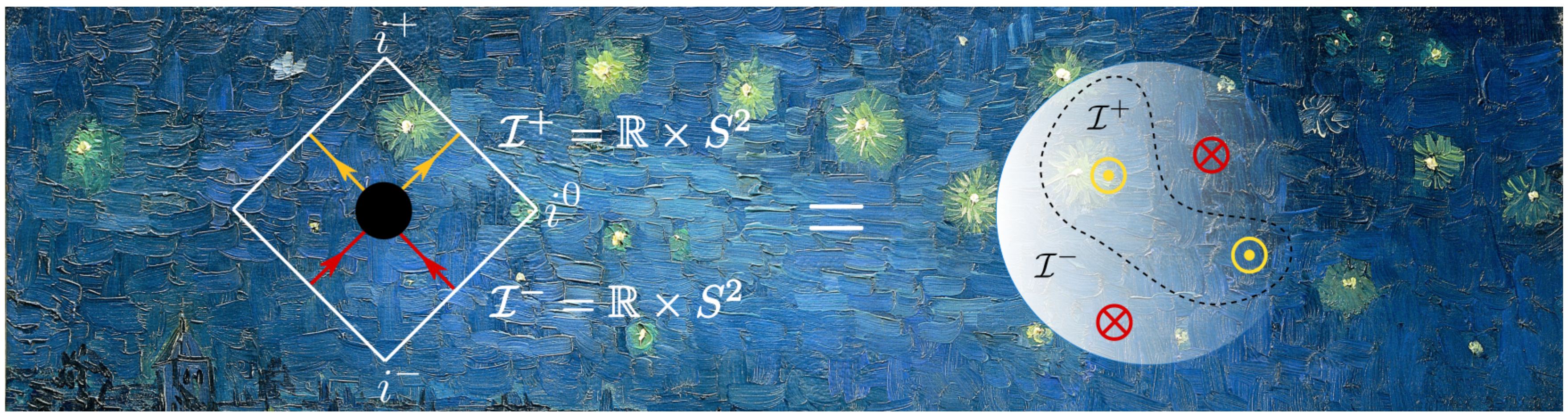
INSTITUT POLYTECHNIQUE DE PARIS

ÉCOLE POLYTECHNIQUE

CELESTIAL AMPLITUDES



ANDREA PUHM

THEORIE, UNIVERS ET GRAVITATION, IHP, 13 DECEMBRE 2021

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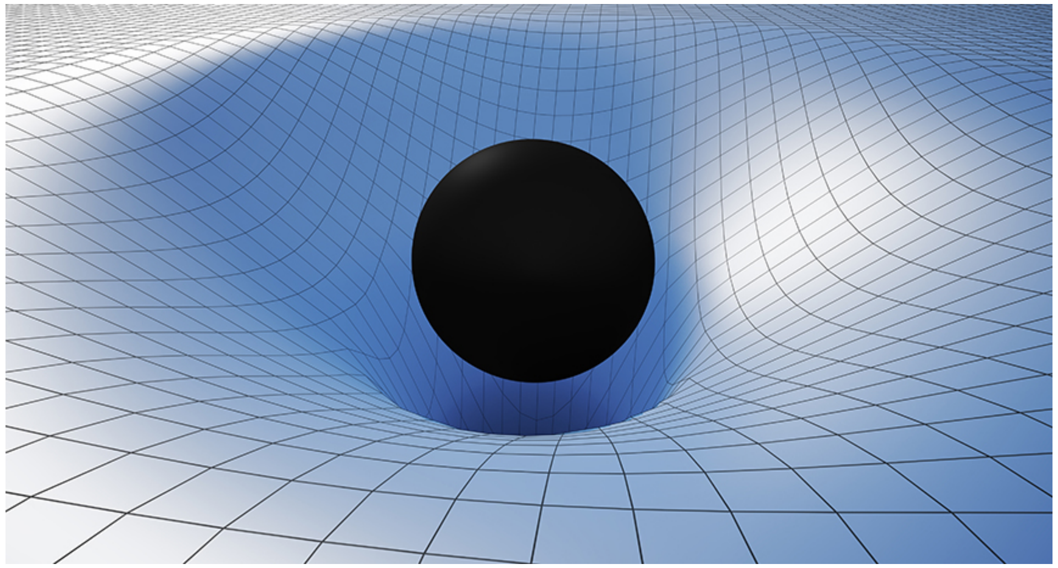
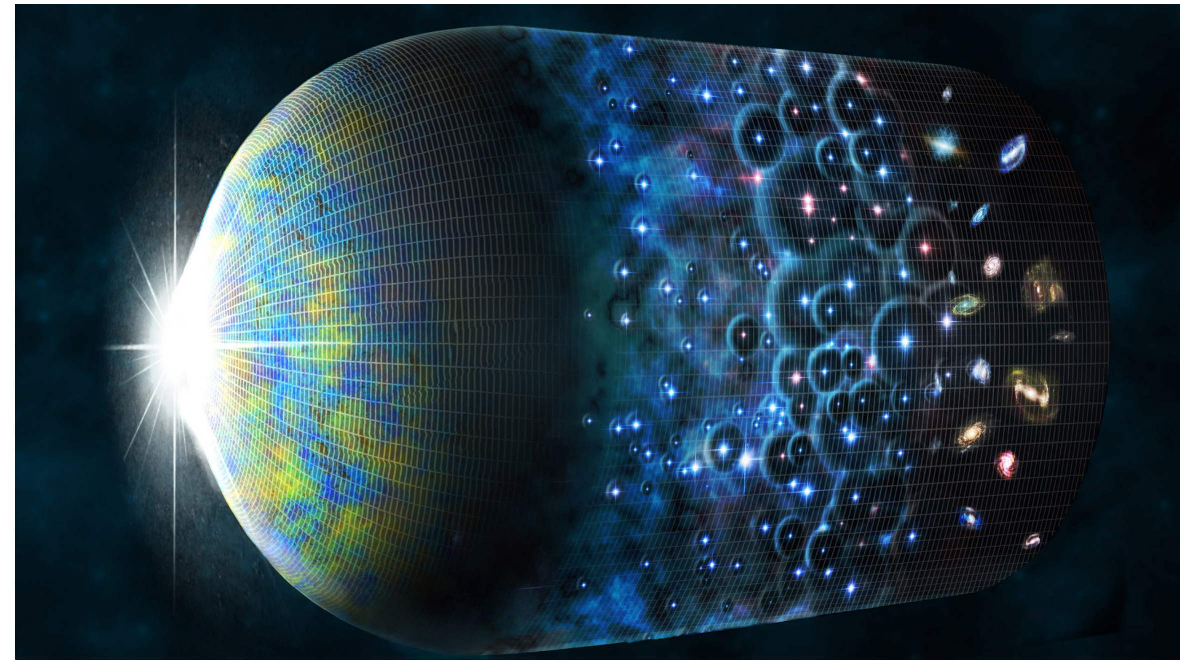
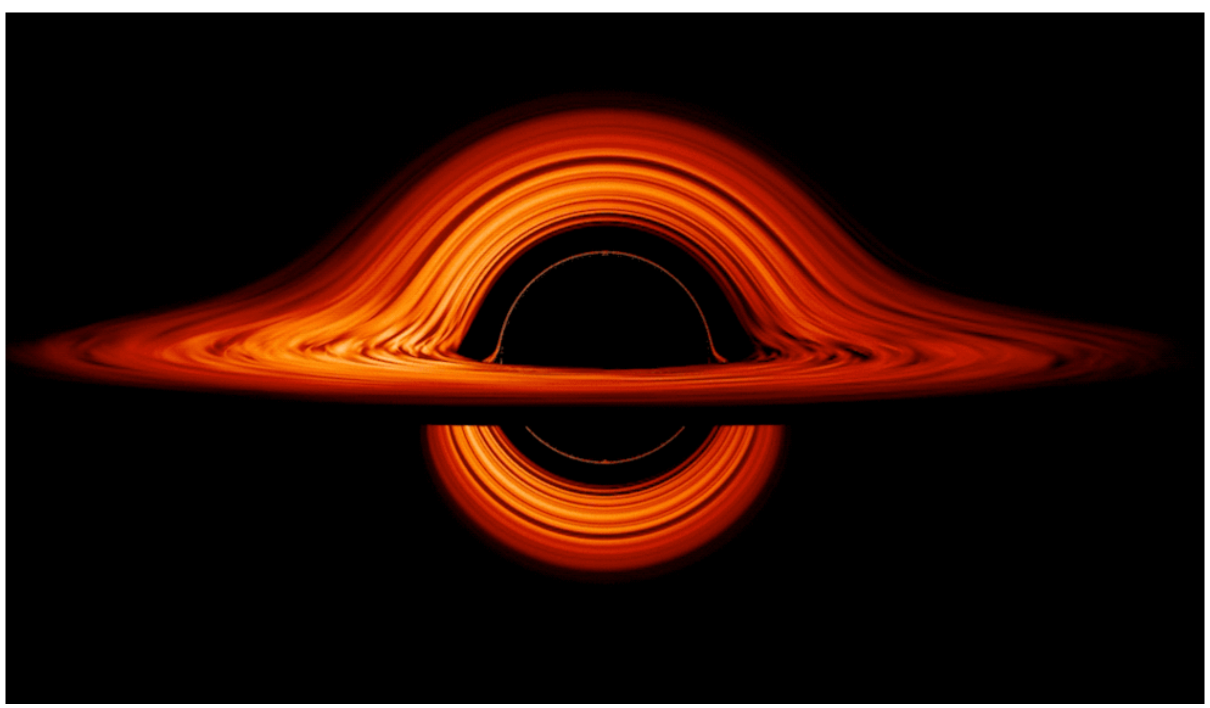
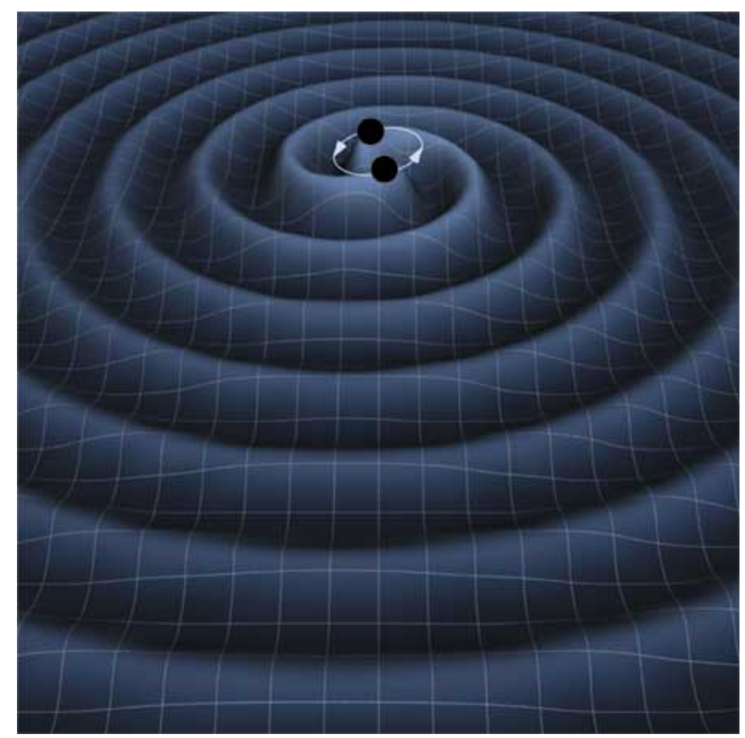
Gravity in our Universe







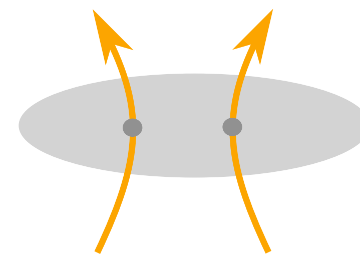


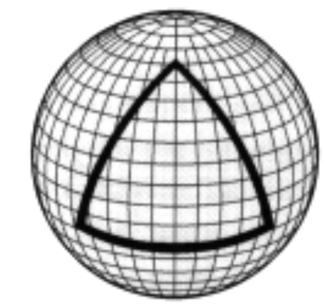
Sources: ScienceNews, NASA, Harvard Gazette , LIGO

