Implications of $B \to D^{(*)}\{K, \pi\}$

IPPP Durham: Beyond the Anomalies III
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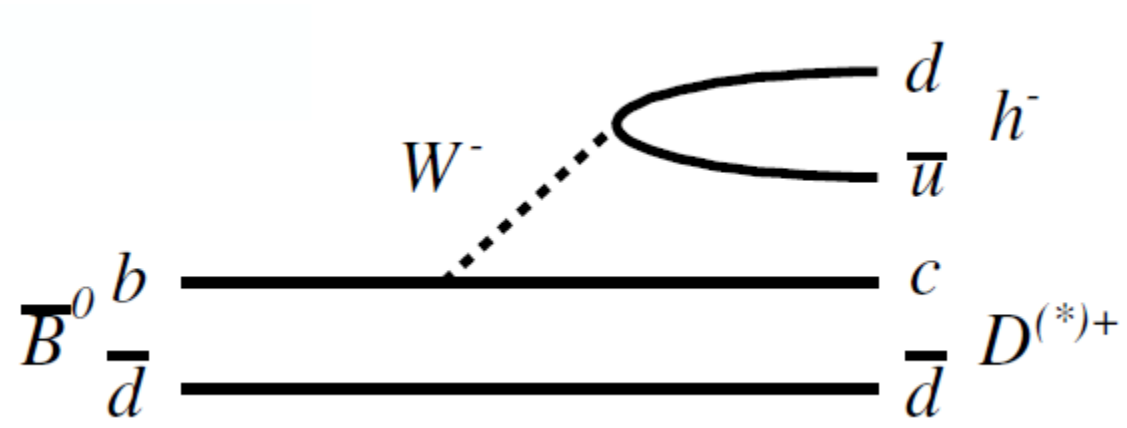
Nico Gubernari
nicogubernari@gmail.com

Daniel Ferlewicz
daniel.ferlewicz@unimelb.edu.au

Non-leptonic $b \to c$ decays

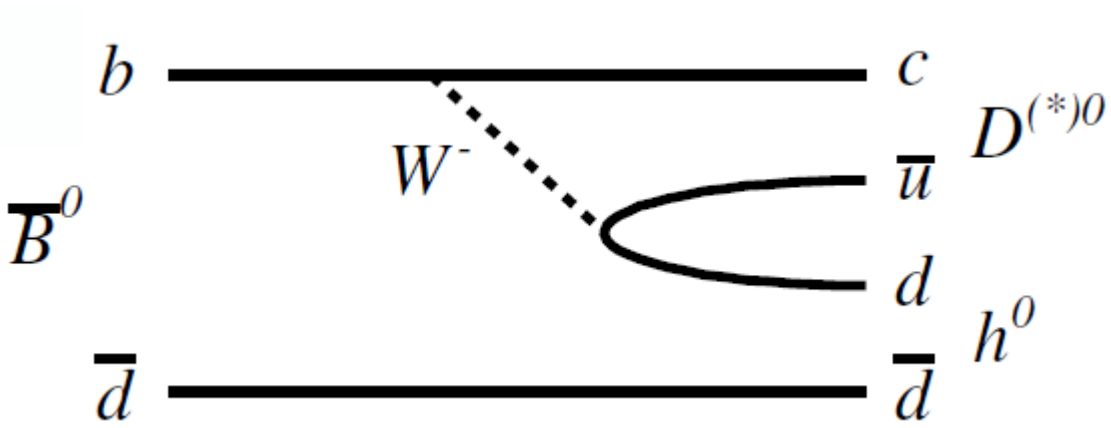
two kinds of non-leptonic B_0 decays mediated by $b \to c$ transitions

color-allowed topology



QCD factorization applicable
no suppression

color-suppressed topology



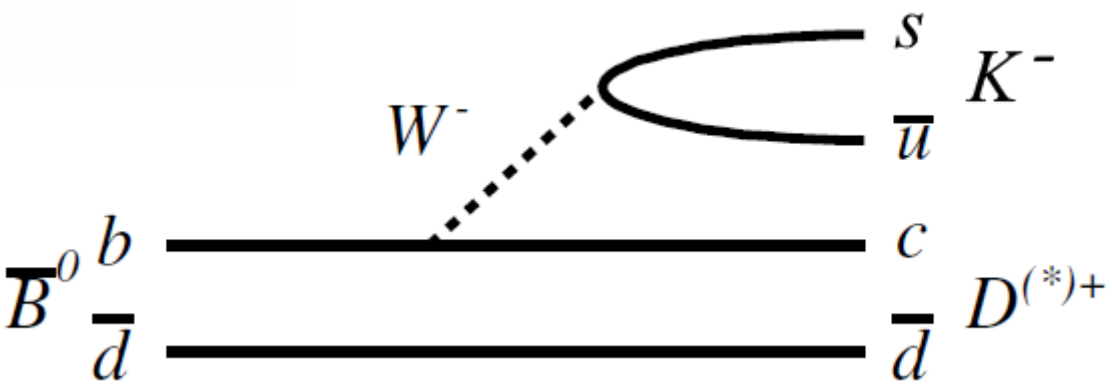
QCD factorization **not** applicable
power-suppressed

The simplest non-leptonic $B_{(s)}$ decays

decays with four different flavors are the simplest and cleanest non-leptonic $B_{(s)}$

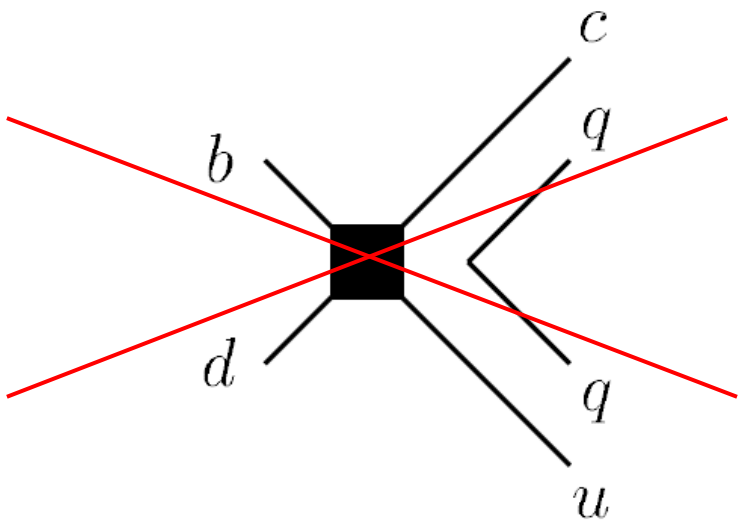
$\Downarrow$

no penguin or annihilation contributions



focus on $\bar{B}_s^0 \to D_s^{(*)+}\pi^-$ and $\bar{B}^0 \to D^{(*)+}K^-$ to test QCD factorization

[Bordone/NG/Huber/Jung/van Dyk '20]



other color allowed decays discussed in Daniel's part of the talk

QCD factorization

QCD factorization: systematic method to compute amplitudes in non-leptonic B decays in heavy-quark limit (leading power in $\frac{\Lambda_{\mathrm{QCD}}}{m_b}$)

$$\mathcal{A}(\bar{B}^0 \to D^+K^-) \propto f_K\, F_0^{B\to D}(M_K^2)\, a_1(D^+K^-) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

improve theoretical predictions for $\bar{B}_s^0 \to D_s^{(*)+}\pi^-$ and $\bar{B}^0 \to D^{(*)+}K^-$ branching fractions

- Wilson coefficients a_1 computed at NNLO [Huber/Kränkl/Li '16]
- update $B \to D^{(*)}$ and $B_s \to D_s^{(*)}$ form factors [Bordone/NG/Jung/van Dyk '19]
- estimate $\frac{\Lambda_{\mathrm{QCD}}}{m_b}$ corrections **for the first time** [Bordone/NG/Huber/Jung/van Dyk '20]

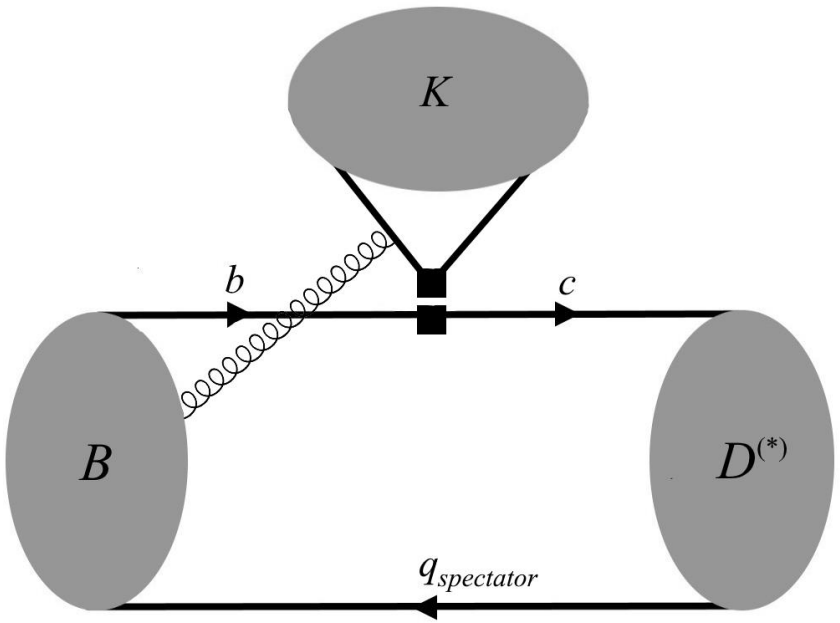
Power corrections

1. no annihilation or penguin topologies in $\bar{B}_s^0 \to D_s^{(*)+}\pi^-$ and $\bar{B}^0 \to D^{(*)+}K^-$

2. no $\frac{\Lambda_{QCD}}{m_c}$ corrections since we use QCD form factors instead of soft form factors

3. no hard-collinear gluon between b or c quarks and the light meson at order $\frac{\Lambda_{QCD}}{m_b}$

4. soft-gluon exchange between the $\bar{B}_{(s)}^0 D_{(s)}^{(*)+}$ system and the light meson L
   we estimate it with light-cone sum rules (LCSRs)



Light-cone sum rules results

our conservative estimates $L = \{\pi, K\}$

$$\frac{\mathcal{A}(\bar{B}_q^0 \to D_q^+ L^-)\big|_{\mathrm{NLP}}}{\mathcal{A}(\bar{B}_q^0 \to D_q^+ L^-)\big|_{\mathrm{LP}}} \simeq [0.06, 0.6]\%$$

$$\frac{\mathcal{A}(\bar{B}_q^0 \to D_q^{*+} L^-)\big|_{\mathrm{NLP}}}{\mathcal{A}(\bar{B}_q^0 \to D_q^{*+} L^-)\big|_{\mathrm{LP}}} \simeq [0.04, 0.4]\%$$

lower value correspond to our central value
upper value obtained by simply multiplying the central value by a factor of 10

update w.i.p. from Piscopo/Rusov

rescattering contribution is negligible [Endo et al '22]

support the fact that $\boldsymbol{\bar{B}_q^0 \to D_q^{(*)+} L^-}$ decays are theoretically clean

