A comparison of spectral reconstruction methods applied to non-zero temperature NRQCD meson correlation functions

FASTSUM collaboration

arXiv:2112.04201

Themes of the talk

QCD

- The QCD phase diagram
- Quark Gluon Plasma

Lattice QCD

- Motivating the lattice
- Non Relativistic QCD

Spectral Reconstruction

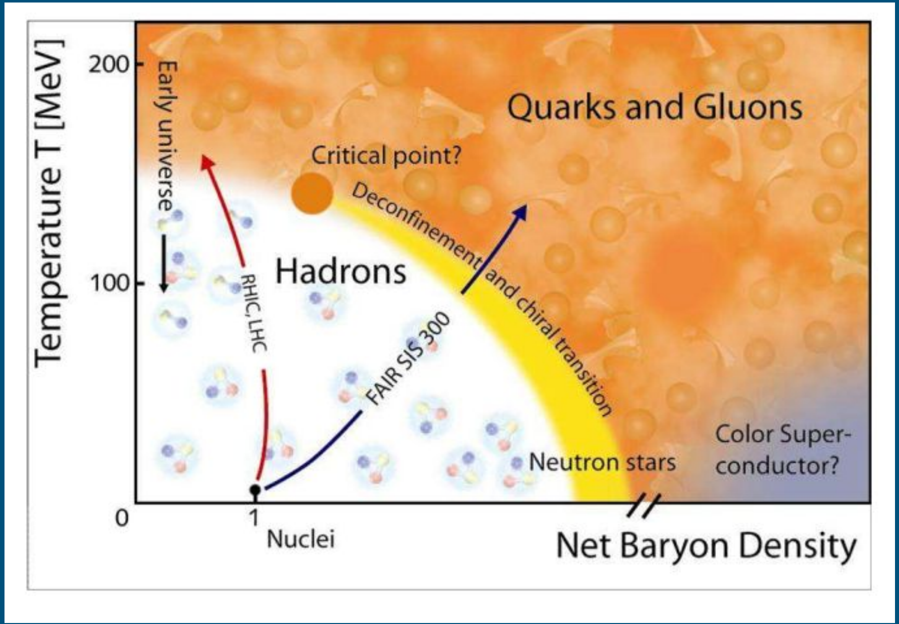
- **Inversion problem**
    - Maximum likelihood estimation
    - Backus Gilbert method
    - Kernel ridge regression

The QCD phase diagram

- Confinement
- Quark Gluon Plasma

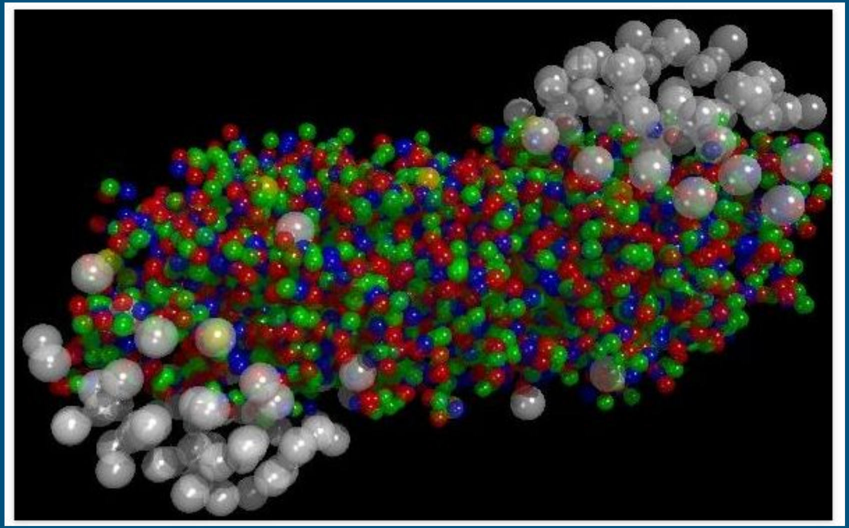
The QCD phase diagram

- Low temperature: quarks are confined into hadrons (mesons, baryons …)
- High temperature: quark gluon plasma
- Transition at T_C
- Better understand this transition



Source: 1512.05145 [hep-lat]