Modeling our Universe

- **$\Lambda > 0$ de Sitter:**

spacetime @ cosmological scale

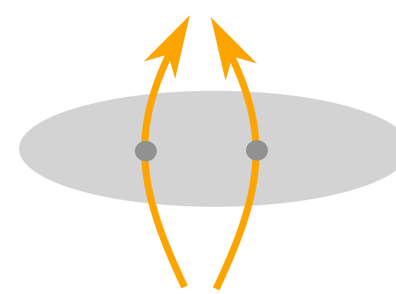


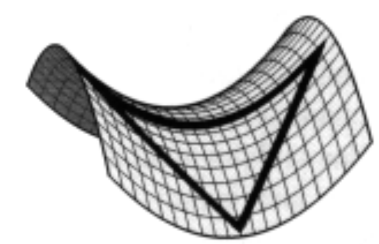


Positive Curvature

- **$\Lambda < 0$ Anti de Sitter:**

spacetime @ throat of highly rotating or charged black holes

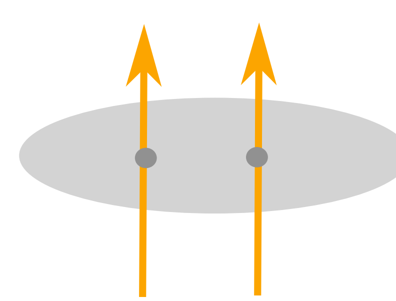


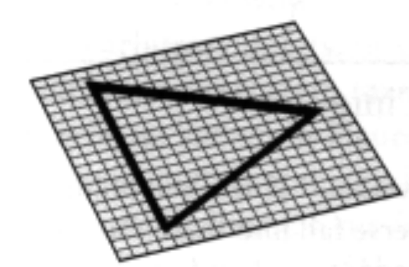


Negative Curvature

- **$\Lambda = 0$ Minkowski:**

spacetime @ intermediate scales



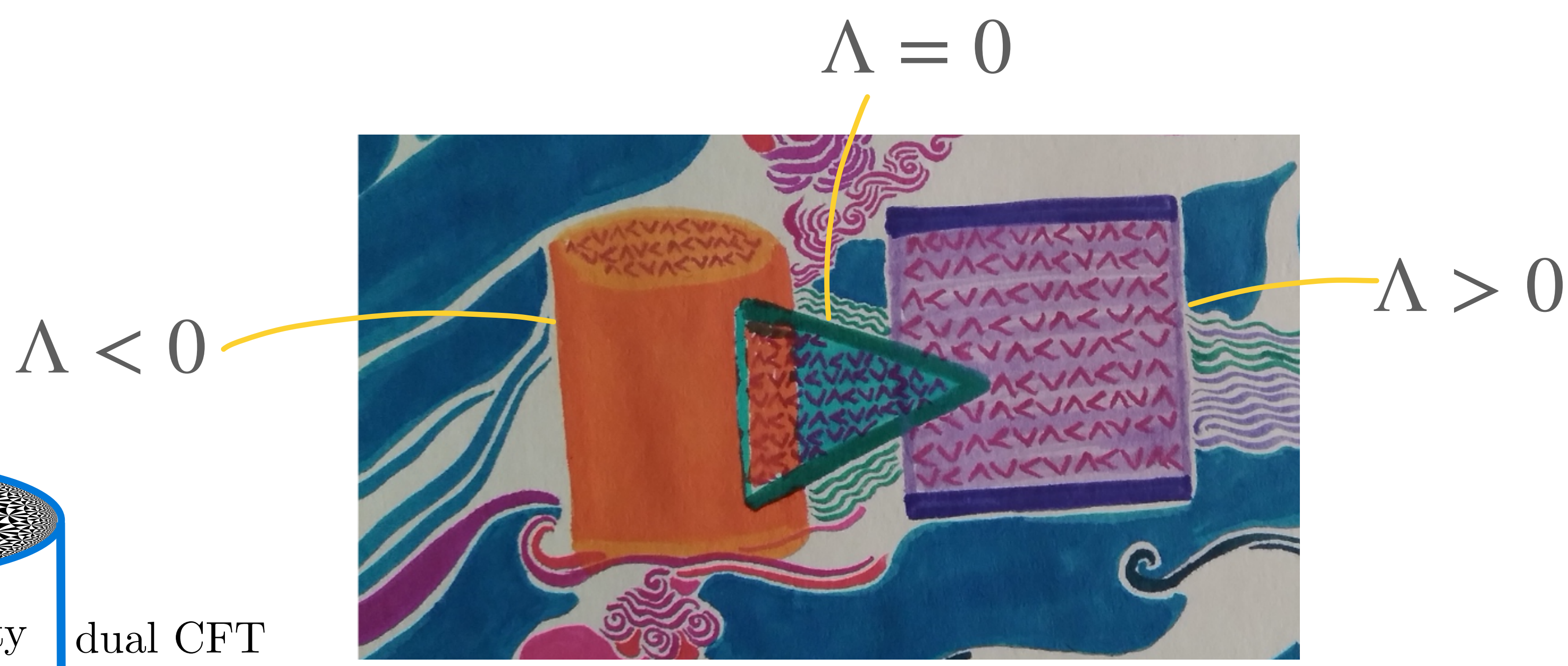


Flat Curvature

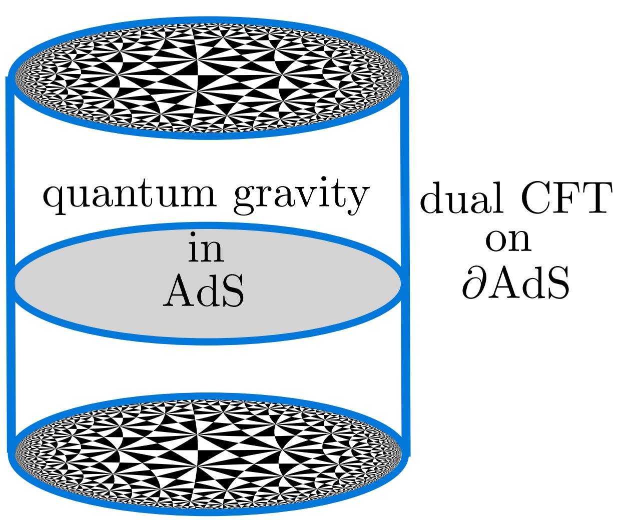
Quantum gravity for $\Lambda \;\substack{<\\=\\>}\; 0$

Powerful tool: **holographic principle**

Duality between quantum gravity on a manifold and a field theory on its boundary.



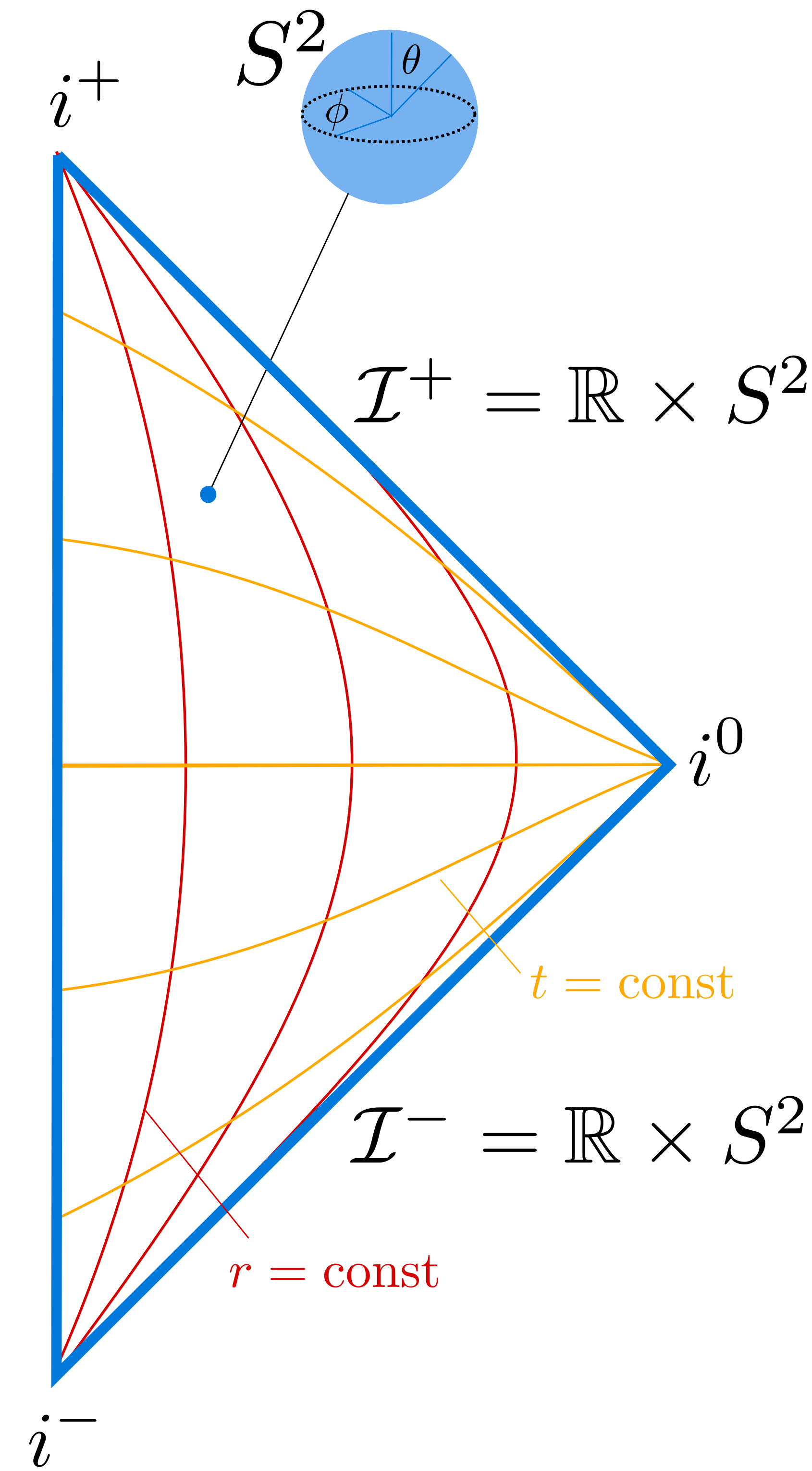
All Λ Holography conference image



How general is it?

$\Lambda = 0$ quantum gravity

All Λ *Holography* conference image



Quantum gravity: metric fluctuates but causal structure of asymptotically flat spacetime same as Minkowski.

Outline

Quantum gravity in *asymptotically flat* spacetimes.

What are the (**asymptotic**) **symmetries**?

Ward identities ⟷ universal behavior of $\mathcal{S}$-matrix in infrared

Poincaré

=

Translations & Lorentz

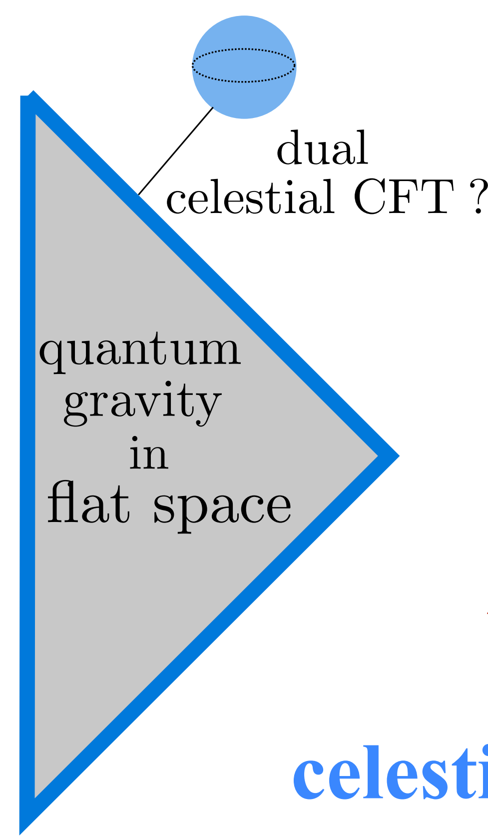
act on celestial sphere as

global conformal

∞ enhancement

BMS

local conformal

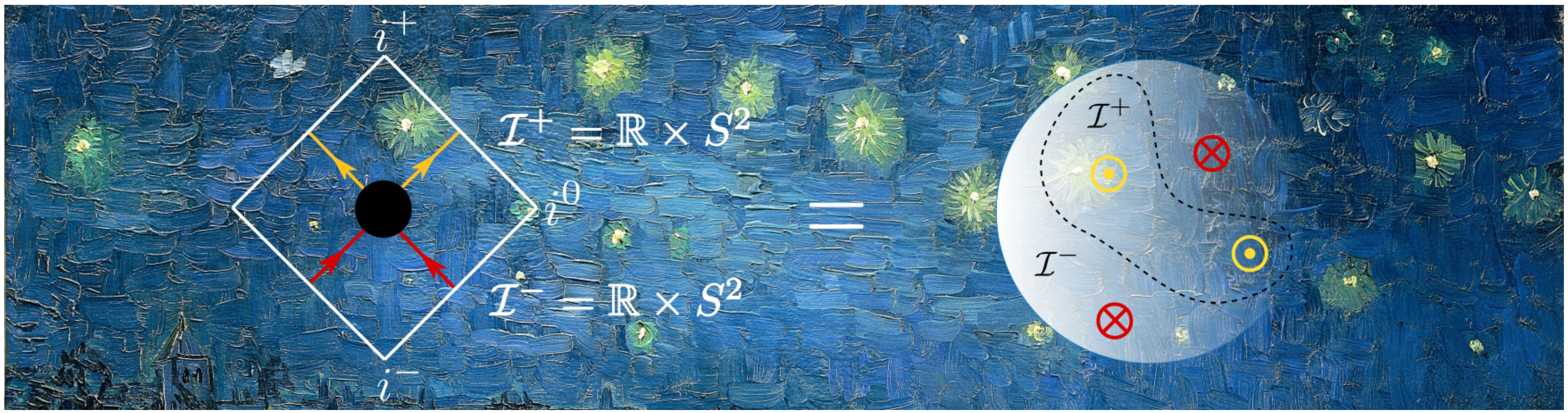


celestial holography programme

celestial amplitudes

$\mathcal{S}$-matrix as celestial amplitude

basic observable of quantum gravity in asymptotically flat spacetimes.



$$_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \prod_j \mathcal{O}^{\pm}_{\Delta_j, J_j}(w_j, \bar{w}_j) \rangle_{\text{celestial CFT}}$$

New framework for QFT scattering:

- makes asymptotic $\supset$ conformal symmetries manifest
- reorganizes (conformally) soft and collinear behavior
- probes all energy scales $\rightarrow$ sensitive to deep UV!