Theory prediction and comparison with data

quantity	unit	this work	ref. [2] (2016)
$F_0^{\bar{B}\to D}(M_K^2)$	—	0.672 ± 0.011	0.670 ± 0.031
$F_0^{\bar{B}_s^0\to D_s}(M_\pi^2)$	—	0.673 ± 0.011	0.700 ± 0.100
$A_0^{\bar{B}\to D^*}(M_K^2)$	—	0.708 ± 0.038	0.654 ± 0.068
$A_0^{\bar{B}_s^0\to D_s^*}(M_\pi^2)$	—	0.689 ± 0.064	0.520 ± 0.060
$\lvert a_1(D_s^+\pi^-)\rvert$	—	$1.0727^{+0.0125}_{-0.0140}$	$1.073^{+0.012}_{-0.014}$
$\lvert a_1(D^+K^-)\rvert$	—	$1.0702^{+0.0101}_{-0.0128}$	$1.070^{+0.010}_{-0.013}$
$\lvert a_1(D_s^{*+}\pi^-)\rvert$	—	$1.0713^{+0.0128}_{-0.0137}$	$1.071^{+0.013}_{-0.014}$
$\lvert a_1(D^{*+}K^-)\rvert$	—	$1.0687^{+0.0103}_{-0.0125}$	$1.069^{+0.010}_{-0.013}$
$\lvert V_{cb}\rvert$	10^{-3}	41.1 ± 0.5	39.5 ± 0.8
$\lvert V_{ud}\rvert f_\pi$	MeV	127.13 ± 0.13	126.8 ± 1.4
$\lvert V_{us}\rvert f_K$	MeV	35.09 ± 0.06	35.06 ± 0.15

improved FFs uncertainties

same results for the WC of QCDF as in Huber/Kränkl/Li

updated remaining inputs

source	PDG	QCDF prediction	
$\mathcal{B}(\bar{B}_s^0 \to D_s^+\pi^-)$	3.00 ± 0.23	4.42 ± 0.21	$\to 4\sigma$
$\mathcal{B}(\bar{B}^0 \to D^+K^-)$	0.186 ± 0.020	0.326 ± 0.015	$\to 5\sigma$
$\mathcal{B}(\bar{B}_s^0 \to D_s^{*+}\pi^-)$	2.0 ± 0.5	$4.3^{+0.9}_{-0.8}$	$\to 2\sigma$
$\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$	0.212 ± 0.015	$0.327^{+0.039}_{-0.034}$	$\to 3\sigma$

discrepancy between measurements and theoretical predictions

Possible explanations

1. large **nonfactorizable contributions** of $O(15-20\%)$ in the amplitude
   $\longrightarrow$ excluded by our estimate at 4.4σ level

2. **experimental issue** $\longrightarrow$ would imply problems in several (consistent) measurements (CLEO, BaBar, LHCb, Belle)

3. shift in the **inputs** (e.g. V_{ud}, V_{us}, V_{cb}) $\longrightarrow$ would probably violate CKM unitarity

4. **BSM physics** only explanation left $\longrightarrow$ see next slide

5. a combination of the effects discussed above

Is NP a viable option?

implies $O(20\%)$ tree-level corrections in
$b \to cu(d/s)$ transitions not observed so far

possibility explored in several works
$\Longrightarrow$ **BSM physics viable option**
(W' models…)

BSM explanation consistent with flavor observables

[Iguro/Kithahara '21]
[Fleischer et al. '20]

however strong constraints from dijet searches

[Bordone/Greljo/Marzocca/Fuentes-Martin '21]

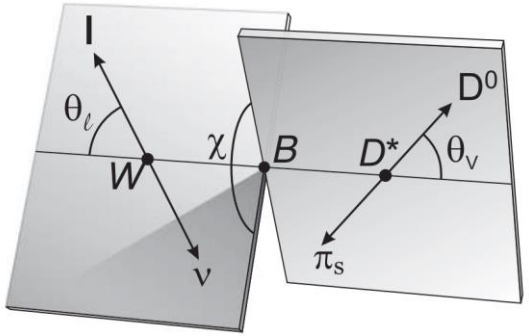
On the experimental side

- Decay widths of $B \to D^{(*)}h$ can be estimated from their semileptonic counterpart

$$\Gamma(\bar{B}^0 \to D^{*+}h^-) = 6\pi^2 \tau_B |V_{uq}|^2 f_h^2 X_h |a_1(q^2)|^2 \times d\Gamma(\bar{B}^0 \to D^{*+}\ell^-\bar{\nu})/dq^2|_{q^2=m_h^2}$$

- Beneke et al: $|a_1| = 1.05$ (10.1016/S0550-3213(00)00559-9)
- Huber et al: $|a_1(\pi)| = 1.071 \pm 0.014$, $|a_1(K)| = 1.069 \pm 0.013$ (10.48550/JHEP09(2016)112)
- Previous studies (Fleischer et al. 10.1103/PhysRevD.83.014017) of $|a_1|$ have not been performed within a single experiment which would cancel many systematic uncertainties
- $SU(3)$ symmetry implies that $|a_1|$ should be consistent for $h = \{\pi, K\}$

$B^0 \to D^{*-}\ell^+\nu_\ell$ measurement



- Semileptonic measurement is required for hadronic factorisation

$$\frac{d\Gamma(B^0 \to D^{*-}\ell^+\nu_\ell)}{dw\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi} = \frac{\eta_{\rm EW}^2 3 m_{B^0} m_{D^{*\pm}}^2}{4(4\pi)^4} G_F^2 |V_{cb}|^2 \sqrt{w^2-1}(1-2wr+r^2)$$
$$\times\{(1-\cos\theta_\ell)^2\sin^2\theta_V H_+^2 + (1+\cos\theta_\ell)^2\sin^2\theta_V H_-^2 + 4\sin^2\theta_\ell\cos^2\theta_V H_0^2 - 2\sin^2\theta_\ell\sin^2\theta_V\cos 2\chi H_+H_-$$
$$-4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi H_+H_0 + 4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi H_-H_0\},$$

$$w = \frac{P_B \cdot P_{D^*}}{m_B m_{D^*}} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}$$

- Helicity amplitudes $H_{\pm,0}$ can be defined by two different parameterisations, BGL and CLN:
  - BGL is defined with three power series
  - No. free parameters set by non-zero coeffs
  - Considered less model-dependent at higher order
    C.G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Rev. D 56, 6895 (1997).

BGL($n_f, n_g, n_{\mathcal{F}_1}$)

$$f(z) = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} a_n^f z^n,$$

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n^g z^n$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} a_n^{\mathcal{F}_1} z^n,$$

$$H_0(w) = \mathcal{F}_1(w)/\sqrt{q^2},$$

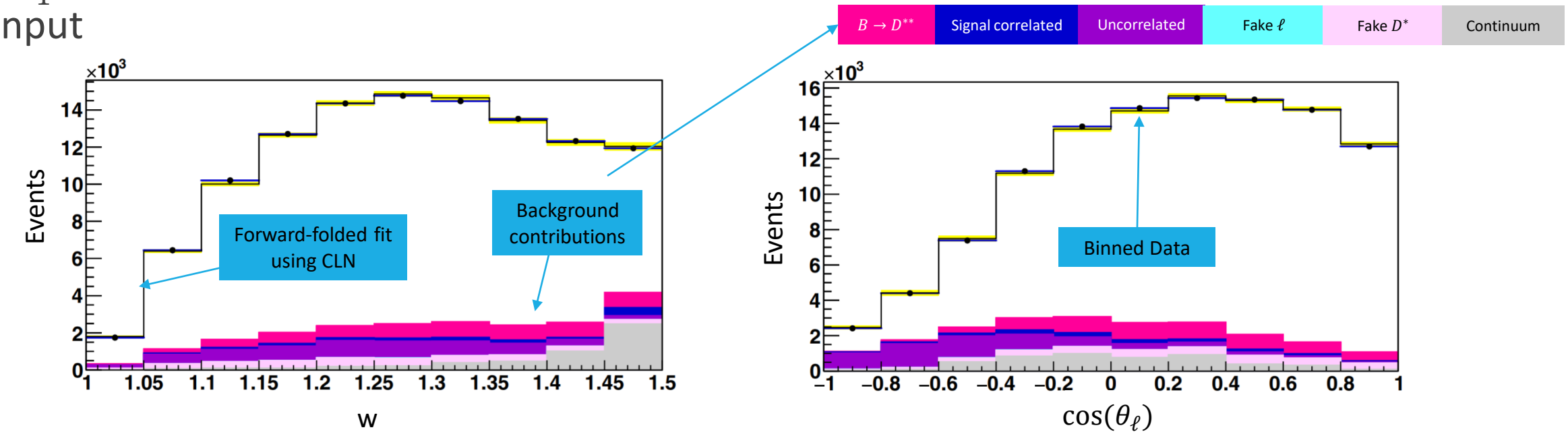
$$H_\pm(w) = f(w) \mp m_{B^0} m_{D^{*\pm}} \sqrt{w^2-1}\, g(w)$$

$B^0 \to D^{*-}\ell^+\nu_\ell$ at Belle

E. Waheed et al. (Belle 2019)
10.1103/PhysRevD.100.052007

- Belle 2019 (711 fb^{-1}) untagged study using a forward-folding approach
- $|V_{cb}|$ consistent between BGL and CLN, form factor parameters compatible
- BGL fit must be truncated to order ($n_f = 1, n_g = 0, n_{\mathcal{F}_1} = 2$), in order to fit to the data alone without lattice input

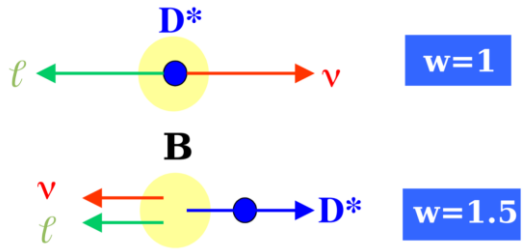
Source	$\eta_{EW}\mathcal{F}(1)\lvert V_{cb}\rvert$ [%]	$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$ [%]
Slow pion efficiency	0.79	1.57
Lepton ID combined	0.67	1.33
$\mathcal{B}(B \to D^{**}\ell\nu)$	0.05	0.10
$B \to D^{**}\ell\nu$ form factors	0.08	0.16
f_{+-}/f_{00}	0.56	1.05
Fake e/μ	0.07	0.11
K/π ID	0.39	0.77
Fast track efficiency	0.53	1.05
$N(\Upsilon(4S))$	0.69	1.37
B^0 lifetime	0.13	0.26
$\mathcal{B}(D^{*+} \to D^0\pi_s^+)$	0.37	0.74
$\mathcal{B}(D^0 \to K\pi)$	0.51	1.02
Total systematic error	1.65	3.26

