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Precision Calculation for quark flavour Physics

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CP-violation Discovery

- Parity violation: Lee and Yang, Wu-experiment in 1956



- Charge-Parity (CP-)violation in Kaon weak decay, first discovered by James Cronin and Val Fitch in 1964



- Fundamental in Standard Model(SM)
- Matter-Antimatter asymmetry

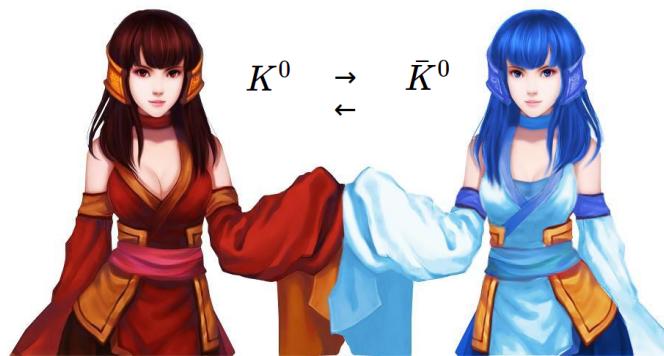


Neutral Kaon System

- Kaon $K^0(\bar{s}d)$ and Anti-kaon $\bar{K}^0(s\bar{d})$ oscillate
- Mass (lifetime) eigen-states in terms of Flavour eigen states

$$|K_S^0\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$
$$|K_L^0\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$$

- If CP conserved, $K_L \rightarrow 2\pi$ cannot happen
- Mixing between mass eigen-states enables $|K_L^0\rangle \rightarrow |K_S^0\rangle \rightarrow 2\pi$



Parameter ϵ_K

- We calculate some contribution to ϵ_K
- ϵ_K relates to probability of the $|K_L^0 \rangle \rightarrow |K_S^0 \rangle \rightarrow 2\pi$

$$\epsilon_K \equiv \frac{1}{2} \arg \left(\frac{-M_{12}}{\Gamma_{12}} \right) e^{i\phi_\epsilon} \sin \phi_\epsilon$$

- $|\epsilon_K| = (2.228 \pm 0.011) \times 10^{-3}$ (PDG)
- M_{12} holds the effective Hamiltonian

$$\mathcal{H} = \frac{g^4}{64\pi^2 m_W^2} (\lambda_t^2 \mathcal{C}^{tt} + \dots) (\bar{s}_L \gamma_\mu d_L) \otimes (\bar{s}_L \gamma^\mu d_L).$$

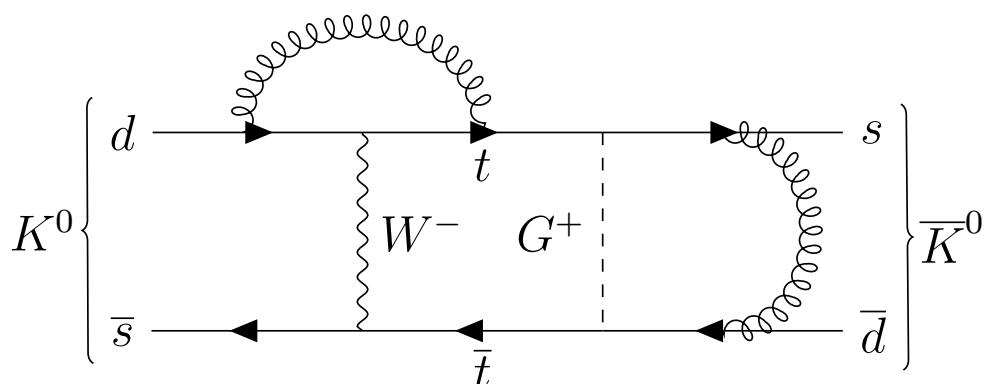
- Wilson Coefficient \mathcal{C}^{tt} bound to $\lambda_t \equiv V_{ts}^* V_{td}$