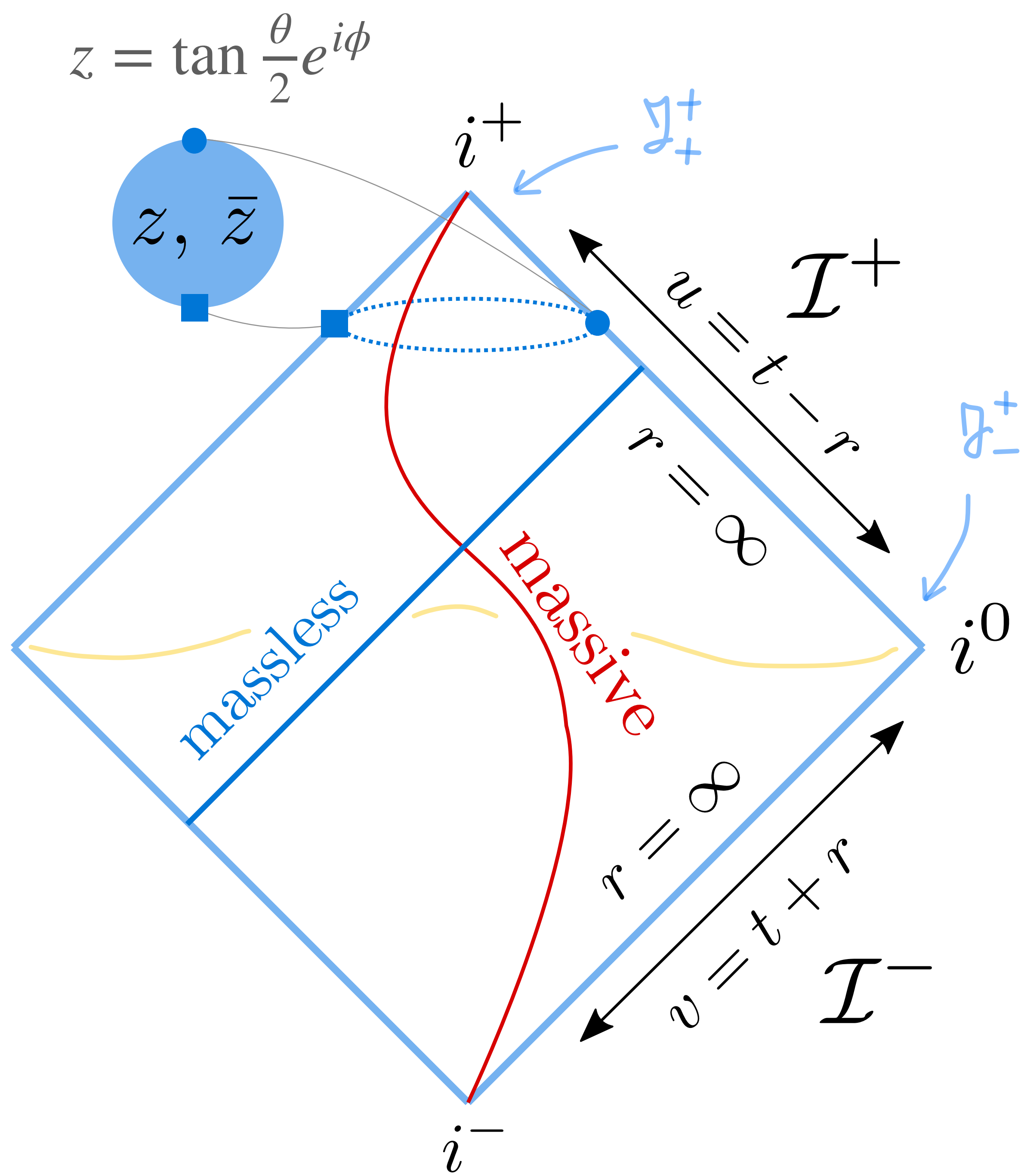
New approach to quantum gravity via celestial holography.

Causal structure of flat space

Massive particles enter at past timelike infinity (i^-) and exit at future timelike infinity (i^+).

Spacelike geodesics extend to spatial infinity (i^0).

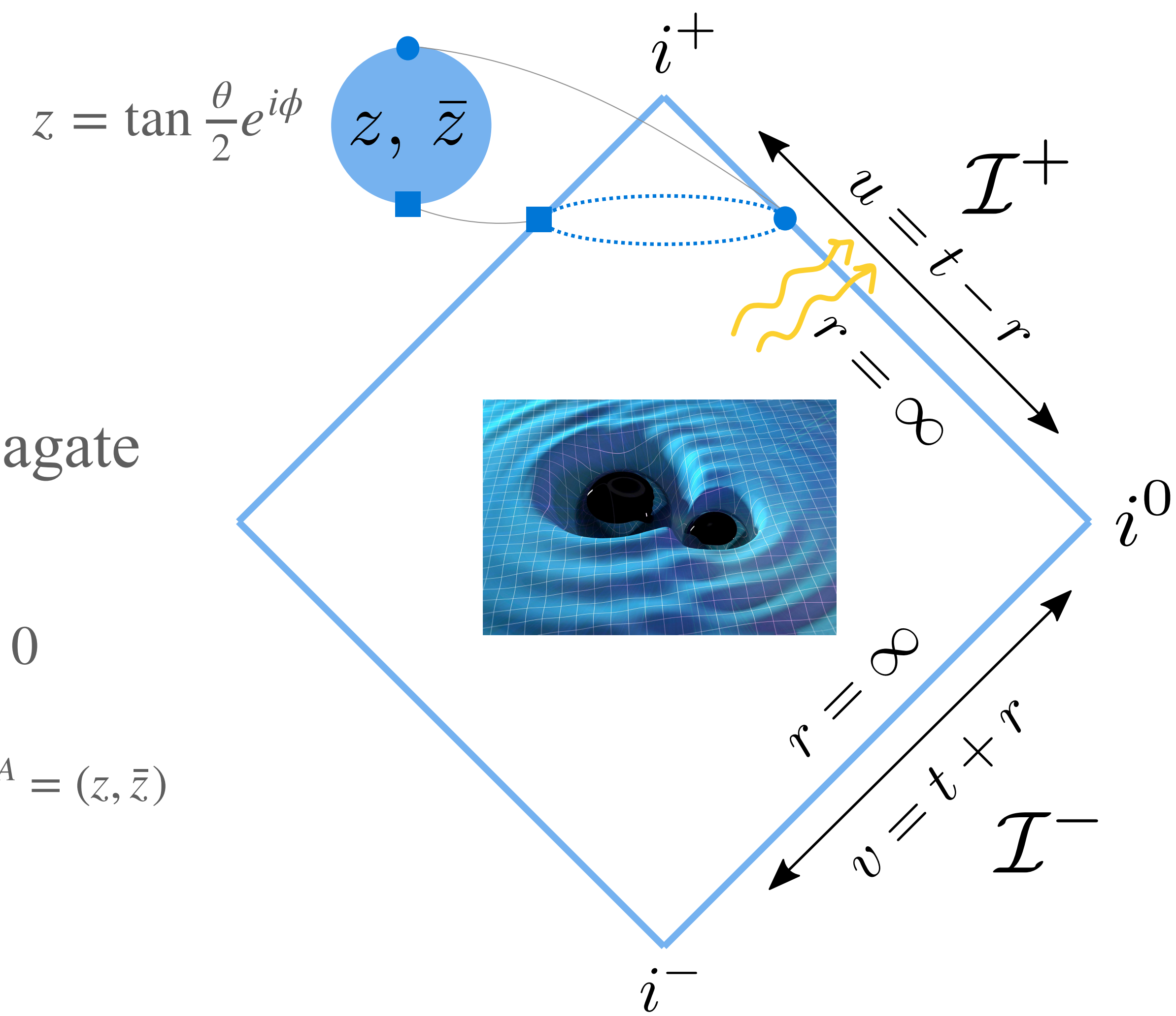
Massless particles enter at past null infinity ($\mathscr{I}^-$) and exit at future null infinity ($\mathscr{I}^+$).



Asymptotically flat spacetime

Quantum gravity: metric fluctuates

$\Rightarrow$ flat + $\frac{1}{r}$ corrections



Coordinates in which gravitational waves propagate radially outwards $r \to \infty$ at fixed u:

Bondi gauge: $g_{rr} = 0\,, \quad g_{rA} = 0\,, \quad \partial_r \det\left(\frac{g_{AB}}{r^2}\right) = 0$

+ specify boundary conditions: $A = z, \bar{z}: \quad x^A = (z, \bar{z})$

mass — angular momentum

$$ds^2 = -(1 + \ldots)du^2 - (2 + \ldots)dudr + (\ldots)dudx^A + (r^2 q_{AB} + r\,C_{AB} + \ldots)dx^A dx^B$$

gravitational waves

[Bondi,van der Burg,Metzner;Sachs'62]

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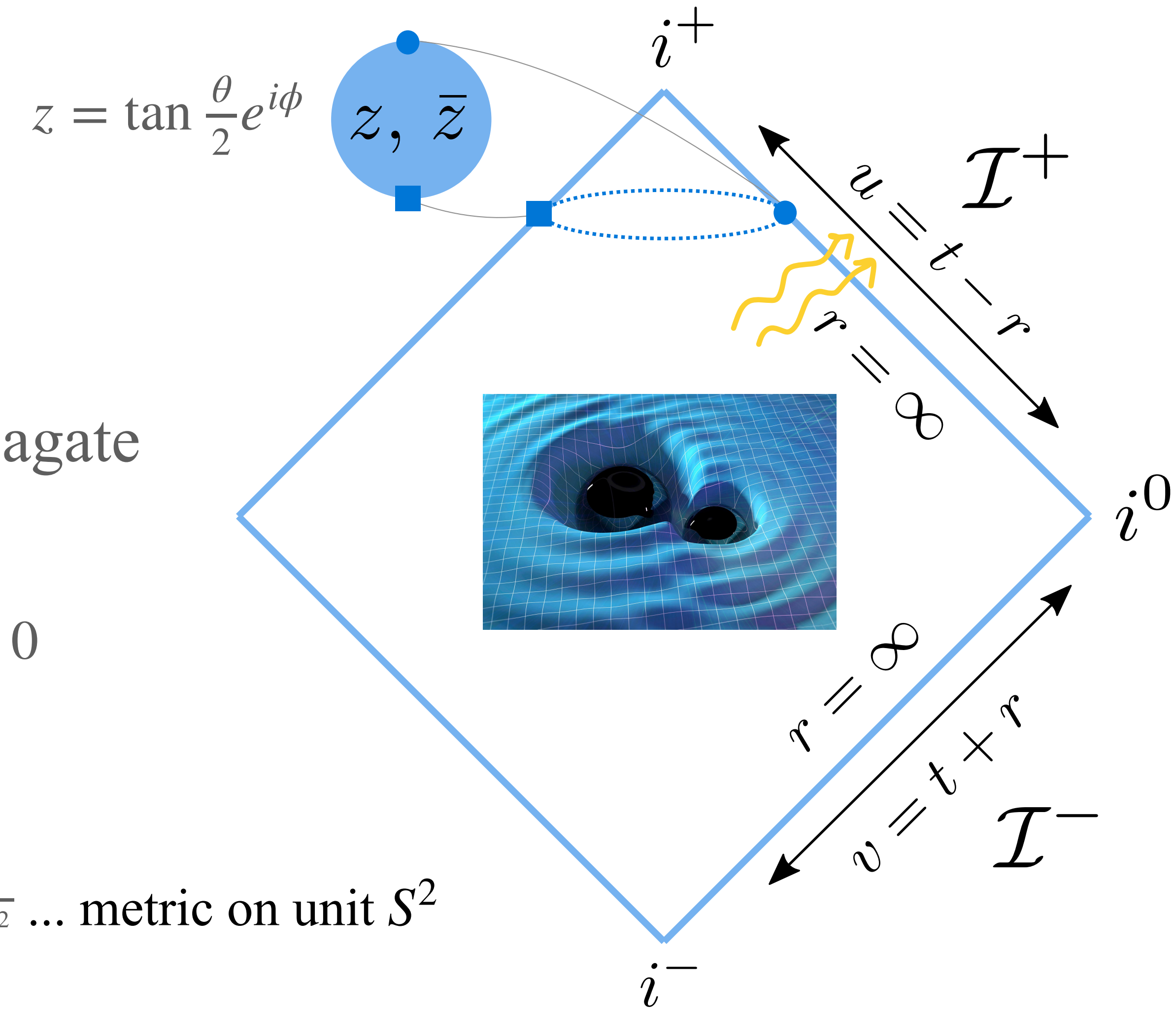

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+ specify boundary conditions:

$\gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$ ... metric on unit S^2

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

$$+\frac{2m}{r}du^2 + rC_{zz}dz^2 + \left[D^zC_{zz} + \frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m) - \frac{1}{4}D_z(C_{zz}C^{zz})\right)\right]dudz + c\,.\,c\,.$$

Radiative data at future null infinity $\mathcal{I}^+$: $\{m, N_A, C_{AB}\}$ [Bondi,van der Burg,Metzner;Sachs'62]

Bondi mass aspect

angular momentum aspect

gravitational data $\Rightarrow$ Bondi news

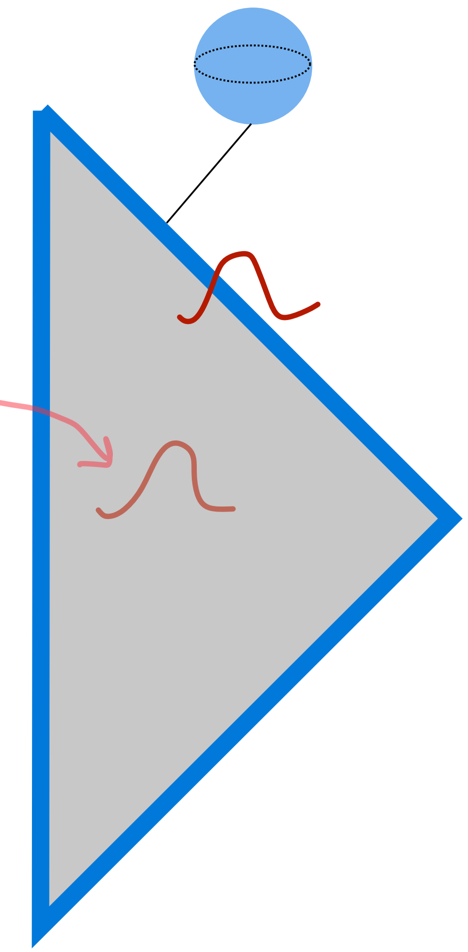
$N_{AB} = \partial_u C_{AB}$

gravitational waves

Asymptotic symmetries

Asymptotic symmetry group = $\dfrac{\text{allowed diffeos}}{\text{trivial diffeos}}$

$\mathscr{L}_{\xi} g_{\mu\nu} \approx 0$ as $r \to \infty$



Diffeos that preserve Bondi gauge and fall-offs:

$$\xi = (1 + \frac{u}{2r})Y^z\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^z\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^z\partial_r + \frac{u}{2}D_zY^z\partial_u + c.c.$$
$$+f\partial_u - \frac{1}{r}(D^zf\partial_z + D^{\bar{z}}f\partial_{\bar{z}}) + D^zD_zf\partial_r$$

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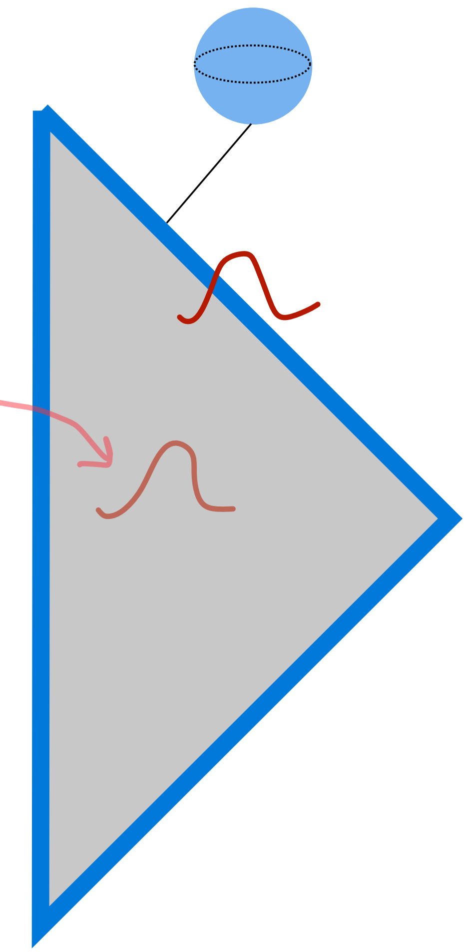

