Thoughts on combining the experimental data

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Including content from

Isidori, Lancierini, Owen, Serra, arXiv:2104.05631

Isidori, Lancierini, Mathad, Owen, Serra, Coutinho, arXiv:2110.09882

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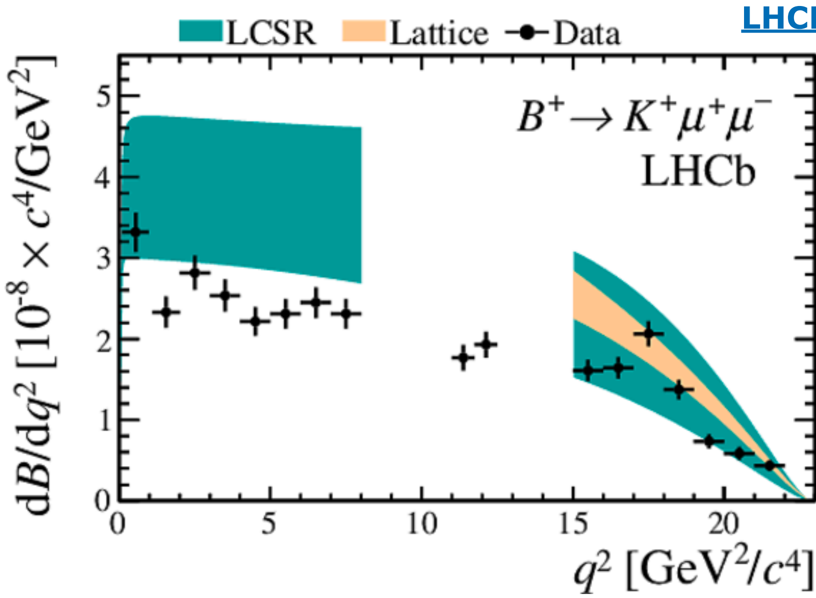
Some general considerations

- Statistical correlations between measurements, if large, will be given in the paper.
- Systematic correlations are instead often implicit.
  - e.g. b—>sμμ branching fraction.

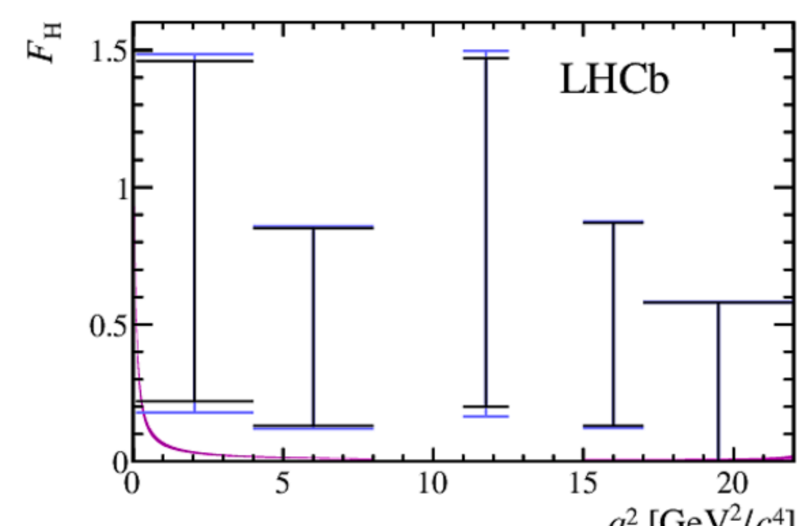


LHCB-PAPER-2014-006

q^2 range (GeV^2/c^4)	central value	stat	syst
$1.1 < q^2 < 6.0$	24.2	0.7	1.2
$15.0 < q^2 < 22.0$	12.1	0.4	0.6

- Less obvious is the correlation across papers. E.g. B_s^0—>ϕμμ.
- What to do if one is given an 68% CL, but nothing else?
  - Full FC scans now included in CDS material.

