An Introduction to Causal Set Theory

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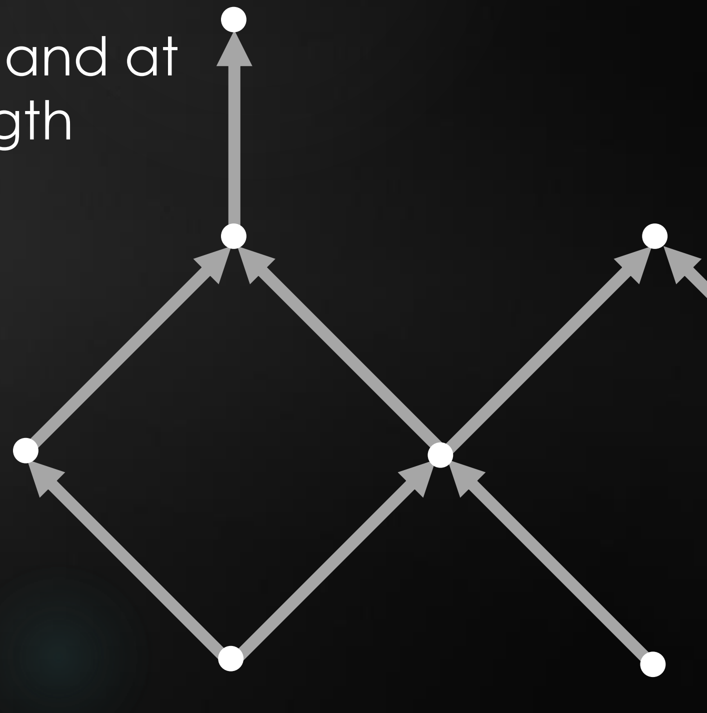
Part 1: The Causal Sets Programme

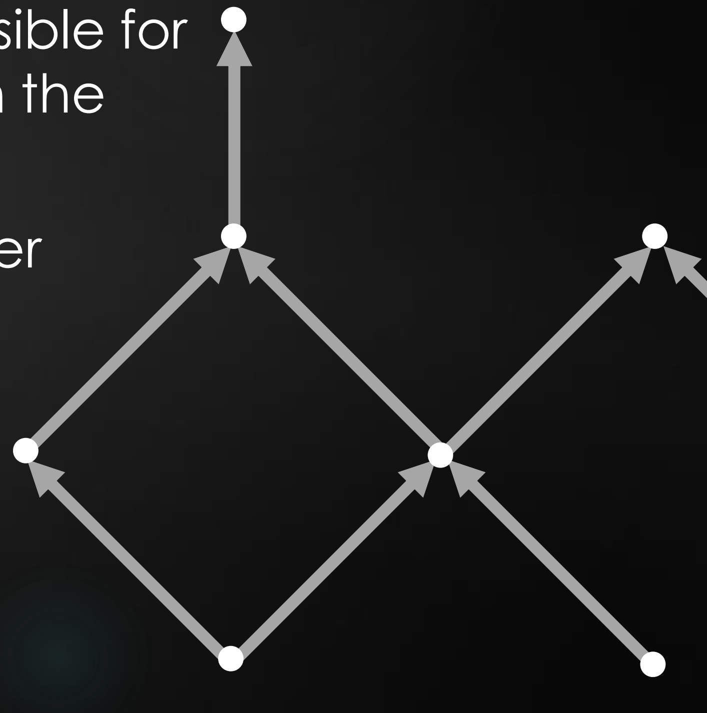
A Continuum Conundrum

- UV divergences – is QFT really valid to such arbitrarily high energy scales?
- Can be dealt with by renormalisation but problem for quantum gravity.
- GR/ΛCDM predicts singularity at the centre of a black hole and at the Big Bang – but continues to provide ‘predictions’ at length scales arbitrarily close to these singularities.
- These are issues associated with a continuous manifold.
- What if spacetime was fundamentally discrete instead?



Motivating the discrete manifold

- Riemann suggested in 1873 that “the reality which underlies space must form a discrete manifoldness” and that “we are quite at liberty to suppose the metric relations of space in the infinitely small do not conform to the hypotheses of geometry”.
- Einstein in 1916 also condemned the continuum as “responsible for the fact that our present means of description miscarry with the quantum theory”.
- If spacetime were indeed a discrete set of points, what other information would we need along with it?
- Causality.