Quark Gluon Plasma



Source: CERN press release, 2000

- Hot, but temperature not well understood
- Created at RHIC and CERN
- Melts before it can reach the detectors
- Hadrons act as probes, presence of excited states can be used as a proxy for temperature [T. Matsui, H. Satz]

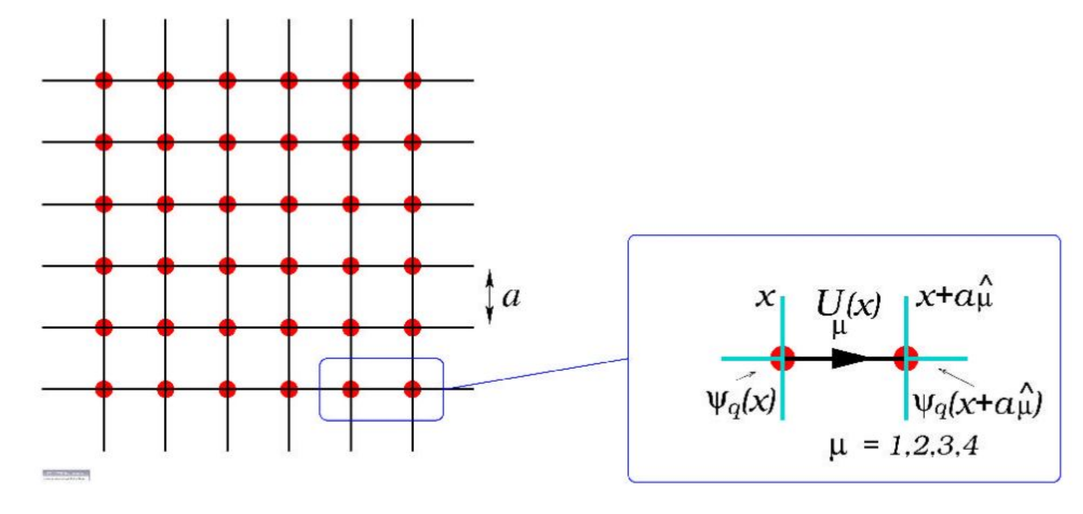
Lattice Quantum Chromodynamics

- QCD is a quantum field theory in need of regularisation
- QCD is a theory of **strong** interactions
    - No perturbation theory

Motivating the lattice

- Exact, non-perturbative, regularisation of a quantum field theory
- High energy cut-off imposed by finite lattice spacing
- Continuum limit recoverable *

*not in the EFT I will use



Source: R.Soualah, EMMI Seminar (2008)