LHCB-PAPER-2014-007



Questions the data can answer

- Two questions to ask of the data:
  - What are values of Wilson Coefficients? and do they deviate from the SM?
  - What is the global statistical significance of the new physics hypothesis?
- 1st question involves disentangling hadronic effects from new physics, requires theoretical calculations, models, amplitude fits etc.
  - While very important, this is not the focus of this talk.
- Instead we focus on the second question, and construct a highly general alternative hypothesis to produce a conservative answer.
  - We use C_9 as a SM nuisance parameter.
  - A case for making the hypothesis general (look-elsewhere-effect).
  - Inclusion of so-called non-exclusive R ratios (connection to Yasmine and Gianluca's talk).

A case for generality

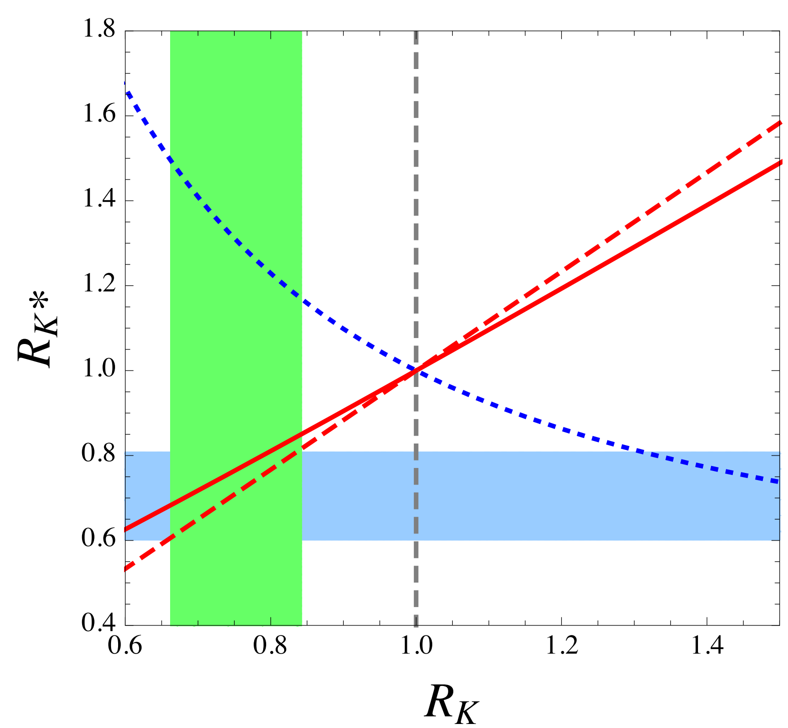
- Want to provide significance which can stand up to the skeptical.
- Original idea: Only combine observables for which there is wide consensus on the SM prediction.
    - Concretely: Combine $B(B_s^0 \to \mu\mu)$ and LFU ratios and fit for $C_{9,10}^{\mu}$.
- However, such a fit smuggles in information about the rest of the system:
    - No new physics in electrons (understandably justified from BF measurements).
    - No right-handed currents or scalar new physics (also reasonable).
- As we argued in [1], constraining the observables and alternative hypothesis in way leads to an overestimation of the significance via the look-elsewhere effect.
    - Only combining the observables that deviate.
    - Fitting with a restricted set of operators.

