

# Unoriented dualities from brane tiling

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YTF, 17 Dec 2021

Based on

A. Antinucci, SM, F. Riccioni *hep-th/ 2007.14749*

A. Antinucci, M. Bianchi, SM, F. Riccioni *hep-th/ 2105.06195*



SAPIENZA  
UNIVERSITÀ DI ROMA

# AdS/CFT

AdS<sub>5</sub> × S<sup>5</sup> (with N unit of flux)

$\mathcal{N} = 4$  SCFT<sub>4</sub> SU(N)

AdS isometry SO(2,4)



SO(2,4) conformal symmetry

Sphere isometry SO(6)



$SO(6) \approx SU(4)$  R-symmetry

$$\frac{R^4}{l_s^4} = g_{YM}^2 N = \lambda$$

$$4\pi g_s = g_{YM}^2$$

Low curvature, weak gravity



Large N, strong coupling

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J. Maldacena, *The large N limit of superconformal field theories and supergravity*, 1997

# AdS/CFT

$\text{AdS}_5 \times S^5$  (with  $N$  unit of flux)

$\mathcal{N} = 4$  SCFT<sub>4</sub> SU( $N$ )

AdS



conformal symmetry

Sphere



Global symmetry

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Deform the sphere: global symmetry changes

$\text{AdS}_5 \times Y_5$



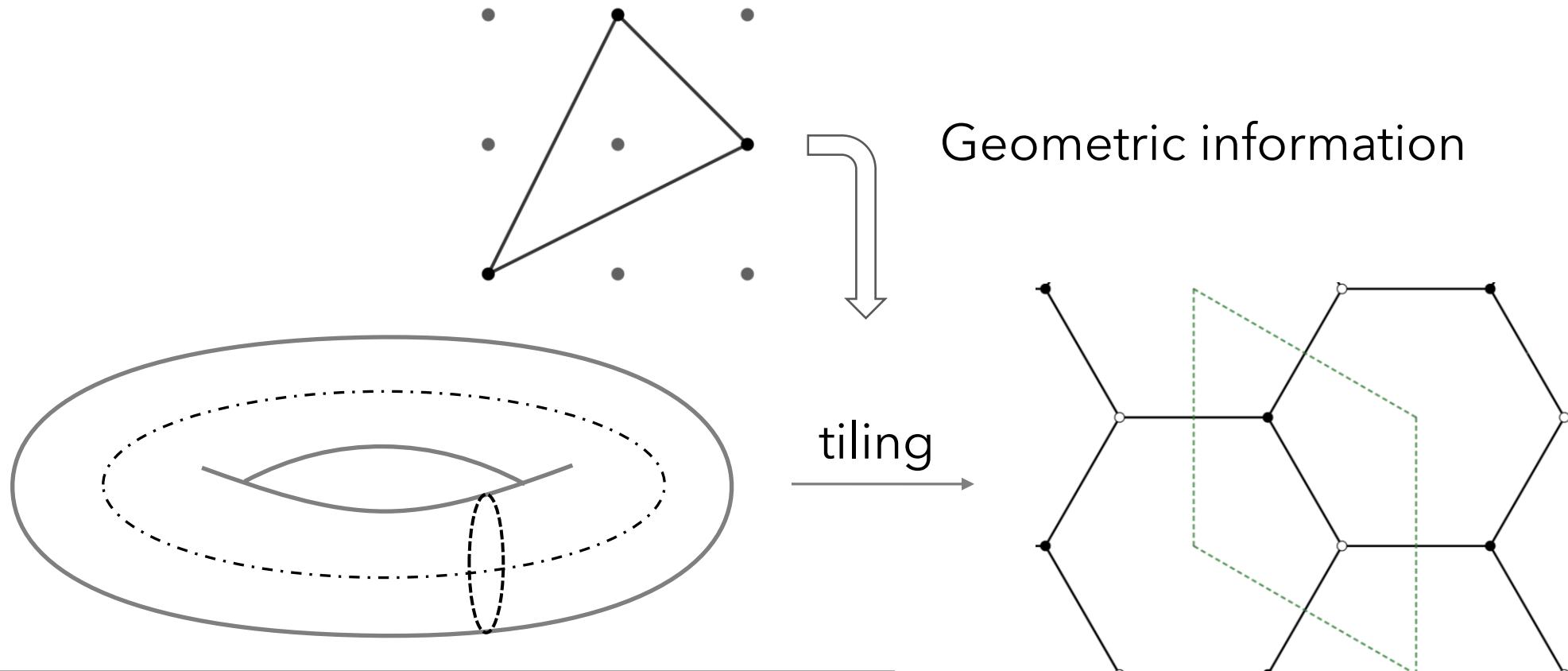
$\mathcal{N} = 1$  SCFT<sub>4</sub> G

$Y_5$  = base of a CY<sub>3</sub> cone

G = Gauge group

# Toric Geometry

We have more control when isometries of transverse space are  $U(1)^2 \times U(1)_R$



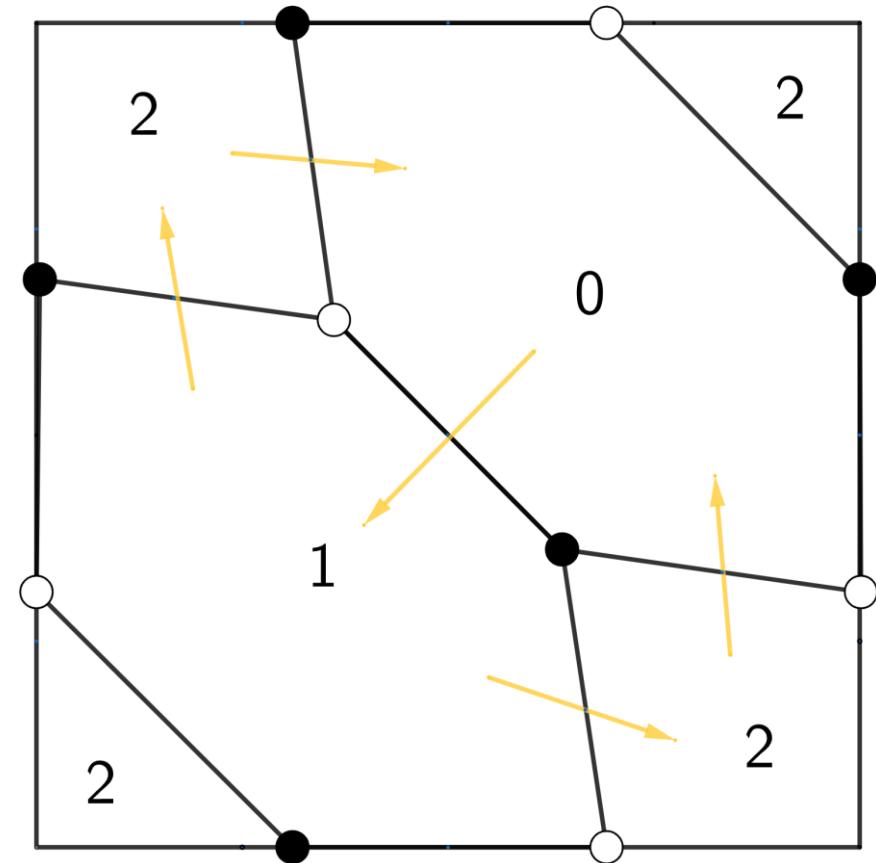
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S. Franco, A. Hanany, K. Kennaway, D. Vegh, B. Wecht, *Brane dimers and quiver gauge theories*, 2005, hep-th/0504110