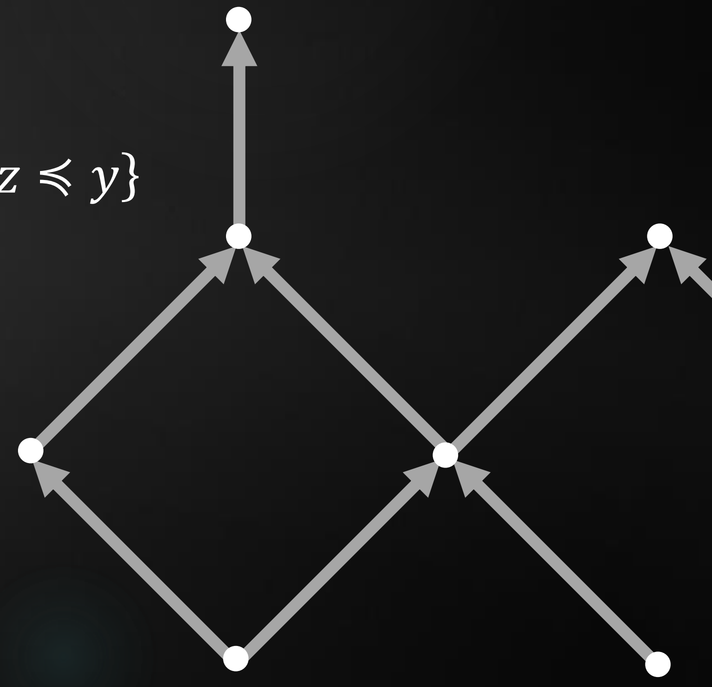



Hold on.
This whole operation was your idea.