[1] Isidori, Lancierini, Owen, Serra, arXiv:2104.05631

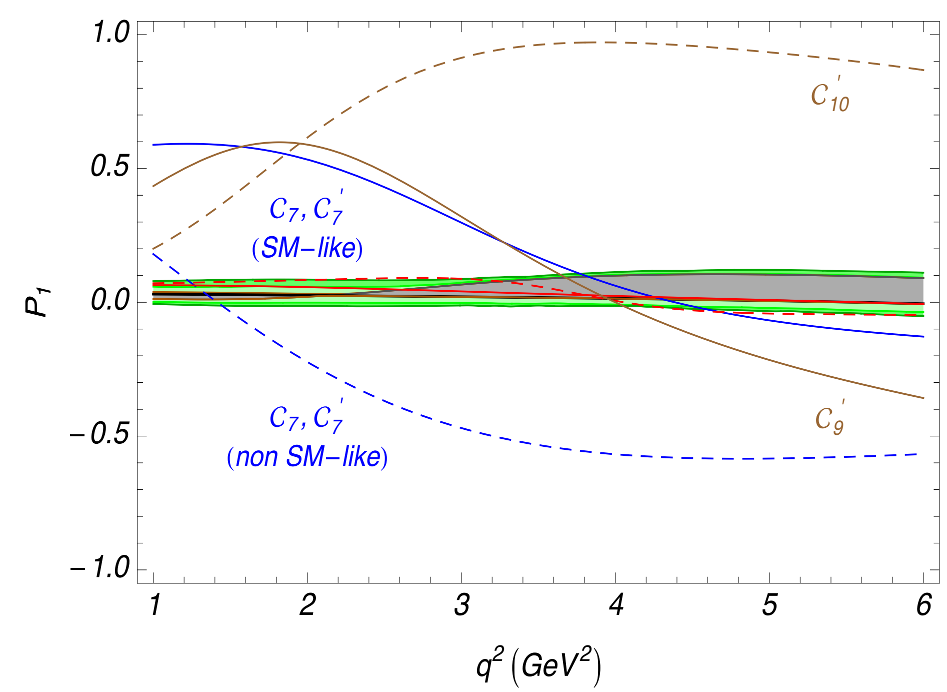
The look elsewhere effect

- The look elsewhere effect (LEE) occurs when the alternative hypothesis test implicitly uses the central values of the data.
    - Example is testing the presence of a resonance of the mass seen in data.
- In our case, floating only left-handed LFUV Wilson Coefficients ignores alternate universes where we saw deviations in K*μμ and/or different values for R_K and R_{K^*}.

Hiller, Nisandzic, 2017



Matias et al, 2012



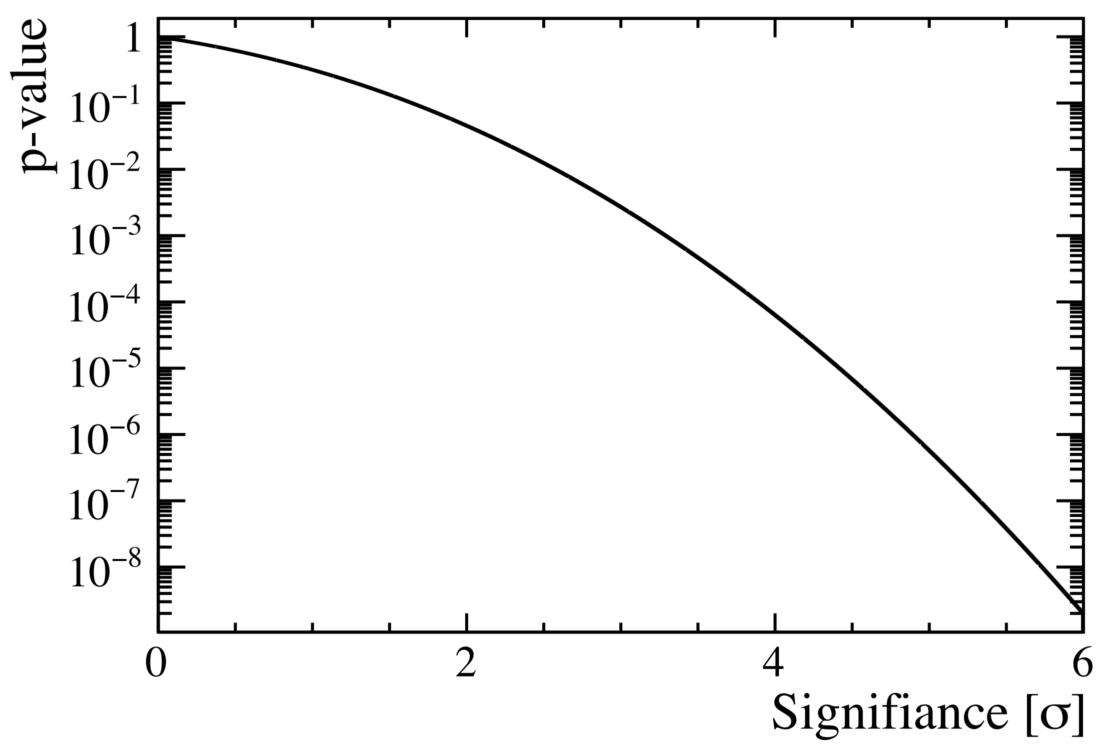
- Fitting only LFU/clean observables are therefore **local** significances, in analogy with local p-value at a particular mass point in a bump hunt.