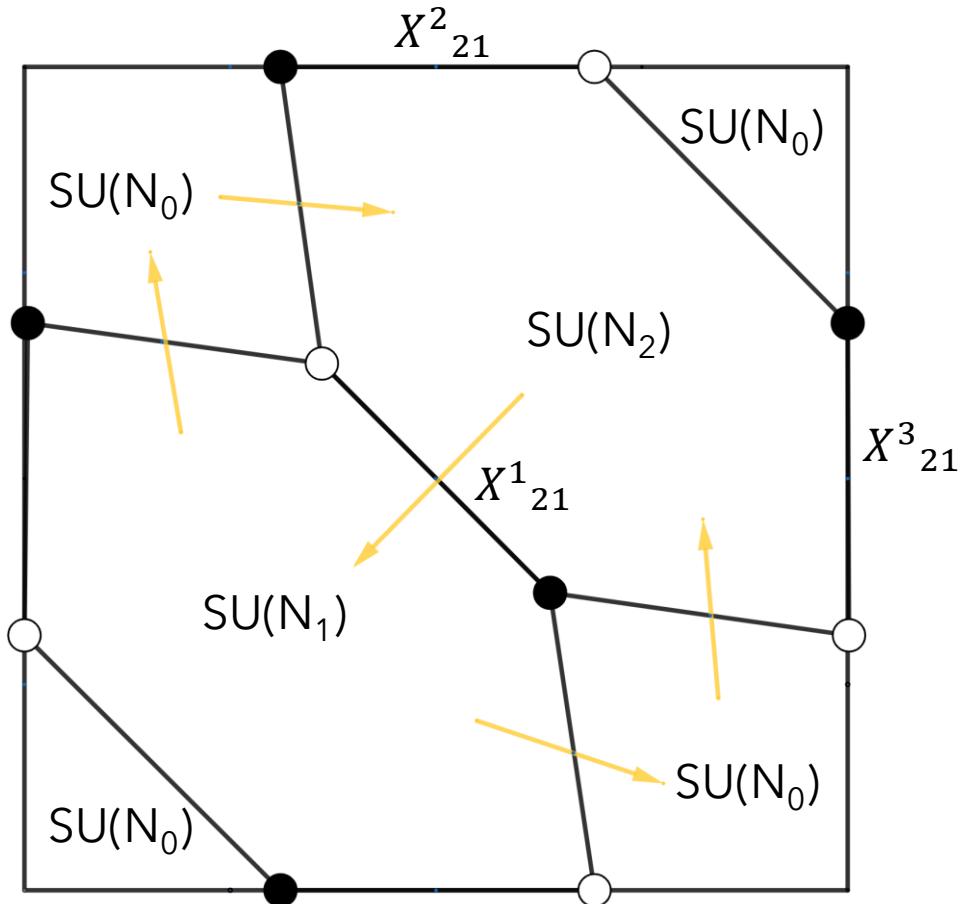
# Brane Tiling

Dictionary:

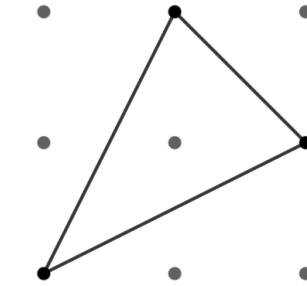
- Face → Gauge group  $SU(N)$
- Edge → Matter field
- White (black) node →  $+(-)$  interaction term