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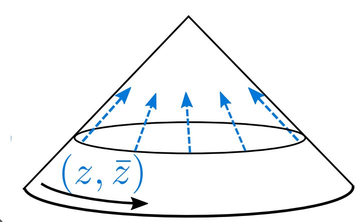
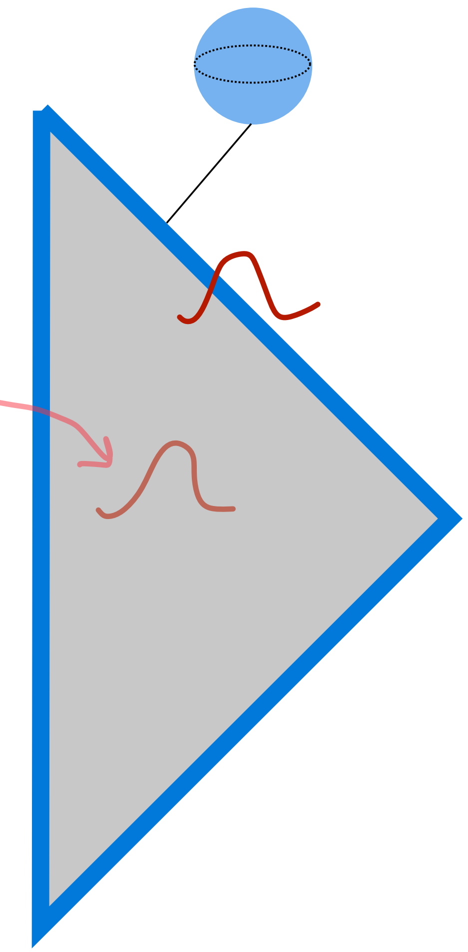


Translations: $\quad f \in \{1, z, \bar{z}, z\bar{z}\}$ e.g. $\xi_{f=1}|_{\mathcal{I}^+} = \partial_u$

Lorentz: $\quad Y^z \in \{1, z, z^2, i, iz, iz^2\}$ global CKVs $\partial_{\bar{z}}Y^z = 0$

4+6=10 Poincare generators

Asymptotic symmetries



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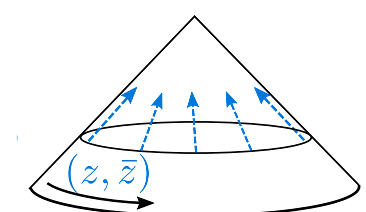

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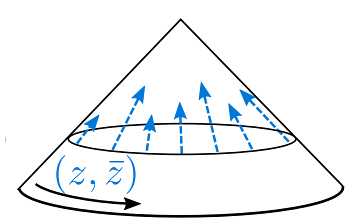
↓

∞ generators!

[Bondi,van der Burg,Metzner,Sachs'62]

↓



BMS = Supertranslations $\ltimes$ Lorentz

$f(z, \bar{z})$

Asymptotic symmetries

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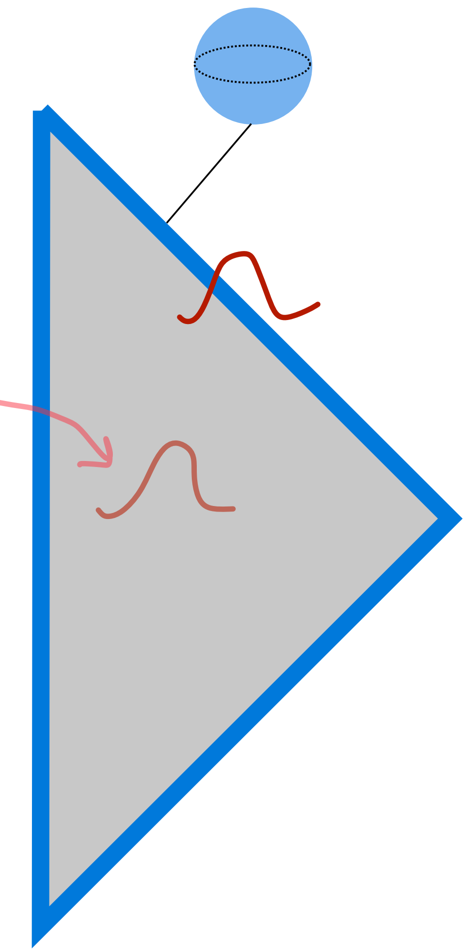

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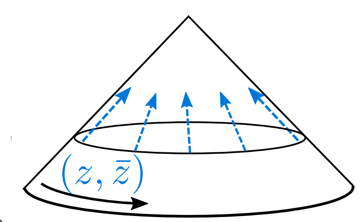

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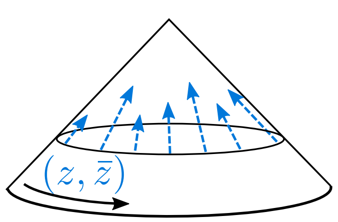


BMS = Supertranslations ⋉ Lorentz

$f(z,\bar z)$

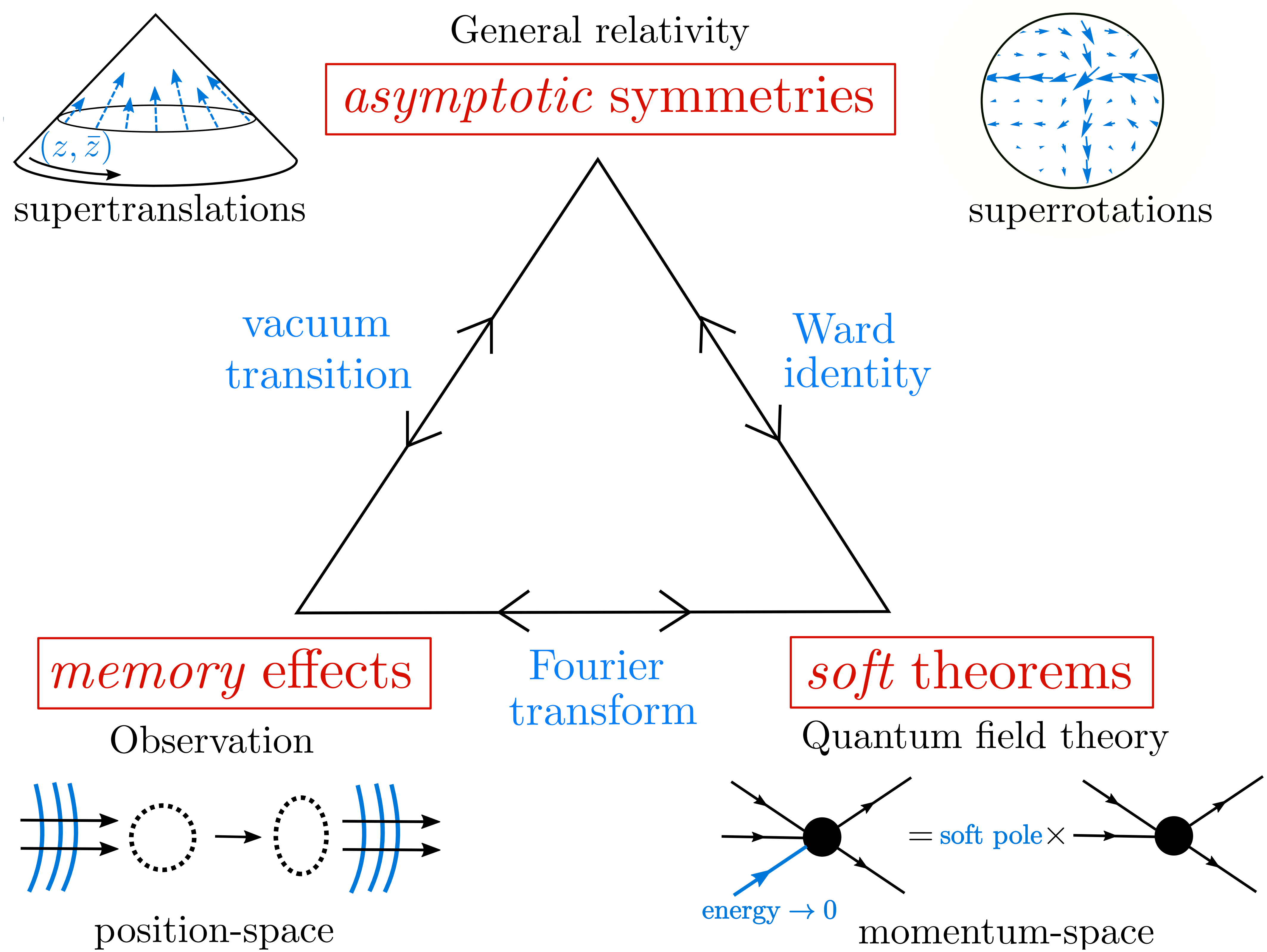
extended / generalized BMS = Supertranslations ⋉ Superrotations

[deBoer,Solodukhin'03]
[Barnich,Troessaert'11-12']
[Campiglia,Laddha'15]

$Y^z(z) \in \{z^n, iz^n\}$ local CKVs → Virasoro

$Y^z(z,\bar z)$ → $\mathrm{Diff}(S^2)$

Gravity in the infrared



BMS symmetry

Supertranslations and superrotations change the physical state.

$\mapsto$ ***memory effect*** [Strominger,Zhiboedov'14] [Pasterski,Strominger,Zhiboedov'15]

BMS charge:

$$Q^+[f,Y] = \frac{1}{8\pi G}\int_{\mathcal{I}^+_-} d^2z\sqrt{\gamma}[2fm + Y^A N_A]$$

$$\parallel$$

$$Q^-[f,Y] = \frac{1}{8\pi G}\int_{\mathcal{I}^-_+} d^2z\sqrt{\gamma}[2fm + Y^A N_A]$$

[Strominger'13]

antipodal matching:

conservation of energy and angular momentum at every angle

Conserved charges commute with the $\mathcal{S}$-matrix:

$$\langle out\,|\,Q^+\mathcal{S} - \mathcal{S}Q^-\,|\,in\rangle \overset{!}{=} 0$$

$$Q^\pm = Q^\pm_S + Q^\pm_H$$

soft hard

$$\langle out\,|\,Q^+_S\mathcal{S} - \mathcal{S}Q^-_S\,|\,in\rangle = -\,\langle out\,|\,Q^+_H\mathcal{S} - \mathcal{S}Q^-_H\,|\,in\rangle$$

soft graviton insertion

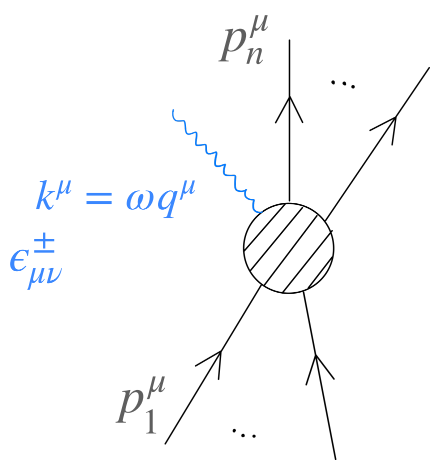
action of hard charges on states

Soft theorems

Universal behavior of gauge theory amplitudes when energy of gauge boson is taken *soft*, i.e. to zero. Let's look at gravity:

$$\langle out\,|\,a_\pm \mathcal{S}\,|\,in\rangle = \mathcal{A}^{\pm}_{n+1} \overset{\omega\to 0}{=} \left[\omega^{-1}S_n^{(0)\pm} + \omega^0 S_n^{(1)\pm} + \mathcal{O}(\omega)\right]\mathcal{A}_n = \langle out\,|\,\mathcal{S}\,|\,in\rangle$$

[Weinberg'65] [Cachazo,Strominger'14]



$$S_n^{(0)\pm} = \frac{\kappa}{2}\sum_{j=1}^{n}\frac{(p_j\cdot\epsilon^\pm)^2}{p_j\cdot q}$$

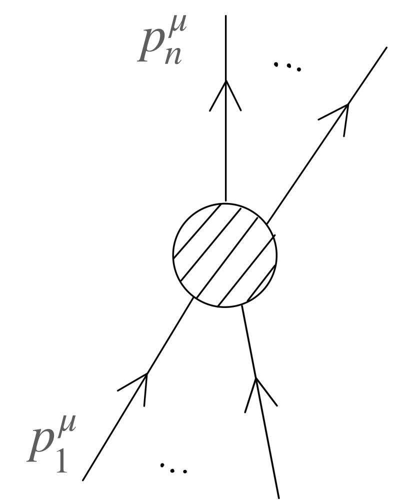
j-th momentum

$$\epsilon^\pm_{\mu\nu} = \epsilon^\pm_\mu\epsilon^\pm_\nu \qquad \epsilon^\pm\cdot q = 0$$

$$\kappa = \sqrt{32\pi G}$$

$$S_n^{(1)\pm} = -i\frac{\kappa}{2}\sum_{j=1}^{n}\frac{(p_j\cdot\epsilon^\pm)(q\cdot J_j\cdot\epsilon^\pm)}{p_j\cdot q}$$

j-th angular momentum



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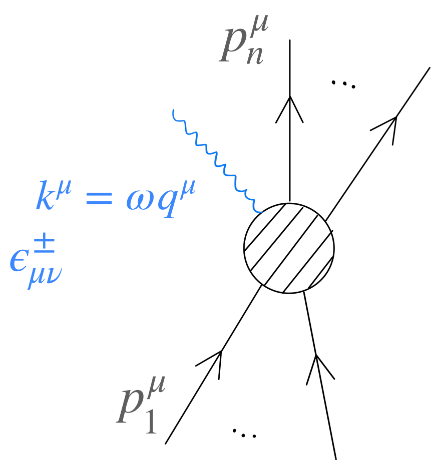