Common talking points on the LEE

1. **The LEE only occurs in mass peak searches.** The look-elsewhere effect originates from the 1950s, known as the *problem of multiple comparisons,* and can effect any situation whereby the data is implicitly used twice. Also known as the post-hoc analysis.

2. **The question can always be bigger, meaning that the LEE is subjective.** The main purpose of the LEE is to take you back to a hypothesis test which is a-piori, it does not always need to be bigger. (Otherwise it would be impossible to claim anything).

3. **The LEE has a smaller effect for large significances.** A trial factor of 10 will dilute a significance of 3.0σ to 2.2σ whereas a 5.0σ will only move to 4.4σ. This is simply due to the non-linear relationship between a Gaussian significance and the p-value.



Approach in arXiv:2104.05631

- Write down all operators to which LHCb measurements were sensitive to.

$$\mathcal{O}_9^\ell = (\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell)\,, \quad \mathcal{O}_{10}^\ell = (\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$
$$\mathcal{O}_9^{\ell\prime} = (\bar{s}_R\gamma_\mu b_R)(\bar{\ell}\gamma^\mu\ell)\,, \quad \mathcal{O}_{10}^{\ell\prime} = (\bar{s}_R\gamma_\mu b_R)(\bar{\ell}\gamma^\mu\gamma_5\ell)\,,$$
$$\mathcal{O}_{\hat{S}}^\ell = (\bar{s}_L b_R)(\bar{\ell}_R\ell_L)\,, \quad \mathcal{O}_{\hat{S}}^{\ell\prime} = (\bar{s}_R b_L)(\bar{\ell}_L\ell_R)\,.$$

- Use amplitudes and form factors from Flavio[1] to translate observables into WC space.
- Include C7$^{(')}$ with a-priori constraint from b—>sγ B-factory results (0.2σ impact).
- Consider one scalar contributes to B_s—>μμ (only one independent contribution assuming Λ_{NP}>VEV) [Alonso, 2014].
- Assume no scalars in Kee.
- Assume that WC are real.
- End up with 9 WC to which the measurements are sensitive.
  - 4 muonic WC: C_9, C_{10}, C_9',C_{10}'
  - 4 LFUV WC: ΔC_9, ΔC_{10}, $\Delta C_9'$,$\Delta C_{10}'$ [Difference in muonic/electronic WC]
  - 1 scalar contribution in muons: C_s-C_s'