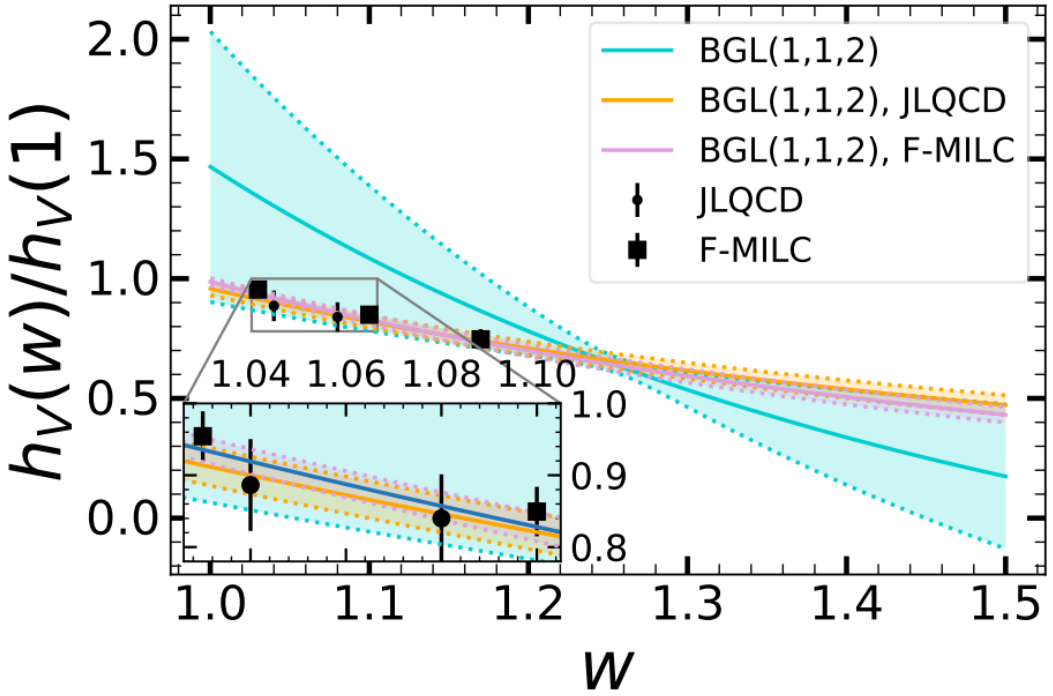


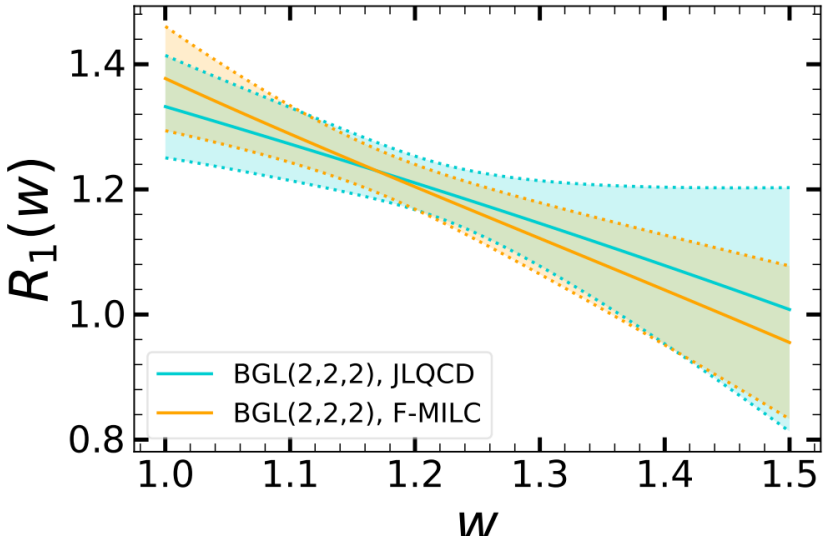
Fitting to the Belle data

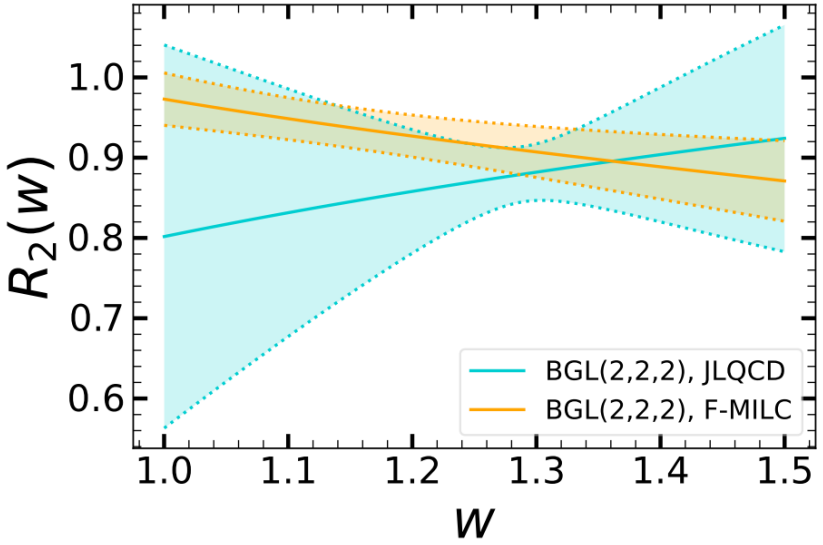
D. Ferlewicz, E. Waheed, P. Urquijo (2021)
10.1103/PhysRevD.103.073005
With updates using F-MILC results

- JLQCD and Fermilab-MILC provide first LQCD form factor calculations with $w \neq 1$ values for this mode.
  - JLQCD (DOI 10.22323/1.363.0139): h_{A_1} and h_V at $w = 1.04, 1.08$
  - Fermilab-MILC (arXiv 2105.14019): $g, f, \mathcal{F}_1$ at $w = 1.03, 1.10, 1.17$
- Belle data is combined with these calculations for new fit

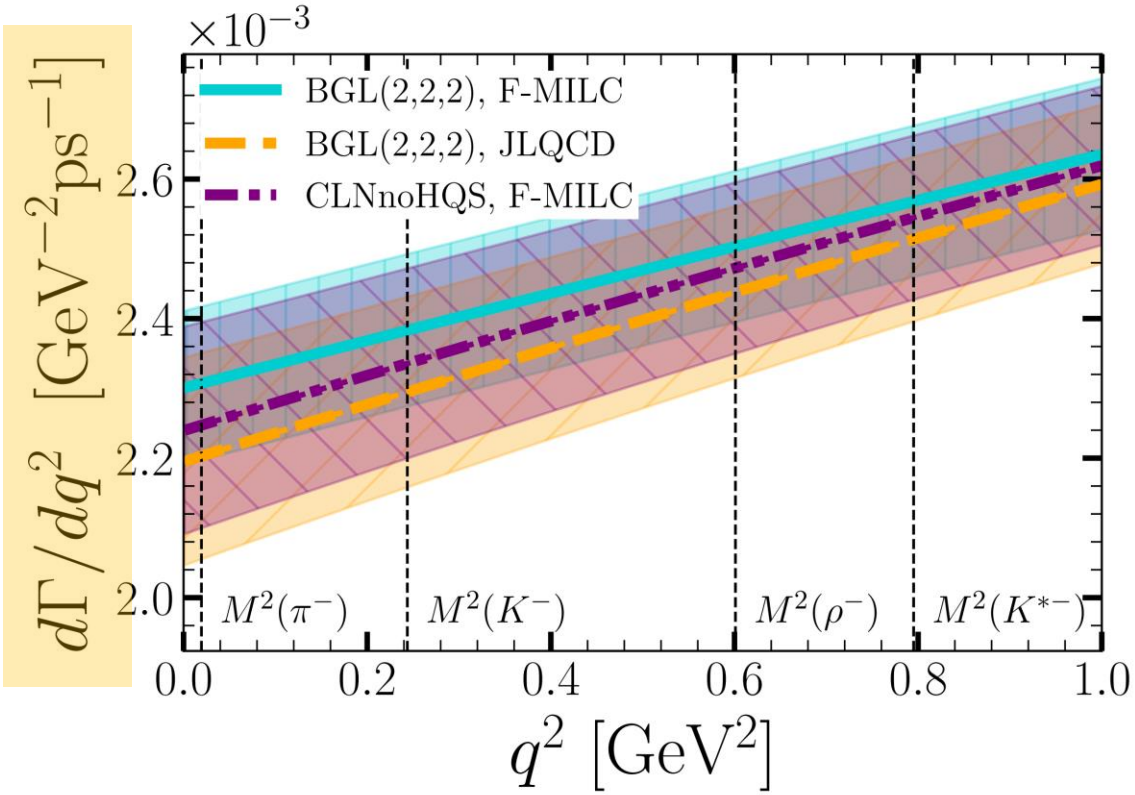


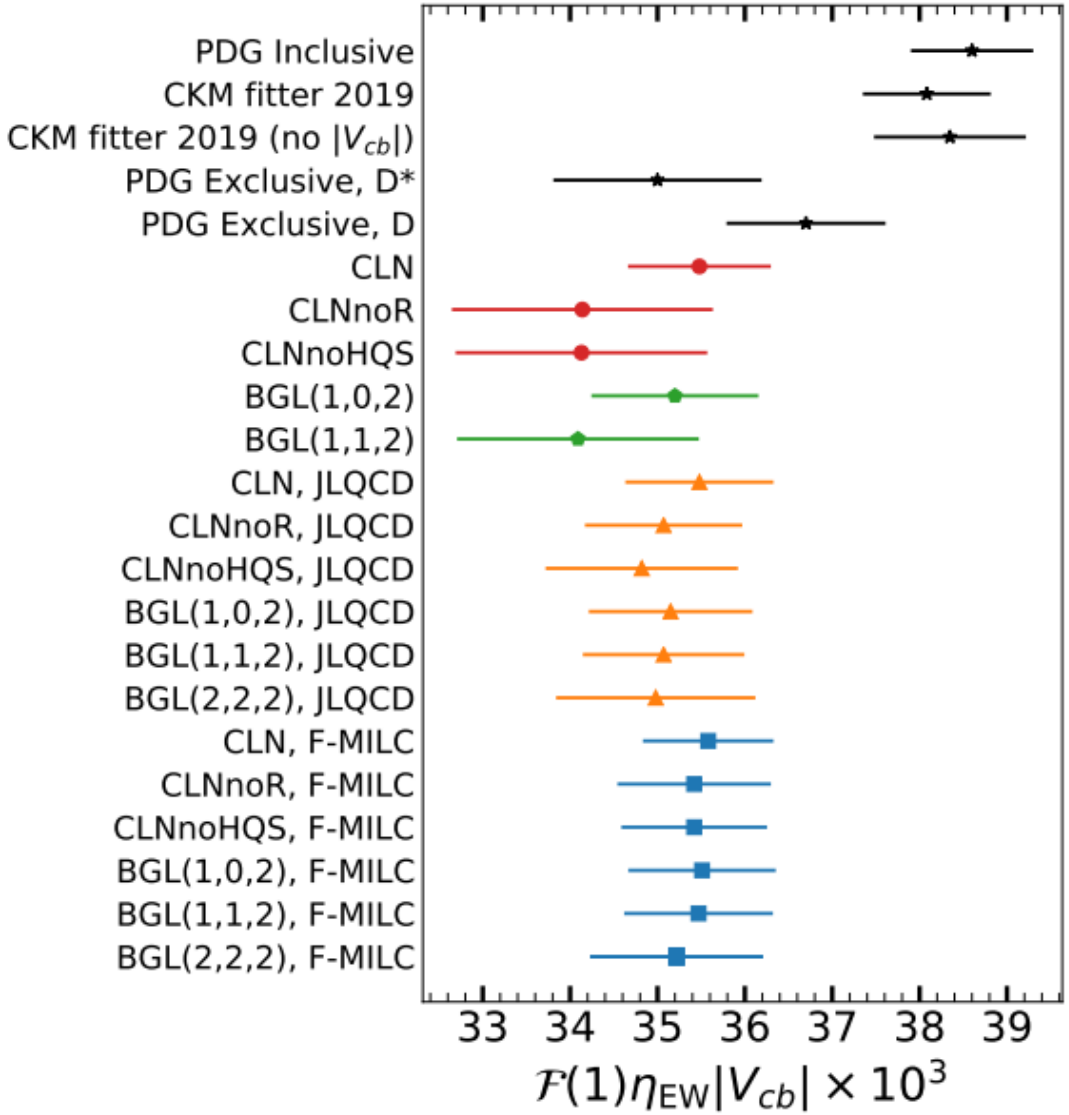






$B^0 \to D^{*-}\ell^+\nu_\ell$ results





- $\mathcal{F}(1)\eta_{EW}|V_{cb}| = 35.22 \pm 0.50 \pm 0.84$
  (Taking BGL(2,2,2), F-MILC as nominal)

$\bar{B}^0 \to D^{*+}h^-$ measurement

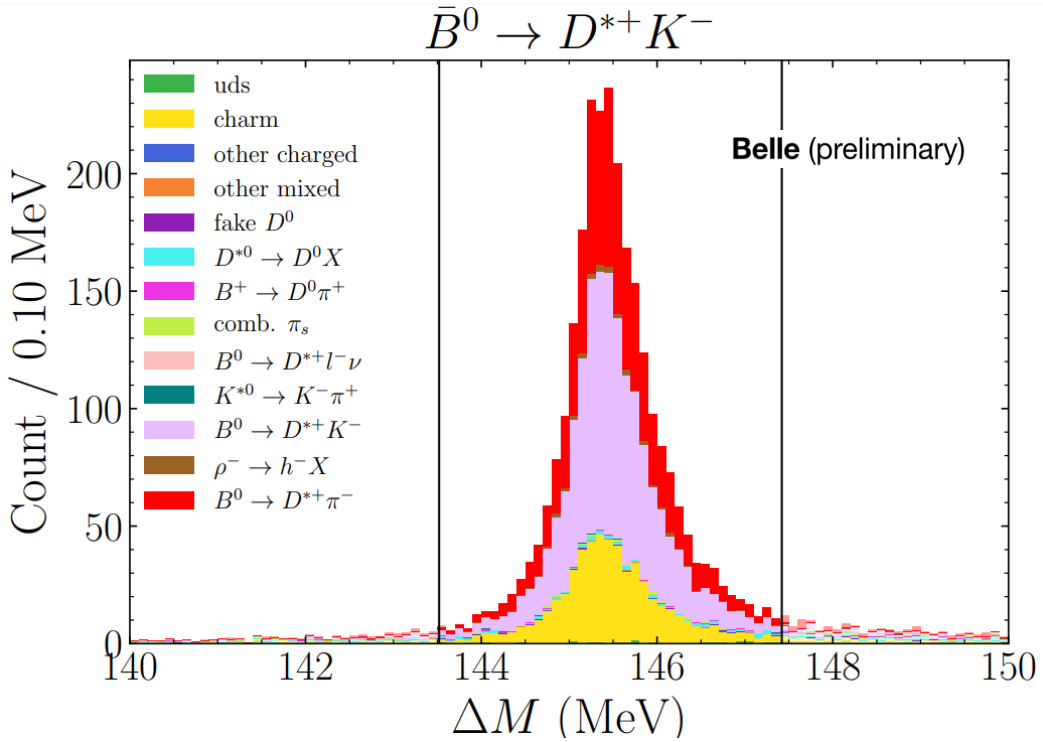
J.F. Krohn et al. (Belle, 2022)

First presentation of results!