State of affairs

QCD phase diagram needs more exploring

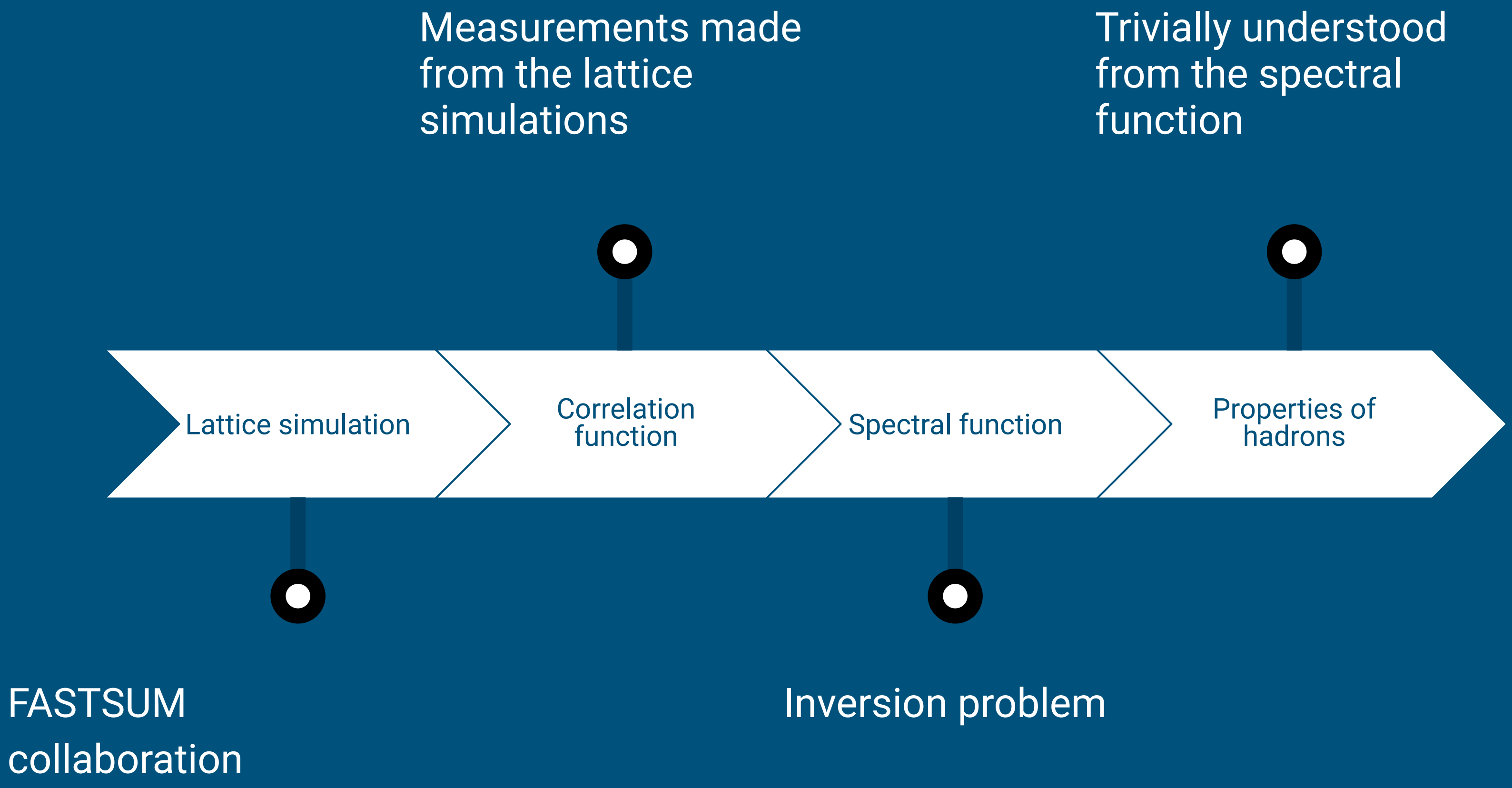
- Study hardons at a range of temperatures

Studying hardons requires non-perturbative solutions

- Lattice QCD

The inversion problem of spectral reconstruction

- ...


Measurements made from the lattice simulations
Trivially understood from the spectral function
Lattice simulation
Correlation function
Spectral function
Properties of hadrons
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Inversion problem

Non-Relativistic QCD (NRQCD)

A technical caveat
(Ignorable)
[Thacker, Lepage Phys.Rev. D43 (1991) 196-208]

- Effective field theory
- Expansion in powers of the bottom quark mass
- Bottom quark mass larger than other scales of the system
- No continuum limit

Spectral Reconstruction

The actual talk

The Inverse Problem

From the lattice we can directly calculate the correlation function

We require the spectral function to infer properties of particles

$$G(\tau) = \int_{\omega_{min}}^{\omega_{max}} \frac{\mathrm{d}\omega}{2\pi} K(\tau, \omega)\rho(\omega), \qquad K(\tau, \omega) = e^{-\omega\tau}$$