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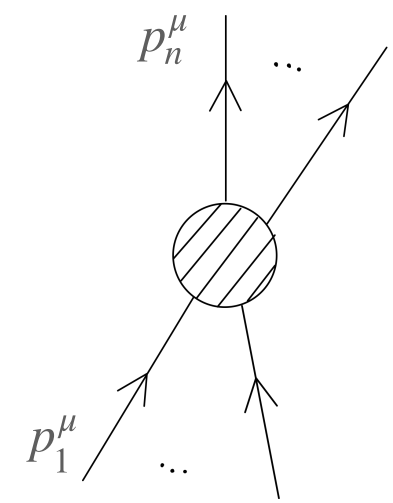

Soft graviton theorem implies BMS symmetry:

$$\langle out\,|\,Q_S^+\mathcal{S} - \mathcal{S}Q_S^-\,|\,in\rangle = -\langle out\,|\,Q_H^+\mathcal{S} - \mathcal{S}Q_H^-\,|\,in\rangle$$

[Strominger'13] [He,Lysov,Mitra,Strominger'14]

Ward identities

[Strominger'13]

Leading soft graviton (supertranslation) gives a Kac-Moody current:

$$\langle out | : \mathcal{P}_z \mathcal{S} : | in \rangle = \sum_j \frac{\omega_j}{z - z_j} \langle out | \mathcal{S} | in \rangle$$

[Kapec,Mitra,Raclariu,Strominger'16]

Subleading soft graviton (superrotation) gives a stress tensor candidate:

$$\langle out | : \mathcal{T}_{zz} \mathcal{S} : | in \rangle = \sum_j \left[\frac{\ell_j - \omega_j \partial_{\omega_j}}{2(z - z_j)^2} + \frac{\partial_{z_j}}{z - z_j} \right] \langle out | \mathcal{S} | in \rangle$$

Ward identities

[Strominger'13]

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$$\langle \mathscr{T}_{zz} \mathscr{O}_1^{+} \dots \mathscr{O}_n^{-} \rangle = \sum_j \left[\frac{\ell_j + h_j}{(z - z_j)^2} + \frac{\partial_{z_j}}{z - z_j} \right] \langle \mathscr{O}_1^{+} \dots \mathscr{O}_n^{-} \rangle$$

need to change scattering basis

$$h_j = \tfrac{1}{2}(-\omega_j \partial_{\omega_j} + s_j)$$

$$\bar{h}_j = \tfrac{1}{2}(-\omega_j \partial_{\omega_j} - s_j)$$

⇒ 4D QFT soft theorems look like Ward identities in a 2D CFT!

4D Lorentz ≅ 2D global conformal

Action of Lorentz group on celestial sphere:

Cartesian

$$X^\mu = \left(u + r, r\frac{z+\bar{z}}{1+z\bar{z}}, r\frac{i(\bar{z}-z)}{1+z\bar{z}}, r\frac{1-z\bar{z}}{1+z\bar{z}}\right)$$

retarded Bondi

$$r \mapsto rF(z,\bar{z}) + \mathcal{O}(1/r)$$

$$F(z,\bar{z}) = \frac{|az+b|^2 + |cz+d|^2}{1+z\bar{z}}$$

angle-dependent rescalings

$$u \mapsto \frac{u}{F(z,\bar{z})} + \mathcal{O}(1/r)$$

$$z \mapsto \frac{az+b}{cz+d} + \mathcal{O}(1/r)$$

$a, b, c, d \in \mathbb{C}$

$ad - bc = 1$

global conformal transformation

Isomorphism: $SO^+(1,3) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$

$i = 1,2,3$ $\quad K_i, J_i \longleftrightarrow D, P_a, K_a^{SCT}, J_{ab}$ $\quad a, b = 1,2$

boosts, rotations

$\downarrow$

$L_0, L_{\pm 1}, \bar{L}_0, \bar{L}_{\pm 1}$ $\qquad [L_m, L_n] = (m-n)L_{m+n}$

global conformal generators

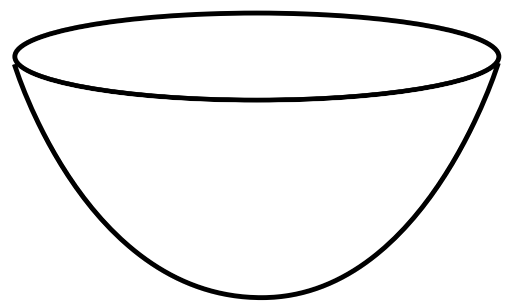
Superrotations: **enhancement** to *local* conformal symmetry $w/\ m, n \in \mathbb{Z}$

Energy-momentum $\mapsto$ conformal basis

[de Boer,Solodhukin'03] [Cheung,de la Fuente,Sundrum'16][Pasterski,Shao,Strominger'17]

For $\mathcal{S}$-matrix elements we specify on-shell momenta for *in* and *out* states:

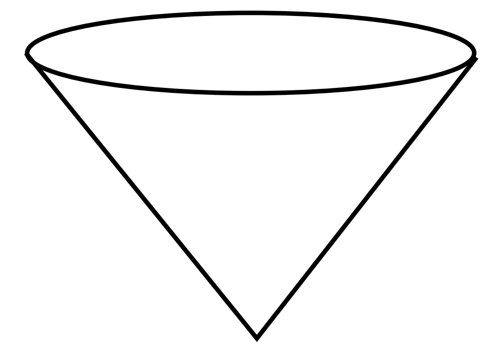
massive particles



$$p^2 = -m^2$$

$$p^\mu = \frac{m}{2y}(1 + y^2 + z\bar{z}, z + \bar{z}, i(\bar{z} - z), 1 - y^2 - z\bar{z})$$

massless particles



$$k^2 = 0$$

$$k^\mu = \omega(1 + w\bar{w}, w + \bar{w}, i(\bar{w} - w), 1 - w\bar{w})$$

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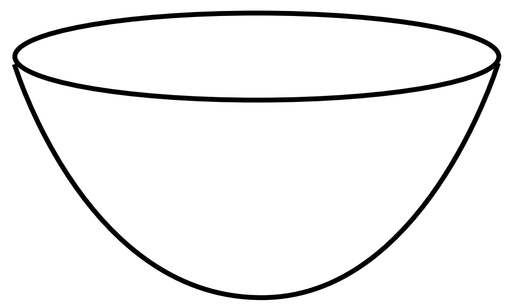

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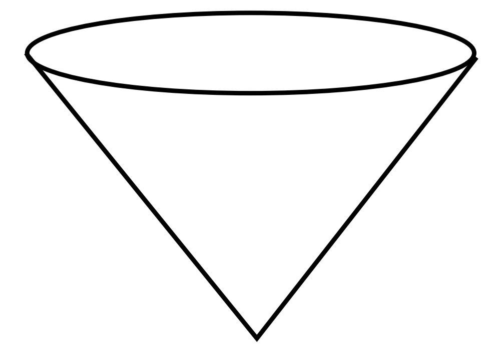

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Holographic map:

$$\int_0^\infty \frac{dy}{y^3} \int d^2z G_\Delta(y, z, \bar{z}; w, \bar{w})(\,.\,)$$

bulk-to-boundary propagator

$$\int_0^\infty \frac{d\omega}{\omega} \omega^\Delta(\,.\,)$$

Mellin transform

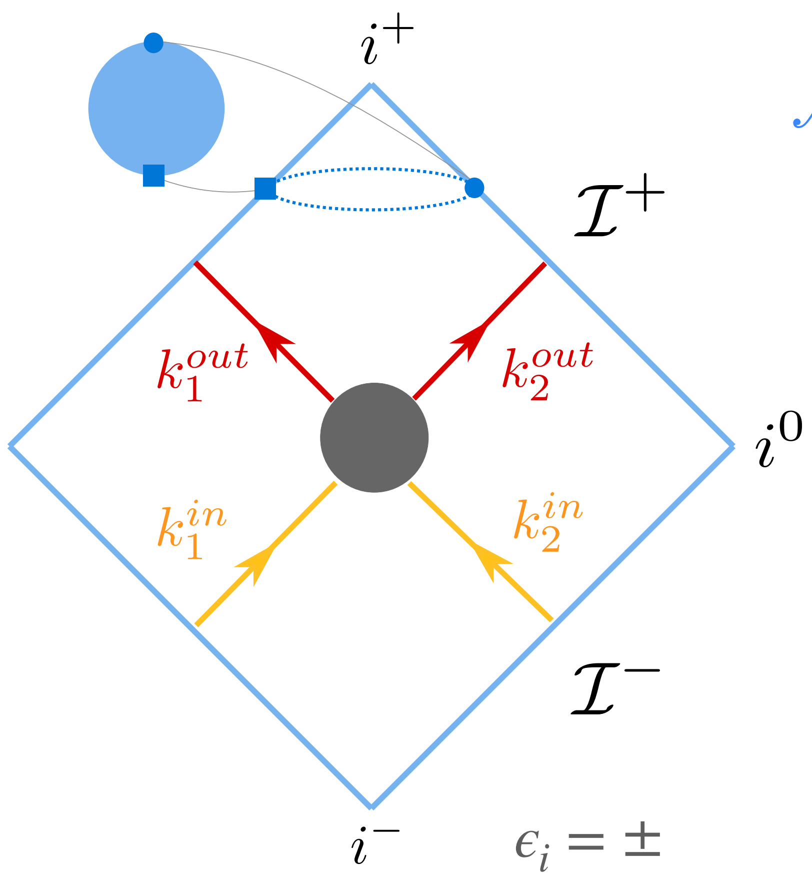
$= h + \bar{h}$ $= h - \bar{h}$

For CFT correlators we specify the conformal dimension Δ and spin J of the operators and the point $(w, \bar{w})$ on S^2 where they are inserted.

4D $\mathscr{S}$-matrix ⇒ 2D correlator

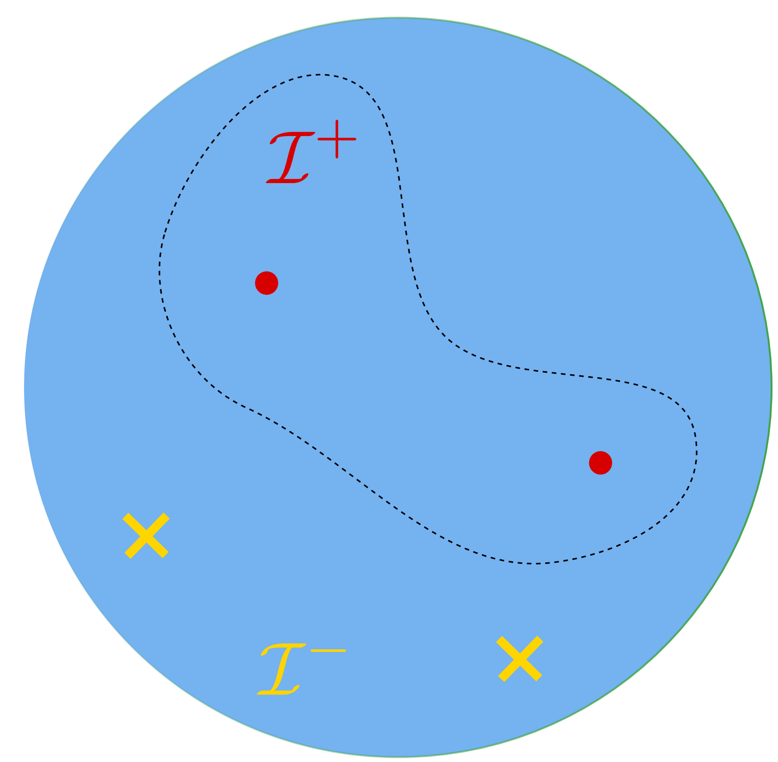
[de Boer,Solodhukin'03] [Cheung,de la Fuente,Sundrum'16][Pasterski,Shao,Strominger'16]

For massless scattering the map is a Mellin transform:



$$\mathscr{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta(\,.\,)$$

$$=$$



$\epsilon_i = \pm$

$$k_i^\mu = \epsilon_i \omega_i q_i^\mu(w_i, \bar{w}_i)$$

$$\mathscr{A}_n(\omega_i, \ell_i, w_i, \bar{w}_i) \equiv \langle out \,|\, \mathscr{S} \,|\, in \rangle$$

momentum-space amplitude

$$\widetilde{\mathscr{A}}(\Delta_i, J_i, w_i, \bar{w}_i) \equiv \langle \prod_{i=1}^n \mathscr{O}^{\epsilon_i}_{\Delta_i, J_i}(w_i, \bar{w}_i) \rangle$$

celestial amplitude

Conformal primary wavefunctions

[Pasterski,Shao'17]

massless: $\Phi^{s=|J|}_{\Delta,J}(X^\mu; w, \bar{w}) \simeq \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta \epsilon_{\mu_1\ldots\mu_s} e^{\pm i\omega q\cdot X}$

on-shell plane waves

4D spin-s field under Lorentz transformations

2D conformal primary with conformal dimension Δ and spin J

Lorentz transformation

bulk point $X^\mu \mapsto \Lambda^\mu{}_\nu X^\nu$

conformal transformation

boundary point $w \mapsto \frac{aw+b}{cw+d}$ $\quad \bar{w} \mapsto \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}$

$ad - bc = 1 = \bar{a}\bar{d} - \bar{b}\bar{c}$

$$\Phi^s_{\Delta,J}\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{\Delta+J}(\bar{c}\bar{w}+\bar{d})^{\Delta-J} \underbrace{D_s(\Lambda)}\Phi^s_{\Delta,J}(X^\mu; w, \bar{w})$$

3+1D spin-s representation of the Lorentz algebra

Radiative: $J = \pm\, s$ & solve the linearized eom for massless spin-s particles

Generalized: $|J| \leq s$ & allow sources and distributions [Pasterski,AP'20]

Conformal primary backgrounds

Conformal primary wavefunctions satisfy Kerr-Schild double copy:

[Pasterski,AP'20]

$$g_{\mu\nu} = \eta_{\mu\nu} + \underbrace{m_\mu m_\nu \varphi^\Delta}_{h_{\Delta,J=\pm 2;\mu\nu}}$$

Kerr-Schild vector m^μ null and geodesic wrt $\eta_{\mu\nu}$ and $g_{\mu\nu}$. [Monteiro,O'Connell,White'14]

$\Rightarrow$ $h_{\Delta,J=\pm 2;\mu\nu}$ gives *exact* radiative *solution to Einstein's equations!*

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Exact *generalized* conformal primary solution to Einstein's equations:

- Aichelburg-Sexl shockwave or ultraboosted Schwarzschild [Aichelburg,Sexl'71]

$$g_{\mu\nu} = \eta_{\mu\nu} - \underbrace{4G_N \alpha q_\mu q_\nu \log(X^2)\delta(q\cdot X)}_{h^{gen}_{\Delta=-1,J=0;\mu\nu}} \qquad E = \alpha q^0$$

generalized conformal primary metric

- Kerr gyraton or ultraboosted Kerr see [Cristofoli'20] [Arkani-Hamed,Huang,O'Connell'20]...

$$g_{\mu\nu} = \eta_{\mu\nu} - 4G_N \alpha q_\mu q_\nu \log(|X^2 - a^2|)\delta(q\cdot X) \qquad a^\mu = s^\mu/m$$

Conformal primary operators

[Pasterski,Shao'17]
[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP,Trevisani'21]

Wavefunction approach connects 2D celestial CFT states to 4D bulk physics via direct construction:

2D celestial CFT operator

4D wavefunction

$$\mathcal{O}^{s,\pm}_{\Delta,J}(w,\bar{w}) \equiv i\left(\hat{O}^s(X^\mu), \Phi^s_{\Delta^*,-J}(X^\mu_\mp; w, \bar{w})\right)_\Sigma$$

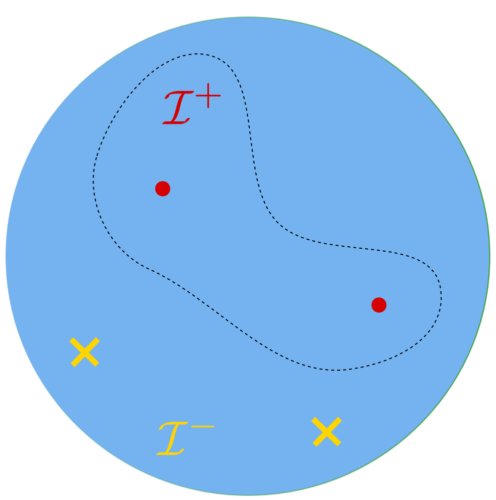
4D bulk operator

expanded into modes

inner product = integral over Cauchy slice Σ in the 4D bulk

e.g. Klein-Gordon norm for $s = 0$

$$i(\phi_1, \phi_2) = \int_{\Sigma_3} d\Sigma^\mu (\phi_1 \partial_\mu \phi_2^* - \phi_2^* \partial \phi_1)$$



Operators inserted on points on the celestial sphere carrying $\pm$ label for *in* / *out* states.

From prescription for analytically continuing the wavefunctions as $X^\mu_\pm = X^\mu \pm i\varepsilon\{-1,0,0,0\}$ or $u \mapsto u \pm i\varepsilon$.

Celestial amplitudes

$\mathcal{S}$-matrix elements constructed as [Pasterski,Shao,Strominger'16+'17]

$$\widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \int_0^\infty \frac{d\omega_i}{\omega_i} \omega_i^{\Delta_i} \mathcal{A}(\omega_i, z_i, \bar{z}_i)$$

transform by construction as correlators of (quasi)-primaries

$J_j = j$-th helicity

$$\widetilde{\mathcal{A}}\left(\Delta_i, \frac{az_i + b}{cz_i + d}, \frac{\bar{a}\bar{z}_i + \bar{b}}{\bar{c}\bar{z}_i + \bar{d}}\right) = \prod_{j=1}^{n} (cz_j + d)^{\Delta_j + J_j} (\bar{c}\bar{z}_j + \bar{d})^{\Delta_j - J_j} \widetilde{\mathcal{A}}(\Delta_i, z_i, \bar{z}_i)\,.$$

since the external particles are in boost eigenstates.

Celestial amplitudes make conformal symmetry manifest.

Spectrum vs Symmetries

[Pasterski,Shao'17]

Basis of finite energy plane waves ⇒ conformal primaries w $\Delta \in 1 + i\mathbb{R}$.

After summing over all energies what are soft particles?

Notion of *soft* particle ↦ ***conformally soft*** particle $\Delta = 1$. [Donnay,AP,Strominger'18]

only tip of ∞ tower of $\Delta \in \frac{1}{2}\mathbb{Z}$!

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only tip of ∞ tower of $\Delta \in \frac{1}{2}\mathbb{Z}$!

How do translations act in the conformal basis?

[Donnay,AP,Strominger'18]
[Stieberger,Taylor'18]

$$\mathscr{P}^\mu = q^\mu e^{\partial_\Delta} \Leftrightarrow \Delta \mapsto \Delta + 1$$

$$\int_0^\infty \frac{d\omega}{\omega} \omega^\Delta \omega = \int_0^\infty \frac{d\omega}{\omega} \omega^{\Delta+1}$$

Translations **shift** the conformal dimension Δ.

Celestial amplitudes obscure translation symmetry.

Conformally soft limits and celestial currents

Soft graviton theorem:

$$\lim_{\omega\to 0} \mathscr{A}^{\pm}_{n+1} = \left[\omega^{-1} S^{(0)\pm}_n + \omega^{0} S^{(1)\pm}_n + \mathcal{O}(\omega)\right] \mathscr{A}_n$$

$$\int_0^{\omega_*} \frac{d\omega}{\omega}\omega^{\Delta+\#} \sim \frac{1}{\Delta+\#}$$

Soft limits at different orders in $\omega \to 0 \Leftrightarrow$ "conformally soft" Δ poles.

Conformally soft limits and celestial currents

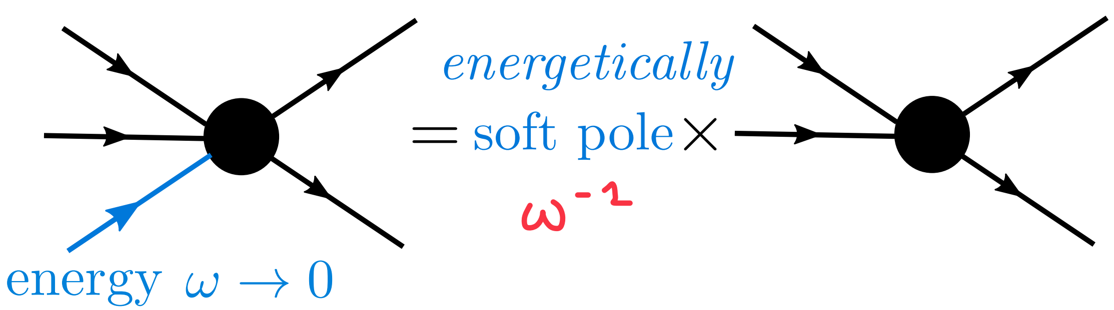
Soft graviton theorem:

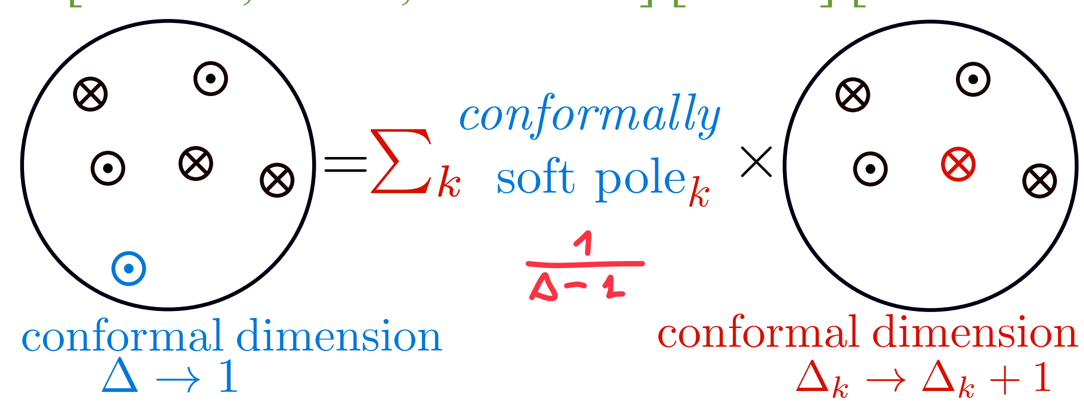
$$\lim_{\omega\to 0} \mathscr{A}^{\pm}_{n+1} = \left[\omega^{-1} S_n^{(0)\pm} + \omega^{0} S_n^{(1)\pm} + \mathscr{O}(\omega)\right] \mathscr{A}_n$$

$$\int_0^{\omega_*} \frac{d\omega}{\omega}\omega^{\Delta+\#} \sim \frac{1}{\Delta+\#}$$

Soft limits at different orders in $\omega \to 0 \Leftrightarrow$ "conformally soft" Δ poles.

Leading soft graviton: [Weinberg'65] [Adamo,Mason,Sharma'19] [AP'19] [Guevara'19]





$$\mathscr{P}_w \mathscr{O}_\omega(z,\bar{z}) \sim \frac{\omega}{w-z}\mathscr{O}_\omega(z,\bar{z}) \quad \xrightarrow{\int_0^\infty \frac{d\omega}{\omega}\omega^{\Delta}} \quad \mathscr{P}_w \mathscr{O}_\Delta(z,\bar{z}) \sim \frac{1}{w-z}\mathscr{O}_{\Delta+1}(z,\bar{z})$$

supertranslation current

[Strominger'13]

unusual looking OPE!

[Donnay,AP,Strominger'18]

Asymptotic symmetries

Continue finite energy $\Delta = 1 + i\mathbb{R}$ to conformally soft $\Delta \in \frac{1}{2}\mathbb{Z}$.

Conformally soft limits of celestial amplitudes as ***asymptotic symmetries***:

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi_{\Delta^*,-J})_\Sigma$$

asymptotic symmetry generator

pure gauge

[Donnay,AP,Strominger'18]
[Donnay,Pasterski,AP'20]
[Pasterski,AP'20]
[Pano,Pasterski,AP'21]

$s = \vert J \vert$	Δ	$\tilde{\Delta} = 2 - \Delta$	energetically soft pole	celestial current	asymptotic symmetry
1	1	1	ω^{-1}	$\mathcal{J}$	large U(1)
$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\omega^{-\frac{1}{2}}$	$\mathcal{S}$ & $\tilde{\mathcal{S}}$	large SUSY
2	1	1	ω^{-1}	$\mathcal{P}$	supertranslation
2	0	2	ω^{0}	$\mathcal{T}$ & $\tilde{\mathcal{T}}$	superrotation

2D stress tensor!

An ∞ of new celestial symmetries?

Continue finite energy $\Delta = 1 + i\mathbb{R}$ to conformally soft $\Delta \in \frac{1}{2}\mathbb{Z}$.

Conformally soft limits of celestial amplitudes as ***new celestial symmetries***:

$$\mathcal{O}_{\Delta,J} \equiv i(\hat{O}, \Phi_{\Delta^*,-J})_\Sigma$$

conformally soft
but ***not*** pure gauge

- $\exists$ more (conformally) soft theorems:
  e.g. subsubleading soft graviton $\Delta = -1$

[Fotopoulos,Taylor,Stieberger,Zhu'20]
[Adamo,Mason,Sharma'19][Guevara,19]

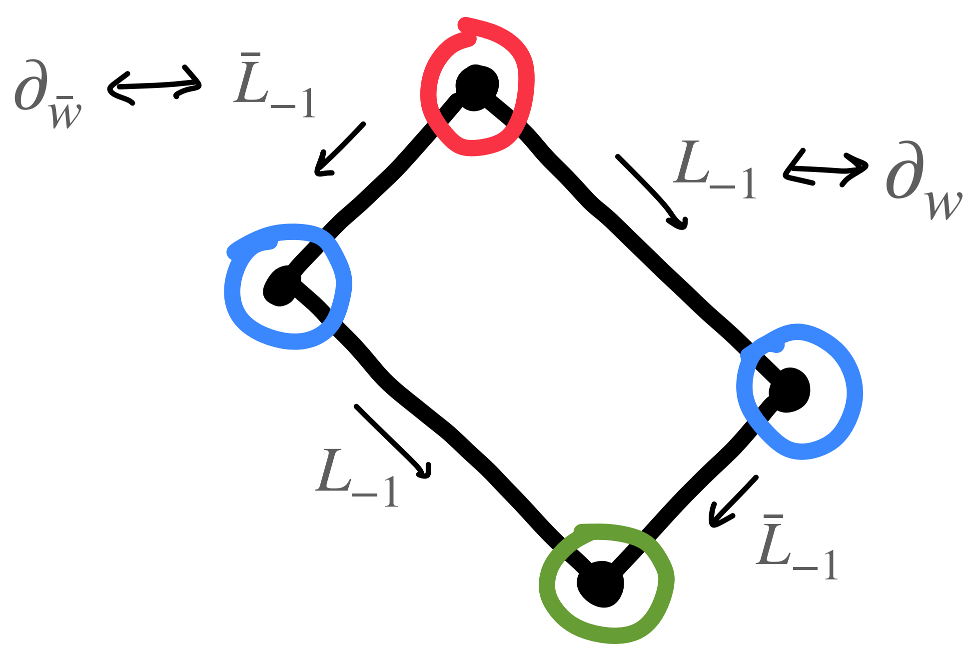
- $\exists$ ∞ many more conformally soft $\mathcal{O}_{\Delta=-n,J}$
  e.g. $w_{1+\infty}$ symmetry

[Strominger'21]
[Guevara,Himwich,Pate,Strominger'21]
[Himwich,Pate,Singh'21] [Jiang'21]

⇒ Conformally soft limits without clear asymptotic symmetry interpretation.