- New Belle (711 fb^{-1}) $\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$ measurement, with $D^{*+} \to D^0\pi^+$ and $D^0 \to K^-\pi^+$ or $D^0 \to K^-2\pi^+\pi^-$ (previous 10.4 fb^{-1})

Selection criteria:

- Pion $\mathcal{L}_{K/\pi}$ < 0.6 (except slow pions)
- Kaon $\mathcal{L}_{K/\pi}$ > 0.6
- D^* candidates have ΔM_{D^*-D} within ≈ 2.1MeV/c^2 of mean
- $M_{bc} > 5.27$ GeV/c^2
- $-150 < \Delta E\ (MeV) < 125$



- Signal yields from simultaneous unbinned maximum-likelihood fit of $\Delta E = E_B - E_{beam}$
  - π signal PDF = double Gaussian + Crystal Ball
  - K signal PDF = Gaussian + Crystal Ball
  - Common resolution factor for widths is used for fits to data: $\sigma_i^{data} = \beta\sigma_i^{width}$

$\bar{B}^0 \to D^{*+}h^-$ measurement

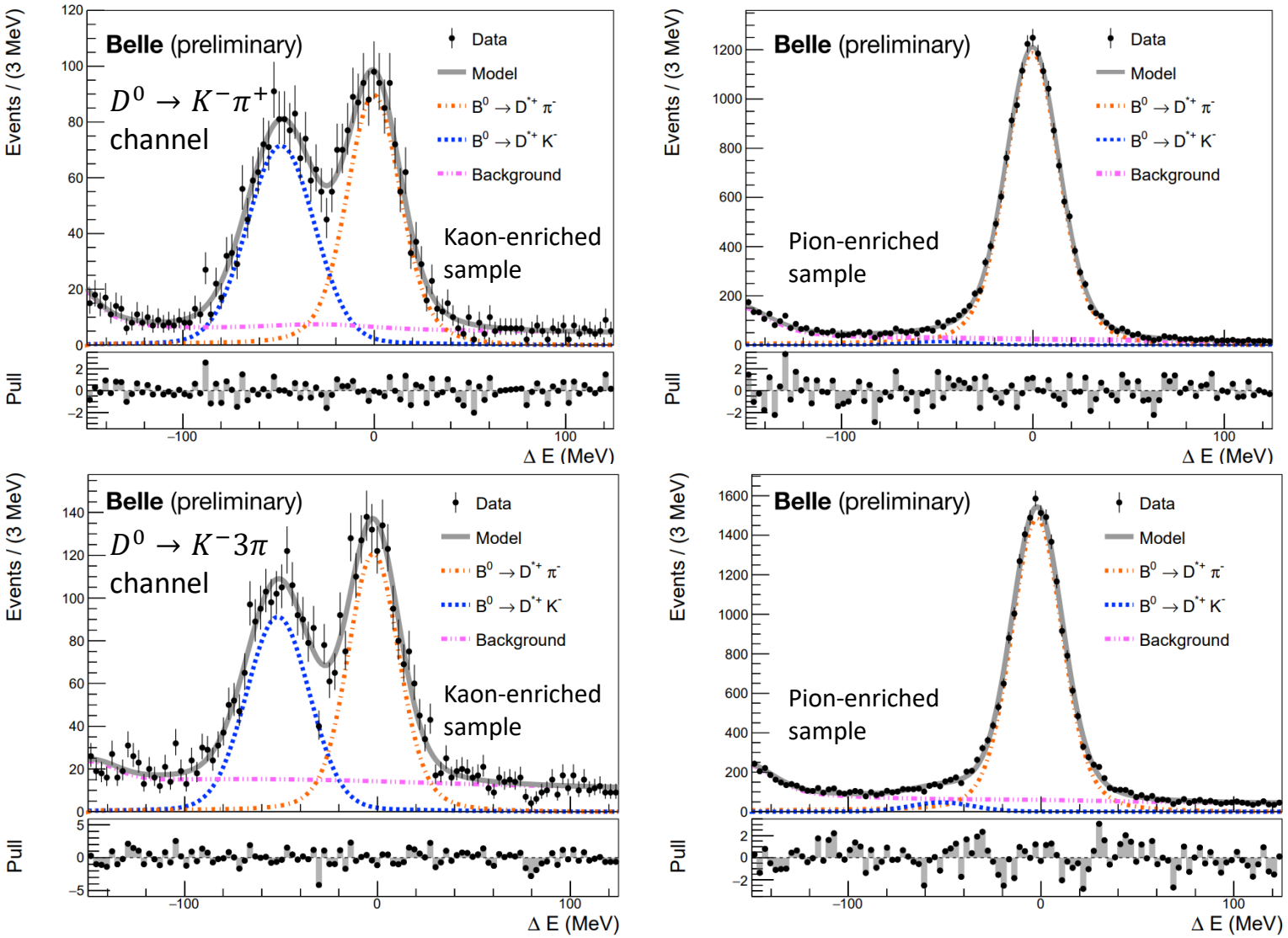
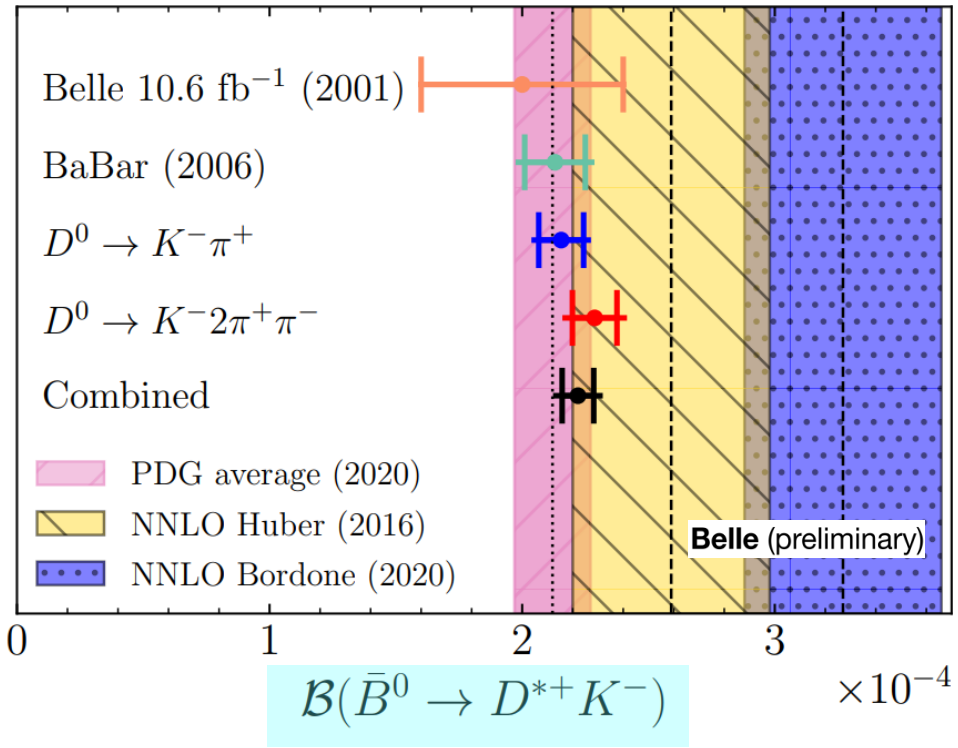


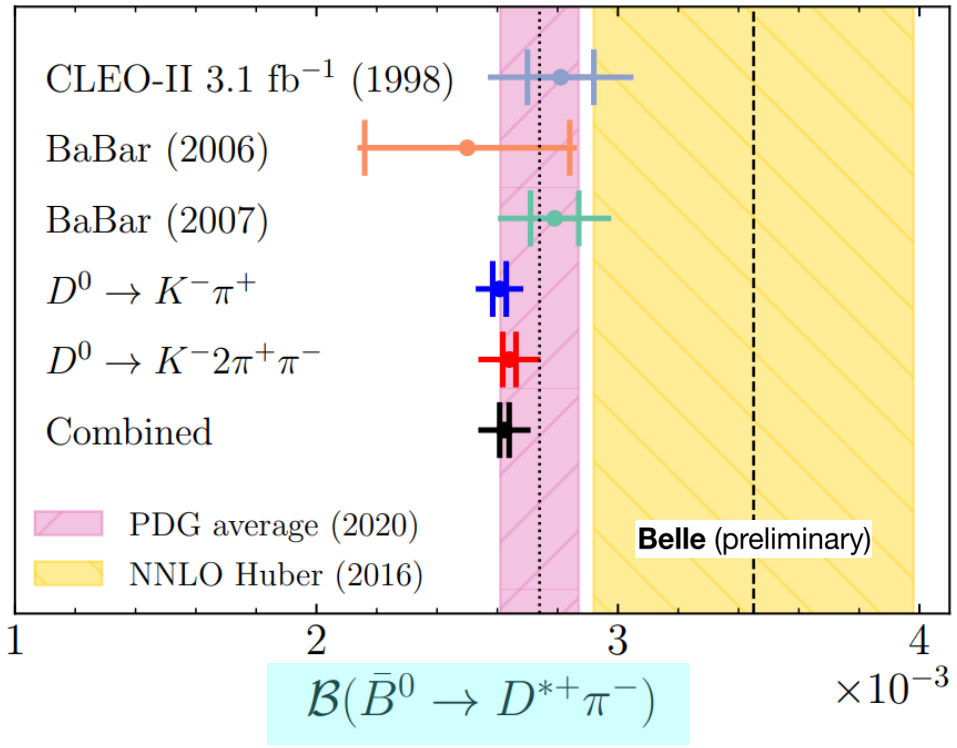
Table of uncertainties for B.F.
Values with † are propagated to R^D,
otherwise are cancelled

type	$\bar{B} \to D^{*+}\pi^-$	$\bar{B} \to D^{*+}K^-$
π-ID stat.	0.75% 0.58%†	0.32%
π-ID sys.	0.49% 0.41%†	0.19%
K-ID stat.	0.74%	1.04% 0.64%†
K-ID sys.	0.55%	0.89% 0.55%†
K-ID run dep. sys.	0.30%	0.30%
π_{slow} stat.	0.79%	0.79%
π_{slow} sys.	0.01%	0.01%
π_{slow} corr.	1.33%	1.33%
Tracking sys.	1.26%	1.26%
Fixed yields bkg. PDF	0.07%†	0.07%†
Fixed shapes bkg. PDF	0.07%†	0.07%†
Fit bias	0.09%†	0.37%†
$N_{\bar{B}^0 B^0}$	1.84%	1.84%
$\mathcal{B}(D^{*+} \to D^0\pi^+)$	0.74%	0.74%
$\mathcal{B}(D^0)$	0.94%	0.94%
MC stat.	0.26%†	0.99%†
Total sys. ($\mathcal{B}$)	3.26%	3.47%
Total sys. (ratio)	1.50%	1.50%
Total stat. err.	0.57%	2.74%

