Constructing the Standard Model from String Theory

YTF 17/12/2021

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2108.07316 and 2110.14029

Overview

1. Heterotic String Theory
2. Calabi-Yau Manifolds
3. Vector Bundles
4. Monad Bundles
5. Modern Computational Methods
6. Conclusion

I will try and be as pedagogical as possible

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- First attempt: “Vacuum Configurations for Superstrings” - Candelas, Horowitz, Strominger and Witten 1985
  - There are other approaches! F-Theory, intersecting D-Branes…

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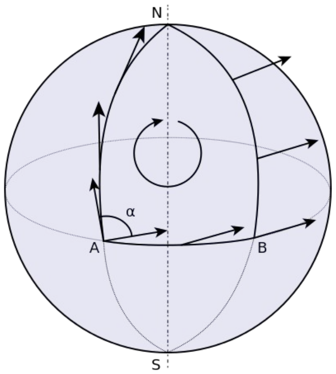

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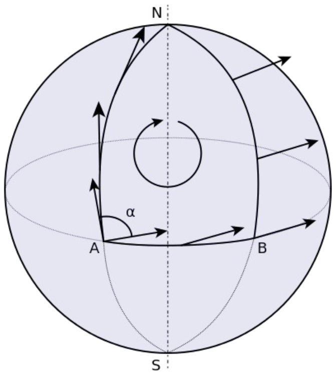

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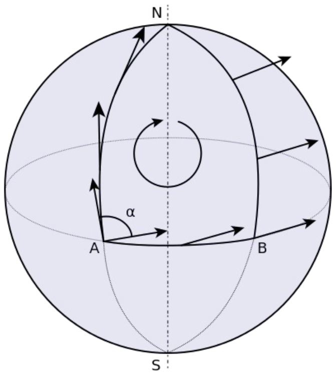

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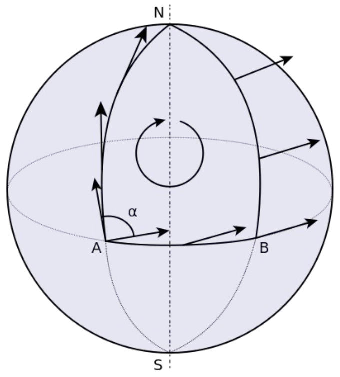

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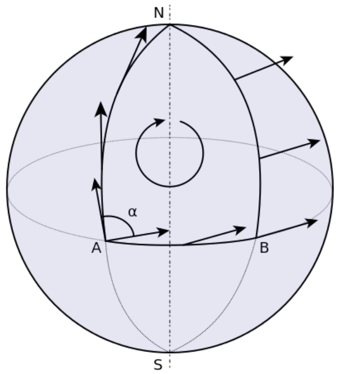
- Note, we will have extra scalar fields in 4D
  - Couplings appear as functions of these scalars
  - “Moduli Stabilisation”



Vector Bundles (Non-Trivial Gauge Fields)

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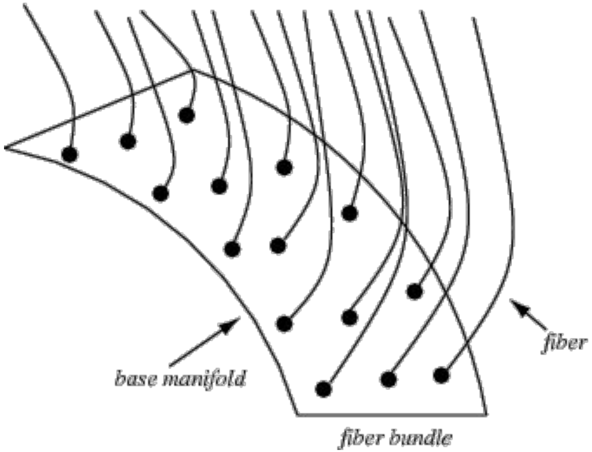