Parameter ϵ_K ϵ_K M_{12} \mathcal{H} \mathcal{C}
Wilson Coefficient

The missing block

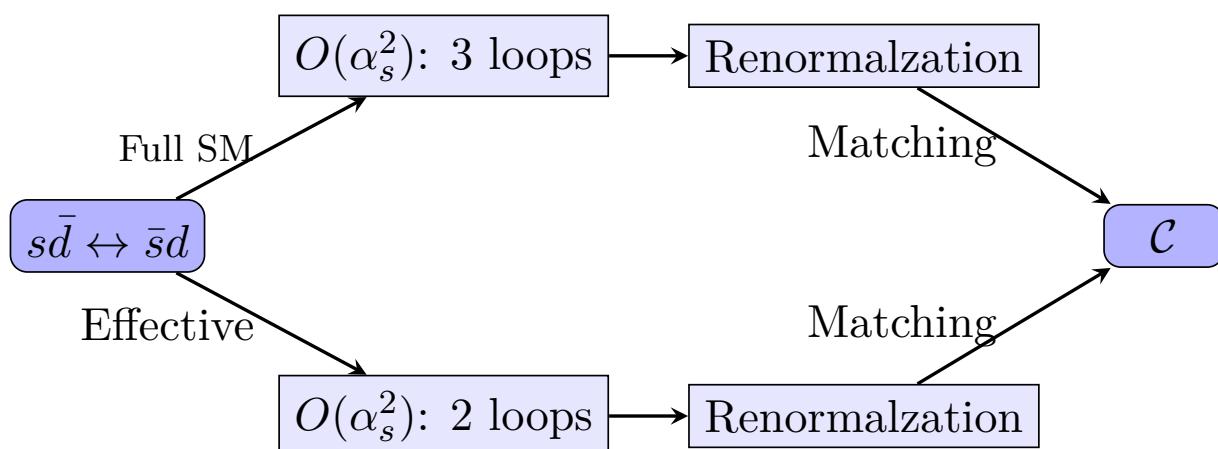
For the Wilson Coefficient

- NNLO Contribution from 3-loop QCD correction ([My Project](#)):
Perturbative



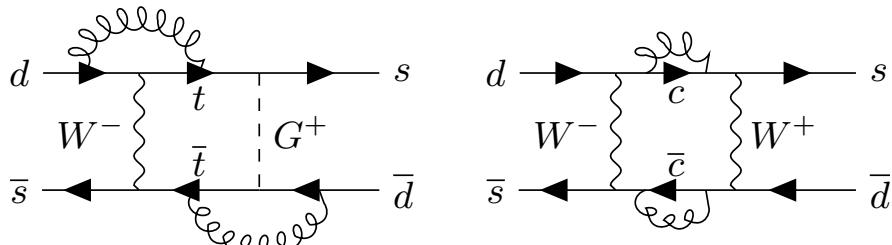
- 2-loop lattice computation : Numerical