Component	$D^0 \to K^-\pi^+$		$D^0 \to K^-2\pi^+\pi^-$	
	$\bar{B}^0 \to D^{*+}\pi^-$	$\bar{B}^0 \to D^{*+}K^-$	$\bar{B}^0 \to D^{*+}\pi^-$	$\bar{B}^0 \to D^{*+}K^-$
$\bar{B}^0 \to D^{*+}\pi^-$	16494 ± 142	1247 ± 46	19500 ± 162	1587 ± 52
$\bar{B}^0 \to D^{*+}K^-$	225 ± 53	1182 ± 49	731 ± 71	1414 ± 55
Background	3390 ± 115	658 ± 61	7067 ± 185	1448 ± 97

$\bar{B}^0 \to D^{*+}h^-$ measurement





$\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$		
$D^0 \to K^-\pi^+$	$(2.154 \pm 0.089 \pm 0.078) \times 10^{-4}$	1.1
$D^0 \to K^-2\pi^+\pi^-$	$(2.287 \pm 0.088 \pm 0.092) \times 10^{-4}$	0.7
Combined	$(2.221 \pm 0.063 \pm 0.077) \times 10^{-4}$	0.9

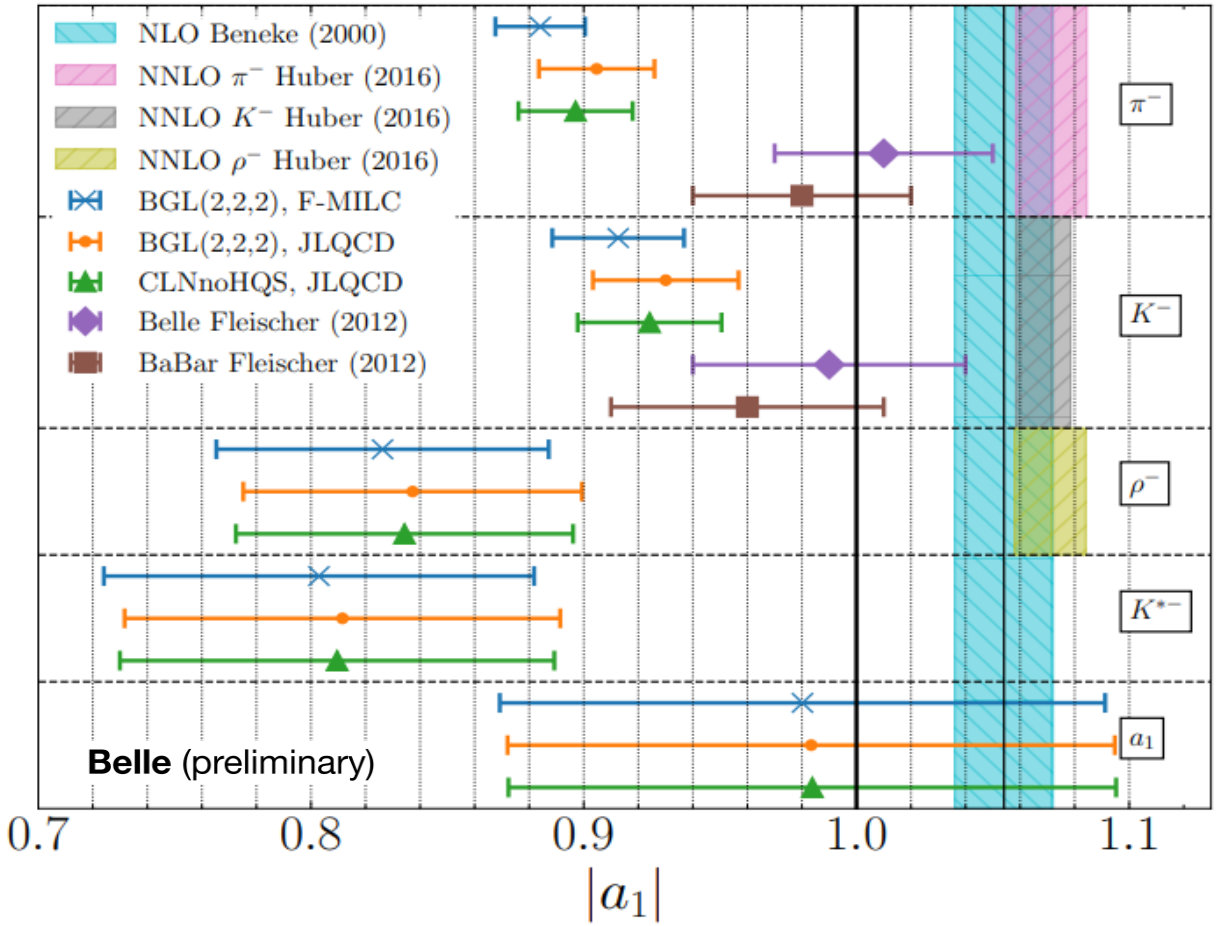
$\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)$	Result	$n\sigma$ meas.−theo.
$D^0 \to K^-\pi^+$	$(2.607 \pm 0.023 \pm 0.083) \times 10^{-3}$	1.8
$D^0 \to K^-2\pi^+\pi^-$	$(2.640 \pm 0.022 \pm 0.101) \times 10^{-3}$	1.7
Combined	$(2.623 \pm 0.016 \pm 0.086) \times 10^{-3}$	1.7

$\mathcal{R}_{K/\pi}$		
$D^0 \to K^-\pi^+$	$(8.26 \pm 0.35 \pm 0.16) \times 10^{-2}$	1.8
$D^0 \to K^-2\pi^+\pi^-$	$(8.56 \pm 0.34 \pm 0.16) \times 10^{-2}$	2.5
Combined	$(8.41 \pm 0.24 \pm 0.13) \times 10^{-2}$	2.7

Hadronic factorisation measurement

Total error contributions

Measurement	Hadronic stat.	Hadronic sys.	Combined Semileptonic BGL, F-MILC	BGL, JLQCD	CLN, JLQCD	SM constants
$\lvert a_1(\pi^-)\rvert$	0.4	0.3	1.5	2.1	2.0	1.1
$\lvert a_1(K^-)\rvert$	2.1	0.9	1.4	1.8	1.8	0.4
$\lvert a_1(\rho^-)\rvert$	6.6		1.3	1.5	1.4	2.8
$\lvert a_1(K^{*-})\rvert$	9.1		1.3	1.4	1.3	3.3
$\lvert a_1(a_1^-)\rvert$	10.4		1.2	1.1	1.1	4.2
$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	4.2	1.8	0.3	0.7	0.7	2.2
$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	13.3		0.7	1.5	1.7	6.0
$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\pi^-)\rvert^2$	18.2		0.9	1.9	2.1	7.0
$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	20.8		1.5	3.0	3.2	8.7
$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	14.0		0.4	0.9	1.0	5.6
$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(K^-)\rvert^2$	18.7		0.6	1.3	1.4	6.7
$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	21.2		1.2	2.3	2.5	8.4
$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\rho^-)\rvert^2$	22.5		0.2	0.4	0.4	8.7
$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\rho^-)\rvert^2$	24.6		0.8	1.4	1.5	10.1
$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^{*-})\rvert^2$	27.6		0.6	1.0	1.1	10.7



- Measuring $|a_1|$ within a single experiment cancels out many systematic uncertainties:
  - K/π-ID, fit parameters, fit bias and MC statistical uncertainty remain for the hadronic mode
  - ℓ-ID, e/μ fake rates, $\mathcal{B}(D^{**})$ and PDF related uncertainties for the semileptonic mode

Hadronic factorisation measurement

	Model	$\lvert a_1 \rvert$	$n\sigma$ meas.-theo
π^-	BGL(2,2,2), F-MILC	0.884 ± 0.016	8.9 $(-18 \pm 2)\%$
π^-	BGL(2,2,2), JLQCD	0.905 ± 0.021	6.7 $(-16 \pm 2)\%$
π^-	CLNnoHQS, JLQCD	0.897 ± 0.021	7.1 $(-16 \pm 2)\%$
K^-	BGL(2,2,2), F-MILC	0.913 ± 0.024	5.8 $(-15 \pm 3)\%$
K^-	BGL(2,2,2), JLQCD	0.930 ± 0.027	4.7 $(-13 \pm 3)\%$
K^-	CLNnoHQS, JLQCD	0.924 ± 0.026	5.0 $(-14 \pm 3)\%$

- Take ratio of $|a_1(q_A)|^2/|a_1(q_B)|^2$ to test if this is a universal factor

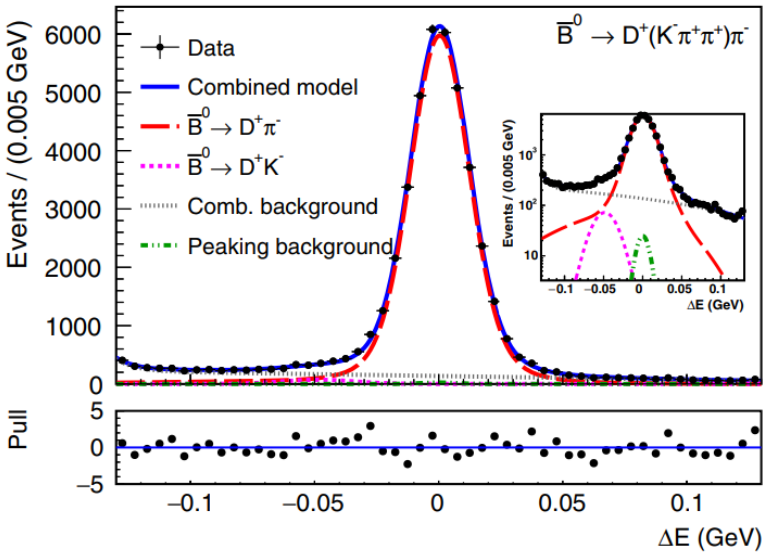
Model	Ratio	Result
BGL(2,2,2), F-MILC	$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.066 ± 0.054
BGL(2,2,2), JLQCD	$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.057 ± 0.054
CLN noHQS, JLQCD	$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.061 ± 0.054

$\bar{B}^0 \to D^+ h^-$ measurement

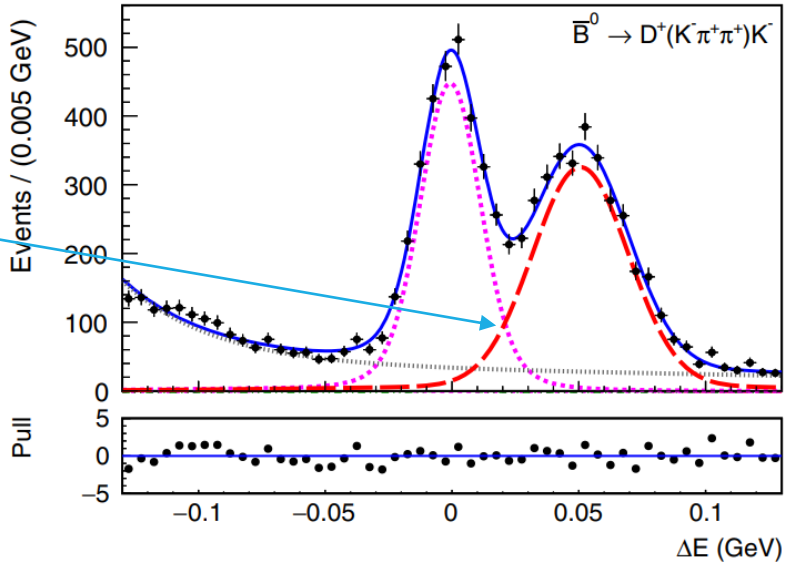
E. Waheed et al. (Belle, 2022)
10.1103/PhysRevD.105.012003

