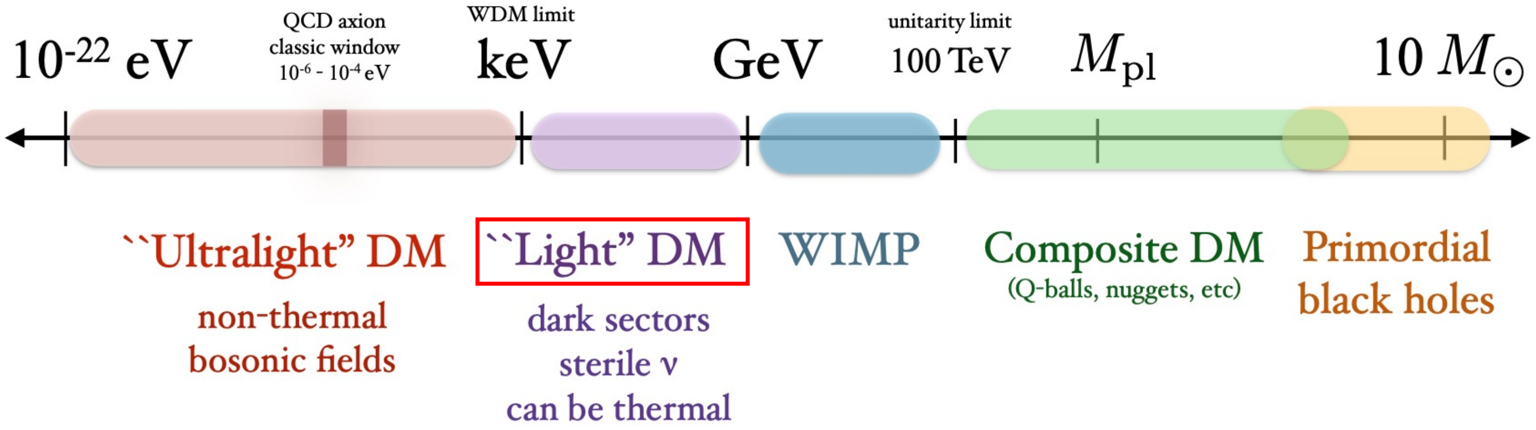
Fuelling the Search for Light Dark Matter – Electron Scattering

Louis Hamaide – YTF 2021

Introduction - Motivation for Dark Matter

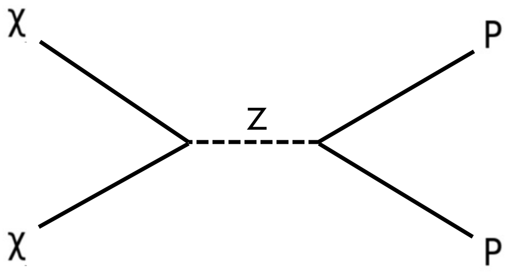
- Many theoretical + observational motivations for dark matter (DM): rotation curves of galaxies, bullet clusters, neutrino oscillations, strong CP problem and more!
- The Standard Model has proven very successful, however experiments show maybe we may need to add new *dark, stable,* and relatively *collisionless* particles
- Large mass range available to explore experiments and theory



Motivating Searches of Light(er) Dark Matter

- DM detection usually aims to constrain models of DM produced during "freeze-out" (WIMPs) and or "freeze-in" (FIMPs) as individual scenarios can be fully tested.

- Lee-Weinberg bound ('77): assume weak interaction-generated (thermal, neutrino-like) WIMPs:



+

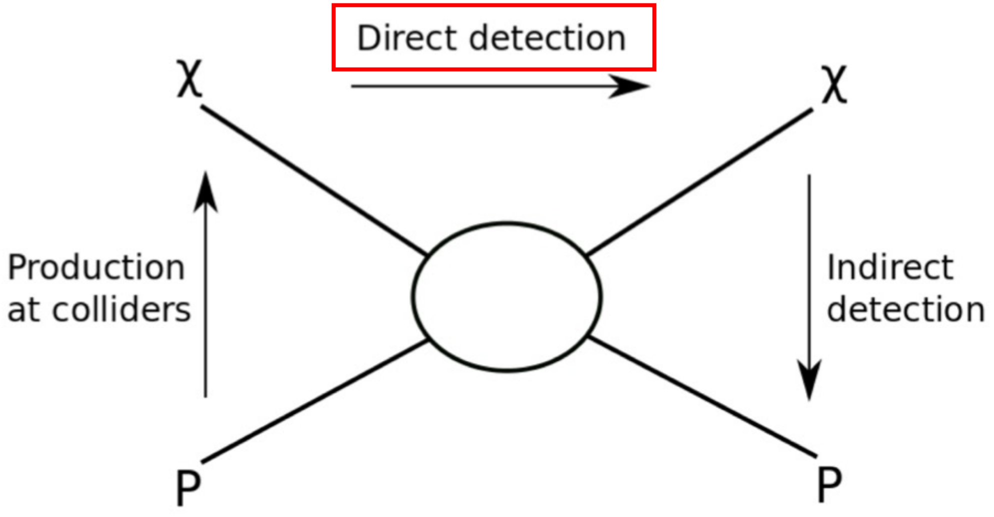
$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27}\,\mathrm{cm^3 s^{-1}}}{\langle \sigma_{\chi\chi} v_{rel} \rangle}$$

- Annihilation cross section $\sim m_\chi^2/m_Z^4$ → sets a lower mass bound ~2GeV

- These assumptions can be relaxed → **search for <1 GeV DM is motivated !**

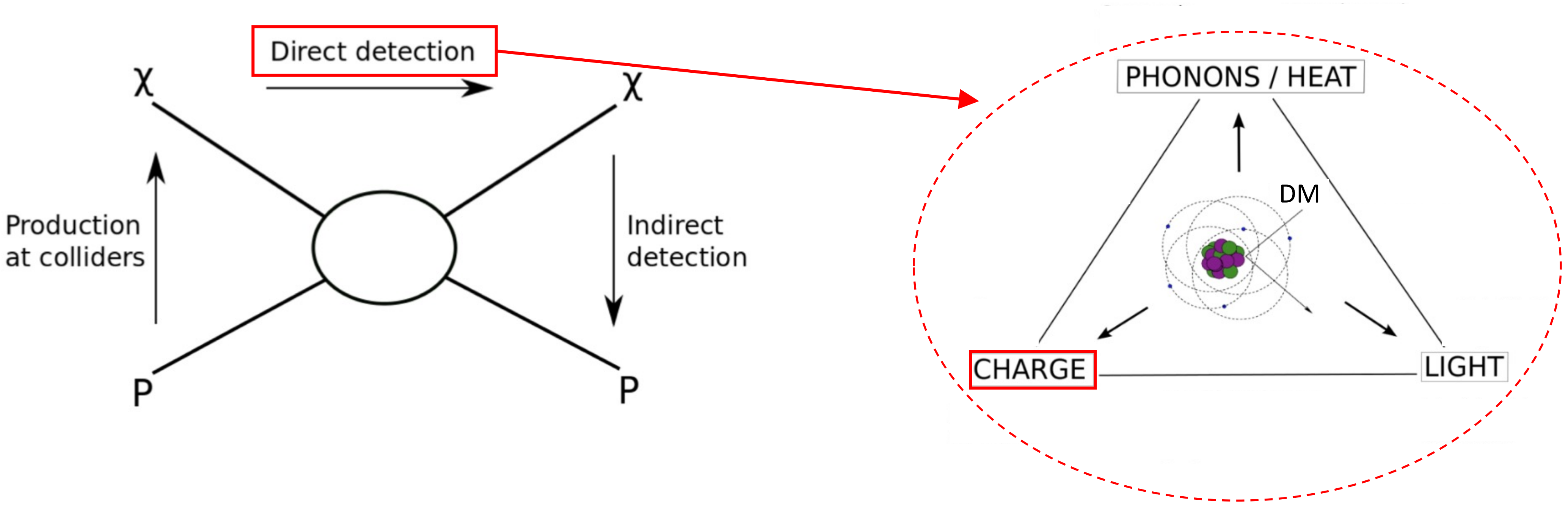
Searches for Light Dark Matter

- Virtually any interaction of DM with the Standard Model can be probed using one of the following methods/signals:



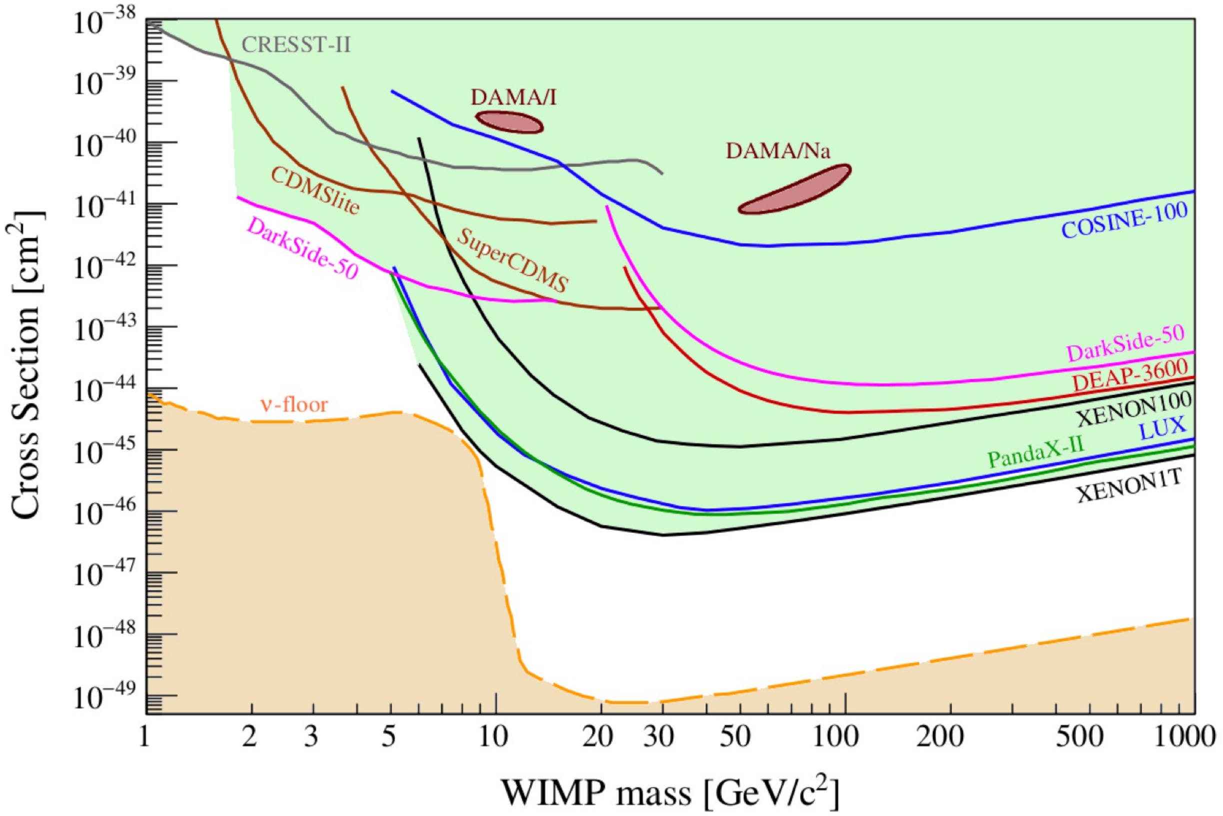
Searches for Light Dark Matter

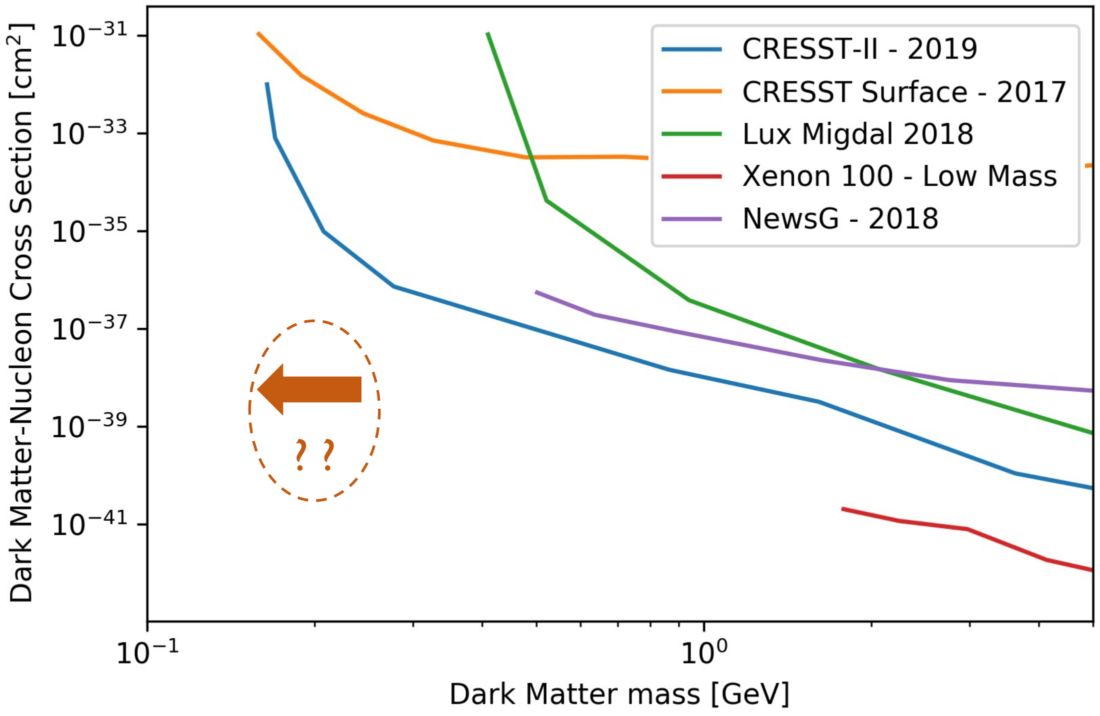
- Virtually any interaction of DM with the Standard Model can be probed using one of the following methods/signals:



Going to lower DM masses (1/2)

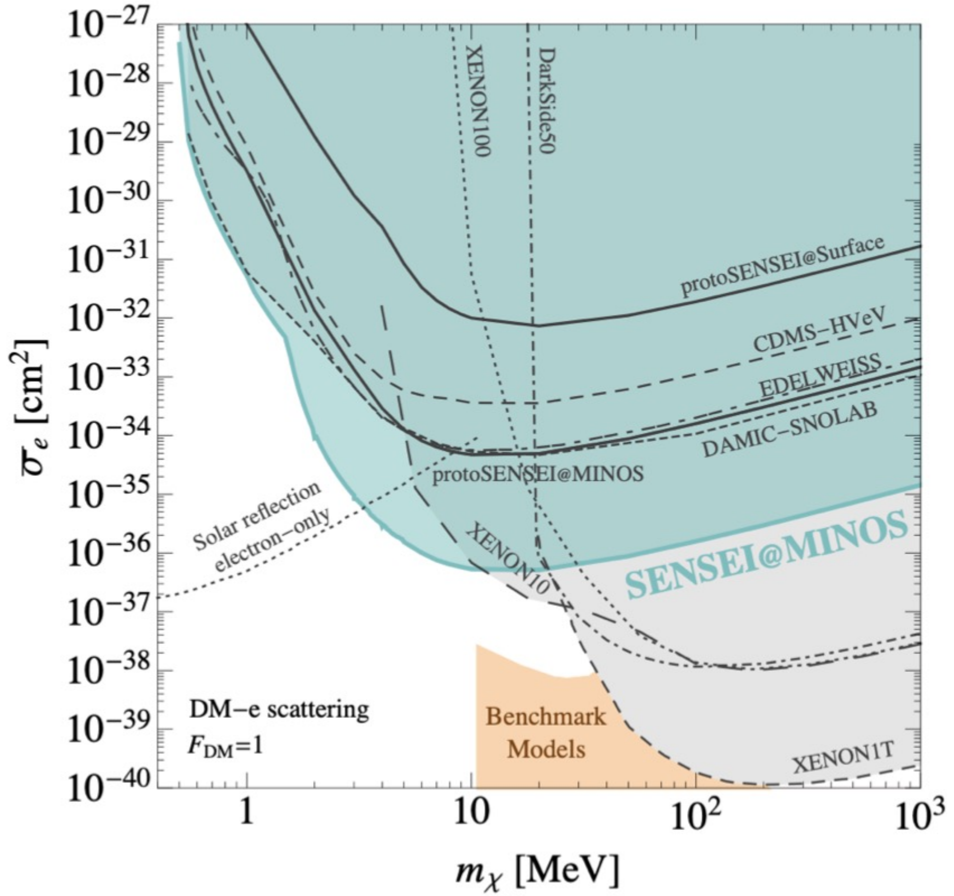
- Xenon nuclear recoil DM detectors are the most competitive at large masses.





