Precision Calculation for quark flavour Physics

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The History

- Parity P-violation in Co-60 weak decay was discovered in Wu-experiment, 1956[1].
- James Cronin and Val Fitch first discovered (charge, parity) CP-violation in kaon ($s\bar{d}$, $\bar{s}d$) decay ,1964[2].
- The C-conjugation cannot compensate P-violation and CP-violation in weak interaction is recognised as a nature in universe.
- A parameter is proposed for CP-violation in $K^0, \bar{K}^0$ mixing and measured as[3]

$$|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}. \tag{1}$$

CP violation and $\bar{s}d \to s\bar{d}$

We are now calculating the NNLO QCD contribution to CP-violation parameter[4]

$$\epsilon_K \equiv \frac{1}{2} \arg\left(\frac{-M_{12}}{\Gamma_{12}}\right) e^{i\phi_\epsilon} \sin\phi_\epsilon. \tag{2}$$

in $K^0, \bar{K}^0$ mixing.The constants ϕ_ϵ is from experiments. Both M_{12} and Γ_{12} describe mixing. M_{12} can be found via effective Hamiltonian

$$\mathcal{H} = \frac{g^4}{64\pi^2 m_W^2}(\lambda_t^2 \mathcal{C}^{tt} + \dots)(\bar{s}_L\gamma_\mu d_L) \otimes (\bar{s}_L\gamma^\mu d_L) \tag{3}$$

with the CKM element product $\lambda_t \equiv V_{ts}^* V_{td}$ and the dots standing for contribution from other CKM elements and other orders of α_s. The Wilson Coefficient $\mathcal{C}^{tt}$ bound to λ_t^2 is calculated.

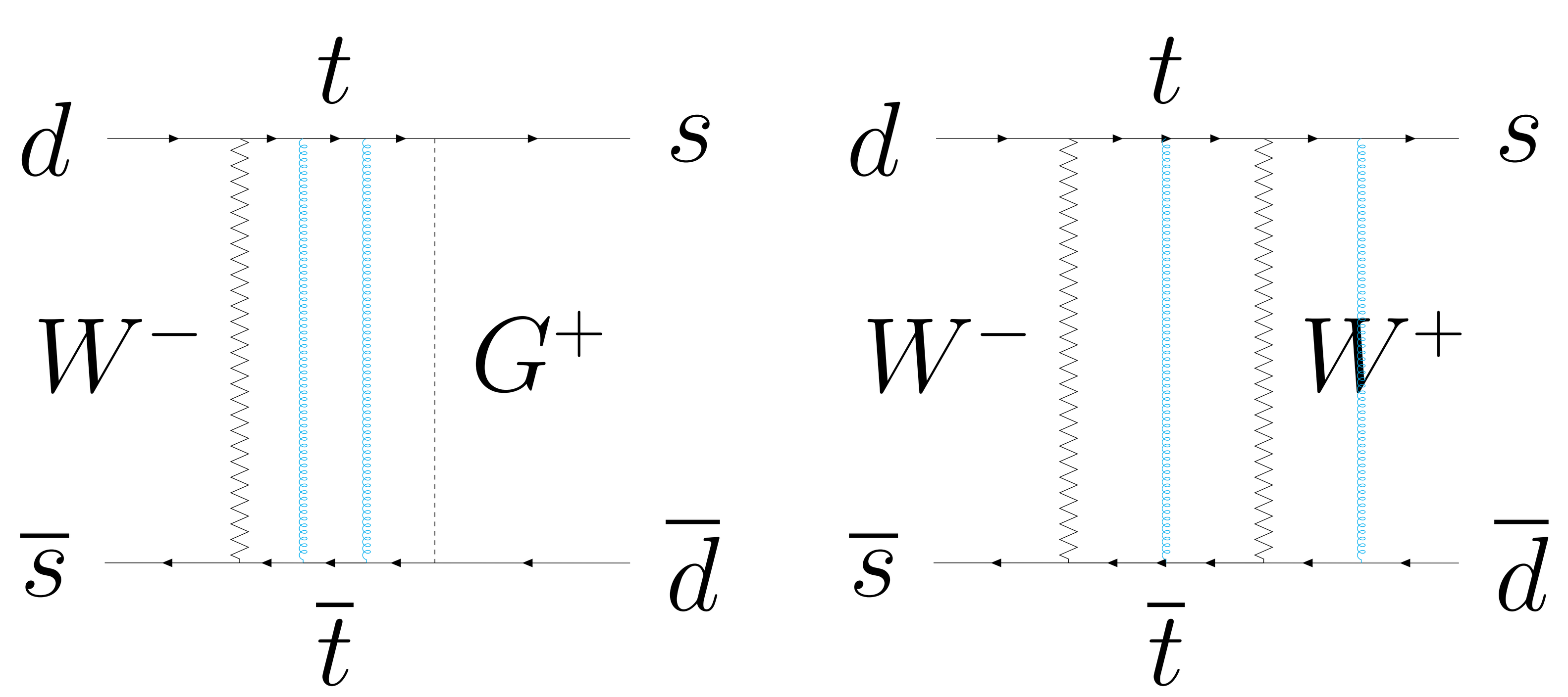
The Diagrams

All the diagrams at $O(\alpha_s^2)$ from full SM and low-energy effective field theory (EFT) are evaluated and can be summarised as:

- 3-loop full-SM diagrams,
- 1,2-loop full-SM-counter-term diagrams,
- 2-loop EFT diagrams,
- 0,1-loop EFT counter-terms diagrams.

The SM and SM counter-term diagrams are combined for removing ultraviolet UV poles. The EFT diagrams are treated in the same way. Then the infrared (IR) poles still remain due to light quark (only $m_t \neq 0$) and vacuum (no external momenta) assumption. The IR poles cancel during matching the SM and EFT amplitudes and $\mathcal{C}^{tt}$ can be deduced.