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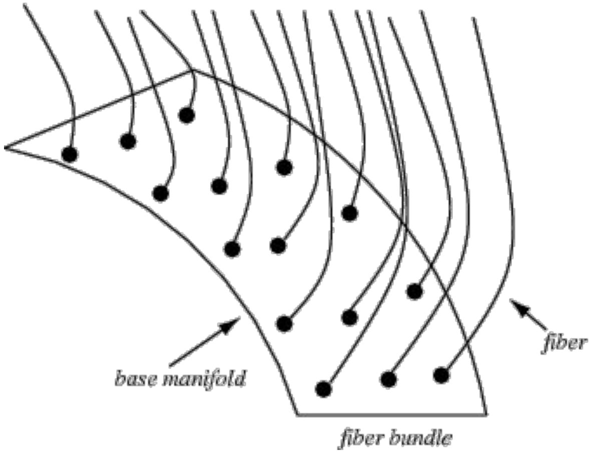

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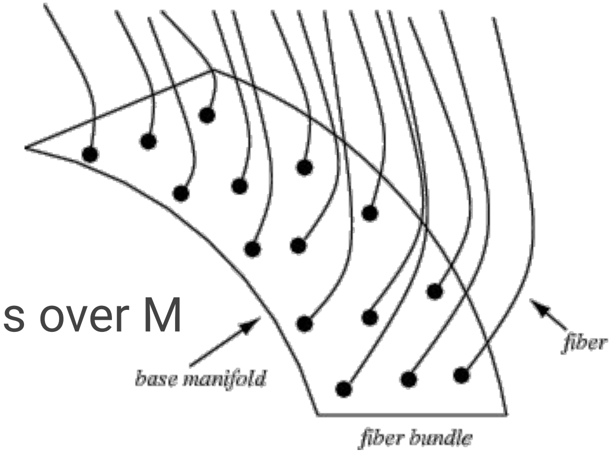

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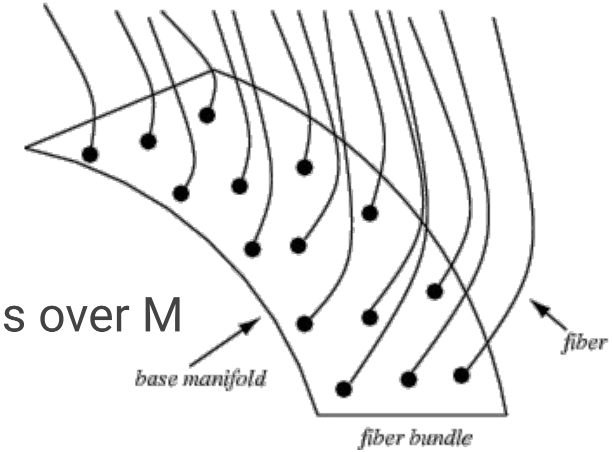

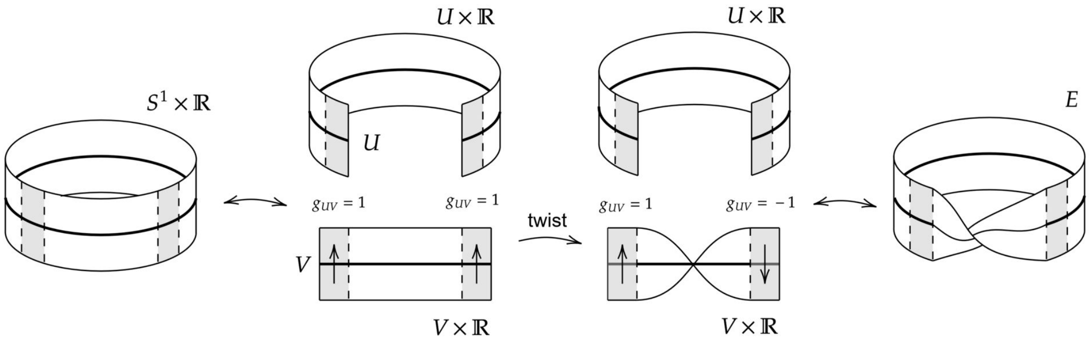

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- Line bundles are the simplest vector bundle
  - Look locally like C x M (or R x M for real)

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- Intuitively: $V \sim B/C$
- V is now given by a large number of integers