- New Belle (711 fb^{-1}) $\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$ and $\mathcal{B}(\bar{B}^0 \to D^+K^-)$ measurement, with $D^+ \to K^-\pi^+\pi^-$, updating the 2001 (10.4 fb^{-1}) result
- Similar selection criteria to $\bar{B}^0 \to D^{*+}h^-$, but with D meson mass selected with $\approx$ 13 MeV/c^2 of known M_{D^+}
- Simultaneous fit to pion- and kaon-enriched samples

Source	R^D	$\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$	$\mathcal{B}(\bar{B}^0 \to D^+K^-)$
$\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$	...	1.71%	1.71%
Tracking	...	1.40%	1.40%
$N_{B\bar{B}}$	...	1.37%	1.37%
f^{00}/f^{+-}	...	1.92%	1.92%
$D^+ \to K^-\pi^+\pi^+$ model	...	0.69%	0.69%
PDF parametrization	2.71%	1.63%	1.79%
PID efficiency of K/π	0.88%	0.68%	0.73%
D^+ mass selection window	0.05%	0.56%	0.64%
J/ψ veto selection	0.12%	0.004%	0.15%
Peaking background yield	0.07%	0.04%	0.00%
MC statistics	< 0.01	0.04%	0.04%
Fit bias	...	0.58%	0.61%
Total	2.85%	3.43%	3.54%



Signal PDFs are double Gaussians with floated resolution factor



$\bar{B}^0 \to D^+ h^-$ results

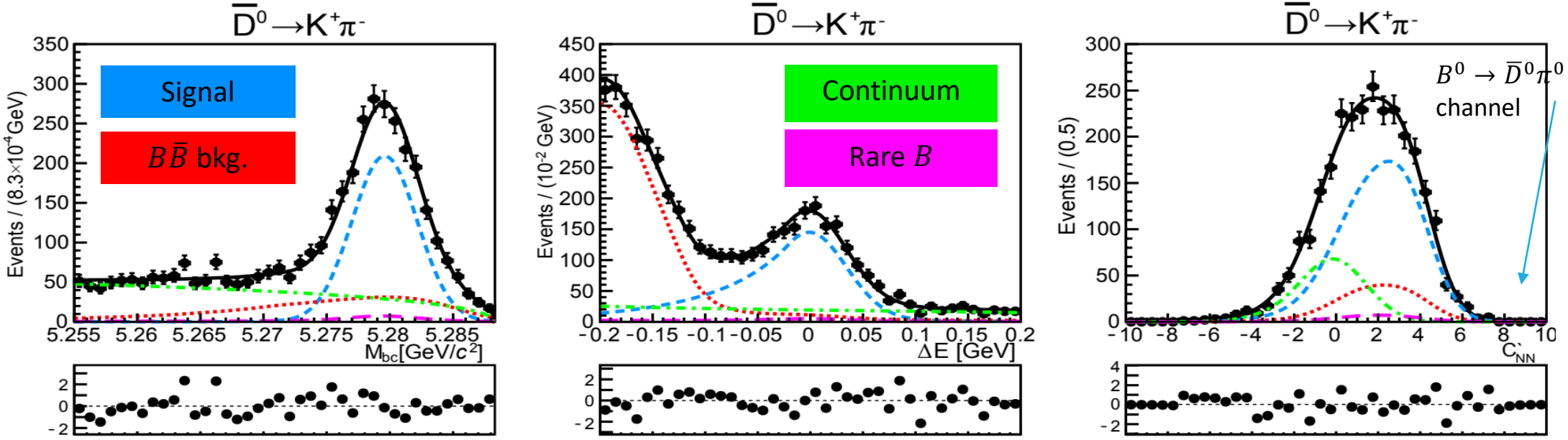
Measurement	Result ($\pm$ *stat.* $\pm$ *sys.* [$\pm$ *D* branching fraction sys.]*)	Theory prediction (Huber 2016)
$\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$	$(2.48 \pm 0.01 \pm 0.09 \pm 0.04^*) \times 10^{-3}$	$(3.93 \pm 0.43) \times 10^{-3}$
$\mathcal{B}(\bar{B}^0 \to D^+K^-)$	$(2.03 \pm 0.05 \pm 0.07 \pm 0.03^*) \times 10^{-4}$	$(3.01 \pm 0.32) \times 10^{-4}$
$R^D \approx \tan^2\theta_C \left(\frac{f_K}{f_\pi}\right)^2$	$0.0819 \pm 0.0020 \pm 0.0023$	0.077 ± 0.002

- Branching fractions and ratio consistent with previous measurements
- This channel is often used in control samples for *CP*-violation and ϕ_3 measurements
- Can be used with the previous Belle $B^0 \to D^-\ell^+\nu$ study to check consistency in $|a_1|$ measurements with reduced systematic uncertainties

$\mathcal{B}$ and $\mathcal{A}_{CP}$ of $B \to \bar{D}^0\pi$

T. Bloomfield et al. (Belle, 2022)
arXiv:2111.12337
(accepted by PRD on 10/03)

- Updated (from 2002 21.3 fb^{-1}) measurements of $\mathcal{B}(B^0 \to \bar{D}^0\pi^0)$ and $\mathcal{B}(B^+ \to \bar{D}^0\pi^+)$ with full Belle data set (711 fb^{-1}), with a 2- and 3-body D meson decay
- $B^0 \to \bar{D}^0\pi^0$ decay is colour-suppressed compared to charged final states
- Yield, $\mathcal{A}_{CP}$ extracted from unbinned extended maximum-likelihood fit to $M_{bc}, \Delta E$ and a neural net output for continuum suppression.



Continuum suppression neural net using event shape variables and B-meson flavour tagging algorithm

$\mathcal{B}$ and $\mathcal{A}_{CP}$ of $B \to \bar{D}^0\pi$ results

- First measurement of $\mathcal{A}_{CP}$ with neutral mode!
- Expect negligible direct *CP*-violation in neutral mode ⇒ good test of BSM

Systematic	$B^0 \to \bar{D}^0\pi^0$	$B^+ \to \bar{D}^0\pi^+$
No. $B\bar{B}$	1.37%	1.37%
$\mathcal{B}(\Upsilon(4S) \to B^0\bar{B}^0)$	1.23%	1.17%
DCS mode correction	0.01%	0.01%
Mean efficiency	2.43%	2.54%
Fixed f_d^s	$^{+0.31}_{-0.38}$%	$^{+0.19}_{-0.08}$%
Cal. Factors ($C'_{\rm NN}$)	0.34%	0.06%
ΔE KEST modification	0.63%	0.24%
KEST PDFs	0.35%	0.05%
Fixed Rare Yields	0.47%	0.03%
Fit bias	0.30%	0.16%
Bkg. $\mathcal{A}_{CP}$	0.01%	0.05%
Total	3.65%	3.32%

Measurement	Result ($\pm$ *stat.* $\pm$ *sys.*)
$\mathcal{B}(B^0 \to \bar{D}^0\pi^0)$	$(2.70 \pm 0.06 \pm 0.10) \times 10^{-4}$
$\mathcal{B}(B^+ \to \bar{D}^0\pi^+)$	$(4.53 \pm 0.02 \pm 0.15) \times 10^{-3}$
$\mathcal{A}_{CP}(B^0 \to \bar{D}^0\pi^0)$	$(0.42 \pm 2.05 \pm 1.22)\%$
$\mathcal{A}_{CP}(B^+ \to \bar{D}^0\pi^+)$	$(0.19 \pm 0.36 \pm 0.57)\%$

Conclusion

- Effort is needed to compute power corrections
- Interpretations of the tension suggest large power-corrections (… or BSM)
- Four papers from Belle (and one phenomenology paper) out recently/soon:

Measurement of $|V_{cb}|$ from $B \to D^* \ell \nu$ at Belle
E. Waheed et al. (Belle 2019)
10.1103/PhysRevD.100.052007

Study of $B \to Dh$ decays at Belle
E. Waheed et al. (Belle, 2022)
10.1103/PhysRevD.105.012003

Measurements of $\mathcal{B}(B \to D^{(*)}h)$ and QCD factorisation
J.F. Krohn et al. (Belle, 2022)
New result, submitting to PRD

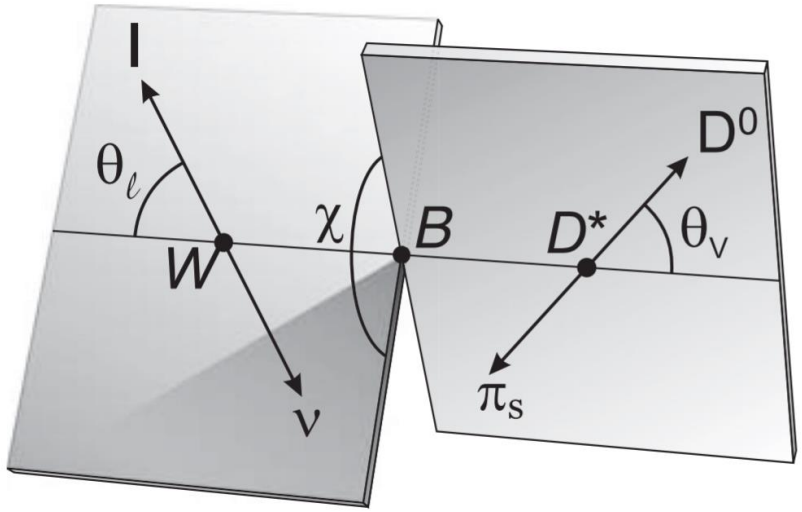
$\mathcal{B}$ and $\mathcal{A}$ of $B \to \bar{D}\pi$
T. Bloomfield et al. (Belle, 2022)
arXiv:2111.12337 (for PRD)

Revisiting fits to $B \to D^* \ell \nu$
D. Ferlewicz, E. Waheed, P. Urquijo (2021)
10.1103/PhysRevD.103.073005

- LQCD calculations of semileptonic decays used to improve fits for $B \to D^* \ell \nu$
- $\mathcal{B}(B \to D^{(*)}h)$ results deviate from predictions
  - $|a_1(\pi)| = 0.884 \pm 0.016$, $|a_1(K)| = 0.913 \pm 0.024$
- New $\bar{B}^0 \to Dh$ Belle results are available to further test SM $|a_1|$ predictions.