# Brane Tiling



Example:  
transverse space  $\mathbb{C}^3/\mathbb{Z}_3$



$$SU(N_0) \times SU(N_1) \times SU(N_2)$$

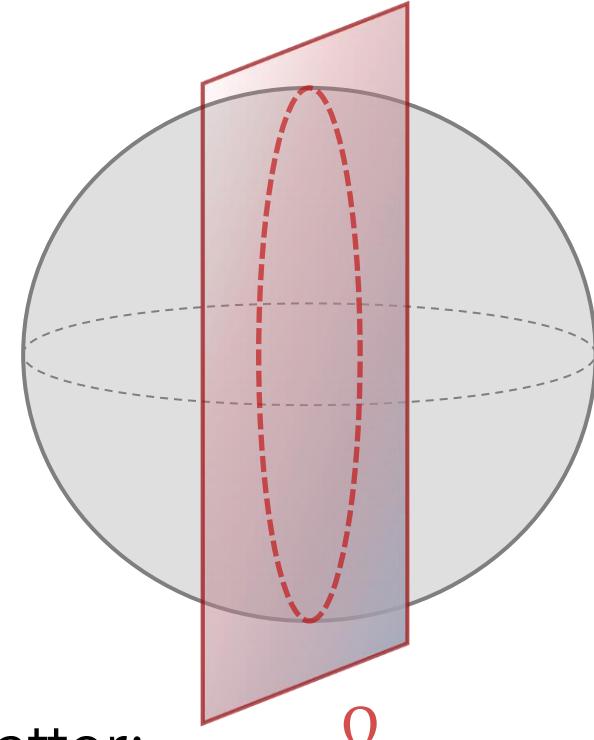
$$\begin{aligned} W = & X^1_{21} X^2_{10} X^3_{02} - X^1_{21} X^3_{10} X^2_{02} \\ & + X^3_{21} X^1_{10} X^2_{02} - X^3_{21} X^2_{10} X^1_{02} \\ & + X^2_{21} X^3_{10} X^1_{02} - X^2_{21} X^1_{10} X^3_{02} \\ = & \varepsilon_{ijk} X^i_{21} X^j_{10} X^k_{02} \end{aligned}$$

# Orientifold projection

$\mathbb{Z}_2$  involution of the transverse space  
Orientation of strings reversed

Why the orientifold  $\Omega$ ?

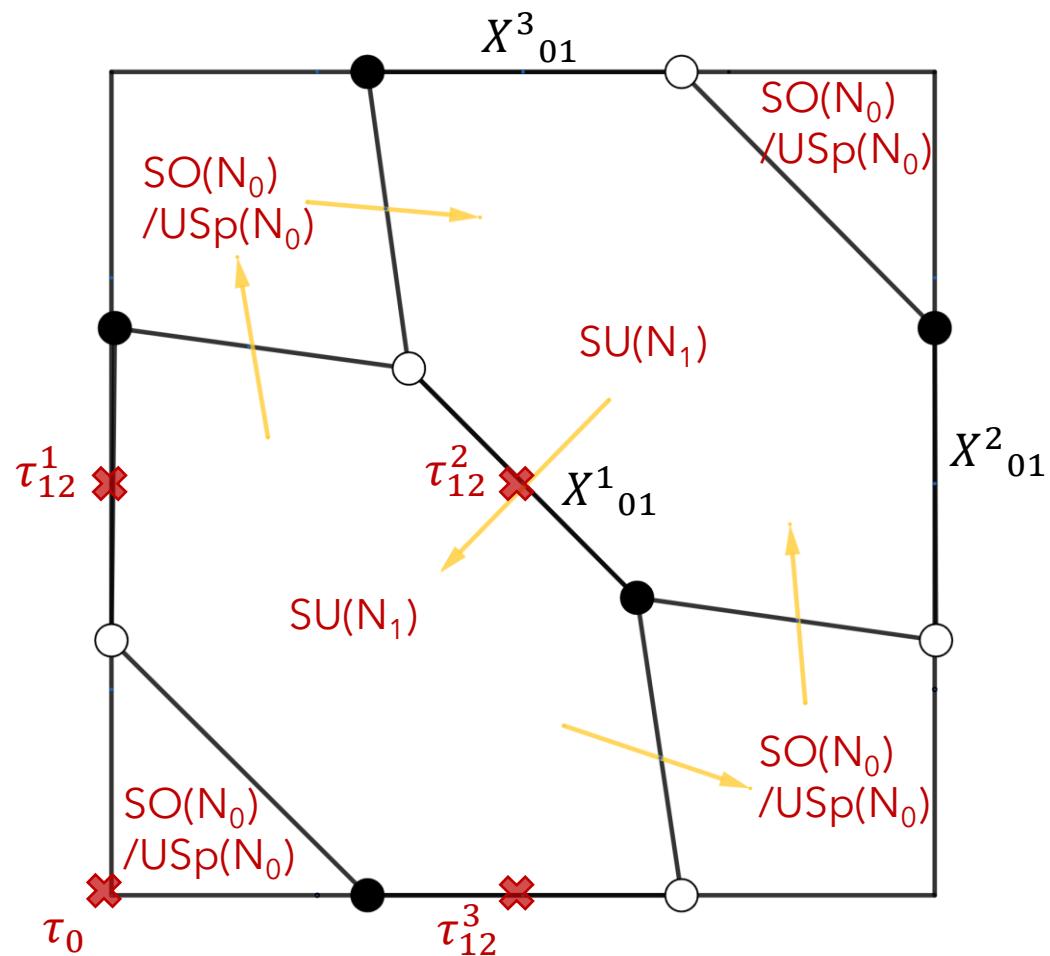
- It allows for SO, Sp gauge groups and tensorial matter;
- Crucial role in phenomenology.