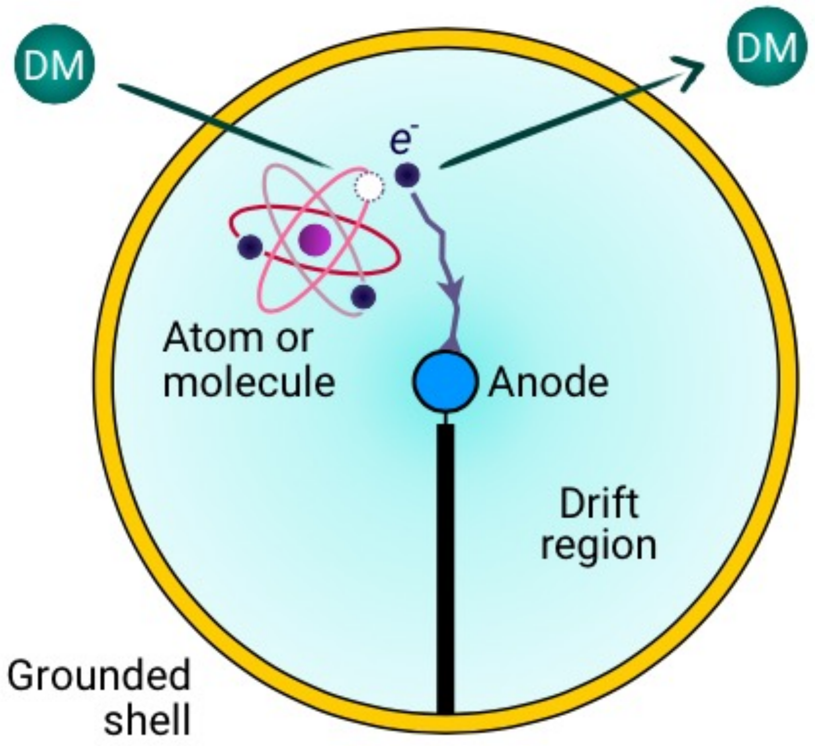
Going to lower DM masses (2/2)

- The same seems to happen in electron scattering: experiments sensitive to single electron events should be competitive for lower mass electron scattering



Spherical Proportional Counters

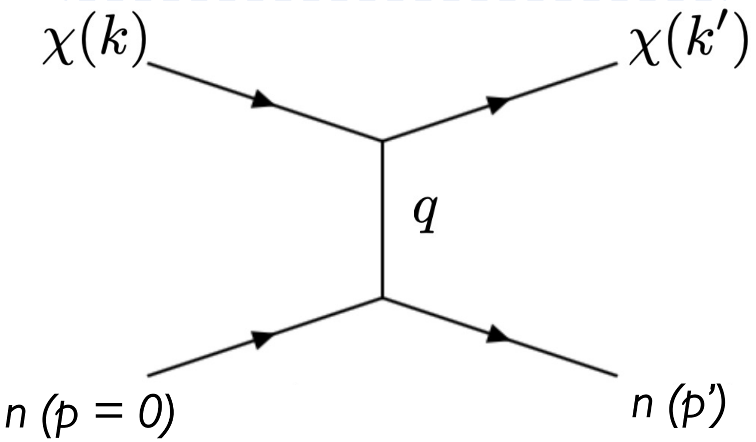
- SPCs consist of gas in a sphere sensitive to 1e- events and exhibit low noise
- We study He and Ne (lighter), Xe (for comparison), CH_4 and C_4H_{10} (quenchers)



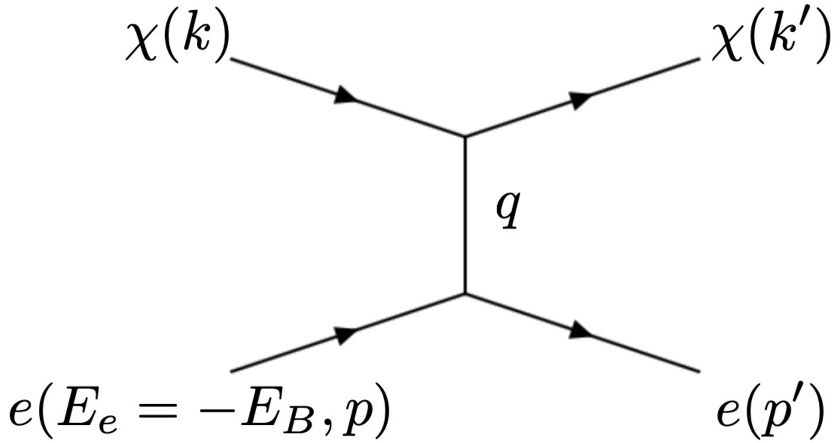
Ionization electrons drift to the anode where the initial energy can be reconstructed

Dark Matter Electron Scattering

Traditional WIMP detector



Electron scattering



$$\mathcal{M} = g_x g_y \frac{1}{q^2 - m^2} \int d^3x\, e^{-i\mathbf{p}\cdot\mathbf{r}} e^{i\mathbf{q}\cdot\mathbf{r}} e^{i\mathbf{p}'\cdot\mathbf{r}} \longrightarrow \mathcal{M}(n, l \to \text{free}\, e^-) = g_x g_y \underbrace{\frac{1}{q^2 - m^2}}_{F_{DM}(q)} \int d^3x\, \boxed{\tilde{\psi}^*_{p'l'm'}(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}} \psi_{nlm}(\mathbf{x})}$$

where $\boxed{d\sigma \;\; \alpha \;\; |\mathcal{M}|^2}$

Bound electron wavefunctions (1/5)

- Hartree-Fock approximation: mean field self-consistent bound states with energies correct to first order

in non-relativistic single particle time independant pertubation theory:

$$H(\mathbf{x}_1, ..., \mathbf{x}_N) = \sum_i \left(-\frac{1}{2}\nabla_i^2 - \frac{Z}{\mathbf{x}_i}\right) + \sum_{i \neq j} \frac{1}{|\mathbf{x}_i - \mathbf{x}_j|} \quad \rightarrow \quad H(\mathbf{x}) = \sum_i \left(-\frac{1}{2}\nabla_i^2 - \frac{Z}{|\mathbf{x}_i|} + U(\mathbf{x}_i) + V_i(\mathbf{x})\right) \ ,$$

$$H\psi(\mathbf{x}_1, ..., \mathbf{x}_N) = E\psi(\mathbf{x}_1, ..., \mathbf{x}_N) \quad \rightarrow \quad \left(\sum_i (H_i^{(0)} + H_i^{(1)})\right) \psi_1(\mathbf{x}_1)...\psi_N(\mathbf{x}_N) = (\epsilon_1^{(0)} + \epsilon_1^{(1)} + ... + \epsilon_N^{(0)} + \epsilon_N^{(1)})\psi_1(\mathbf{x}_1)...\psi_N(\mathbf{x}_N)$$

$$\text{where } H^{(0)}(\mathbf{x}_i) = -\tfrac{1}{2}\nabla^2 - \tfrac{Z}{|\mathbf{x}_i|} + U(\mathbf{x}_i) \text{ and } H_i^{(1)}(\mathbf{x}) = V_i(\mathbf{x}) = \textstyle\sum_{j \neq i} \frac{1}{|\mathbf{x_i} - \mathbf{x_j}|} - U(\mathbf{x}_i)$$

and $\epsilon_i^{(1)} = \left\langle \psi^{(0)} \middle| H_i^{(1)} \middle| \psi^{(0)} \right\rangle$

- HF wavefunctions variationally minimize energy: guarantees correct result (to first order). However does not guarantee orthogonality!

Bound electron wavefunctions (2/5)

- Hartree-Fock approximation: mean field self consistent bound states:

$$-\frac{1}{2}\frac{d^2 P_{n_a l_a}}{dr^2} + \frac{l_a(l_a+1)}{2r^2} P_{n_a l_a}(r) - \frac{Z}{r} P_{n_a l_a}(r) + \sum_{n_b l_b} (4l_b+2)\left(v_0(n_b l_b, r) P_{n_a l_a}(r) - \sum_l \Lambda_{l_a l l_b} v_l(n_b l_b, n_a l_a, r) P_{n_b l_b}(r) \right)$$

$$= \epsilon_{n_a l_a} P_{n_a l_a}(r) + \sum_{n_b \neq n_a} \epsilon_{n_a l_a, n_b l_a} P_{n_b l_a}(r)$$

Self-consistent approach required

Expect accuracy of O(30%) in event rates/bounds

Bound electron wavefunctions (2/5)

- Hartree-Fock approximation: mean field self consistent bound states:

$$-\frac{1}{2}\frac{d^2P_{n_al_a}}{dr^2} + \frac{l_a(l_a+1)}{2r^2}P_{n_al_a}(r) - \frac{Z}{r}P_{n_al_a}(r) + \sum_{n_bl_b}(4l_b+2)\left(v_0(n_bl_b,r)P_{n_al_a}(r) - \sum_l \Lambda_{l_all_b} v_l(n_bl_b,n_al_a,r)P_{n_bl_b}(r)\right)$$

$$= \epsilon_{n_al_a}P_{n_al_a}(r) + \sum_{n_b\neq n_a}\epsilon_{n_al_a,n_bl_a}P_{n_bl_a}(r)$$

Self-consistent approach required

Expect accuracy of O(30%) in event rates/bounds

$$AO_{nlm} \propto \sum_i c_i^n \exp(-\alpha_i^n r^2) Y_{lm}(\theta,\phi)$$

$$MO_{nlm} = \bar{v}_{nlm} AO_{nlm}$$

$$H_{ij} = \int AO_i(\mathbf{r}) H_j(\mathbf{r}) AO_j(\mathbf{r})\, d^3\mathbf{r}\ .$$

Treats atoms and molecules, relativistic treatments, molecular dipoles, and more.