[1] arXiv:1810.08132

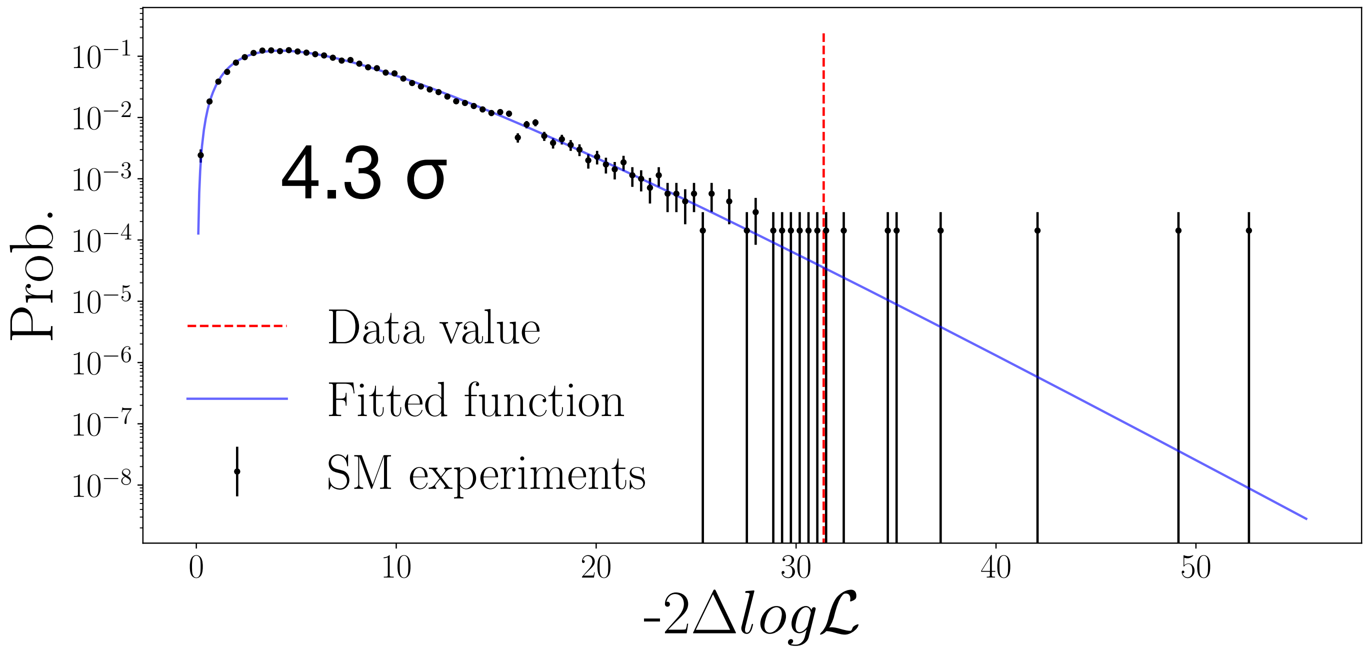
Approach in arXiv:2104.05631

- Observables included: R_K, R_{K^*}, $B_s \rightarrow \mu\mu$ and $B^0 \rightarrow K^{*0}\mu\mu$ angular analysis.
- Generate toys based from SM predictions and experimental uncertainties.
  - R_K and R_{K^*} uses full likelihood but assumed to be independent.
  - $K^*\mu\mu$ observables generated/fit with full experimental correlations.
- For each toy calculate test statistic.

$$\Delta\chi^2 = -2\log\frac{\mathcal{L}(X|\Delta\hat{C}_9^U, C_i^{\mathrm{SM}})}{\mathcal{L}(X|\hat{C}_i)}$$

- Compare distribution of SM toys to one seen in data. Integrate to get significance.

Isidori, Lancierini, Owen, Serra, arXiv:2104.05631



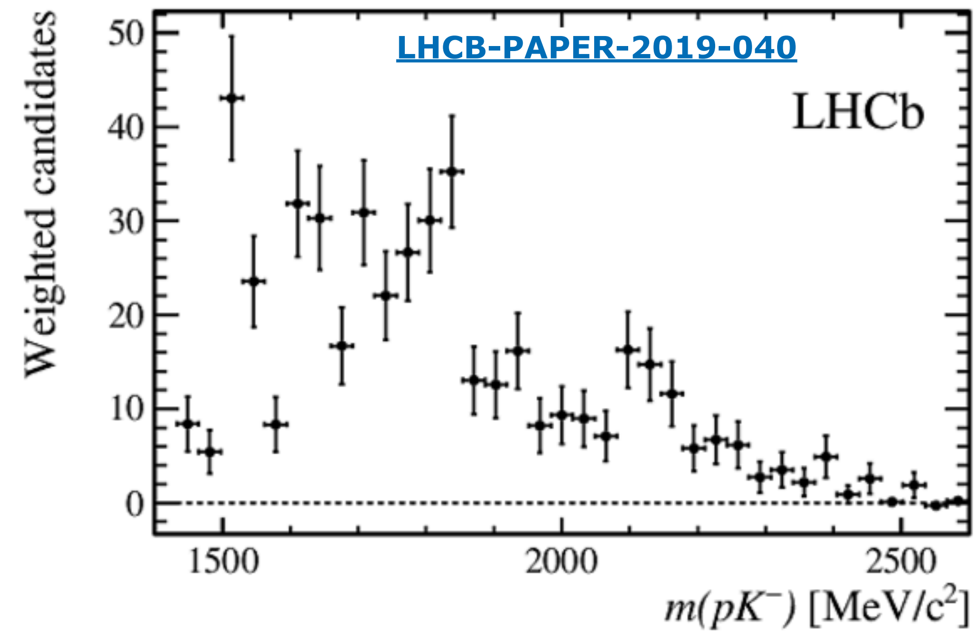
- Why use toys and not Wilk's theorem?
  - Can get flat directions in WC space.
  - Effective degrees of freedom in system not necessarily integer number.
  - E.g. $B_s \rightarrow \mu\mu$ and C_{10} vs C_S.