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A. Sagnotti, *Open strings and their symmetry groups*, 1987  
M. Wijnholt, *Geometry of Particle Physics*, 2007

# Orientifold projection



Again  $\mathbb{C}^3/\mathbb{Z}_3$

$$\Omega : SO/USp(N_0) \times SU(N_1)$$

$$\prod \tau = (-1)^{N_W/2}$$

$$W = \varepsilon_{ijk} X^i {}_{21} X^j {}_{10} (X^k {}_{02})^T$$

S. Franco, A. Hanany, D. Krefl, J. Park, A. Uranga, D. Vegh, *Dimers and orientifolds*, 2007, hep-th/0707.0298

# a-maximization

$$a = \frac{3}{32} (3\text{Tr } R^3 - \text{Tr } R)$$

'Counting' of degrees of freedom, a-theorem  $a_{UV} > a_{IR}$

Non-R abelian global symmetry mix with  $U(1)_R$

The local maximum of  $a$ ,  $a_{SCFT}$ , uniquely determines superconformal  $R$ -charges:

$$\frac{3}{2} R_{\mathcal{O}} = \Delta = 1 + \frac{1}{2} \gamma_{\mathcal{O}}$$

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B. Wecht, K. Intriligator, *The Exact superconformal R symmetry maximizes a*, 2003, hep-th/0304128

# a-maximization

$$a_{SCFT} = \max \left\{ \frac{3}{32} (3 \text{Tr } R^3 - \text{Tr } R) \right\}$$

Relation to geometry

$$ds^2 = \frac{r^2}{R^2} (-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} dr^2 + \underbrace{R^2 ds_Y^2}_{Y^5}$$

