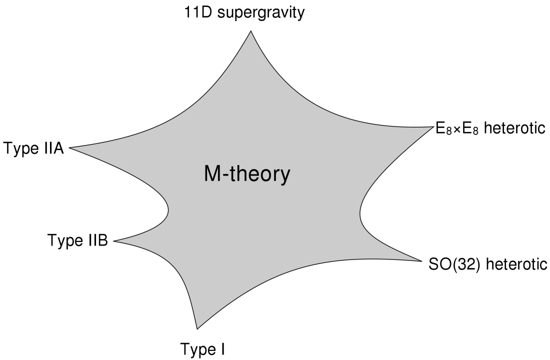
# Type II Calabi-Yau compactifications in general spacetime signature

Maxime Médevielle

December 17 2021

# Introduction

- String Theory is a quantum gravity theory where the fundamental objects are one-dimensional.
  We have 5 different theories related to each other :



# Spacelike T-duality

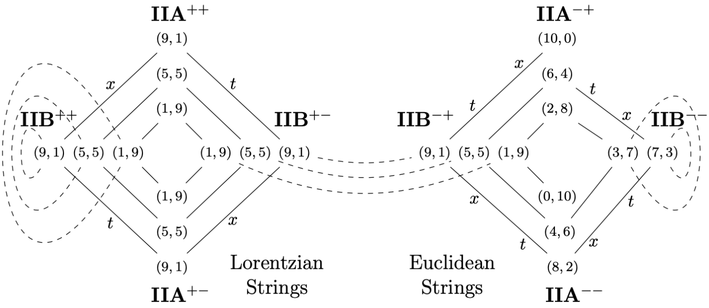
- When we consider theories on certain backgrounds they are undistinguishable, this is T-duality.
- The simplest example is to consider the circle compactification $\mathcal{R}^{1,8} \times \mathcal{S}^1$ : Type IIA on a circle of radius R and Type IIB on a circle of radius 1/R are equivalent.

# Spacelike T-duality

- When we consider theories on certain backgrounds they are undistinguishable, this is T-duality.
- The simplest example is to consider the circle compactification $\mathcal{R}^{1,8} \times \mathcal{S}^1$ : Type IIA on a circle of radius R and Type IIB on a circle of radius 1/R are equivalent.

# Timelike T-duality

Doing a timelike T-duality gives new theories : IIA* and IIB*. (Hull '98)
With S-duality we uncover a web of theories realising all maximal susy algebra and spacetime signatures.



# Why study signature change ?

- GR tells us that spacetime is dynamic. Einstein equations work for D>1 and arbitrary signature and topology.
- In a Quantum Gravity can these properties be explained ? by consistency or dynamics ?
- A symmetry based argument : analogy with the Higgs.

# Why study signature change ?

- GR tells us that spacetime is dynamic. Einstein equations work for D>1 and arbitrary signature and topology.
- In a Quantum Gravity can these properties be explained ? by consistency or dynamics ?
- A symmetry based argument : analogy with the Higgs.

# Why study signature change ?

- GR tells us that spacetime is dynamic. Einstein equations work for D>1 and arbitrary signature and topology.
- In a Quantum Gravity can these properties be explained ? by consistency or dynamics ?
- A symmetry based argument : analogy with the Higgs.

# Supergravity

- We work at the level of Supergravity. Type IIA theories are :

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g|} \Big( e^{-2\Phi} \left[ \mathcal{R} - \frac{\alpha}{2} |H_3|^2 + 4(\nabla\Phi)^2 \right] - \frac{\alpha\beta}{2} |F_2|^2 - \frac{\beta}{2} \left| \tilde{F}_4 \right|^2 \Big)$$

and Type IIB theories :

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g|} \Big( e^{-2\Phi} \left[ \mathcal{R} - \frac{\alpha}{2} |H_3|^2 + 4(\nabla\Phi)^2 \right] - \frac{1}{2}\alpha\beta |F_1|^2 - \frac{1}{2}\beta \left| \tilde{F}_3 \right|^2 - \frac{\alpha\beta}{4} \left| \tilde{F}_5 \right|^2 \Big)$$

- Ghosts are not necessarily inconsistent : YM on a timelike circle.

## Supergravity

- We work at the level of Supergravity. Type IIA theories are :

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|g|}\Big(e^{-2\Phi}\left[\mathcal{R} - \frac{\alpha}{2}\left|H_3\right|^2 + 4(\nabla\Phi)^2\right] - \frac{\alpha\beta}{2}\left|F_2\right|^2 - \frac{\beta}{2}\left|\tilde{F}_4\right|^2\Big)$$

and Type IIB theories :

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{|g|}\Big(e^{-2\Phi}\left[\mathcal{R} - \frac{\alpha}{2}\left|H_3\right|^2 + 4(\nabla\Phi)^2\right] - \frac{1}{2}\alpha\beta\left|F_1\right|^2 - \frac{1}{2}\beta\left|\tilde{F}_3\right|^2 - \frac{\alpha\beta}{4}\left|\tilde{F}_5\right|^2\Big)$$

- Ghosts are not necessarily inconsistent : YM on a timelike circle.

# going to 4D

- Calabi-Yau are manifolds that preserve some supersymmetry. Interesting because in the case of Type II it is possible to have exact non perturbative results with rich a dynamic.
- The kind of theories we obtain are N=2 D=4 Supergravity theories coupled to vectormultiplets (1 gauge field and 1 complex scalar) and hypermultiplets (4 real scalars).

# going to 4D

- Calabi-Yau are manifolds that preserve some supersymmetry. Interesting because in the case of Type II it is possible to have exact non perturbative results with rich a dynamic.
- The kind of theories we obtain are N=2 D=4 Supergravity theories coupled to vectormultiplets (1 gauge field and 1 complex scalar) and hypermultiplets (4 real scalars).