The Route

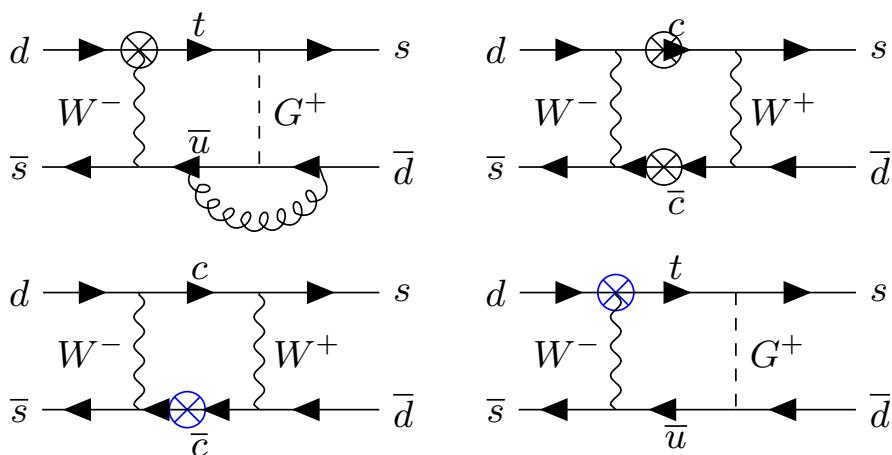


Full SM Amplitude

- Bare diagrams



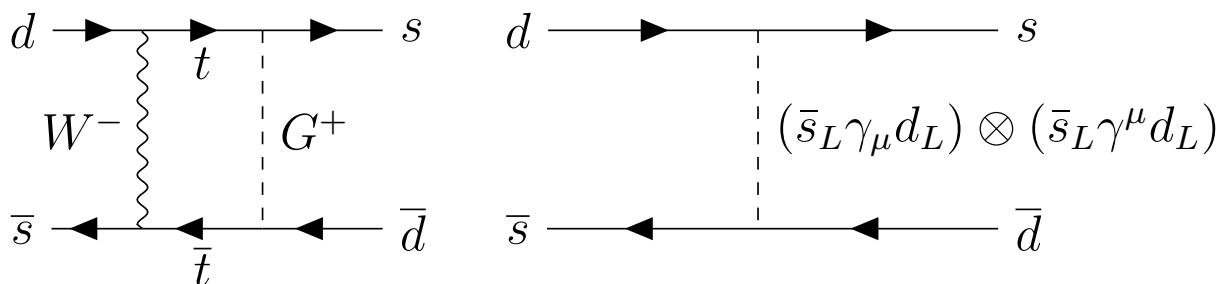
- Counter-term diagrams: (α_s and α_s^2)



- Ultraviolet poles cancel but Infrared (IR) poles remain

Effective Amplitude

- EFT: Effective Field Theory
- Low-energy approximation of Full SM
- Electroweak contribution is integrated out



- Gluonic correction is added to Effective diagrams
- Counter-term diagrams calculated, IR-poles remain

Matching between SM and EFT

- Equating Full SM and EFT amplitudes
- IR-poles cancel during matching
- SM and EFT share the same low-energy behaviour

$$\mathcal{H} = \frac{g^4}{64\pi^2 m_W^2} (\lambda_t^2 \mathcal{C}^{tt} + \dots) (\bar{s}_L \gamma_\mu d_L) \otimes (\bar{s}_L \gamma^\mu d_L).$$

- Wilson Coefficient \mathcal{C}^{tt} extracted from matching
- Improved precision to ϵ_K

Computational Tools

1 Main Programming

- *2loopmass=QGRAF+FORM* (Joachim Brod, Emmanuel Stamou and Lorenz Hüdepohl): Diagram evaluation up to 2 loops
- *REDUCE*: Converting 3-loop integrals into master integrals

2 Subsidiary Programming

- *FORM & Mathematica*: Onward Simplification and Matching
- *Python3 & Bash* : Configuration

Light Quark and Vacuum assumption

Challenge in 3-loop Calculation:

- Large number ($\sim 1.3 \times 10^4$)
- Complexity of each diagram

Assumptions:

- 1 $m_u, m_d, m_c, m_s, m_b \rightarrow 0$
 - $m_W, m_t \gg m_u, m_d, m_c, m_s, m_b$
 - High energy scale: Perturbative QCD is valid.
- 2 Vacuum diagrams (no external momenta)
 - Wilson Coefficient invariant with external states

- Less complexity of individual diagram
- Smaller number of diagrams

e.g.

- Simplified Feynman rules with light quarks

$$c \xrightarrow{\quad} : \quad \frac{i(\not{p} + m)}{p^2 - m^2} \rightarrow \frac{i\not{p}}{p^2}$$

- No Goldstone,light-quark,light-quark coupling

$$d \xrightarrow{\quad} u$$

$$G^+ \vdash$$

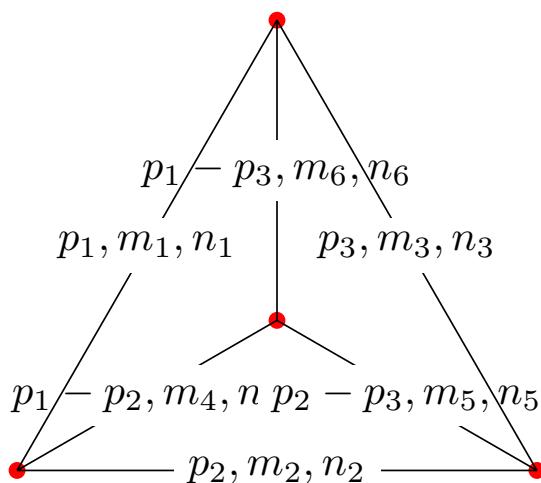
$$\frac{g}{2\sqrt{2}} \left(\frac{m_u}{m_W} (1 - \gamma^5) - \frac{m_d}{m_W} (1 + \gamma^5) \right) V_{ud} \rightarrow 0$$