The Inverse Problem

J. Hadamard,
Princeton University Bulletin 13
(1902) 49.

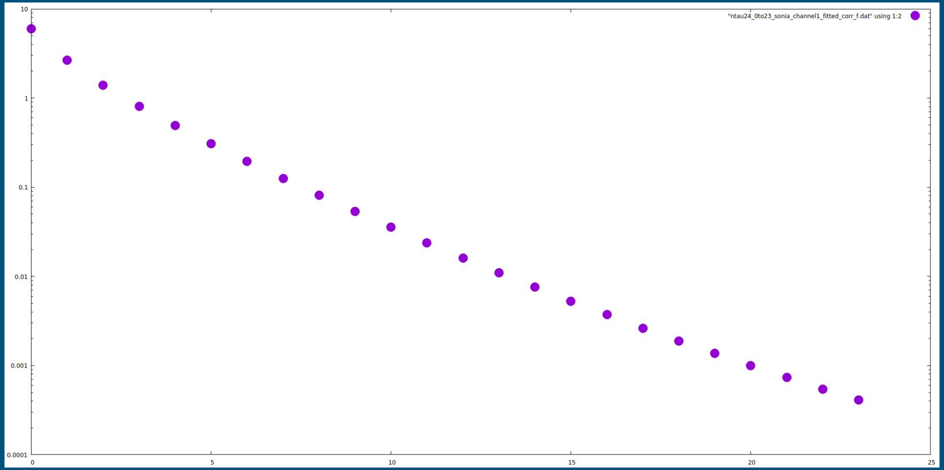
O(10-100) points

O(1000) points

Ill-posed.
No unique solution

$$G(\tau) = \int_{\omega_{min}}^{\omega_{max}} \frac{\mathrm{d}\omega}{2\pi} K(\tau,\omega)\rho(\omega), \qquad K(\tau,\omega) = e^{-\omega\tau}$$

The Inverse Problem



Correlation function

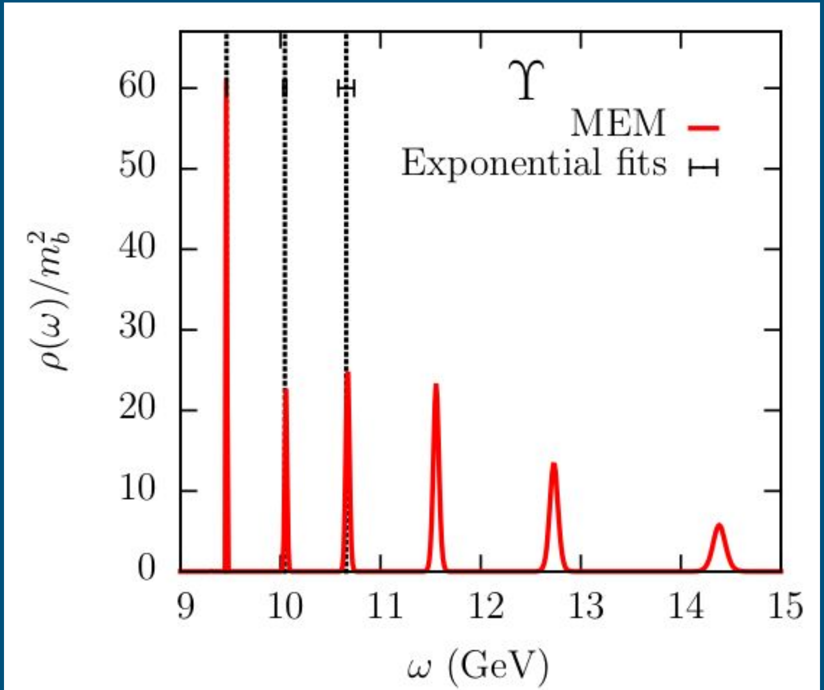
- Laplace transform of a delta function is an exponential

<- Log scale (axis values not important)

- Not a flat line in log scale
    - Not pure exponential

A pure exponential correlation function would imply there is only a single state possible (no excited states), as well as no finite lifetime

The Inverse Problem



1402.6210 [hep-lat]

Spectral function

- Mass of states given by peak location
- Width gives understanding of lifetime

Three proposed resolutions (put a footer about other methods)

Maximum likelihood

- Reduce the number of parameters by fitting a function rather than each point in the data
- Parametric
- Biased to ansatz

Backus Gilbert

- Create a local average of the spectral function built from only the data and kernel
- Resolution bound

Kernel ridge regression

- Supervised machine learning procedure
- Learns from training data to map correlation functions to spectral functions
- Biased to training data

Maximum Likelihood

Minimise the difference between the data and a curve:

- Extract ground state mass and width
- The amplitudes are less important
- The excited state parameters are heavily influenced by systematic effects

In practise:

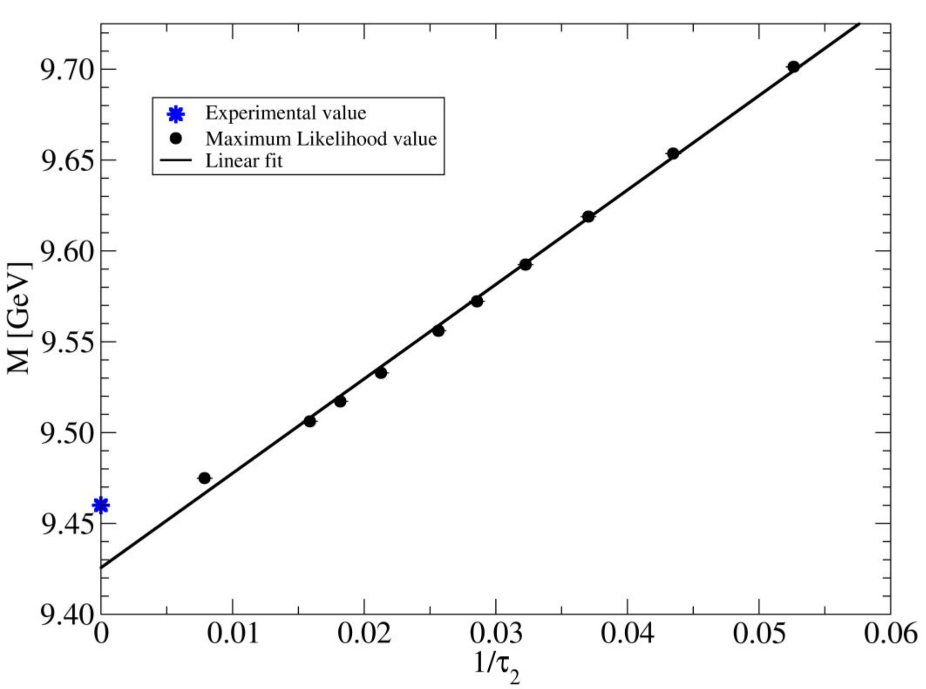
We have intuition about the spectral function, we want to use this to fit the correlation function

$$\rho_{\text{Ansatz}}(\omega) = \mathcal{A}_{\text{ground}}\; e^{-\frac{(\omega - M_{\text{ground}})^2}{2\sigma^2}} + \mathcal{A}_{\text{excited}}\;\delta(\omega - M_{\text{excited}}).$$

$$\downarrow$$

$$G(\tau) = A_{\text{ground}}\; e^{-(M_{\text{ground}} - \frac{\sigma^2 \tau}{2})\tau} + A_{\text{excited}}\; e^{-M_{\text{excited}}\tau}.$$

Maximum Likelihood



2112.04201 [hep-lat]

Issues:

- Bias to choice of ansatz
- Inflexible
- Prone to systematic effects

Backus Gilbert

Create a local average of the spectral function

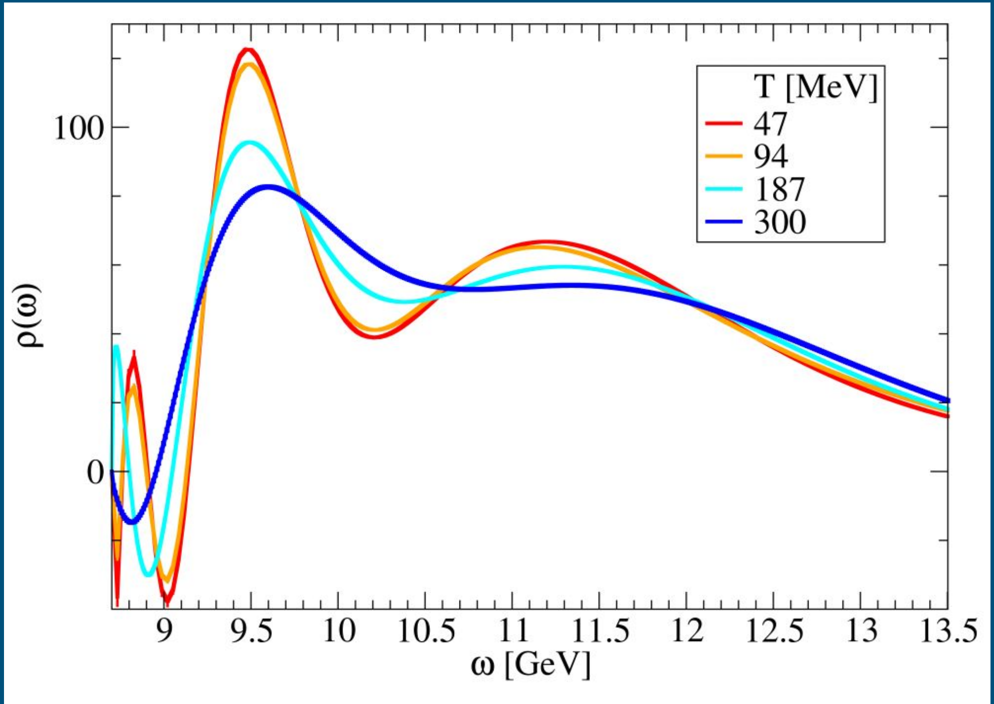
$$\hat{\rho}(\omega_0) = \int_{\omega_{min}}^{\omega_{max}} A(\omega, \omega_0)\rho(\omega)d\omega.$$