Importance of non-exclusive LFU ratios

- Several LFU ratios contain decays with broad, overlapping resonances whose hadronic structure is unknown.

- Most prominent example: R_{pK}

$$\boxed{R_{pK} = \frac{\mathcal{B}(\Lambda_b^0 \to pK^-\mu^+\mu^-)}{\mathcal{B}(\Lambda_b^0 \to pK^-e^+e^-)}}$$



- If R_{pK} deviates significantly from unity then its NP. However, translating that into into WC space is tricky for such a large mass range.
    - Experimental/theoretical progress would help in this particular case.
- However, the LFU ratio is likely to stay inclusive, as it maximises the precision.
    - Can we include these measurements already in a combination?

Inclusion of non-exclusive LFU ratios

Isidori et al, arXiv:2110.09882

- Neglecting lepton masses ($q^2 >> m_l^2$), no interference between left and right handed lepton currents. [Hiller, Schmaltz, 2014]

$$\frac{d\Gamma^\ell_X}{dq^2} = \frac{d\Gamma^\ell_{X,L}}{dq^2} + \frac{d\Gamma^\ell_{X,R}}{dq^2}$$

$$\frac{d\Gamma^\ell_{X,R}}{dq^2} = \frac{d\Gamma^\ell_{X,L}}{dq^2}\Bigg|_{\{C^\ell_L \to C^\ell_R,\ C^{\ell\prime}_L \to C^{\ell\prime}_R\}}$$

$$C^\ell_L = C^\ell_9 - C^\ell_{10}\,, \qquad C^{\ell\prime}_L = C^{\ell\prime}_9 - C^{\ell\prime}_{10}$$
$$C^\ell_R = C^\ell_9 + C^\ell_{10}\,, \qquad C^{\ell\prime}_R = C^{\ell\prime}_9 + C^{\ell\prime}_{10}$$

Get general formula, applicable to any LFU ratio, but only used for non-exclusive modes.

$$\boxed{R_X = \frac{\left\{\left(C^\mu_L\right)^2 + \left(C'^\mu_L\right)^2 + \langle\eta^0_X\rangle C^\mu_L C'^\mu_L + C_7 \cdot \left(\langle\eta^{77}_X\rangle C_7 + \langle\eta^{79}_X\rangle C^\mu_L + \langle\eta'^{79}_X\rangle C'^\mu_L\right)\right\} + (L \to R)}{\left\{\left(C^e_L\right)^2 + \left(C'^e_L\right)^2 + \langle\eta^0_X\rangle C^e_L C'^e_L + C_7 \cdot \left(\langle\eta^{77}_X\rangle C_7 + \langle\eta^{79}_X\rangle C^e_L + \langle\eta'^{79}_X\rangle C'^e_L\right)\right\} + (L \to R)}}$$