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See “Data science applications to string theory” by Fabian Ruehle for a review

https://www.sciencedirect.com/science/article/pii/S0370157319303072

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property	term in $v(B,C)$	comment
index match	$-\frac{2\lvert\mathrm{ind}(V)-\tau\rvert}{hM^3}$	$\tau = -3\lvert\Gamma\rvert$ is the target index, $\mathrm{ind}(V)$ computed from Eq. (2.20)
anomaly	$\frac{1}{hM^2}\sum_{i=1}^{h}\min\left(c_{2i}(TX)-c_{2i}(V),0\right)$	no penalty if anomaly condition satisfied, $c_{2i}(V)$ computed from Eq. (2.20)
bundleness	$-(d_{\mathrm{deg}}+1)$	d_{deg} = dimension of degeneracy locus as discussed in Sec. 2.4; if the degeneracy locus is empty, d_{deg} is to be taken as -1
split bundle	$-n_{\mathrm{split}}$	n_{split} = number of splits in V
equivariance	$-\sum_{U\subset B,C}\mathrm{mod}(\mathrm{ind}(U),\lvert\Gamma\rvert)$	U runs over all line bundles in B, C or blocks of same line bundles, as discussed in Sec. 2.4
trivial bundle	$-n_{\mathrm{trivial}}$	n_{trivial} = number of trivial line bundles
stability V	$-\frac{\max(0,h^0(X,B)-h^0(X,C))}{hM^3}$	tests Hoppe's criterion for V, cohomologies from formulae in Sec. 2.3
stability V^*	$-\frac{\max(0,h^0(X,B^*)-h^0(X,C^*))}{hM^3}$	tests Hoppe's criterion for V^*, cohomologies from formulae in Sec. 2.3

Table 2: Contributions to the intrinsic value for the monad environment. The intrinsic value $v(B,C)$ is the sum of all eight terms and $M = \max(b_{\max}, c_{\max})$.

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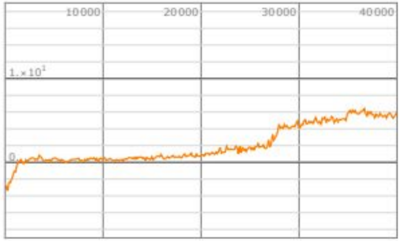
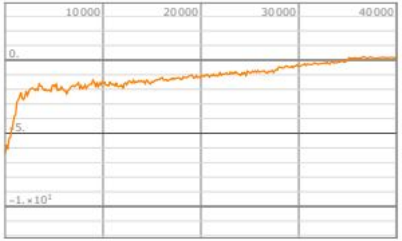
- These were hugely successful, on sizes up to 10^22!
  - 28 unique models found for the rank-(6,2) bicubic
    - after removing redundancies and further checks
  - Only one was known before!

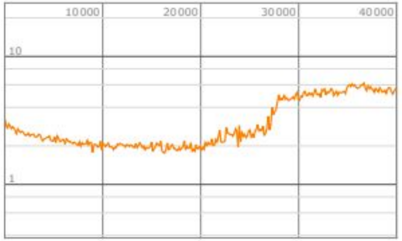
Reinforcement Learning Plots



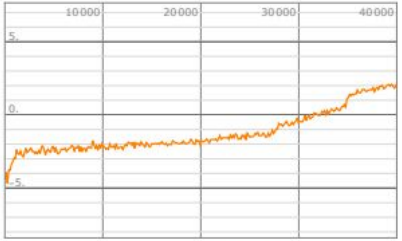
(a) Loss vs batch number.



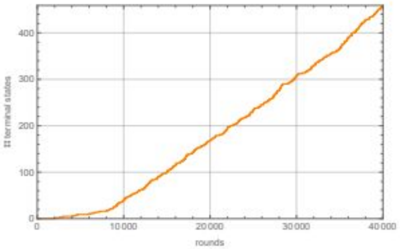
(b) Policy loss vs batch number.



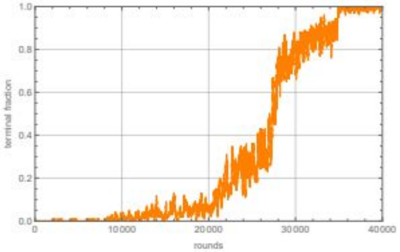
(c) Value loss vs batch number.



(d) TD return vs batch number.