Made of only the data given by the problem

$$A(\omega, \omega_0) = \sum_{\tau} c_\tau(\omega_0)K(\omega, \tau),$$

$$\hat{\rho}(\omega_0) = \sum_{\tau} c_\tau(\omega_0)G(\tau).$$

Backus Gilbert



2112.04201 [hep-lat]

- Artifacts below ~9GeV
- Mass and width increase with temperature
- Widths are huge

Kernel Ridge Regression

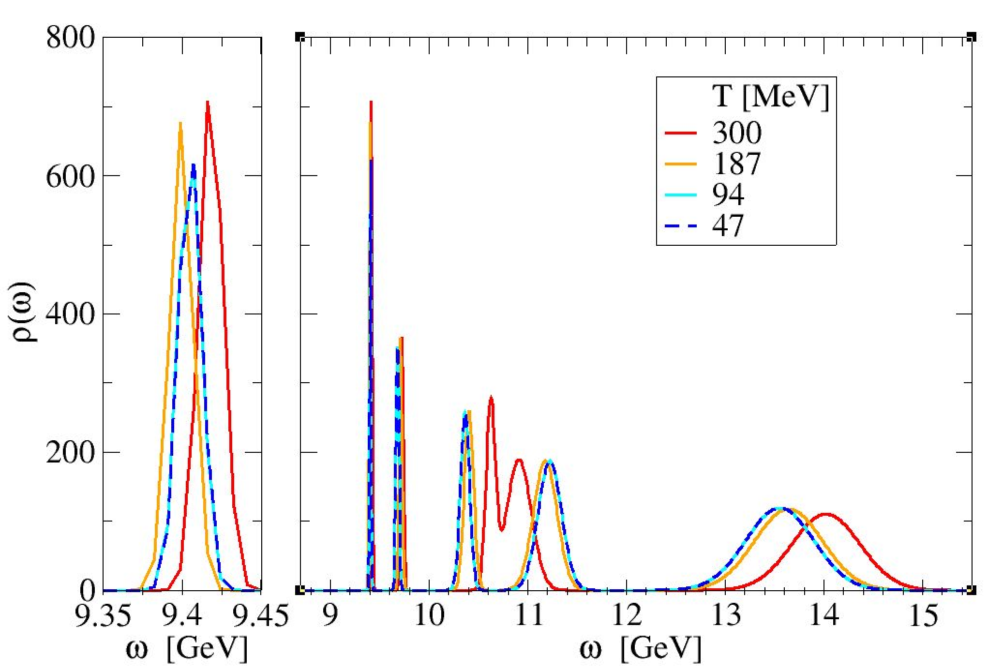
- Generalisation of linear regression
  - Input data is transformed
  - Weights are penalised for being too large

$$C_{ij} = \exp\left(-\gamma \sum_n \left[\frac{G_i(\tau_n) - G_j(\tau_n)}{\overline{G}(\tau_n)}\right]^2\right)$$

- Fake data is required
  - Training is biased to this
  - Functional form of the training data is <u>fixed</u>

$$\mathbf{Y} = \mathbf{C}\alpha$$

Kernel Ridge Regression



2112.04201 [hep-lat]

- All underestimate the ground and excited state mass
- Non trivial temperature dependence of mass
- No apparent temperature dependence of the width

Conclusion

- QCD phase diagram / transition need exploring
- Temperature of the quark gluon plasma requires us to know more about excited states of mesons
- Lattice QCD one tool for the job
- Inverse problem is a challenge
- No physics yet (from the three methods mentioned here)
- Many more methods [arXiv:1307.6106, arXiv:1402.6210, arXiv:1908.08437, many more..]

Thank you