Bound electron wavefunctions (2/5)

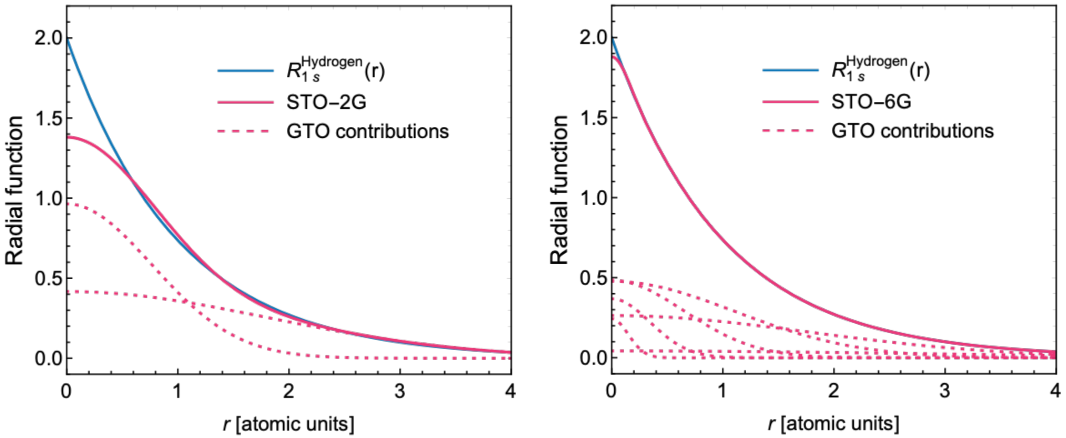
- Hartree-Fock approximation: mean field self consistent bound states:

$$-\frac{1}{2}\frac{d^2P_{n_al_a}}{dr^2}+\frac{l_a(l_a+1)}{2r^2}P_{n_al_a}(r)-\frac{Z}{r}P_{n_al_a}(r)+\sum_{n_bl_b}(4l_b+2)\left(v_0(n_bl_b,r)P_{n_al_a}(r)-\sum_{l}\Lambda_{l_all_b}v_l(n_bl_b,n_al_a,r)P_{n_bl_b}(r)\right)$$

$$=\epsilon_{n_al_a}P_{n_al_a}(r)+\sum_{n_b\neq n_a}\epsilon_{n_al_a,n_bl_a}P_{n_bl_a}(r)$$

Self-consistent approach required

Expect accuracy of O(30%) in event rates/bounds



Gaussian basis choice important at small/large r

pyscf

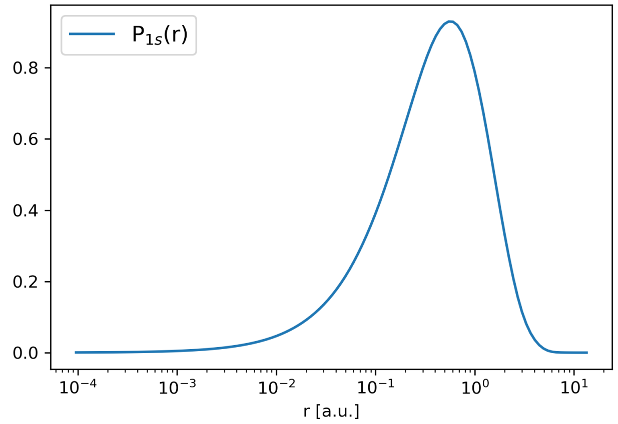
$$AO_{nlm}\propto\sum_i c_i^n\exp(-\alpha_i^n r^2)Y_{lm}(\theta,\phi)$$

$$MO_{nlm}=\bar{v}_{nlm}AO_{nlm}$$

$$H_{ij}=\int AO_i(\mathbf{r})H_j(\mathbf{r})AO_j(\mathbf{r})\,d^3\mathbf{r}\ .$$

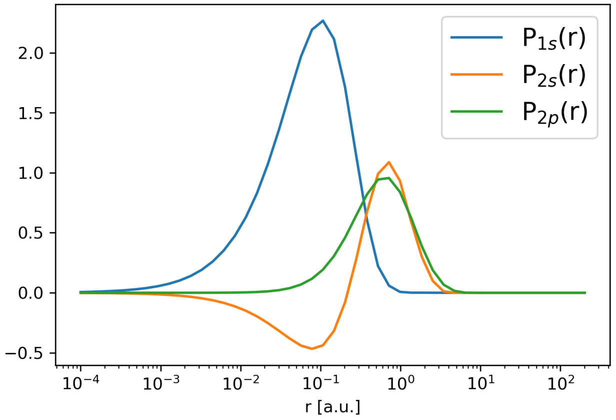
Treats atoms and molecules, relativistic treatments, molecular dipoles, and more.

Bound electron wavefunctions (3/5)



HELIUM

$E_{bound}(1s)$ =
(0.025) keV
→ 1s events!

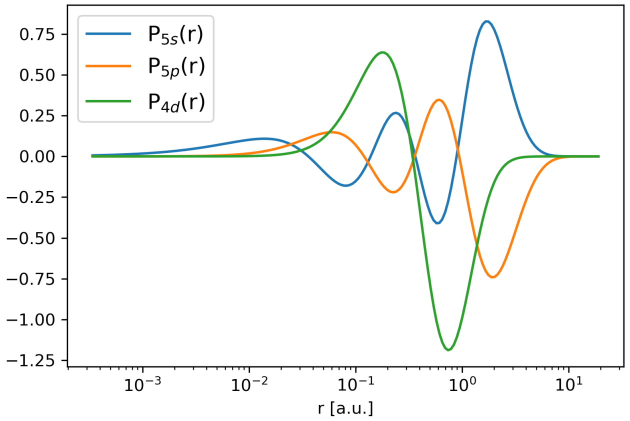


NEON

$E_{bound}(1s,2s,2p)$ =
(0.89, 0.052, 0.023)
keV
→ Only n=2 events!

Cutoff condition:

$KE_{DM} > E_B$



XENON

$E_{bound}(4s,4p,4d,5s,5p)$=
(0.21,0.16,0.075,0.025
0.012) keV
→ Only n=4, 5 events!

Bound electron wavefunctions (4/5)

- Molecular orbitals are no longer eigenfunctions (spherical harmonics) of the SO(3) generators.

$$\psi(\mathbf{r}) = \frac{P(r)}{r} Y_{lm}(\theta, \phi) \quad \rightarrow \quad \psi(x, y, z)$$

- We need new classification of orbitals :

irreducible representations of SO(3) → irreducible representation of point group

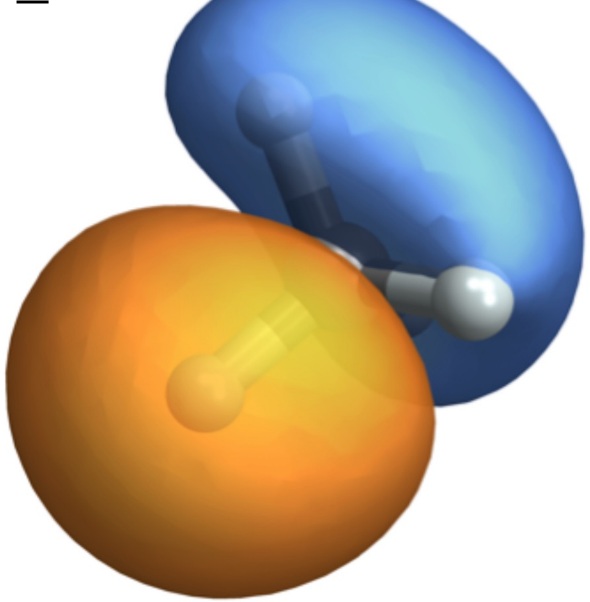
$\mathbf{T_d}$	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	linear functions, rotations	quadratic functions	cubic functions
A_1	+1	+1	+1	+1	+1	-	$x^2+y^2+z^2$	xyz
A_2	+1	+1	+1	-1	-1	-	-	-
E	+2	-1	+2	0	0	-	$(2z^2-x^2-y^2, x^2-y^2)$	-
T_1	+3	0	-1	+1	-1	(R_x, R_y, R_z)	-	$[x(z^2-y^2), y(z^2-x^2), z(x^2-y^2)]$
T_2	+3	0	-1	-1	+1	(x, y, z)	(xy, xz, yz)	(x^3, y^3, z^3) $[x(z^2+y^2), y(z^2+x^2), z(x^2+y^2)]$