3-Loop Scalar Integral: Representation

- The Integral $I(m_1, m_2, m_3, m_4, m_5, m_6; n_1, n_2, n_3, n_4, n_5, n_6)$

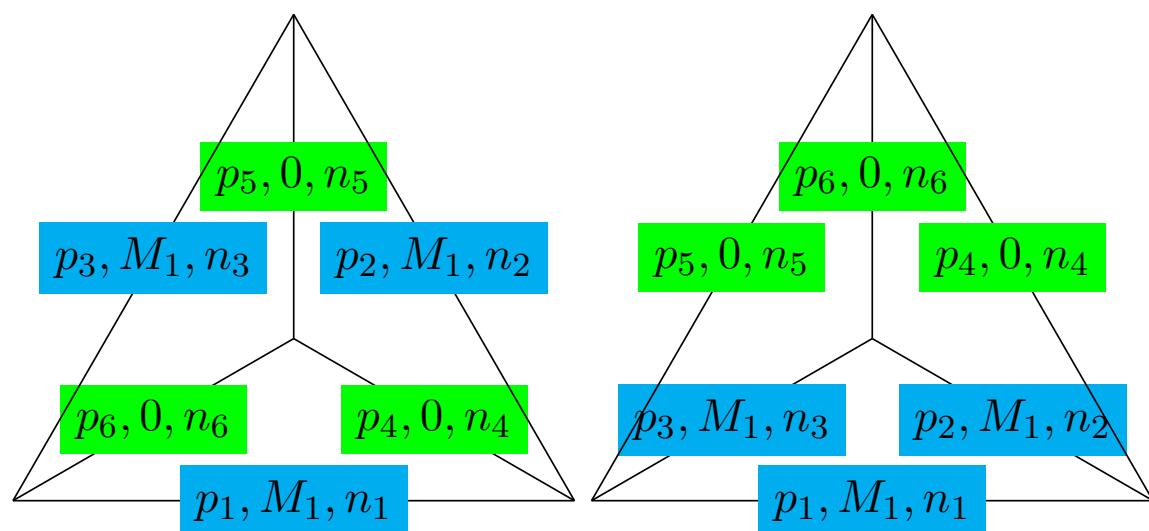
$$\int \frac{d^d p_1 d^d p_2 d^d p_3}{(p_1^2 - m_1^2)^{n_1} (p_2^2 - m_2^2)^{n_2} (p_3^2 - m_3^2)^{n_3} ((p_1 - p_2)^2 - m_4^2)^{n_4} ((p_2 - p_3)^2 - m_5^2)^{n_5} ((p_1 - p_3)^2 - m_6^2)^{n_6}}$$

- Tetrahedron Representation



3-Loop Scalar Integral: Symmetry

- Unification of Equivalent Integrals by Tetrahedron Symmetry
- S_4 Group among the 4 Vertices



3-Loop Scalar Integral: Representation

- Fewer Scalar Integrals after Unification
- More Efficient Reduction by REDUZE
- Master Integrals with Known Values, e.g.

$$I(m_1, m_2, m_3, m_4, m_5, m_6; 1, 1, 1, 0, 0, 0) = \int \frac{d^d p}{(p^2 - m_1^2)} \int \frac{d^d p}{(p^2 - m_2^2)} \int \frac{d^d p}{(p^2 - m_3^2)}$$

Summary

- Separate FORM Codes for Reduction
- Employed the *2loopmass* for 3-Loop Computation
- Finished the Computation of:
 - Full SM Counter-term Diagrams
 - Effective Bare and Counter-term Diagrams
- Next Steps
 - Simplify the Full SM Diagrams with Master Integrals in
 - Match the Amplitudes and Work out Wilson Coefficient

Thank you!