$$\text{Vol}(Y) \sim \frac{1}{a_{SCFT}}$$

# Orientifold projection

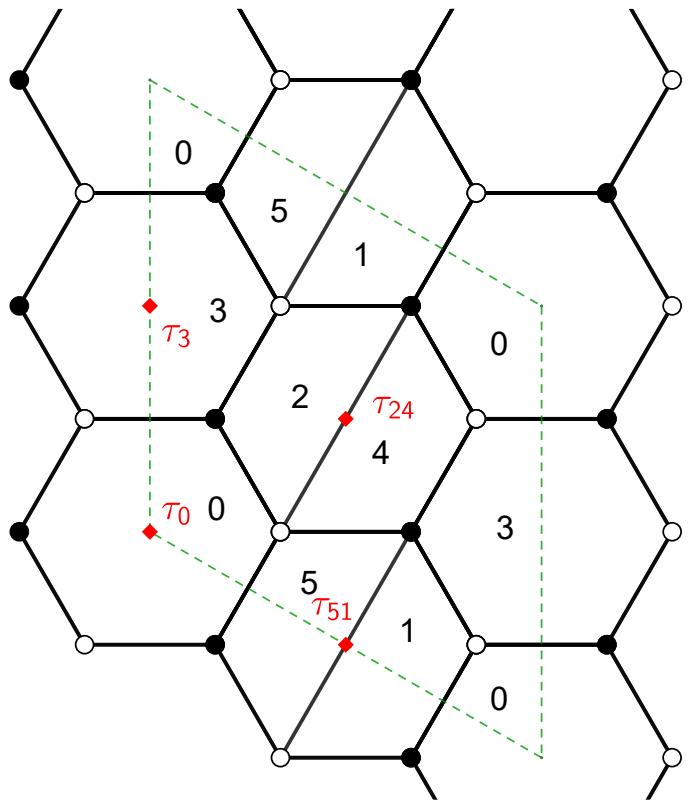
What is the fate of the conformal point after the involution  $\Omega$ ?

1.  $\frac{a_{SCFT}^\Omega}{a} = \frac{1}{2}$

2. No conformal point

3.  $\frac{a_{SCFT}^\Omega}{a} < \frac{1}{2}$        $\longleftrightarrow$  New!

# A third scenario



Consider Pseudo del Pezzo 3c (PdP<sub>3c</sub>)  
 $SU(N_0) \times SU(N_1) \times SU(N_2) \times SU(N_3) \times SU(N_4) \times SU(N_5)$

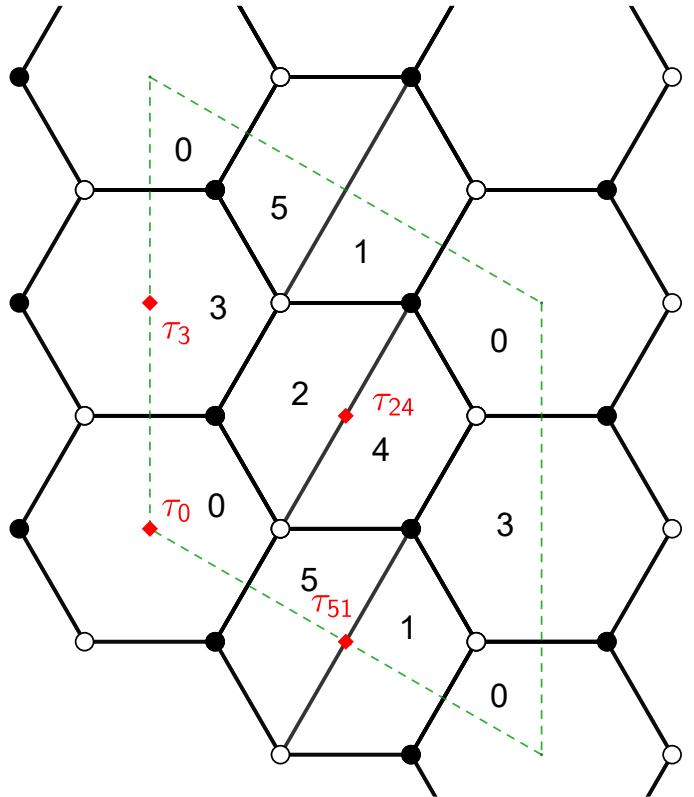
$$W = X_{01}X_{13}X_{30} + X_{34}X_{40}X_{03} - X_{02}X_{23}X_{30} - X_{35}X_{50}X_{03} + X_{02}X_{24}X_{45}X_{50} \\ + X_{35}X_{51}X_{12}X_{23} - X_{01}X_{12}X_{24}X_{40} - X_{34}X_{45}X_{51}X_{13}$$