Tetrahedral group (Methane): T_d

Bound electron wavefunctions (5/5)

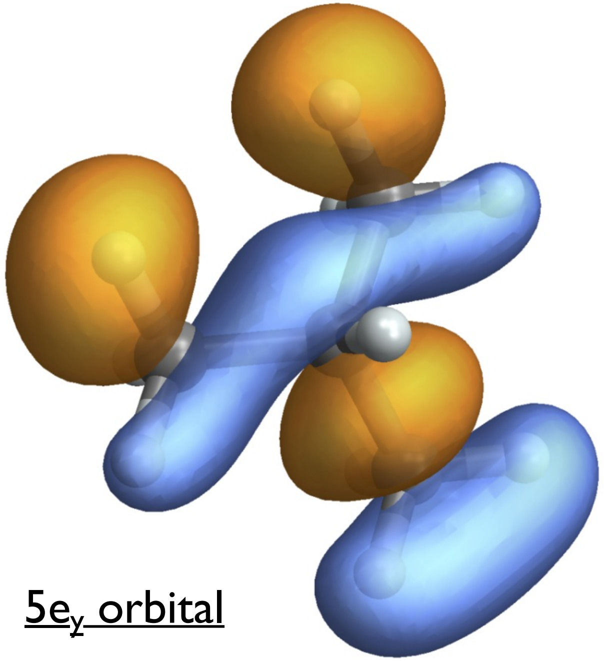
- Molecular orbitals don't enjoy the spherically symmetric Hamiltonian of noble gas. Instead the gaussian basis is multi-centered:

$2t_{2x}$ orbital



METHANE (CH_4)

E_{bound} = (−304.9 eV,−24.8 eV,−14.8 eV)
→ Only 2a and 2t events!



$5e_y$ orbital

ISOBUTHANE (C_4H_{10}))

E_{bound}= (−305.429eV, −304.941eV, … −12.4723eV, −12.318eV)
→ Only $E_{>2a}$ events!

Unbound electron wavefunctions

- Continuum limit: Hartree-Fock integrated, approximate the self-consistent piece of the potential:

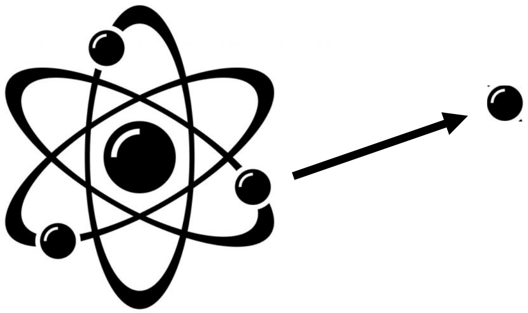
$$-\sum_{i\neq j} \frac{1}{|\mathbf{r_i}-\mathbf{r_j}|} = V_{dir}(r) + V_{exch}(r)$$

$$V_{dir}(r) = \sum_{n_b l_b}(4l_b+2)\int_0^\infty \left(\frac{P^2_{n_b l_b}(r_1)}{\max(r_1,r)}\right) dr_1$$

$$V_{\mathrm{exch}} = k_x\left(\frac{24\rho(r)}{\pi}\right)^{1/3}$$

→ « Hartree-Plus-Statistical-Exchange » potential (Cowan) gives accurate wavefunctions and energies (checked against bound WFs).

- Use frozen core approximation: electrons don't have time to react (ie change wavefunction) to ejected electron.

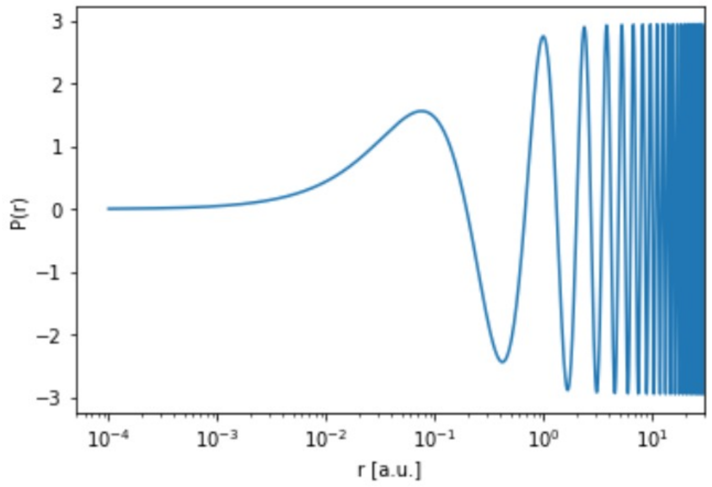


Potential:

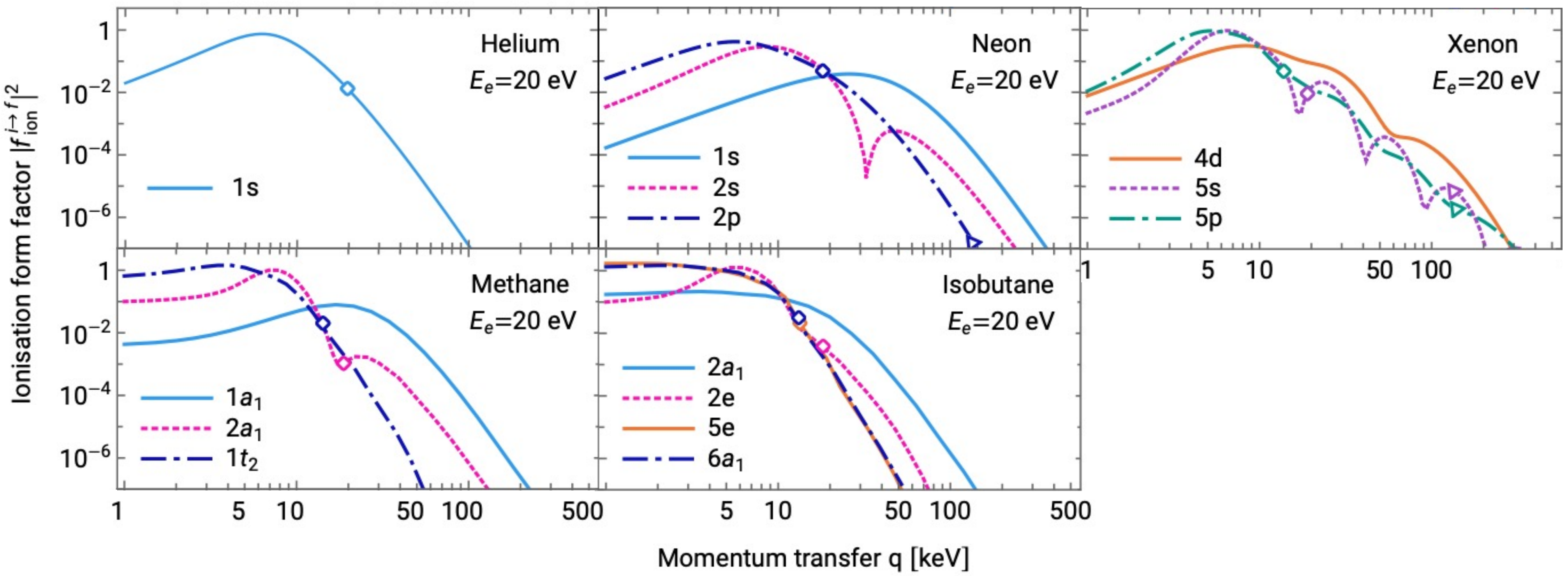
- Point nucleus : -Z/r
- Electron "cloud": $V_{dir}(r)$
- Electron "exchange": V_{exch}

→ 1/r at large r

Neon 2p → l_R=0, E = 250eV



Form Factors



Form factor ($\boldsymbol{\alpha}|\mathcal{M}|^2$): $|f_{\rm ion}^{nl}(q)|^2 = \frac{(2m_e E_R)^{3/2}}{\pi^2} \sum_{\text{states:i,f}} \left| \int d^3x \, \tilde{\psi}^*_{p'l'm'}(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}} \psi_{nlm}(\mathbf{x}) \right|^2$

arxiv 2110.02985

Event rates

- The dark event rate can be calculated using:

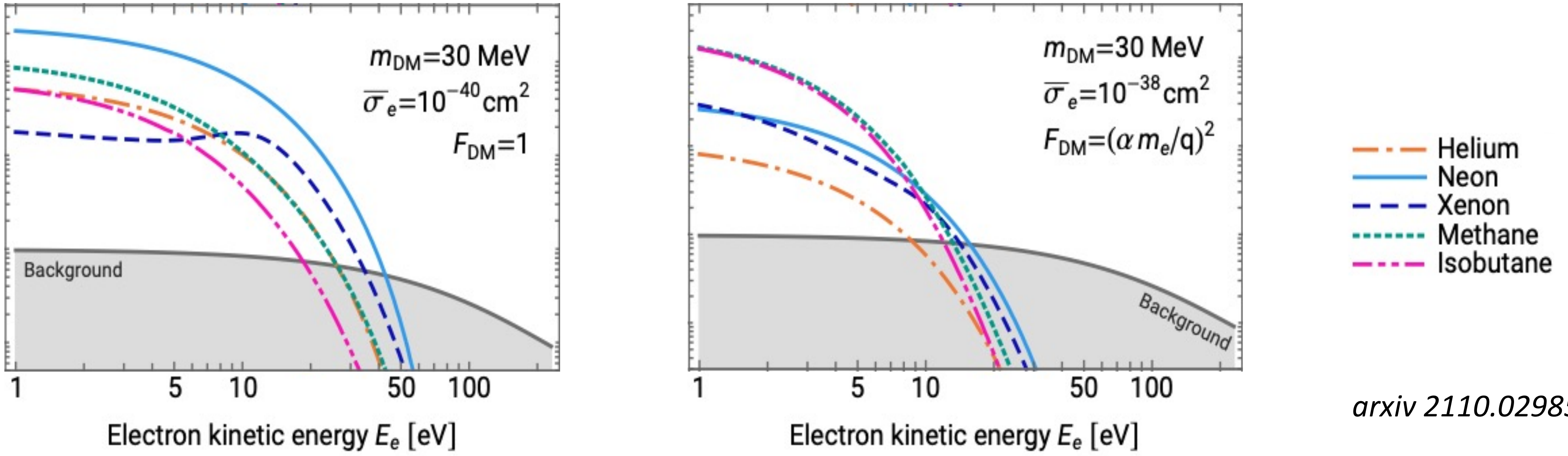
$$\boxed{\frac{\mathrm{d}R}{\mathrm{d}E_e}} = \frac{1}{m_A}\frac{\rho_{\mathrm{DM}}}{m_{\mathrm{DM}}}\sum_{nl} w_{nl}\frac{\mathrm{d}\langle\sigma_{\mathrm{ion}}^{nl} v_{\mathrm{DM}}\rangle}{\mathrm{d}E_e}$$

, where

$$\frac{\mathrm{d}\langle\sigma_{\mathrm{ion}}^{nl} v\rangle}{\mathrm{d}\ln E_e} = \frac{\sigma_e}{8\mu_e^2}\int_{q_-}^{q_+} q\mathrm{d}q\left|f_{\mathrm{ion}}^{nl}\right|^2 |F_{\mathrm{DM}}|^2 g(v_{\min}^{nl})$$

- Assume phenomenological background provided by NEWS-G, $F_{DM}=1$
- 10% Methane or Isobutane (Coulomb wave) contribution

Heavy mediator ⟷ Light mediator

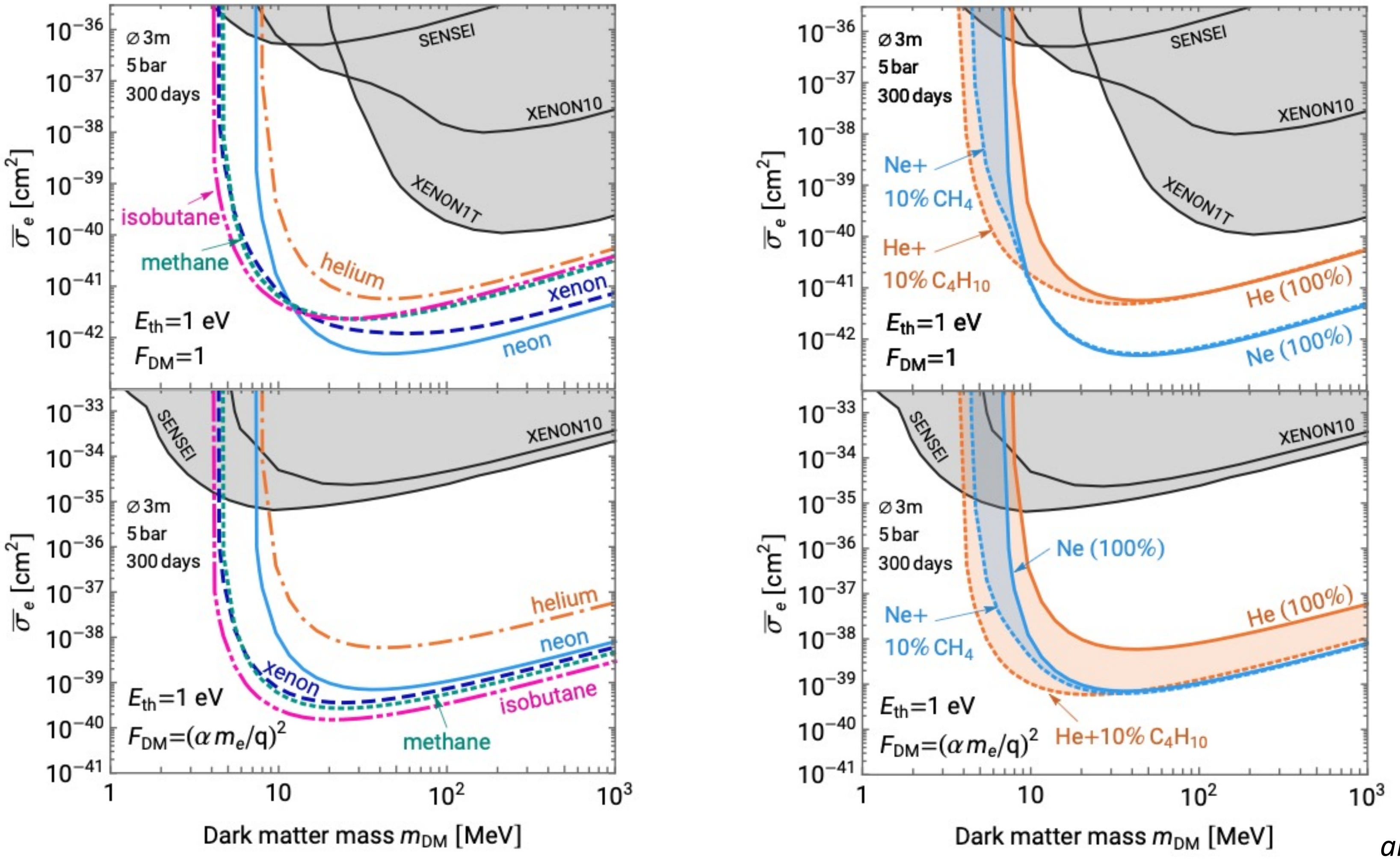


arxiv 2110.02985

Sensitivities

Heavy mediator

Light mediator



arxiv 2110.02985

Summary

- Dedicated direct detection of DM-electron scattering good probe of light DM (<1GeV)
- Atomic calculation under control – good accuracy & understanding (vs other HF & experiment)
- Molecular calculation more difficult (Coulomb approximation used), but can confidently be used to set bounds (i.e. with mixing) for the first time
- We learned what makes a a good DM – electron scattering target: could explore for more!
- Seems promising → more sensitive than current bounds / comparable to other proposed experiments
- SPC good probe of light DM-electron scattering ! Large scale experimental proposal coming soon: DARKSPHERE

Thank you!

Back up - Motivation for Light(er) Dark Matter (1/2)

- Dark matter has many ways of appearing in the present day universe:

Thermal Production (e.g. freeze-out WIMP), Freeze-in (FIMP), Inflaton decay, Gravitino…

WIMPs annihilate until $T \sim m_\chi/20$, leaving relic density:

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27} \text{cm}^3\text{s}^{-1}}{\langle \sigma_{\chi\chi} v_{rel} \rangle}$$

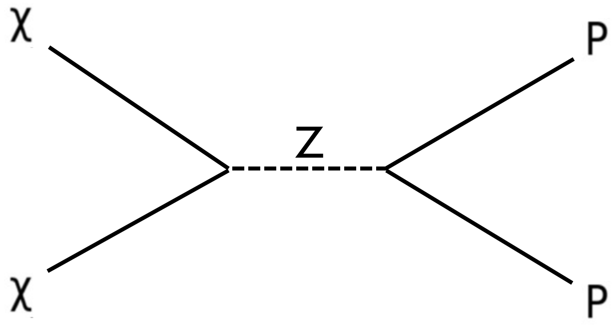
SM particles annihilate near $T \sim m_\chi$, leaving relic density:

$$\Omega_\chi h^2 \propto \langle \sigma_{\chi\chi} v_{rel} \rangle$$

- Individual models of freeze-out and or freeze-in DM can be fully tested (even for unknown details of UV cosmology).
- Without knowledge of T_R, we cannot fully test inflaton decay or gravitino

Back up - Motivation for Light(er) Dark Matter (2/2)

- Lee-Weinberg bound: assume weak interaction-generated (thermal, neutrino-like) WIMPs



- Annihilation cross section ~m_χ^2/m_Z^4 → sets a lower mass bound ~2GeV so as not to overclose the universe.