Global conformal multiplets

All celestial symmetries can be described in unified celestial CFT framework via $SL(2,\mathbb{C})$ conformal multiplets or "celestial diamonds". [Pasterski,AP,Trevisani'21] [Banerjee'18] [Banerjee,Pandey,Paul'19]



● $SL(2,\mathbb{C})$ primary annihlated by $L_1, \bar{L}_1$

↙↘ $L_{-1}, \bar{L}_{-1}$ generate descendants

$\Rightarrow$ classify all primary descendants

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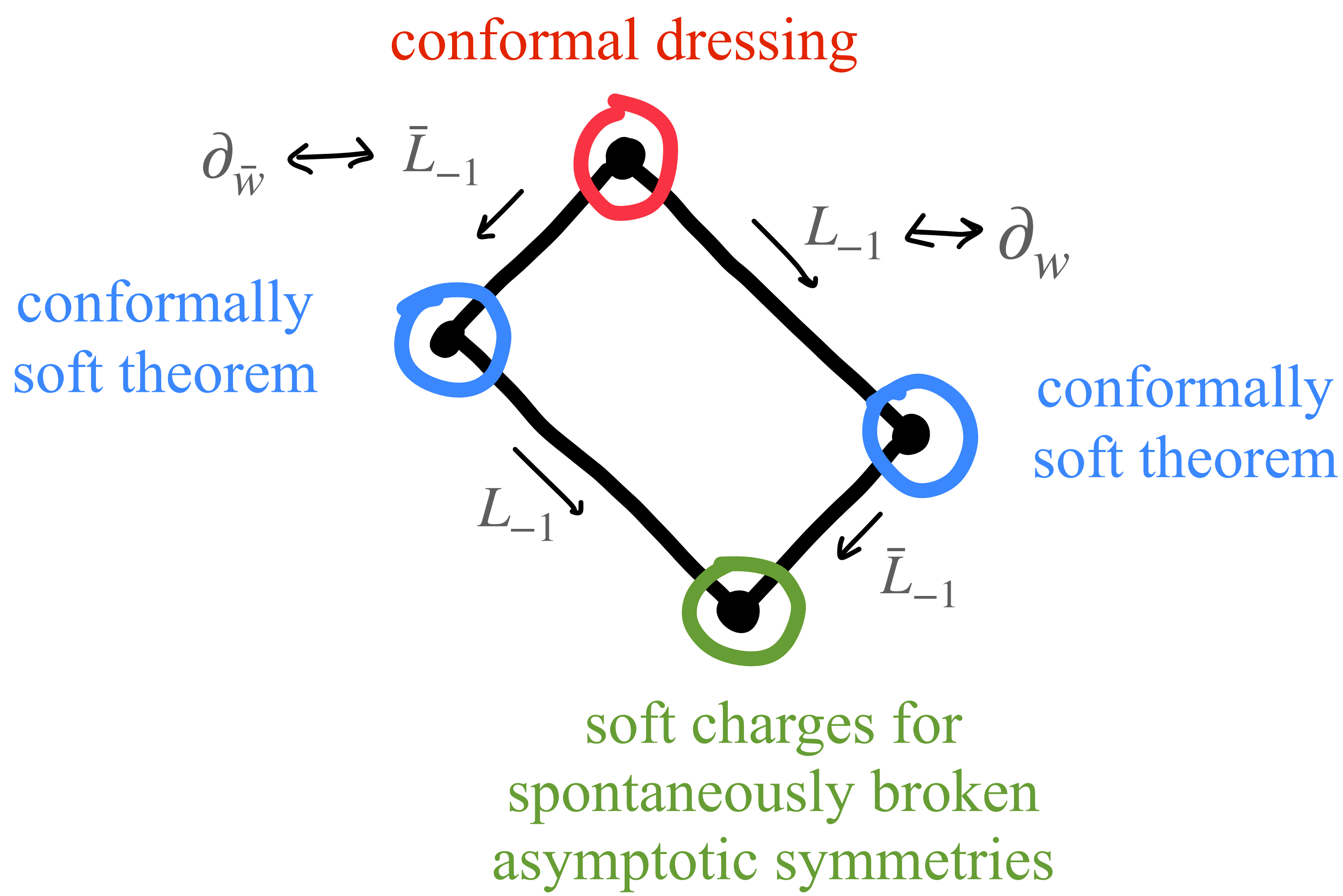


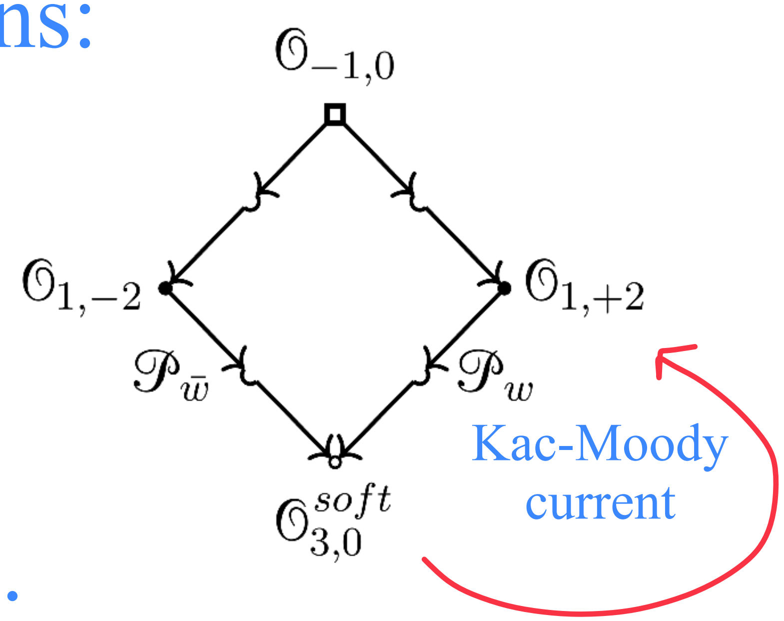
- $SL(2,\mathbb{C})$ primary annihlated by $L_1, \bar{L}_1$
- ↙↘ $L_{-1}, \bar{L}_{-1}$ generate descendants

$\Rightarrow$ classify all primary descendants

BMS current and stress tensor

Supertanslations:

[Banerjee'18]
[Banerjee,Pandy,Paul'19]
[Pasterski,AP,Trevisani'21]

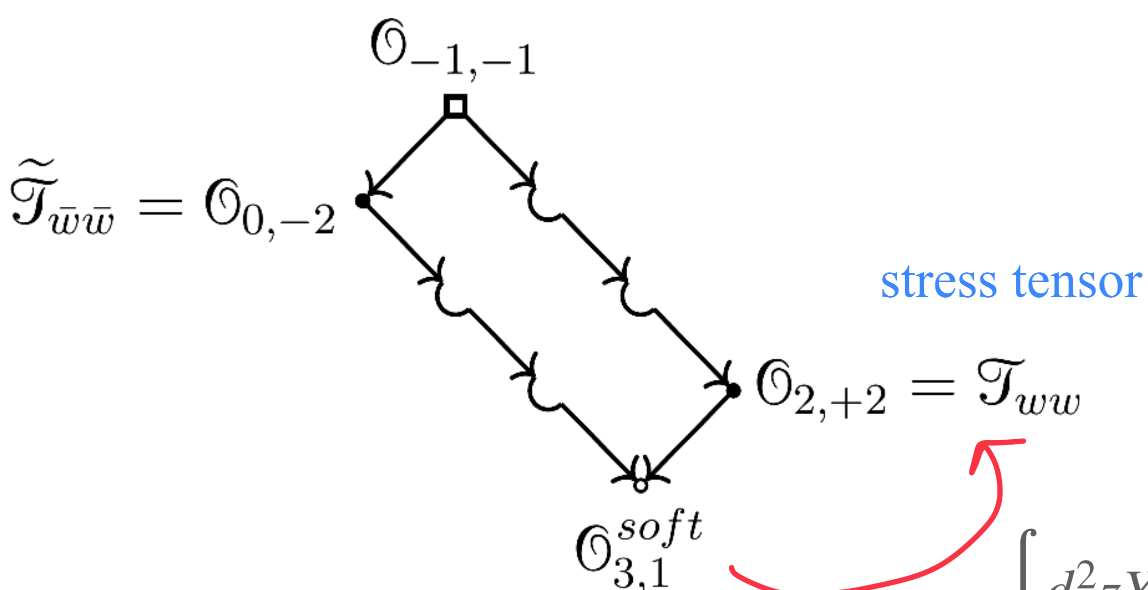


(conformally) soft theorems $(\Delta, J) = (1, +2)$ $(h, \bar{h}) = (\frac{3}{2}, -\frac{1}{2})$

$$\int d^2z f_{ww} \mathcal{O}^{soft}_{3,0} = Q_S[f_{ww}]$$

soft part of supertranslation charge

Superrotations:



(conformally) soft theorems $(\Delta, J) = (2, +2)$ $(h, \bar{h}) = (2, 0)$

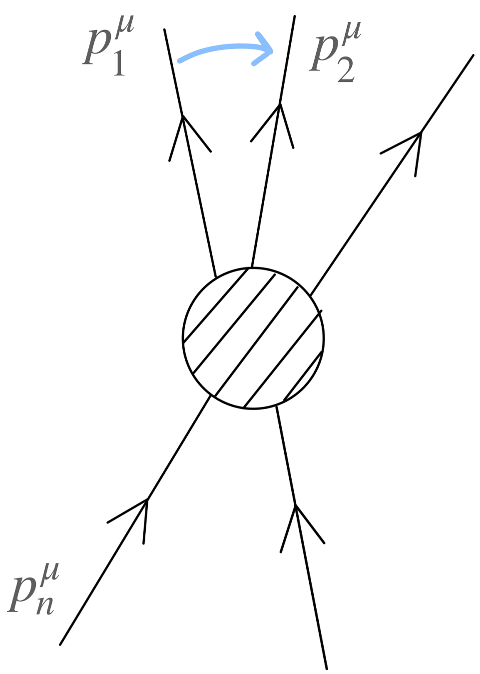
$$\int d^2z Y^z_{ww} \mathcal{O}^{soft}_{3,+1} = Q_S[Y^z_{ww}]$$

soft part of superrotation charge

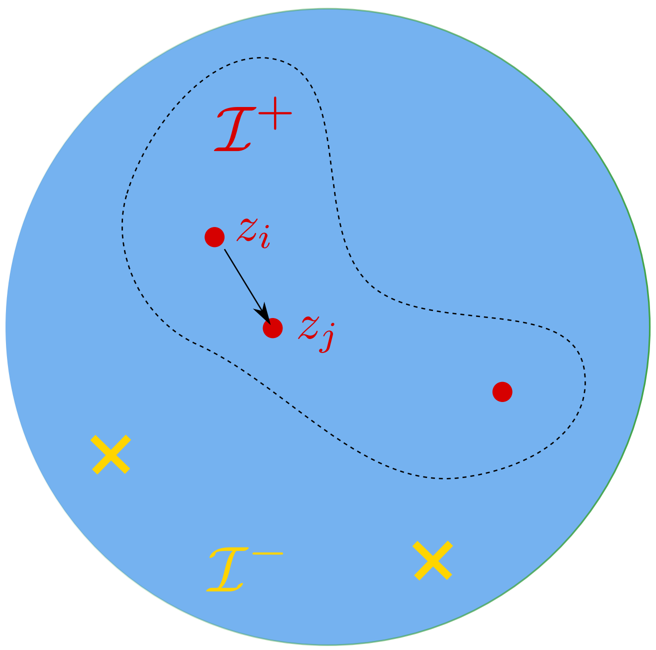
Soft operators part of super-momentum and super-angular momentum of BMS flux algebra.

[Barnich,Ruzziconi'21] [Donnay,Ruzziconi'21]

Collinear limits ⇒ celestial OPEs



The collinear limit of 4D amplitudes is captured by the 2D celestial OPE:



$$p_i^\mu \,||\, p_j^\mu \qquad \leftrightarrow \qquad z_{ij} \equiv z_i - z_j \to 0$$

Brute force from Mellin transform of collinear amplitudes limit:

$$\mathcal{A}_{\ell_1,\ldots,\ell_n}(p_1,\ldots,p_n) \overset{z_i \to z_j}{\rightarrow} \sum_{\ell} \mathrm{Split}^{\ell}_{\ell_i \ell_j}(p_i,p_j)\mathcal{A}_{\ell_1\ldots\ell\ldots\ell_n}(p_1,\ldots,P,\ldots,p_n)$$

$$P^\mu = p_i^\mu + p_j^\mu \qquad \omega_p = \omega_i + \omega_j$$

$$\lim_{z_i \to z_j} \mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)\mathcal{O}_{\Delta_j,J_j}(z_j,\bar{z}_j) = \int_0^\infty \frac{d\omega_i}{\omega_i}\omega_i^{\Delta_i}\int_0^\infty \frac{d\omega_j}{\omega_j}\omega_j^{\Delta_j}\mathrm{Split}^{\ell}_{\ell_i\ell_j}(p_i,p_j)\,|\,P,\ell\rangle + \ldots$$

[Fan,Fotopoulos,Taylor'19]
[Fotopoulos,Stieberger,Taylor,Zhu'19]

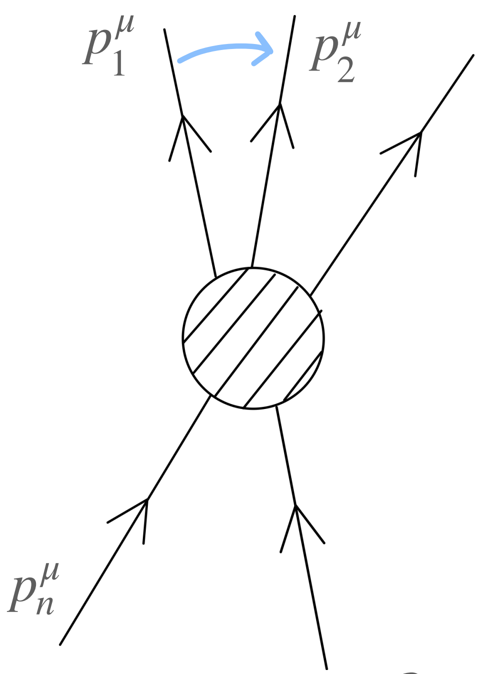
$$\mathrm{Split}^2_{22}(p_i,p_j) = -\frac{\kappa}{2}\frac{\bar{z}_{ij}}{z_{ij}}\frac{\omega_P^2}{\omega_i\omega_j}$$

Euler beta fct $\quad B(x,y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

celestial OPE of gravitons:

$$\mathcal{O}_{\Delta_i,+2}(z_i)\mathcal{O}_{\Delta_j,+2}(z_j) \sim -\frac{\kappa}{2}\frac{\bar{z}_{ij}}{z_{ij}}B(\Delta_i - 1,\Delta_j - 1)\mathcal{O}_{\Delta_i+\Delta_j;+2}(z_j)$$

