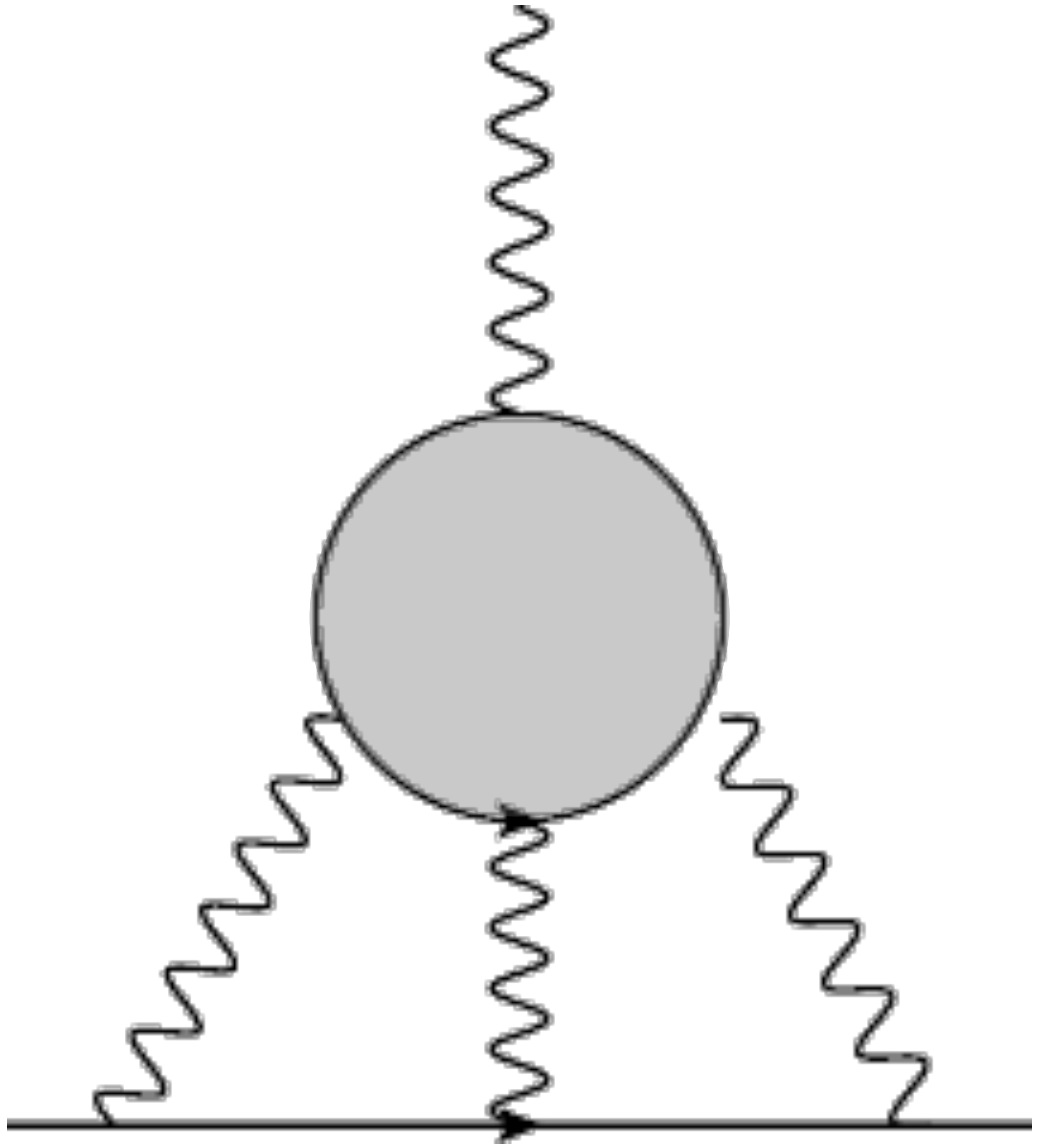
[Theory Initiative 20]

- More complex analytical structure than HVP makes data-driven analysis more involved
- Representation in terms of dispersion integrals only recently
  [Colangelo, Hoferichter, Procura, Stoffer 2014]
- Dominant contributions from π^0, η, η' and π and K loops

Contribution	TI 21
π^0, η, η'-poles	93.8(4.0)
π, K-loops/boxes	−16.4(2)
S-wave $\pi\pi$ rescattering	−8(1)
subtotal	69.4(4.1)
scalars	} − 1(3)
tensors	
axial vectors	6(6)
u, d, s-loops / short-distance	15(10)
c-loop	3(1)
total	92(19)

[Theory Initiative 20]