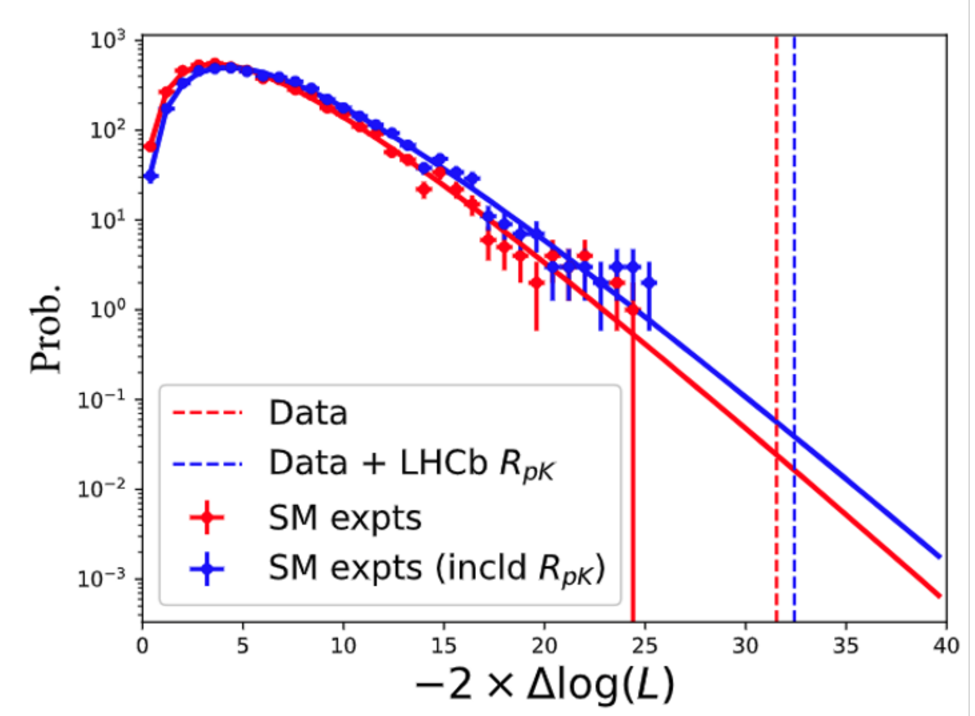
The η parameters encode hadronic information

$$F^\ell_X = \int_{q^2_{\rm min}}^{q^2_{\rm max}} f^\ell_X(q^2)dq^2$$
$$\left\langle\eta^{i,\ell}_X\right\rangle = \frac{1}{F^\ell_X}\int_{q^2_{\rm min}}^{q^2_{\rm max}} f^\ell_X(q^2)\eta^i_X(q^2)dq^2.$$

Parameter	Limits	
	default	4q-ops
$\langle\eta^0_X\rangle$	[-2,2]	
$\langle\eta^{79}_X\rangle$	[-12,12]	[-20,20]
$\langle\eta^{79\prime}_X\rangle$	[-4,4]	[-10,10]
$\langle\eta^{77}_{pK}\rangle$	[0,120]	[0, 160]
$\langle\eta^{77}_{K\pi,K\pi\pi}\rangle$	[0,60]	[0,100]

Impact on the significance

Isidori, Lancierini, Mathad, Owen, Serra, Coutinho, arXiv:2110.09882



- Inclusion of R_{pK} slightly decreases significance.
- Why?
  - Not perfectly aligned with other R ratios.
  - Quite consistent with SM.
  - Lots of hadronic uncertainty.
- Projecting precision from full run II shows substantial improvement in discovery potential.
  - Non-exclusive ratios can have a large impact here.
- Fixing the hadronic parameters has reasonably small impact on significance (<0.5σ).

Linearising the expression w.r.t. $\Delta(C_i)$ and neglecting interference with suppressed amplitudes (e.g. C_i').

$$R_X - 1 \approx \frac{\mathrm{Re}\left(2\frac{\Delta C_L}{C_L^{\mathrm{SM}}} + \langle\eta_X^0\rangle\frac{\Delta C_L'}{C_L^{\mathrm{SM}}}\right)}{1 + \langle\eta_X^{77}\rangle\left|\frac{C_7^{\mathrm{SM}}}{C_L^{\mathrm{SM}}}\right|^2 + \mathrm{Re}\left[\langle\eta_X^{79}\rangle\frac{C_7^{\mathrm{SM}}}{C_L^{\mathrm{SM}}}\right]}$$

Discussion points

- Inclusion of upper limits on $B_q \longrightarrow ee$ and $b \longrightarrow s\tau\tau$
- How to treat C_9?
    - Currently included as SM nuisance parameter.
    - Comments to make it q^2/helicity dependent.
- Finally, a more general one…

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