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27}\,\mathrm{cm^3 s^{-1}}}{\langle \sigma_{\chi\chi} v_{rel} \rangle}$$

- These assumptions can be relaxed:
  - DM interacts via more than Z-boson (new light bosons)
  - DM doesn't interact via weak scale mediator

→ **Search for <1 GeV dark matter is motivated !**

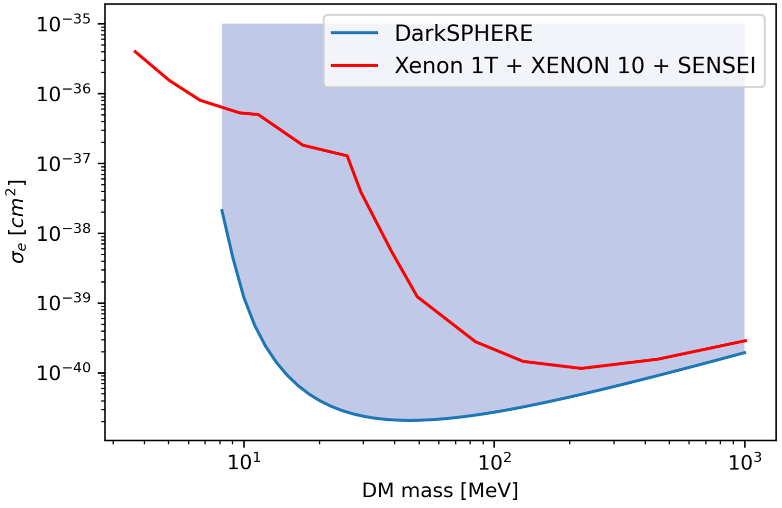
Back up - Sensitivities of bounds to choices

- Likelihood analysis:

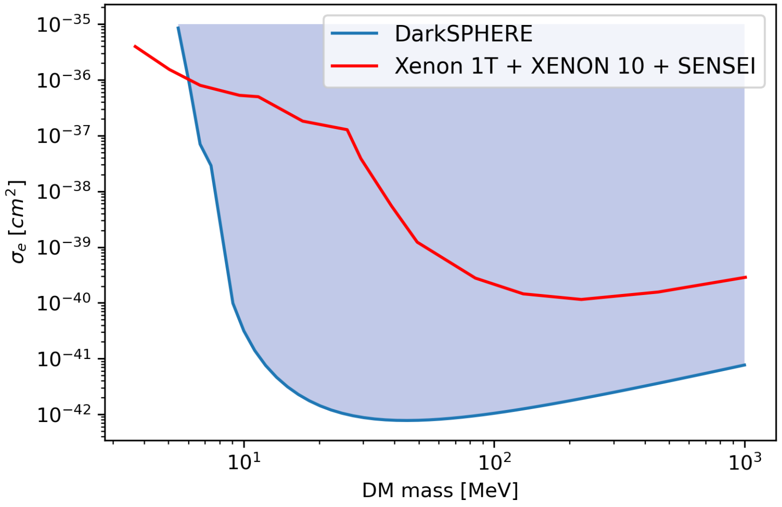
$$\Lambda = \frac{\mathcal{L}(0)}{\mathcal{L}(\sigma_e)} \qquad -2\ln(\Lambda) \sim \chi^2_1 \qquad \mathcal{L}(\sigma_e) = \prod_{i=0}^{N_{bins}} \mathcal{P}\left(N^i_{obs} \mid N^i_\chi(\sigma_e) + N^i_{bg}\right)$$

- Sensitivities of bounds to choices:
  - ➢ ~30-50% Gaussian basis choice
  - ➢ ~50-100% exchange potential choice, orthogonalization
  - ➢ ~10-20% analysis of recoil energy profile vs. deposited energies
  - ➢ ~30% astrophysical parameter choices
  - ➢ Linear with background

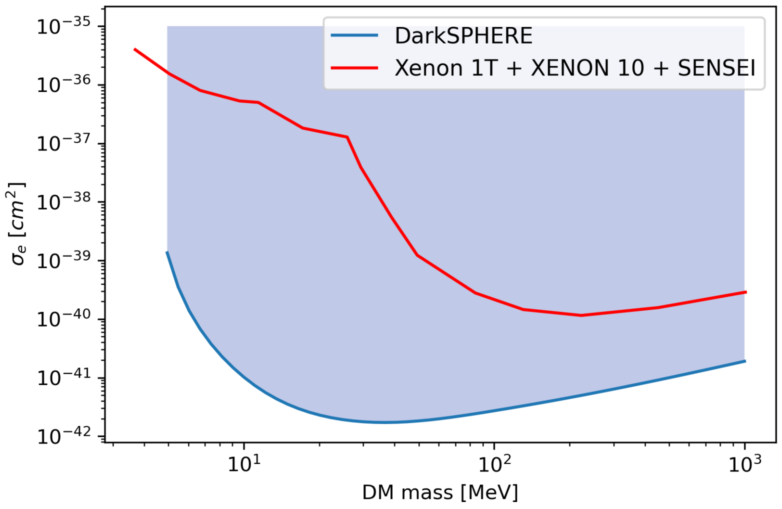
<u>Helium</u>



<u>Neon</u>



<u>Xenon</u>



Back up – Hartree Fock

- Hartree-Fock approximation: self-consistent bound states with energies correct to first order:

$$-\frac{1}{2}\frac{d^2P_{n_al_a}}{dr^2}+\frac{l_a(l_a+1)}{2r^2}P_{n_al_a}(r)-\frac{Z}{r}P_{n_al_a}(r)+$$
$$\sum_{n_bl_b}(4l_b+2)\left(v_0(n_bl_b,r)P_{n_al_a}(r)-\sum_l\Lambda_{l_all_b}v_l(n_bl_b,n_al_a,r)P_{n_bl_b}(r)\right)$$
$$=\epsilon_{n_al_a}P_{n_al_a}(r)+\sum_{n_b\neq n_a}\epsilon_{n_al_a,n_bl_a}P_{n_bl_a}(r),\quad(3.71)$$

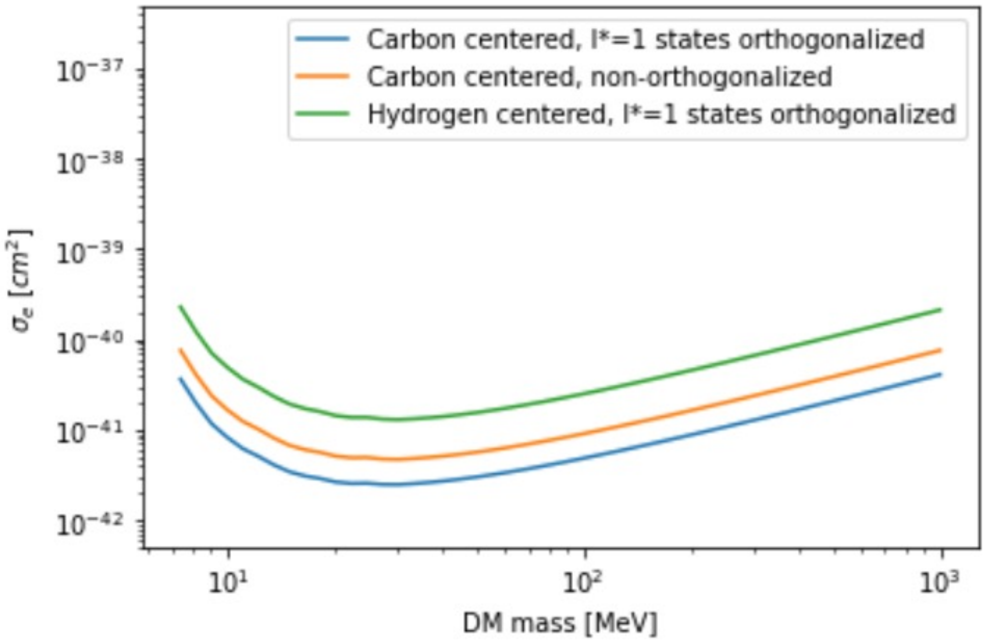
Easy to solve Hamiltonians for full shell atoms