Two choices for  $\Omega = (\tau_0, \tau_3, \tau_{24}, \tau_{51})$

$$\Omega_1 = (+, -, -, +)$$

$$\Omega_2 = (-, +, -, +)$$

# A third scenario



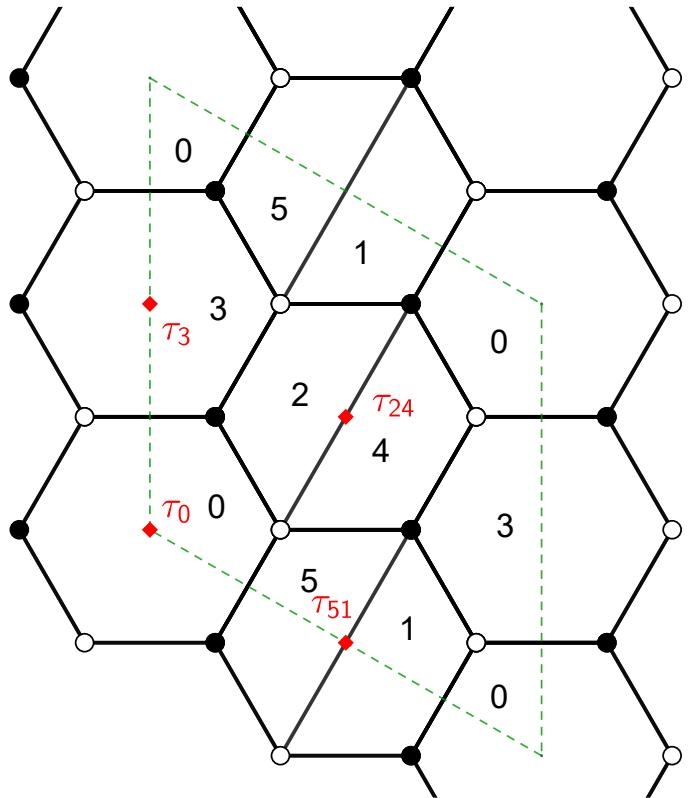
$$a_{SCFT} = \frac{3\sqrt{3}}{4} N^2$$

$$a_{SCFT}^{\Omega_1} = \frac{3\sqrt{3}}{8} N^2$$

$$a_{SCFT}^{\Omega_2} = \frac{27}{8} (5\sqrt{5} - 11) N^2$$

$$\frac{a_{SCFT}^{\Omega_2}}{a_{SCFT}} \sim 0.47 < \frac{1}{2}$$

# A third scenario

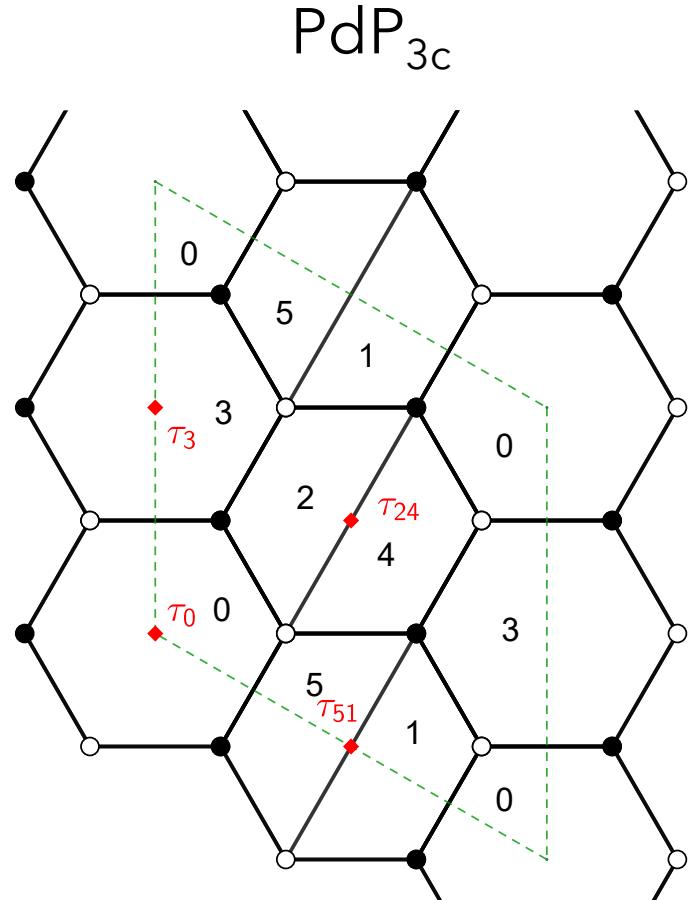


For  $\Omega_2$  the central charge  $a$  is maximized over a smaller set of variables:

we are losing an  $U(1)$

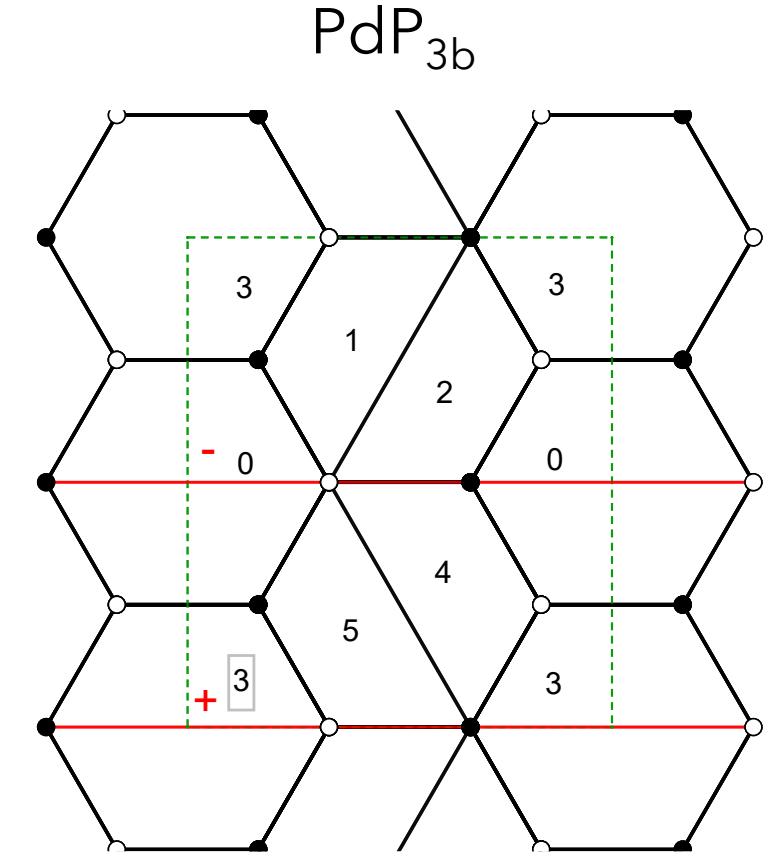
$$\frac{a_{SCFT}^{\Omega_2}}{a_{SCFT}} \sim 0.47 < \frac{1}{2}$$

# A new unoriented IR duality



$$a_{3c}^{\Omega_2} = a_{3b}^{\Omega}$$

$$= \frac{27}{8} (5\sqrt{5} - 11) N^2$$



Different parent, same orientifolded theories

# Recap & Conclusion

- Third Scenario: 'losing' an U(1) causes  $a_{SCFT}$  to decrease
- Duality: Third scenario orientifold dual to a first scenario model
- Only one chiral example, infinitely many non-chiral:  
 $(SPP/\mathbb{Z}_n)^{\Omega} \longleftrightarrow (L^{aaa})^{\Omega}, \quad a = 3n/2$
- In all non-chiral theories, S-duality at works  
A. Amariti, M. Fazzi, S. Rota, A. Segati, 2021, hep-th/2108.05397
- Different toric geometries, the same after the orientifold projection. How?

# Thank you