Collinear limits ⇒ celestial OPEs



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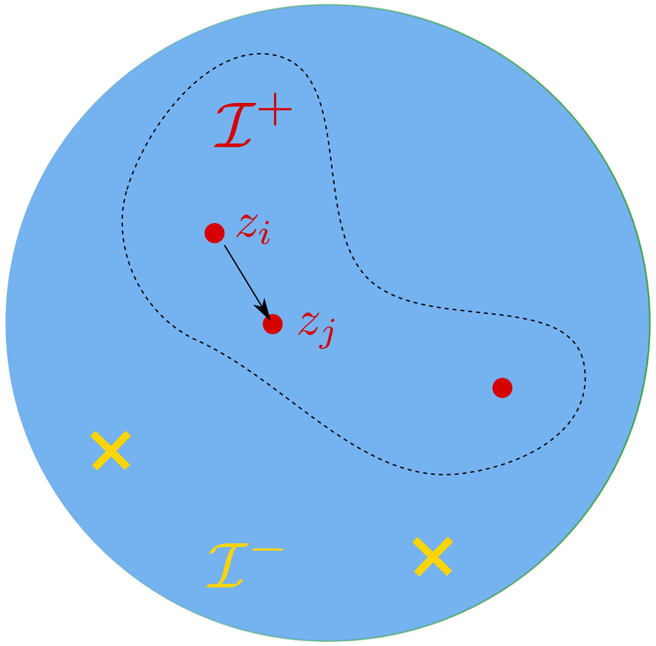


$$p_i^\mu || p_j^\mu \quad \leftrightarrow \quad z_{ij} \equiv z_i - z_j \to 0$$

From symmetry considerations:

[Pate,Raclariu,Strominger,Yuan'19]

$$\mathcal{O}_{\Delta_i,+2}(z_i,\bar{z}_i)\mathcal{O}_{\Delta_j,+2}(z_j,\bar{z}_j) \sim \frac{\bar{z}_{ij}}{z_{ij}} C(\Delta_i,\Delta_j)\mathcal{O}_{\Delta_i+\Delta_j,+2}(z_j,\bar{z}_j)$$

- Translation invariance: $C(\Delta_i,\Delta_j) = C(\Delta_i+1,\Delta_j) + C(\Delta_i,\Delta_j+1)$
- Residue of pole at $\Delta_i \to 1$ fixed by leading soft graviton theorem: $\lim_{\Delta_i\to 1} C(\Delta_i,\Delta_j) \sim -\frac{\kappa}{2}\frac{1}{\Delta_i - 1}$
- Subsubleading soft graviton: extra recursion relation

celestial OPE of gravitons:

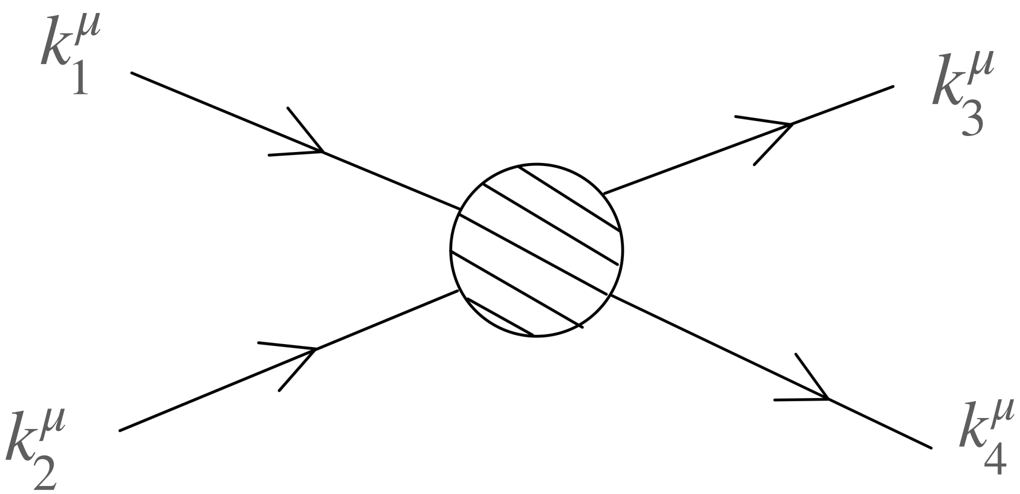
$$\mathcal{O}_{\Delta_i,+2}(z_i)\mathcal{O}_{\Delta_j,+2}(z_j) \sim -\frac{\kappa}{2}\frac{\bar{z}_{ij}}{z_{ij}} B(\Delta_i - 1,\Delta_j - 1)\mathcal{O}_{\Delta_i+\Delta_j,+2}(z_j)$$

Celestial amplitudes from UV to IR

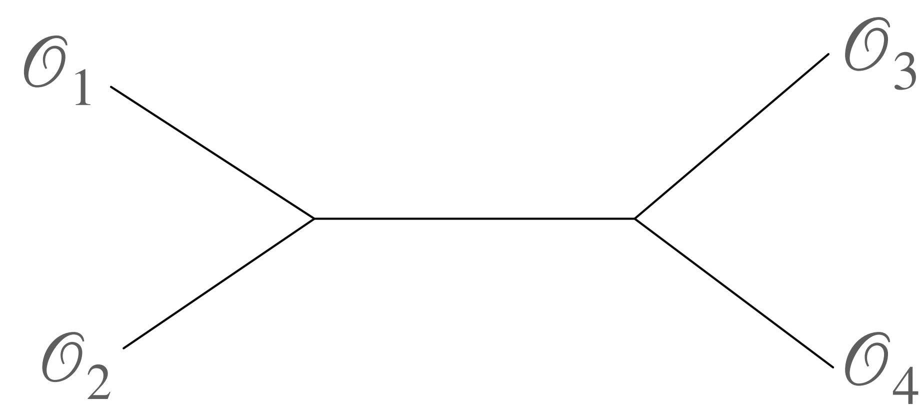
Anti-Wilsonian paradigm: ***celestial amplitude probes all energy scales***

E.g. classical gravity amplitudes divergent $\Rightarrow$ string theory [Stieberger,Taylor'18]

[Arkani-Hamed,Pate,Raclariu,Strominger'20]



$\longleftrightarrow$



$$z = \frac{z_{13}z_{24}}{z_{12}z_{34}}$$

$$\mathcal{A}(k_j^\mu) = M(k_j^\mu)\,\delta^{(4)}\Big(\sum_j k_j^\mu\Big)$$

$$\widetilde{\mathcal{A}}(\Delta_j, z_j, \bar{z}_j) = \mathrm{X}\,\widetilde{\mathcal{A}}(\beta, z) \qquad \beta \equiv \sum_{j=1}^{4}(\Delta_j - 1)$$

$$\mathrm{X} \propto \prod_{i<j}^{4} z_{ij}^{h/3-h_i-h_j}\,\bar{z}_{ij}^{\bar{h}/3-\bar{h}_i-\bar{h}_j}\,\delta(i(z-\bar{z}))$$

kinematic factor

Stripped amplitude highly UV sensitive:

$$\widetilde{\mathcal{A}}(\beta, z) = \int_0^\infty \frac{d\omega}{\omega}\omega^\beta M(\omega^2, -z\omega^2)$$

Poles at $\beta \in 2\mathbb{Z}_>$ in field theory $\Rightarrow$ disappear in quantum gravity

$M \sim \sum a_m^{UV}\omega^{-2m}$ $\qquad$ $M \sim e^{-\alpha'\omega^2}$

Gravity = Gauge Theory^2

[Bern,Carrasco,Johansson'10] [Casali,AP'20]

$$\mathcal{M}(\,.\,) = \int_0^\infty \frac{d\omega}{\omega}\omega^\Delta(\,.\,)$$

manifest translation symmetry $\mapsto$ *manifest conformal symmetry*

gluon amplitude

$e^{ik\cdot X}$ ↓

color, kinematic

$$\mathcal{A}_n^{YM} = \delta^{(4)}\Big(\sum_{i=1}^n k_i^\mu\Big)\sum_{\gamma\in\Gamma}\frac{c_\gamma n_\gamma}{\Pi_\gamma}$$

propagator

double copy

$$c \mapsto n$$

$$\mathcal{A}_n^{G} = \delta^{(4)}\Big(\sum_{i=1}^n k_i^\mu\Big)\sum_{\gamma\in\Gamma}\frac{n_\gamma^2}{\Pi_\gamma}$$

graviton amplitude

kinematic functions $\mapsto$ kinematic operators

$n(\omega)$ $\qquad$ $\mathcal{N}(e^{\partial_\Delta})$

celestial gluon amplitude

$\Phi_{\Delta,J}$ ↓

$$\widetilde{\mathcal{A}}_n^{YM} = \sum_{\gamma\in\Gamma} c_\gamma \mathcal{N}_\gamma \widetilde{\mathcal{A}}_n^{scalar}$$

$$c \mapsto \mathcal{N}$$

celestial double copy

$$\widetilde{\mathcal{A}}_n^{G} = \sum_{\gamma\in\Gamma} \mathcal{N}_\gamma^2 \widetilde{\mathcal{A}}_n^{scalar}$$

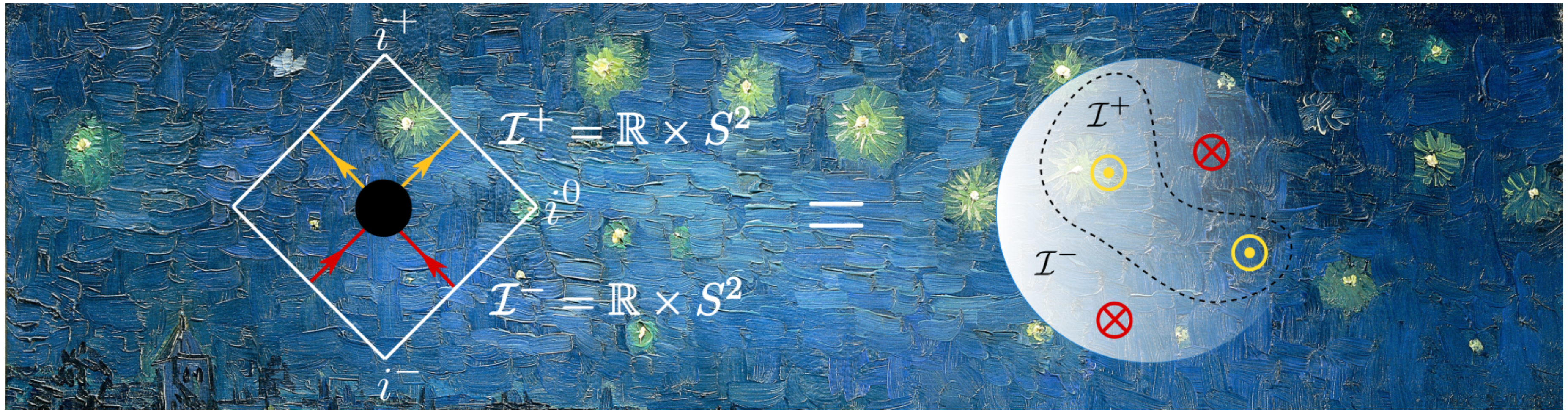
celestial graviton amplitude

$\Rightarrow$ *Operator-valued celestial double copy.*

Similar results for ambitwistor strings in AdS. [Roehrig,Skinner'07] [Eberhardt,Komatsu,Mizera'07]

Also celestial double copy from ambitwistor string worldsheet. [Casali,Sharma'20]

Outlook: celestial amplitudes



S-matrix as celestial correlator:

- makes asymptotic $\supset$ conformal symmetries manifest
- reorganizes (conformally) soft and collinear behavior:

Collinear limits $\Rightarrow$ OPE data for celestial CFT.

Conformally soft sector: ∞ (asymptotic) symmetries & conformal dressings.

- anti-Wilsonian paragidm: probes all energies, sensitive to UV

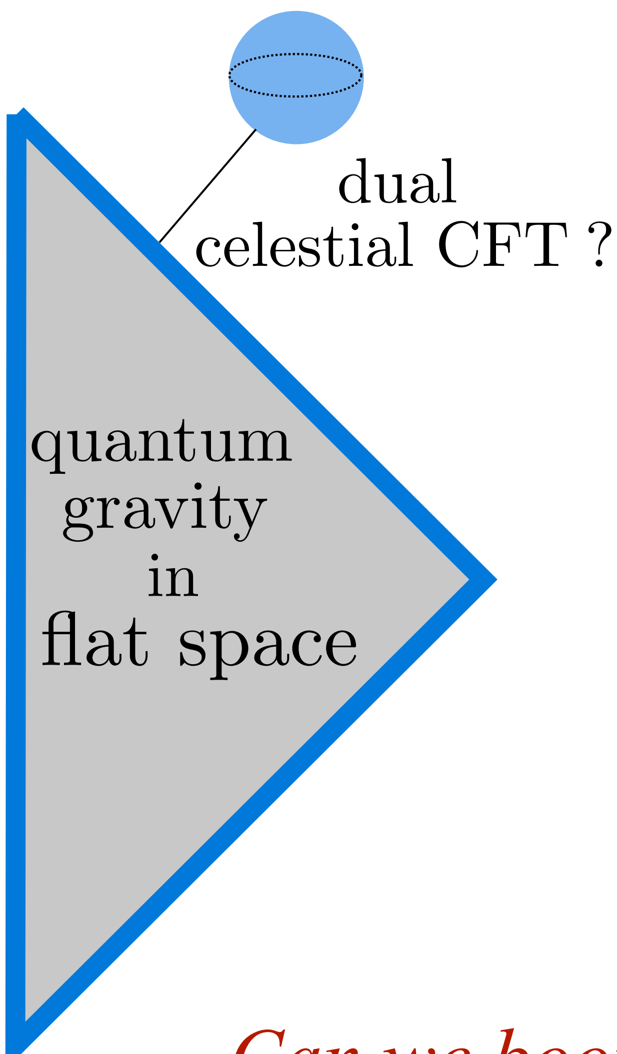
Novel framework for studying scattering amplitudes!

Outlook: celestial holography

Novel approach to bootstrapping quantum gravity in flat space.

$$_{\text{boost}}\langle out \,|\, \mathcal{S} \,|\, in \rangle_{\text{boost}} = \langle \mathcal{O}^{\pm}_{\Delta_1, J_1}(w_1, \bar{w}_1) \ldots \mathcal{O}^{\pm}_{\Delta_n, J_n}(w_n, \bar{w}_n) \rangle_{\text{celestial CFT}}$$

What is this CFT?



What is the spectrum?

What are all the symmetries?

What is the organizing principle?

Can we bootstrap the (conformally) soft sector?

...

Can we bootstrap quantum gravity in asymptotically flat space?

Exploration of celestial territory has only begun!

Thank you!