Non-Rel approx. good for q < 200keV

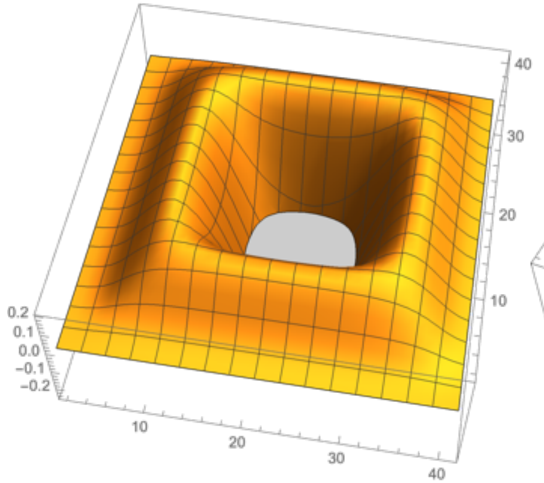
Accuracy of ~30% in event rates/bounds

Back up - Methane orbitals

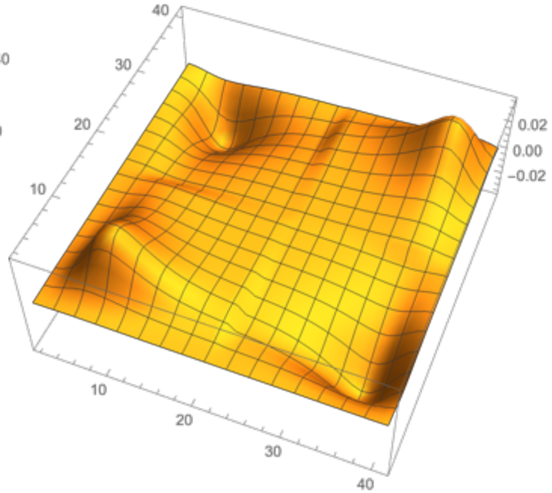
- Methane has tetrahedral symmetry → averaging necessary for potential and wavefunction
  - Similar to averaging over molecular orientations
  - We check averaging yields approx. correct cross sections
  - Need to choose between Hydrogen and Carbon center:
    → 2t state seems hydrogen-like.



2a orbital



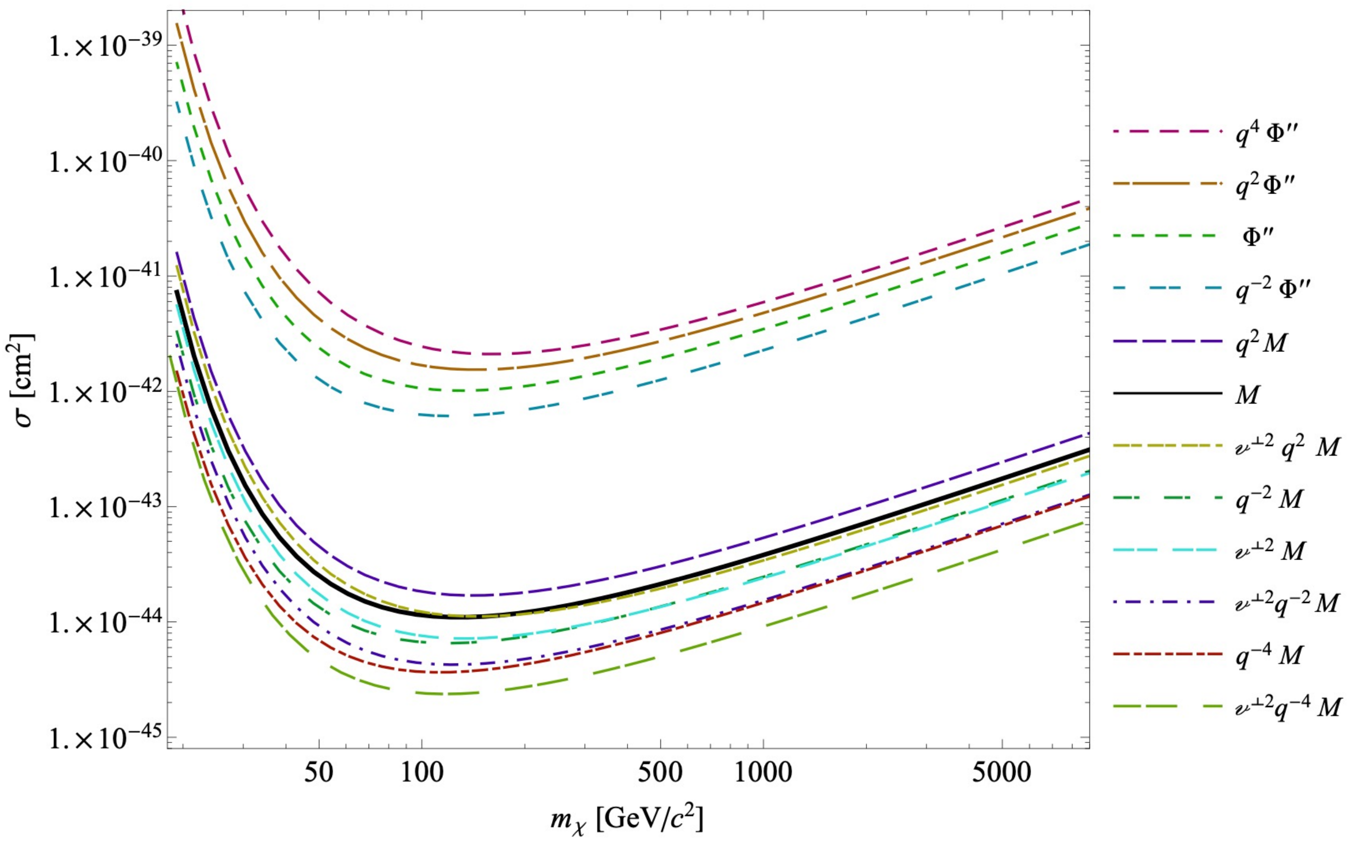
2t orbital



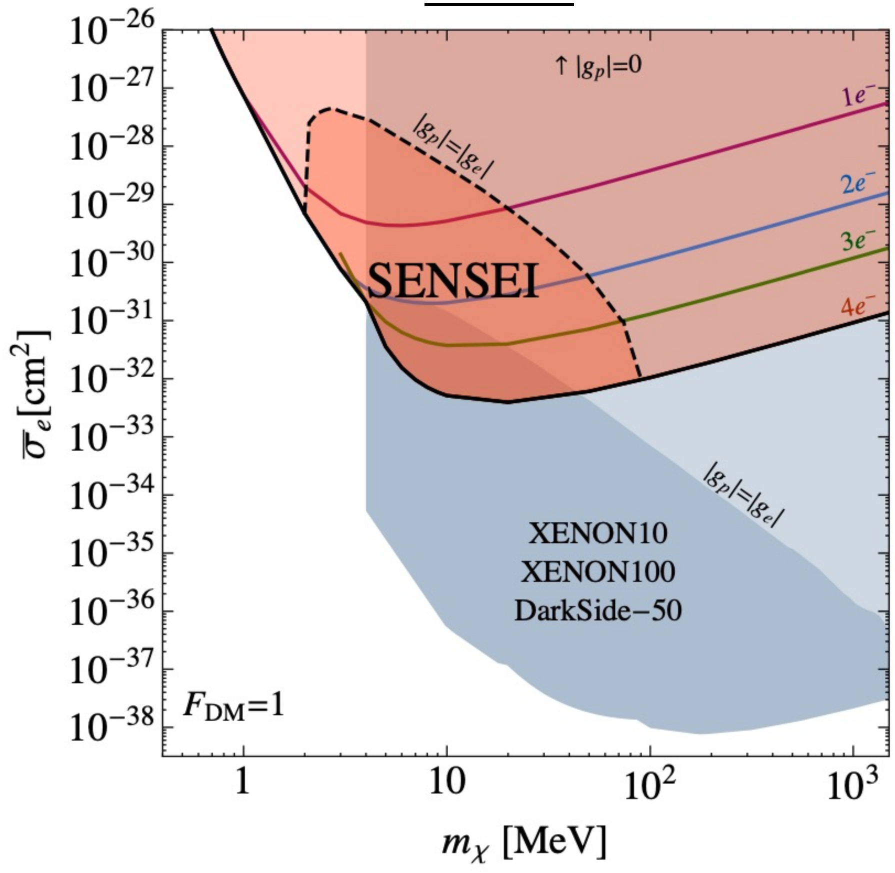
a,t,… orbitals are the tetrahedral harmonic equivalent of s,p,…

Back up – More On Constraints

Nuclear Recoil Argon DarkSide 50



SENSEI



Back up – More On Constraints

<u>Electron Recoil Argon SuperCDMS</u>