Order + Number = Geometry

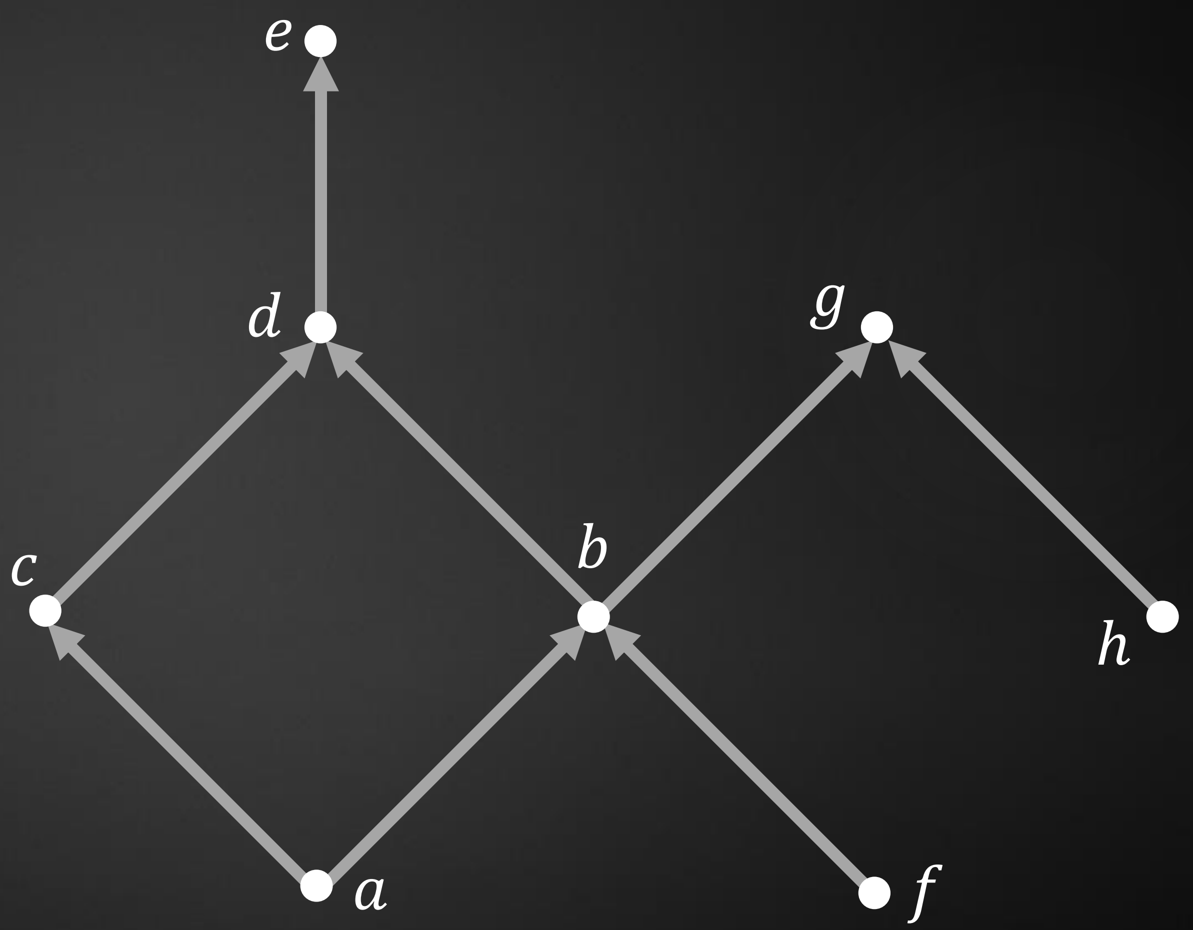
Defining a causal set

- From the seminal Bombelli et al. paper of 1987:
- A causal set C is a partially ordered set with a binary order relation $\preccurlyeq$ which is
- 1. Transitive: If $x \preccurlyeq y$ and $y \preccurlyeq z$ then $x \preccurlyeq z, \forall x, y, z \in C$.
- 2. Acyclic: If $x \preccurlyeq y$ and $y \preccurlyeq x$ then $x = y, \forall x, y \in C$.
- 3. Locally finite: $|I(x, y)| < \infty, \forall x, y \in C$, where $I(x, y) = \{z | x \preccurlyeq z \preccurlyeq y\}$



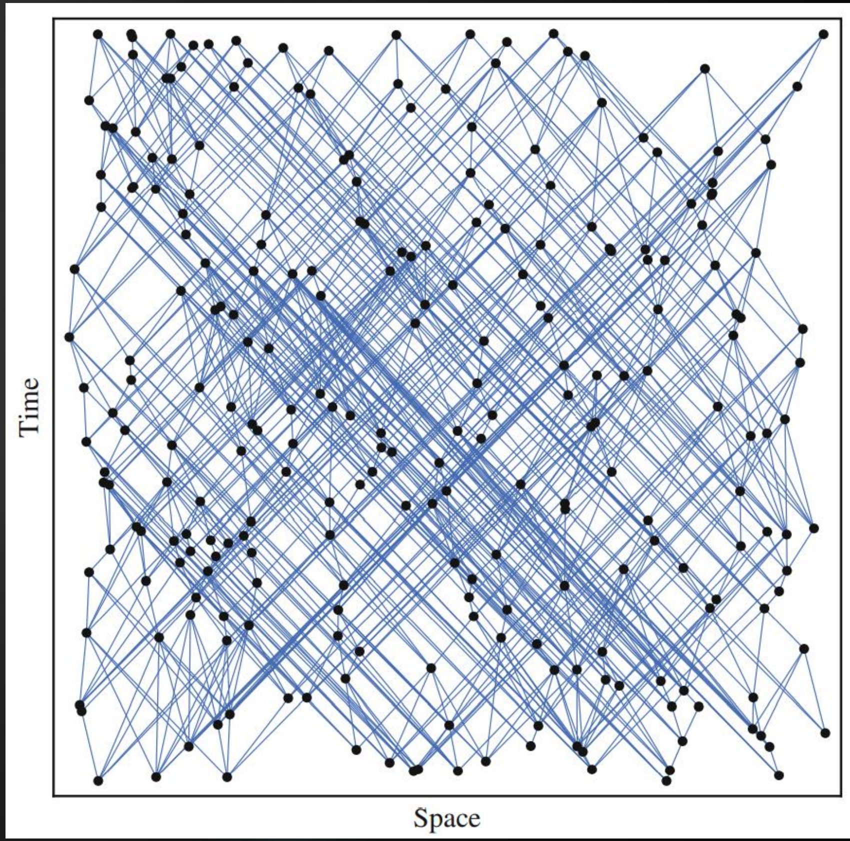
Depicting a causal set

- Hasse diagrams:



Emergence of the continuum

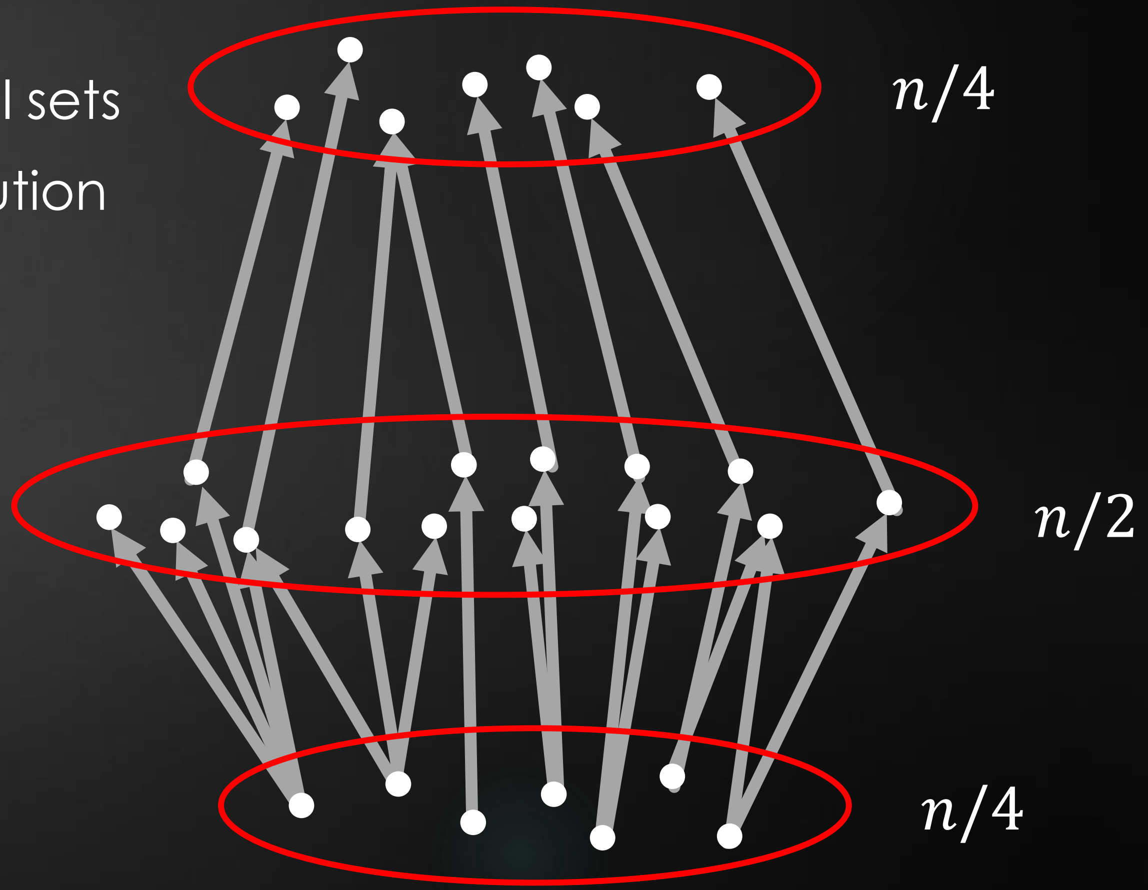
- How can we connect an underlying causal set to an emergent continuum?
- Generating a causal set by ‘sprinkling’ in points in a Lorentz covariant way.
- Can be done by a Poisson distribution.
- If a causal set could plausibly be generated by the sprinkling of a manifold, it is considered to be ‘well approximated’ by that manifold.
- Causal sets at this scale are considered to be ‘microstates’ corresponding to the ‘microstate’ that is the manifold.