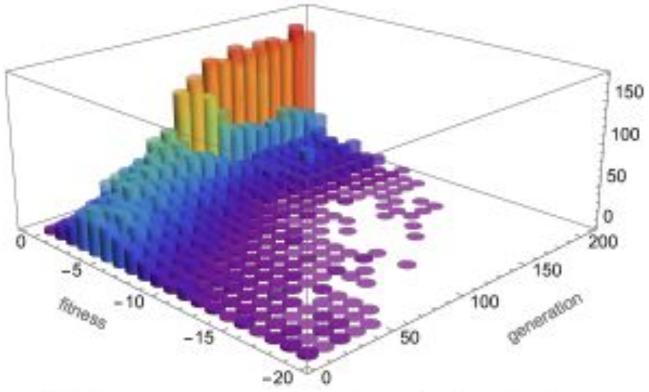
(e) Number of terminal states vs episode number.



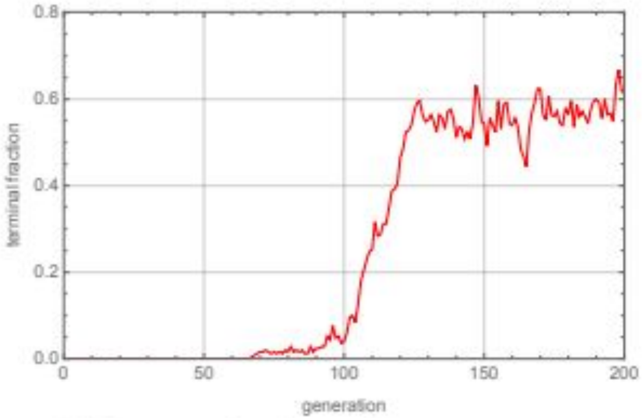
(f) Terminal fraction vs episode number.

Figure 6: Training metrics for the bicubic monad environment with $(r_B, r_C) = (6, 2)$.

Genetic Algorithms Plots



(a) Fitness histogram: number of individuals as a function of generation and fitness.

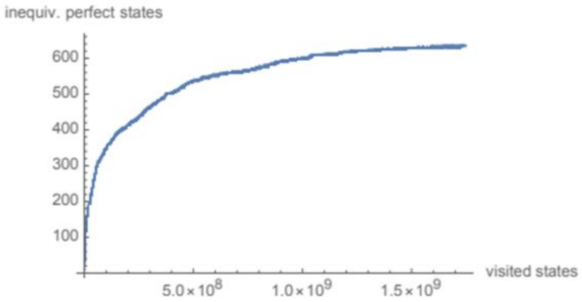


(b) Fraction of perfect models vs generation.

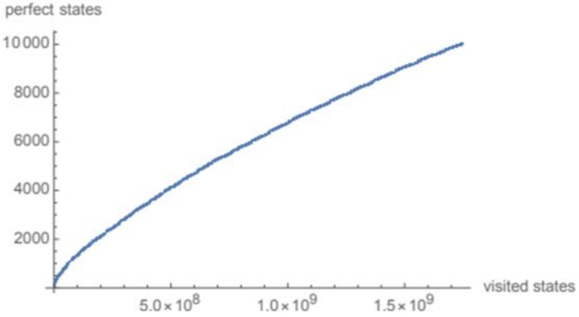
Figure 1: Performance measures for a typical GA initialisation on the bicubic.

RL Vs GA

RL



(a) Number of inequivalent perfect states found as a function of the number of states visited.



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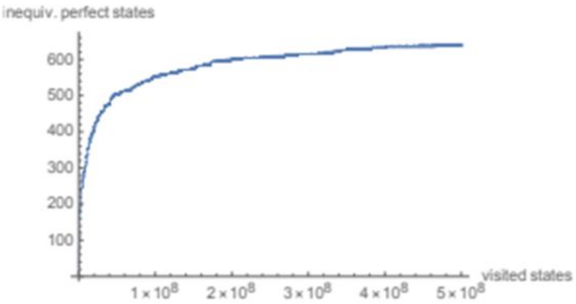

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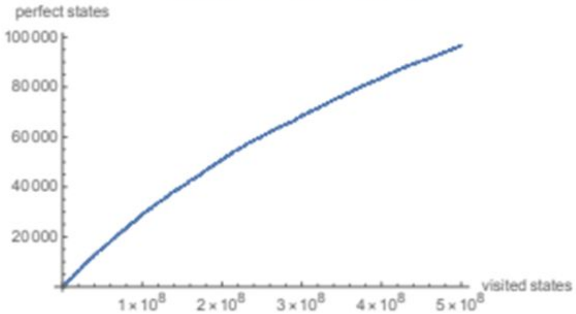

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