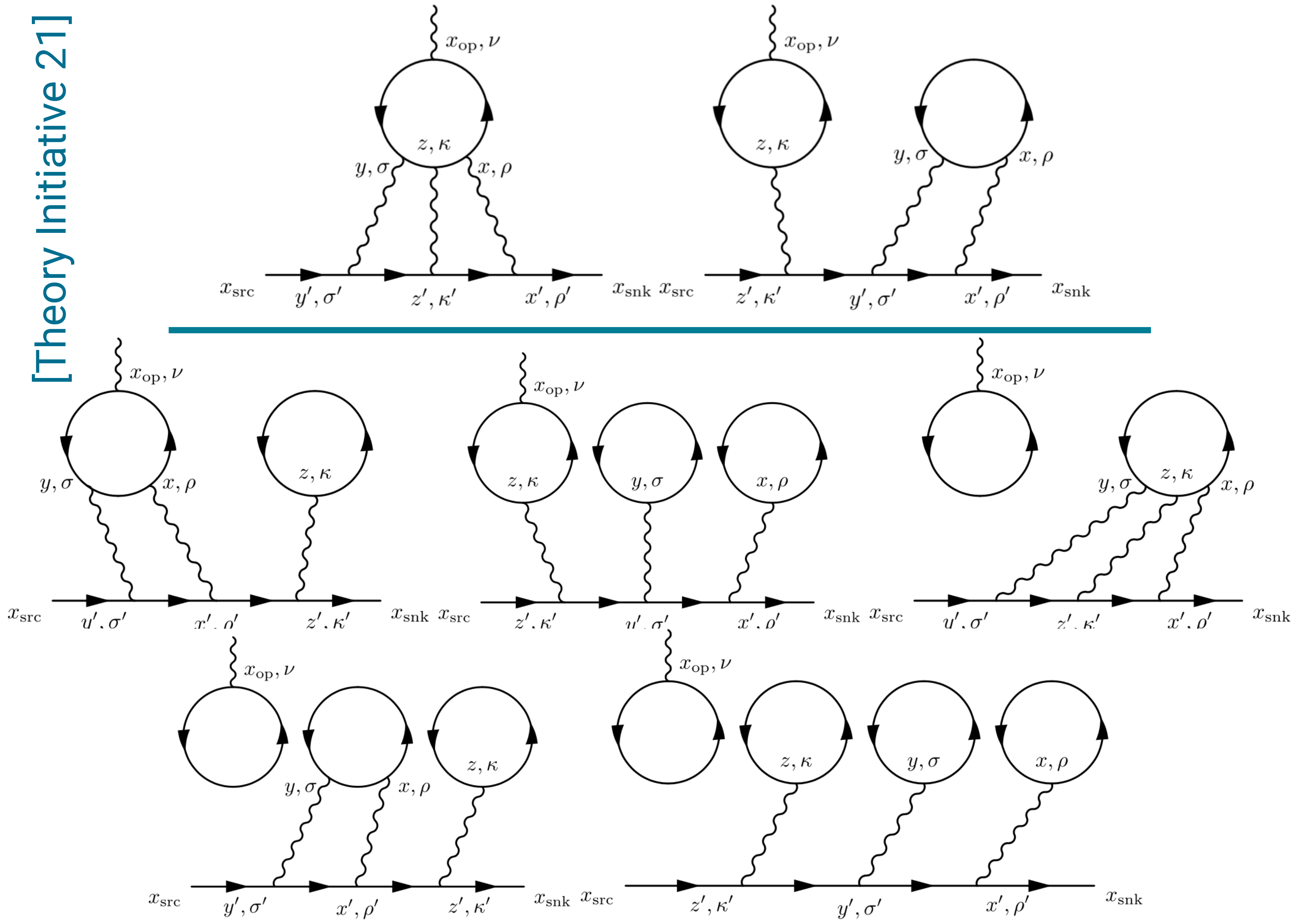
HLbL in the SM



lattice QCD

HLbL — lattice

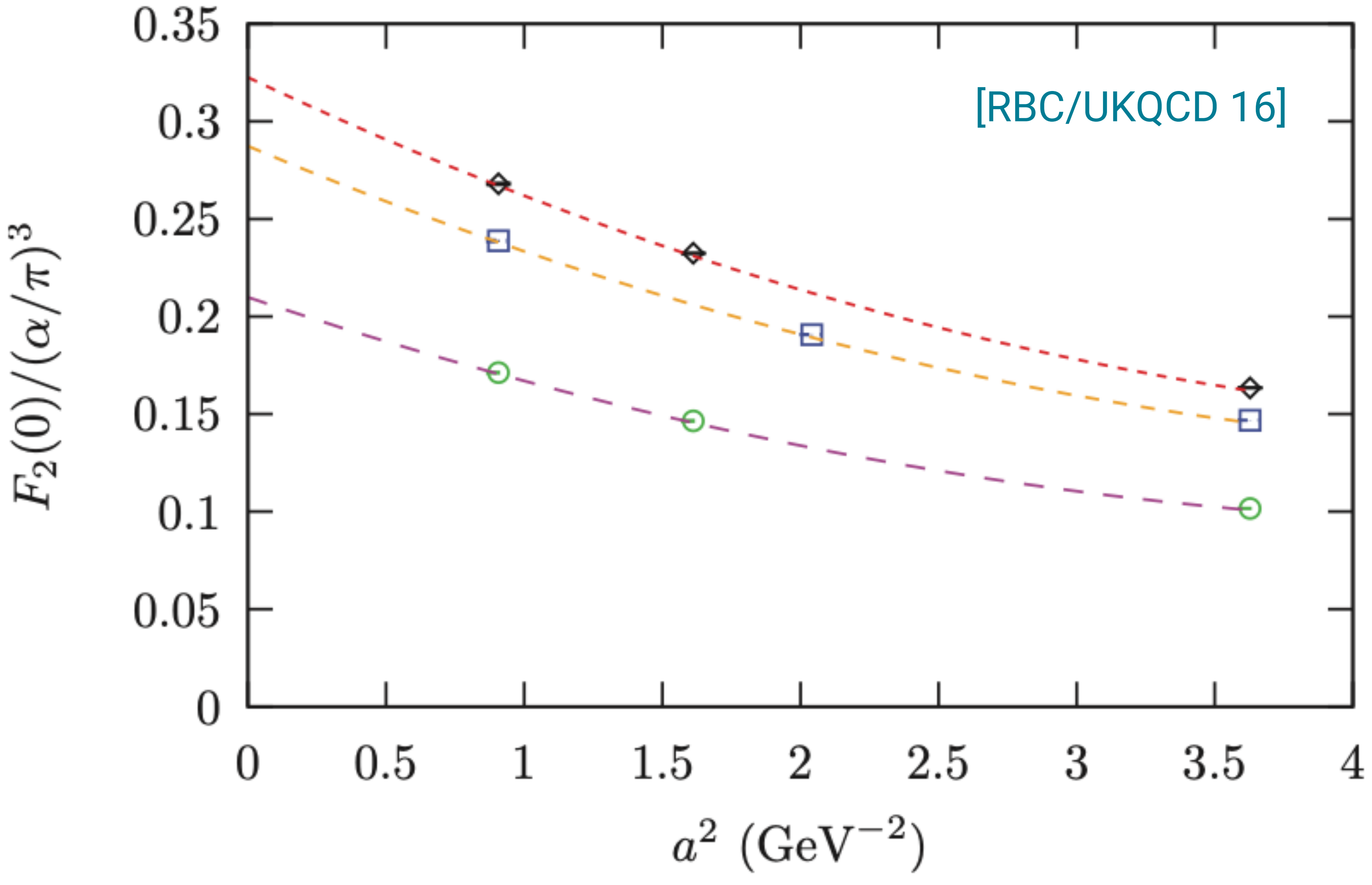
[Theory Initiative 21]



- Many new conceptual and computational techniques
- *Infinite-volume-QED* method:
  QED part: semi-analytical infinite-volume continuum pQED