3-loop Diagram Techniques

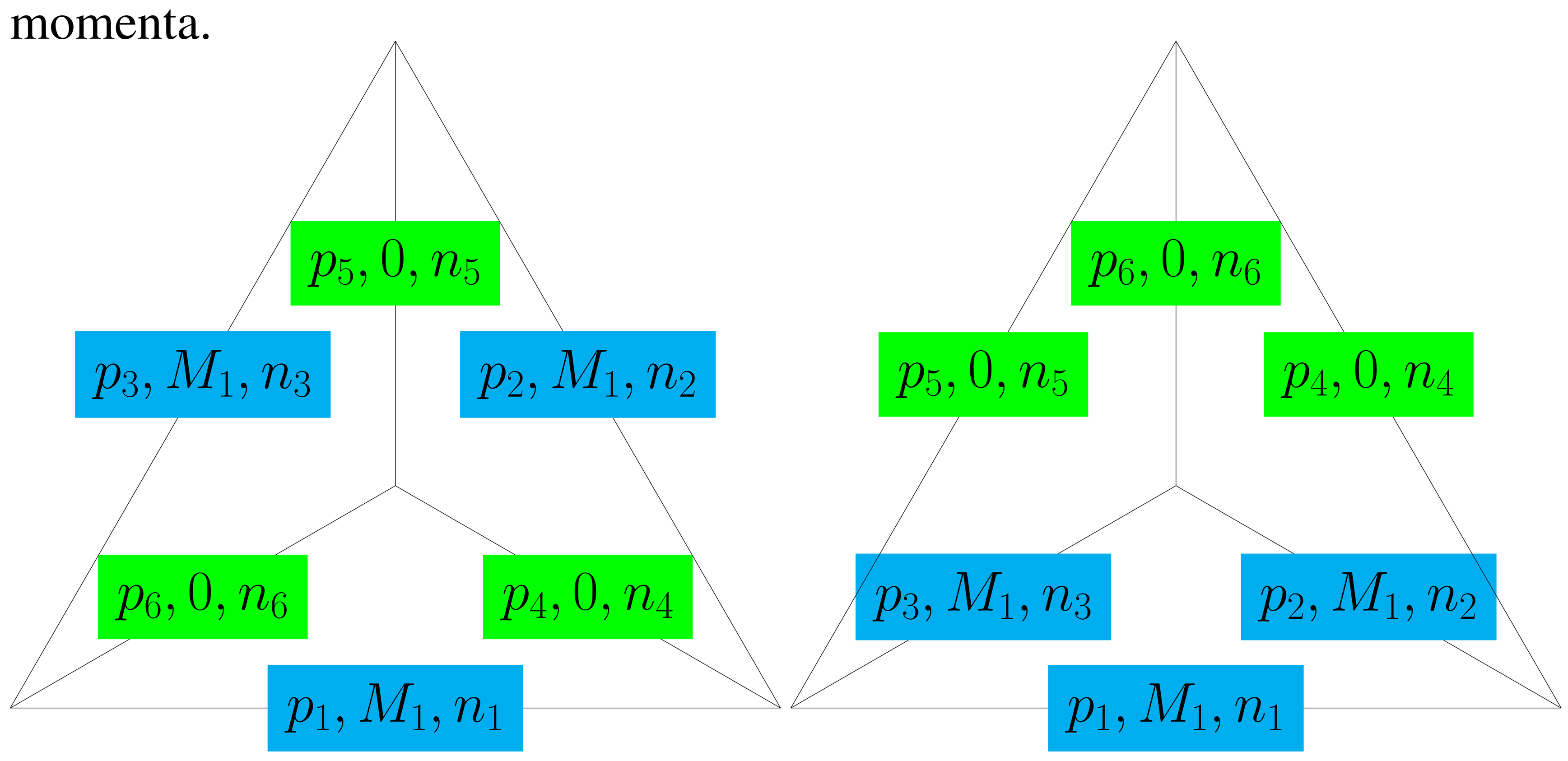


Evaluating the 3-loop full-SM diagrams is the hardest task for the huge number ($\sim 10^4$) and the complexity of each diagram. The above diagrams are 2 examples with gluonic (cyan) correction to the box made of internal quarks , weak and/or Goldstone bosons. All the diagrams have mass dimension -2 and the weak vertices contribute the prefactor in (3). Here are some techniques

- We have ignored the light quark masses except top as

$$m_t, m_W \gg m_u, m_d, m_c, m_s, m_b \tag{4}$$

  and the $\mathcal{C}$ associated with λ_t in (3) contains no quarks masses except top.
- We unified the 3-loop scalar integrals according to tetrahedron symmetry. These integrands can be represented as tetrahedrons with each of the 6 sides being $(p_i^2 - m_i^2)^{n_i}$ with mass m_i and some integer power n_i. The 6 momenta p_i in loops are $\{p_1, p_2, p_3, p_4 \equiv p_1 - p_2, p_5 \equiv p_1 - p_3, p_6 \equiv p_2 - p_3\}$. All the external momenta are ignored as the matching between full-SM and EFT does not depend on external momenta.



The above 2 diagrams represent the same scalar integrals according to 24-order tetrahedron symmetry group $\cong S_4$. The masses are marked with different colours. The unification of equivalent integrals can shrink the size of the output and enhance the efficiency.

Summary

At the moment, I have created the FORM[5] codes for tetrahedron-symmetry operation on the 3-loop scalar integrals. After the unification, the integrals undergoes reduction into master integrals (with known value) by REDUZE[6]. With the above codes, I have enabled *2loopmass*[7] for 3-loop calculation.

References

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