Back up slides

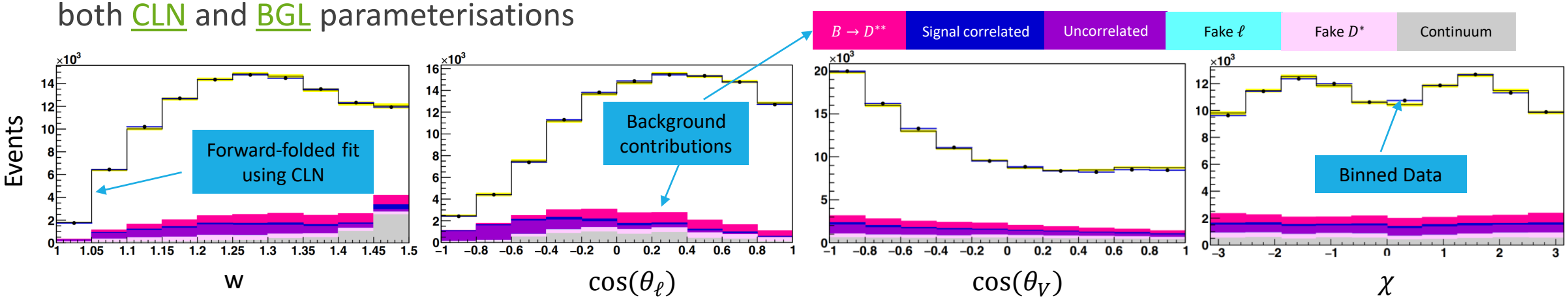
B→D*lν Decay



- Exclusive $|V_{cb}|$ measurements have historically had some model dependence
- Differential decay rate for l = e, μ, is a function of four observables: hadronic recoil (w) and three angles ($\theta_\ell, \theta_V, \chi$)

$$\frac{d\Gamma(B^0 \to D^{*-}\ell^+\nu_\ell)}{dw\,d\cos\theta_\ell\,d\cos\theta_V\,d\chi} = \frac{\eta_{\rm EW}^2 3 m_{B^0} m_{D^{*\pm}}^2}{4(4\pi)^4} G_F^2 |V_{cb}|^2 \sqrt{w^2-1}(1-2wr+r^2)$$
$$\times\{(1-\cos\theta_\ell)^2\sin^2\theta_V H_+^2 + (1+\cos\theta_\ell)^2\sin^2\theta_V H_-^2 + 4\sin^2\theta_\ell\cos^2\theta_V H_0^2 - 2\sin^2\theta_\ell\sin^2\theta_V\cos 2\chi H_+H_-$$
$$-4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi H_+H_0 + 4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_V\cos\theta_V\cos\chi H_-H_0\},$$

- Belle 2019 untagged study (doi.org/10.1103/PhysRevD.100.052007) performed fits for {$H_\pm$, H_0} defined in both CLN and BGL parameterisations



CLN parameterisation

$$\frac{d\Gamma}{dw} \propto (\text{phase space})|V_{cb}|^2\mathcal{F}^2(w)$$

$$\mathcal{F}^2(w) = h_{A_1}^2(w)(1+4\frac{w}{w+1}\frac{1-2wr+r^2}{(1-r^2)})^{-1} \times \left[2\frac{1-2wr+r^2}{(1-r)^2}\left(1+R_1^2(w)\frac{w-1}{w+1}\right) + (1+(1-R_2(w))\frac{w-1}{1-r})^2\right],$$

- Helicity amplitudes defined in terms of form factors and form factor ratios to represent full form factor expression $\mathcal{F}^2(w)$
- Standard CLN has four free parameters

$[\eta_{EW}h_{A_1}(1)]|V_{cb}|$: Overall normalisation
ρ^2: Slope term
$R_1(1)$: Ratio 1 term at w=1
$R_2(1)$: Ratio 2 term at w=1

$$h_{A_1}(w) = h_{A_1}(1)[1-8\rho^2 z(w) + (53\rho^2-15)z(w)^2 - (231\rho^2-91)z(w)^3],$$

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = \frac{h_{A_3}(w) + r h_{A_2}(w)}{h_{A_1}(w)} = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

- QCD sum rule constraints can be removed to add $R'_1(1)$ and $R'_2(1)$ as new free parameters in ratios (**CLNnoR**)
- Heavy quark symmetry can then also be removed to add curvature term c_{D^*} in $h_{A_1}(w)$ (**CLNnoHQS**)

BGL parameterisation

- The helicity amplitudes in BGL are defined by power series

$$H_0(w) = \mathcal{F}_1(w)/\sqrt{q^2},$$

$$H_\pm(w) = f(w) \mp m_{B^0} m_{D^{*\pm}} \sqrt{w^2 - 1}\, g(w)$$

BGL($n_f, n_g, n_{\mathcal{F}_1}$)

$$f(z) = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} a_n^f z^n,$$

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n^g z^n$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} a_n^{\mathcal{F}_1} z^n,$$

- Each coefficient a_n^i is a free parameter
- Setting $\tilde{a}_n^i = \eta_{EW}|V_{cb}|a_n^i$ reduces normalisation D.O.F.
- Another free parameter removed through $\mathcal{F}_1(0) = (m_B - m_{D^*})f(0)$
- $\mathcal{F}(1)\eta_{\mathrm{EW}}|V_{cb}| = \dfrac{1}{2m_{B^0}m_{D^{*\pm}}} \dfrac{|\tilde{a}_0^f|}{P_f(0)\phi_f(0)}$, where $\mathcal{F}(1)$ = $h_{A_1}(1)$

Variation from covariance

- Standard χ^2 minimisation algorithm for fitting expected results from theory, N_{exp}, to observed data, N_{obs}, given covariance in data, C:

$$\chi^2 = \sum_{i,j} \left(N_i^{\text{obs}} - N_i^{\text{exp}}\right) \mathcal{C}_{ij}^{-1} \left(N_j^{\text{obs}} - N_j^{\text{exp}}\right)$$

- High systematic correlations related to the overall normalisation may cause a bias in the fit results

- The measured systematic covariance matrix cannot be added to the statistical covariance matrix in a naïve way

Source	$\eta_{\text{EW}}\mathcal{F}(1)\lvert V_{cb}\rvert$ [%]	$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$ [%]
Slow pion efficiency	0.79	1.57
Lepton ID combined	0.67	1.33
$\mathcal{B}(B \to D^{**}\ell\nu)$	0.05	0.10
$B \to D^{**}\ell\nu$ form factors	0.08	0.16
f_{+-}/f_{00}	0.56	1.05
Fake e/μ	0.07	0.11
K/π ID	0.39	0.77
Fast track efficiency	0.53	1.05
$N(\Upsilon(4S))$	0.69	1.37
B^0 lifetime	0.13	0.26
$\mathcal{B}(D^{*+} \to D^0\pi_s^+)$	0.37	0.74
$\mathcal{B}(D^0 \to K\pi)$	0.51	1.02
Total systematic error	1.65	3.26

Covariance:	$\mathcal{C} = \mathcal{C}_{\text{stat.}}$	$\mathcal{C} = \mathcal{C}_{\text{stat.}} + \mathcal{C}_{\text{sys.}}$
CLN		
ρ^2	1.09 ± 0.04	1.16 ± 0.04
$R_1(1)$	1.20 ± 0.03	1.18 ± 0.04
$R_2(1)$	0.86 ± 0.02	0.85 ± 0.03
$\mathcal{F}(1)\eta_{\text{EW}}\lvert V_{cb}\rvert \times 10^3$	35.3 ± 0.2	36.4 ± 0.7
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$	4.93%	5.09%
χ^2/ndf	41/36	24/36
BGL(1,0,2)		
$\tilde{a}_0^f \times 10^3$	0.506 ± 0.004	0.53 ± 0.01
$\tilde{a}_1^f \times 10^3$	0.6 ± 0.2	0.2 ± 0.3
$\tilde{a}_1^{F_1} \times 10^3$	0.30 ± 0.07	0.2 ± 0.1
$\tilde{a}_2^{F_1} \times 10^3$	-3.8 ± 1.4	-2.6 ± 1.8
$\tilde{a}_0^g \times 10^3$	0.92 ± 0.02	0.92 ± 0.03
$\mathcal{F}(1)\eta_{\text{EW}}\lvert V_{cb}\rvert \times 10^3$	34.9 ± 0.3	36.4 ± 0.8
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$	4.93%	5.09%
χ^2/ndf	39/35	24/35

PDG 2018: 4.9%

Lower χ^2 values

- Need a way to avoid bias by separating the covariance matrices while maintaining covariance properties
⇒ Cholesky decomposition

Belle semileptonic results

TABLE II. Fitted parameters for the CLN, CLNnoR, and CLNnoHQS scenarios. The uncertainties listed are statistical and systematic, respectively. The branching ratios are obtained from the fit. No additional input from LQCD has been used.

Parameter	CLN	CLNnoR	CLNnoHQS
ρ^2	$1.09 \pm 0.04 \pm 0.05$	$0.89 \pm 0.09 \pm 0.15$	$0.93 \pm 0.38 \pm 0.39$
$R_1(1)$	$1.20 \pm 0.03 \pm 0.02$	$2.01 \pm 0.43 \pm 0.46$	$2.10 \pm 0.61 \pm 0.53$
$R_2(1)$	$0.86 \pm 0.02 \pm 0.01$	$0.83 \pm 0.05 \pm 0.04$	$0.77 \pm 0.42 \pm 0.38$
$R_1'(1)$	-0.12 (fixed)	$-3.50 \pm 1.77 \pm 1.85$	$-3.86 \pm 2.50 \pm 2.15$
$R_2'(1)$	0.11 (fixed)	$0.28 \pm 0.14 \pm 0.16$	$0.50 \pm 1.45 \pm 1.32$
c_{D^*}	ρ^2 (fixed)	ρ^2 (fixed)	$1.00 \pm 1.73 \pm 1.59$
$\mathcal{F}(1)\eta_{\rm EW}\lvert V_{cb}\rvert \times 10^3$	$35.63 \pm 0.18 \pm 0.80$	$34.29 \pm 0.67 \pm 1.33$	$34.23 \pm 0.63 \pm 1.30$
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$	5.04	5.04	5.04
χ^2/n.d.f.	40.8/36	34.8/34	34.7/33

Parameter $\times 10^3$	BGL(1,0,2)	BGL(1,1,2)
$\tilde{a}_0^f$	$0.512 \pm 0.004 \pm 0.013$	$0.496 \pm 0.010 \pm 0.020$
$\tilde{a}_1^f$	$0.64 \pm 0.19 \pm 0.33$	$1.38 \pm 0.40 \pm 0.67$
$\tilde{a}_2^f$	0.0 (fixed)	0.0 (fixed)
$\tilde{a}_0^g$	$0.93 \pm 0.02 \pm 0.01$	$1.56 \pm 0.31 \pm 0.33$
$\tilde{a}_1^g$	0.0 (fixed)	$-22.63 \pm 11.03 \pm 11.62$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)
$\tilde{a}_1^{\mathcal{F}_1}$	$0.30 \pm 0.07 \pm 0.09$	$0.38 \pm 0.10 \pm 0.14$
$\tilde{a}_2^{\mathcal{F}_1}$	$-3.88 \pm 1.46 \pm 1.34$	$-3.75 \pm 1.53 \pm 1.59$
$\mathcal{F}(1)\eta_{\rm EW}\lvert V_{cb}\rvert$	$35.34 \pm 0.27 \pm 0.87$	$34.23 \pm 0.66 \pm 1.40$
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$	5.04	5.04
χ^2/n.d.f.	38.6/35	34.8/34

The Cholesky decomposition toy MC

- Separate statistical and systematic covariance matrices
- Include systematic uncertainty via toy MC method and Cholesky decomposition

$$\mathcal{C}_{40\times40} = LL^T = [\ L\ ]\ [\ L^T\ ]$$

Positive definite covariance matrix

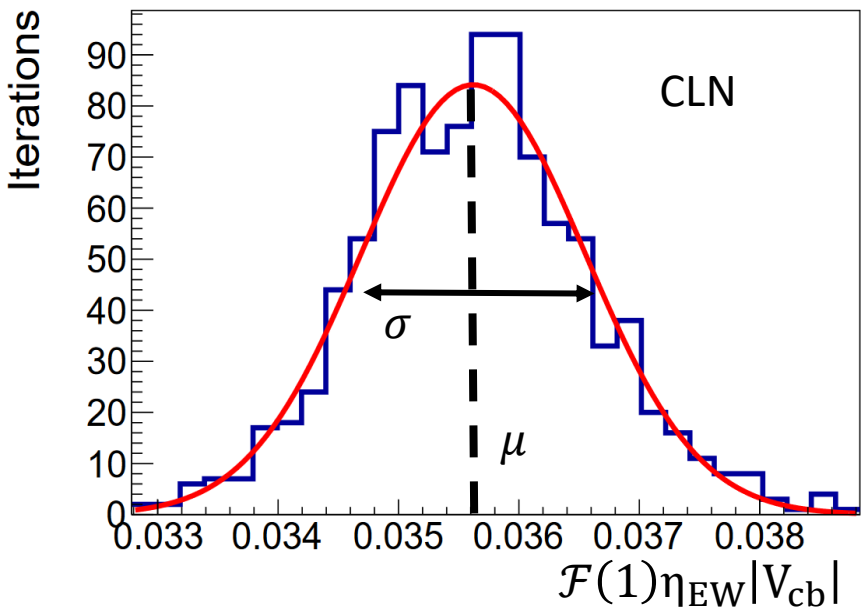
Lower triangular matrix

Upper triangular matrix

$$u = gaus(\mu = 0, \sigma = 1)_{1\times40}$$

$$N'_{obs} = N_{obs} + Lu$$

- Generate N'_{obs} and minimise χ^2 many times to obtain distribution of free parameter values ($\mathcal{C} = \mathcal{C}_{stat.}$ in χ^2 formula)
- Assume a Gaussian to obtain mean value and sys. uncertainty