  QCD part: lattice QCD

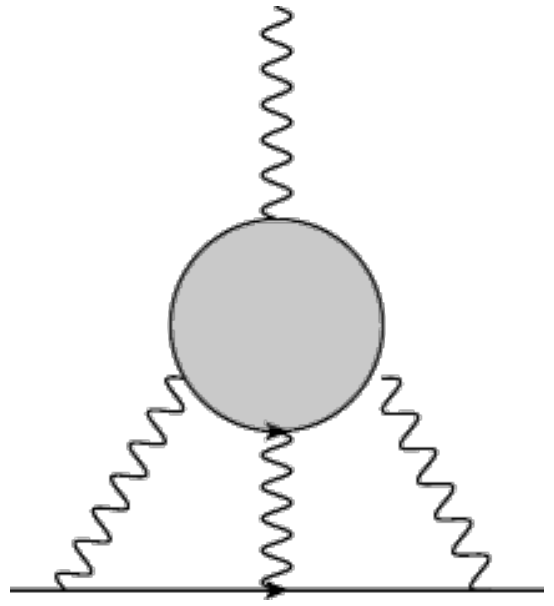
$$a_\mu^{\text{HLbL}} = F_2(0) = \frac{m_\mu e^6}{3} \int_y \int_x \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\, i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)$$