Part 2: Causal Set Dynamics

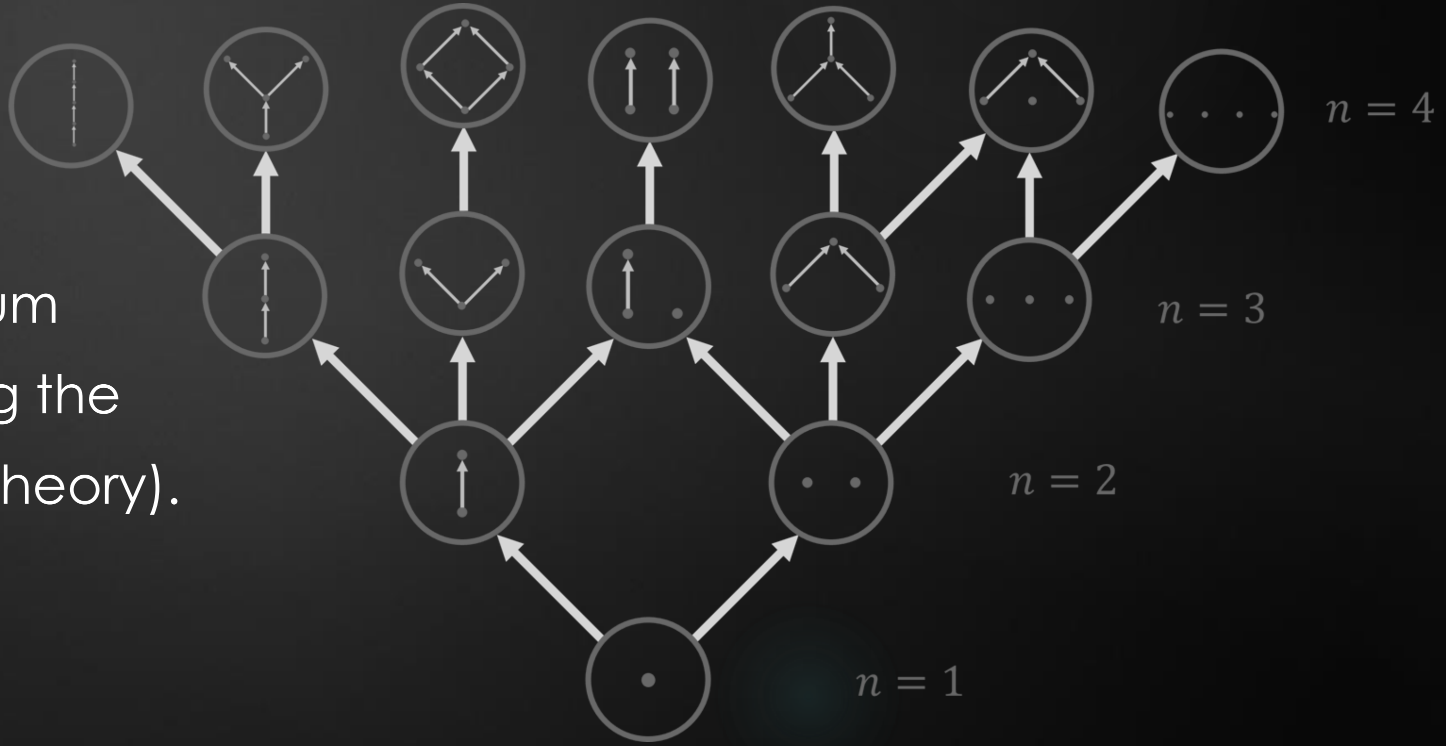
The space of causal sets

- The space of all causal sets includes many non-manifold like causal sets.
- Dominated by Kleitman-Rothschild causal sets
- Bilayer causal sets also a massive contribution



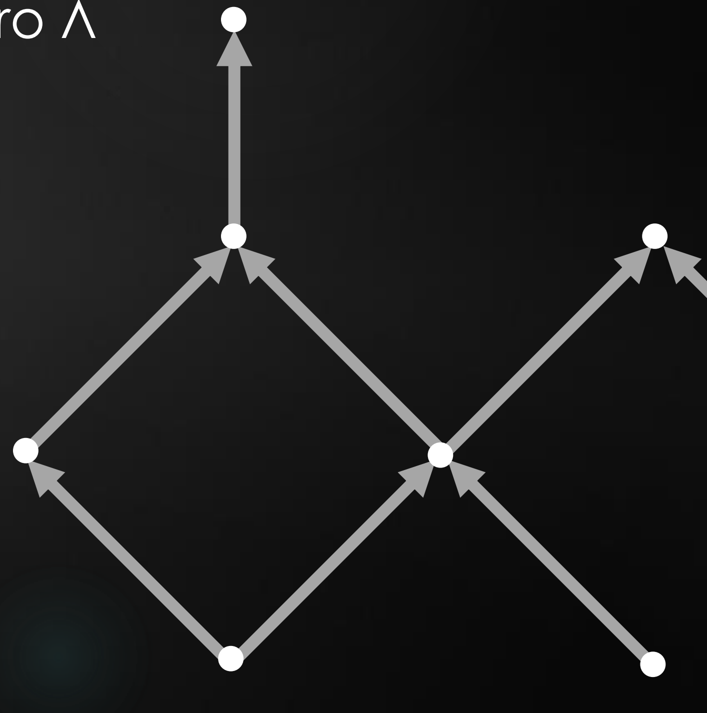
Classical Sequential Growth (CSG)

- Dynamics may make it more likely that manifold-like causal sets dominate over the non-manifold-like causal sets.
- Can grow causal sets with transitive percolation.
- Should be path independent
- Can be shown as a
  Hasse diagram of a 'poscau'
- Forms a basis for a fully quantum
  growth dynamics, considering the
  quantum sum rule (measure theory).



Things I missed

- Intermediate sum-over-histories dynamics
- HKMM Theorem
- Fundamental conjecture of CST
- Causal set phenomenology – swerves, prediction of non-zero Λ
- Dimension estimators for causal sets
- Quantum Sequential Growth
- Quantum fields on causal sets



Thanks for listening!