- Results obtained through this method agree with 2019 Belle study while using the measured systematic covariance matrix

Updated input values

Common Input	
$\eta_{\rm EW}$	1.0066
$h_{A_1}(1) = \mathcal{F}(1)$	0.909 ± 0.017
$h_V(1)$	1.270 ± 0.046
m_{B^0}	5.27963 ± 0.00015 GeV/c^2
$m_{D^{*\pm}}$	2.01026 ± 0.00005 GeV/c^2
τ_{B^0}	$(1.520 \pm 0.004) \times 10^{-12}$ s
$\mathcal{B}(D^{*+} \to D^0\pi^+)$	0.677 ± 0.005
$\mathcal{B}(D^0 \to K^-\pi^+)$	0.0391 ± 0.0003
G_F	1.16637×10^{-5} GeV$^{-2}(\hbar c)^3$
$2n_{B\bar{B}}f_{00}$ (Belle)	$(750 \pm 11) \times 10^6$
BGL Input	
B_c^* 1^+ masses	6.739 GeV/c^2
	6.750 GeV/c^2
	7.145 GeV/c^2
	7.150 GeV/c^2
B_c^* 1^- masses	6.329 GeV/c^2
	6.920 GeV/c^2
	7.020 GeV/c^2
n_I	2.6
$\chi^T(+u)$	5.131×10^{-4} (GeV/c^2)$^{-2}$
$\chi^T(-u)$	3.894×10^{-4} (GeV/c^2)$^{-2}$

$B^0 \to D^{*-}\ell^+\nu_\ell$ results

- In all cases for the Fermilab-MILC results, the χ^2 goodness-of-fit parameter has increase compared to JLQCD, and particularly so in CLN.
- This phenomenon was also present in the original study by the collaboration.
- On investigation, it appears that the cause of this is the presence of large terms in the inverse of the covariance matrix of LQCD points
- This has little effect on the value of $|V_{cb}|$, and comparisons of form factor parameters, with an overall agreement with JLQCD within uncertainty.

TABLE II. Fitted parameters for the CLN, CLNnoR and CLNnoHQS scenarios using data from Fermilab-MILC calculations of form factors at non-zero recoil as additional constraints. The uncertainties listed are statistical and systematic, respectively. The branching ratio is obtained from the fit.

Parameter	CLN	CLNnoR	CLNnoHQS
ρ^2	$1.17 \pm 0.03 \pm 0.04$	$1.10 \pm 0.03 \pm 0.06$	$1.15 \pm 0.05 \pm 0.05$
$R_1(1)$	$1.29 \pm 0.02 \pm 0.01$	$1.43 \pm 0.01 \pm 0.01$	$1.43 \pm 0.01 \pm 0.01$
$R_2(1)$	$0.81 \pm 0.02 \pm 0.01$	$0.87 \pm 0.05 \pm 0.04$	$0.78 \pm 0.08 \pm 0.06$
$R_1'(1)$	-0.12 (fixed)	$-0.92 \pm 0.07 \pm 0.03$	$-0.94 \pm 0.07 \pm 0.03$
$R_2'(1)$	0.11 (fixed)	$0.02 \pm 0.13 \pm 0.12$	$0.32 \pm 0.26 \pm 0.20$
c_{D*}	ρ^2 (fixed)	ρ^2 (fixed)	$1.21 \pm 0.25 \pm 0.15$
$\mathcal{F}(1)\eta_{\rm EW}\lvert V_{cb}\rvert \times 10^3$	$35.58 \pm 0.16 \pm 0.73$	$35.42 \pm 0.22 \pm 0.85$	$35.42 \pm 0.20 \pm 0.81$
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$	4.99%	5.0%	5.0%
χ^2/ndf	379.6/45	361.2/43	362.3/42
p-value	0.0	0.0	0.0

TABLE III. Fitted parameters for the BGL(1,0,2), BGL(1,1,2) and BGL(2,2,2) configurations using data from Fermilab-MILC calculations of form factors at non-zero recoil as additional constraints. The uncertainties listed are statistical and systematic, respectively. The branching ratio is obtained from the fit and $\mathcal{F}(1)\eta_{\rm EW}|V_{cb}|$ is calculated from Eq. 2.

Parameter $\times 10^3$	BGL(1,0,2)	BGL(1,1,2)	BGL(2,2,2)
$\tilde{a}_0^f$	$0.480 \pm 0.003 \pm 0.011$	$0.479 \pm 0.003 \pm 0.011$	$0.476 \pm 0.007 \pm 0.011$
$\tilde{a}_1^f$	$0.41 \pm 0.14 \pm 0.24$	$0.52 \pm 0.13 \pm 0.24$	$1.00 \pm 0.48 \pm 0.50$
$\tilde{a}_2^f$	0.0 (fixed)	0.0 (fixed)	$-12.70 \pm 8.36 \pm 7.08$
$\tilde{a}_0^g$	$1.10 \pm 0.02 \pm 0.02$	$1.27 \pm 0.01 \pm 0.02$	$1.26 \pm 0.03 \pm 0.04$
$\tilde{a}_1^g$	0.0 (fixed)	$-6.92 \pm 0.45 \pm 0.66$	$-5.14 \pm 1.24 \pm 1.25$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)	$-36.54 \pm 13.30 \pm 10.89$
$\tilde{a}_1^{\mathcal{F}_1}$	$0.09 \pm 0.04 \pm 0.05$	$0.08 \pm 0.04 \pm 0.05$	$0.12 \pm 0.08 \pm 0.08$
$\tilde{a}_2^{\mathcal{F}_1}$	$-0.01 \pm 0.72 \pm 0.60$	$0.37 \pm 0.69 \pm 0.60$	$-0.06 \pm 1.21 \pm 1.13$
$\mathcal{F}(1)\eta_{\rm EW}\lvert V_{cb}\rvert$	$35.51 \pm 0.20 \pm 0.82$	$35.47 \pm 0.19 \pm 0.83$	$35.22 \pm 0.50 \pm 0.84$
$\mathcal{B}(B^0 \to D^{*-}\ell^+\nu_\ell)$	5.0%	5.0%	5.0%
χ^2/ndf	89.1/44	62.4/43	58.6/41
p-value	0.0	0.03	0.04

Hadronic factorisation measurement

	Model	$\lvert a_1 \rvert$	$n\sigma$ meas.-theo
π^-	BGL(2,2,2), F-MILC	0.884 ± 0.016	8.9 $(-18 \pm 2)\%$
π^-	BGL(2,2,2), JLQCD	0.905 ± 0.021	6.7 $(-16 \pm 2)\%$
π^-	CLNnoHQS, JLQCD	0.897 ± 0.021	7.1 $(-16 \pm 2)\%$
K^-	BGL(2,2,2), F-MILC	0.913 ± 0.024	5.8 $(-15 \pm 3)\%$
K^-	BGL(2,2,2), JLQCD	0.930 ± 0.027	4.7 $(-13 \pm 3)\%$
K^-	CLNnoHQS, JLQCD	0.924 ± 0.026	5.0 $(-14 \pm 3)\%$
ρ^-	BGL(2,2,2), F-MILC	0.826 ± 0.061	3.6 $(-22 \pm 5)\%$
ρ^-	BGL(2,2,2), JLQCD	0.837 ± 0.062	3.4 $(-21 \pm 6)\%$
ρ^-	CLNnoHQS, JLQCD	0.834 ± 0.062	3.4 $(-21 \pm 6)\%$
K^{*-}	BGL(2,2,2), F-MILC	0.803 ± 0.079	3.1 $(-24 \pm 8)\%$
K^{*-}	BGL(2,2,2), JLQCD	0.812 ± 0.080	3.0 $(-23 \pm 8)\%$
K^{*-}	CLNnoHQS, JLQCD	0.810 ± 0.080	3.0 $(-23 \pm 8)\%$
a_1^-	BGL(2,2,2), F-MILC	0.980 ± 0.111	0.7 $(-7 \pm 11)\%$
a_1^-	BGL(2,2,2), JLQCD	0.983 ± 0.111	0.6 $(-7 \pm 11)\%$
a_1^-	CLNnoHQS, JLQCD	0.984 ± 0.111	0.6 $(-7 \pm 11)\%$

$SU(3)$ symmetry test

- Take ratio of $|a_1(q_A)|^2/|a_1(q_B)|^2$ to test if this is a universal factor

Model	Ratio	Result
BGL(2,2,2), F-MILC	$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.066 ± 0.054
BGL(2,2,2), JLQCD	$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.057 ± 0.054
CLN noHQS, JLQCD	$\lvert a_1(K^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.061 ± 0.054
BGL(2,2,2), F-MILC	$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	0.87 ± 0.13
BGL(2,2,2), JLQCD	$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	0.86 ± 0.13
CLN noHQS, JLQCD	$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	0.87 ± 0.13
BGL(2,2,2), F-MIL	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\pi^-)\rvert^2$	0.83 ± 0.16
BGL(2,2,2), JLQCD	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\pi^-)\rvert^2$	0.81 ± 0.16
CLN noHQS, JLQCD	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\pi^-)\rvert^2$	0.82 ± 0.16
BGL(2,2,2), F-MILC	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.23 ± 0.28
BGL(2,2,2), JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.18 ± 0.27
CLN noHQS, JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\pi^-)\rvert^2$	1.20 ± 0.27
BGL(2,2,2), F-MILC	$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	0.82 ± 0.12
BGL(2,2,2), JLQCD	$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	0.81 ± 0.12
CLN noHQS, JLQCD	$\lvert a_1(\rho^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	0.82 ± 0.12

Model	Ratio	Result
BGL(2,2,2), F-MILC	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(K^-)\rvert^2$	0.77 ± 0.15
BGL(2,2,2), JLQCD	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(K^-)\rvert^2$	0.76 ± 0.15
CLN noHQS, JLQCD	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(K^-)\rvert^2$	0.77 ± 0.15
BGL(2,2,2), F-MILC	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	1.15 ± 0.26
BGL(2,2,2), JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	1.12 ± 0.26
CLN noHQS, JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^-)\rvert^2$	1.13 ± 0.26
BGL(2,2,2), F-MILC	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\rho^-)\rvert^2$	0.94 ± 0.23
BGL(2,2,2), JLQCD	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\rho^-)\rvert^2$	0.94 ± 0.23
CLN noHQS, JLQCD	$\lvert a_1(K^{*-})\rvert^2/\lvert a_1(\rho^-)\rvert^2$	0.94 ± 0.23
BGL(2,2,2), F-MILC	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\rho^-)\rvert^2$	1.41 ± 0.36
BGL(2,2,2), JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\rho^-)\rvert^2$	1.38 ± 0.37
CLN noHQS, JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(\rho^-)\rvert^2$	1.39 ± 0.37
BGL(2,2,2), F-MILC	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^{*-})\rvert^2$	1.49 ± 0.44
BGL(2,2,2), JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^{*-})\rvert^2$	1.47 ± 0.44
CLN noHQS, JLQCD	$\lvert a_1(a_1^-)\rvert^2/\lvert a_1(K^{*-})\rvert^2$	1.48 ± 0.44