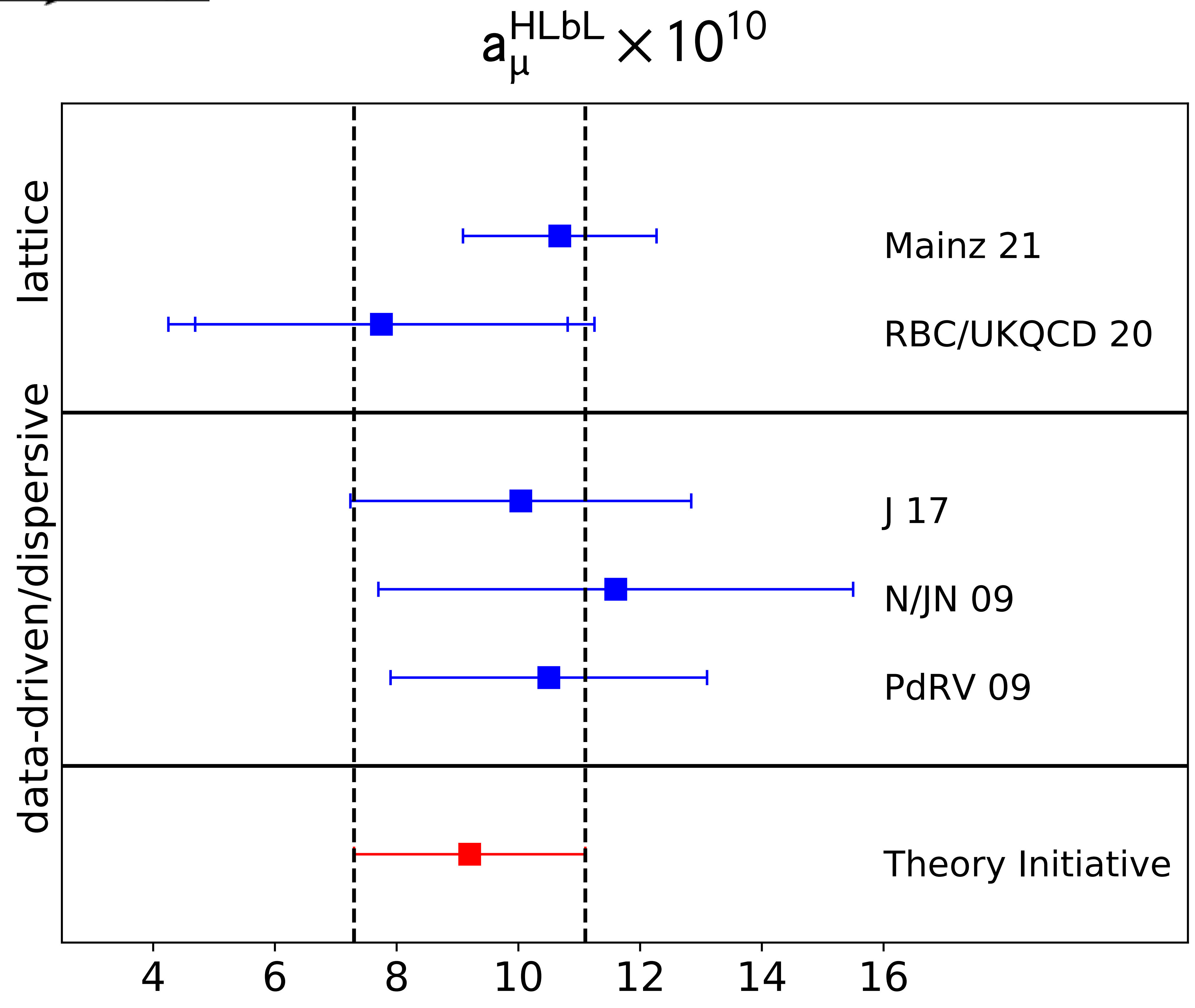
[Mainz 16,21, RBC/UKQCD 17]



Test: muon leptonic LbL
analytical result: 0.371

HLbL in the SM — summary





- Consistent results from variety of approaches
- increased stats. and improved methods make %-level precision feasible on lattice
- "It now appears conclusive that the hadronic light-by-light contribution cannot explain the current tension between theory and experiment for the muon $g - 2$." [Mainz 21]

Summary

QED and EW contributions *are known*

QCD

HVP

- **for data-driven HVP** there is a new consensus (Theory Initiative 20) but an underlying tension in the data needs to be understood
- **for HVP from the lattice** — a lot of activity and novel developments mainly on
  - statistical error
  - scale setting
  - QED
  - infinite-volume and continuum extrapolations

  expect impoved precision and new ideas
- **window method** allows high-precision test lattice vs. dispersion relation vs. experiment
- novel ideas impacting other lattice computations

HLbL

- **for data-driven HLbL** there is a new combined result (Theory Initiative 20)
- **for HLbL on the lattice** there are new results
- SM predictions for HLbL in good agreement — expect improved precision and new ideas but likely not solely responsible for explaining tension