# The theories in 4D

- We obtain the following lagrangian :

$$L_{G+VM} = \frac{1}{2}\star R_4 - g_{\alpha\bar{\beta}}(z,\bar{z})dz^{\alpha}\wedge\star d\bar{z}^{\bar{\beta}} - \frac{\lambda}{4}\mathcal{I}_{IJ}F^I\wedge\star F^J + \frac{1}{4}\mathcal{R}_{IJ}F^I\wedge F$$

$$\begin{aligned} L_{HM}^{(\epsilon_1,\epsilon_2)} = & -G_{\alpha\bar{\beta}}(q,\bar{q})dq^{\alpha}\wedge\star d\bar{q}^{\bar{\beta}} - \frac{1}{4}d\phi\wedge\star d\phi \\ & + \epsilon_1 e^{-2\phi}\left[d\tilde{\phi} + \frac{1}{2}\left(\zeta^I d\tilde{\zeta}_I - \tilde{\zeta}_I d\zeta^I\right)\right]^2 \\ & - \frac{\epsilon_2}{2}e^{-\phi}\left[\mathcal{I}_{IJ}d\zeta^I\wedge\star d\zeta^J - \epsilon_1\mathcal{I}^{IJ}\left(d\tilde{\zeta}_I + \mathcal{R}_{IK}d\zeta^K\right)^2\right] \end{aligned}$$

where $X^2$ really means $X\wedge\star X$.

# Scalar geometries.

- $g_{\alpha\bar{\beta}}$ for Vector multiplets, the geometry is special (para-)Kahler.
- $G_{\alpha\bar{\beta}}$ for Hypermultiplets, the geometry is (para-) quaternionic Kahler.
- We classify all these geometries

# Scalar geometries.

- $g_{\alpha\bar{\beta}}$ for Vector multiplets, the geometry is special (para-)Kahler.
- $G_{\alpha\bar{\beta}}$ for Hypermultiplets, the geometry is (para-) quaternionic Kahler.
- We classify all these geometries

# Scalar geometries.

- $g_{\alpha\bar{\beta}}$ for Vector multiplets, the geometry is special (para-)Kahler.
- $G_{\alpha\bar{\beta}}$ for Hypermultiplets, the geometry is (para-) quaternionic Kahler.
- We classify all these geometries

# The c-map

- We want to repeat the T-duality story. Reducing and dualizing the G+VM lagrangian gives the following

$$
\begin{aligned}
e^{-1}L_3 = &\frac{1}{2}R_3 - \frac{1}{4}\partial_\mu\phi\partial^\mu\phi - g_{\alpha\bar{\beta}}\partial_\mu z^\alpha\partial^\mu\bar{z}^{\bar{\beta}} \\
&+ \epsilon_1 e^{-2\phi}\left[\partial^\rho\tilde{\phi} + \frac{1}{2}\left(\zeta^I\partial^\rho\tilde{\zeta}_I + \tilde{\zeta}_I\partial^\rho\zeta^I\right)\right]^2 \\
&+ \frac{\lambda\epsilon}{2}e^{-\phi}\left[\mathcal{I}_{IJ}\partial_\mu\zeta^I\partial^\mu\zeta^J - \epsilon_1\mathcal{I}^{IJ}\left(\partial^\rho\tilde{\zeta}_I - \mathcal{R}_{IK}\partial^\rho\zeta^K\right)^2\right]
\end{aligned}
$$

This defines a map from the S(para-)K to the (para-)QK manifolds called the c-map.

# the 4D duality web

- If two 4D theories reduce to the same 3D theory, you have a T-duality between these two theories !
- This maps out the complete duality web of theories in 4D, that has pure spacelike, timelike and mixed T-dualities.
- The 4D duality web is a projection of the 10D one, however we have generically two orbits : One Lorentzian and one with signature change.

# the 4D duality web

- If two 4D theories reduce to the same 3D theory, you have a T-duality between these two theories !
- This maps out the complete duality web of theories in 4D, that has pure spacelike, timelike and mixed T-dualities.
- The 4D duality web is a projection of the 10D one, however we have generically two orbits : One Lorentzian and one with signature change.

# the 4D duality web

- If two 4D theories reduce to the same 3D theory, you have a T-duality between these two theories !
- This maps out the complete duality web of theories in 4D, that has pure spacelike, timelike and mixed T-dualities.
- The 4D duality web is a projection of the 10D one, however we have generically two orbits : One Lorentzian and one with signature change.

# Conclusion

- Recap : String theories provides a framework to ask questions about dimensionality and signature of spacetime. We saw how these give rise to interesting theories in 4D, introduced their scalar geometries, and we gave a full description of the duality web relating them.
- Solutions of exotic 4D-theories can now be studied from a microscopic point of view (duality between cosmological and black hole solutions.)

# Conclusion

- Recap : String theories provides a framework to ask questions about dimensionality and signature of spacetime. We saw how these give rise to interesting theories in 4D, introduced their scalar geometries, and we gave a full description of the duality web relating them.
- Solutions of exotic 4D-theories can now be studied from a microscopic point of view (duality between cosmological and black hole solutions.)

Thank you :)

For details : "Type II Calabi-Yau compactifications, T-duality and special geometry in general spacetime signature"
arXiv : 2111.09017

# extra slide

- Signature (0,4) : instantons, solitons, Hartle-Hawking, complex metrics as saddle points of the EQG path integral, ...
- Signature (2,2) : amplitudes/BH in Klein space, Topological String theory, Magical String, Twistors,...
- Negative branes related to non unitarity, negative energy states, super gauge groups, etc...