How well do we know the equation of state in neutron stars?

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May 25, 2022

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Background

Motivation

- The era of multi-messenger astronomy is here

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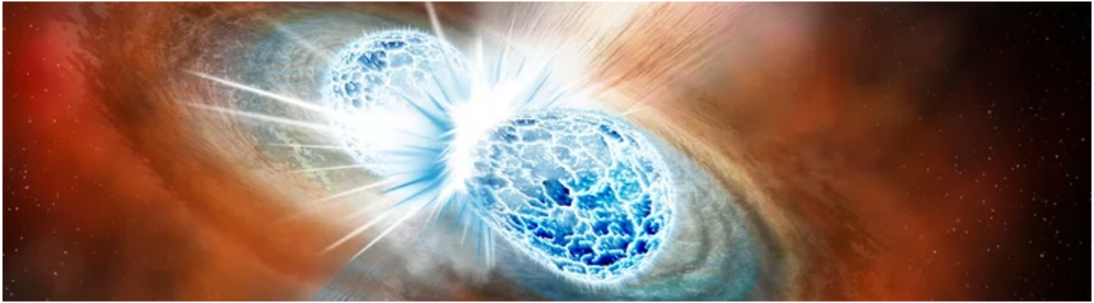
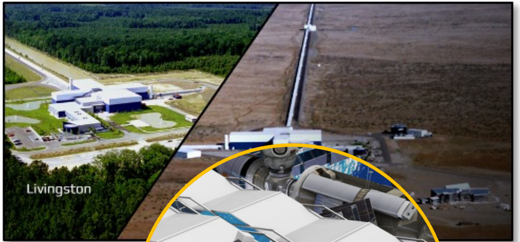
- LIGO/Virgo are measuring black hole and neutron star collisions



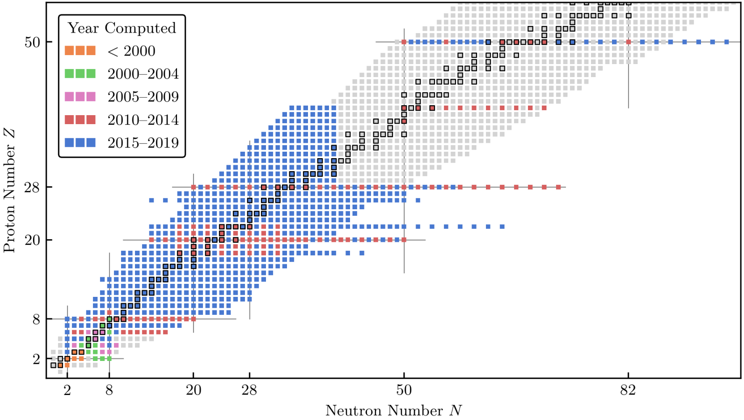
Motivation

- The era of multi-messenger astronomy is here
- LIGO/Virgo are measuring black hole and neutron star collisions
- NICER x-ray telescope
- Where does gold come from?



New facilities

- Requires understanding of nuclei “far from stability”
- Bottom right of nuclear chart; many more neutrons than protons



New facilities

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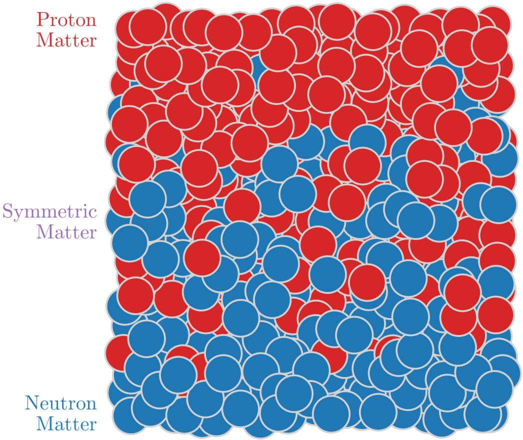
- Bottom right of nuclear chart; many more neutrons than protons
- FRIB just opened to study these rare isotopes

Key nuclear input: the equation of state

- Nuclear matter: infinite in extent, no electromagnetism, and can be made up of any ratio of neutrons/protons



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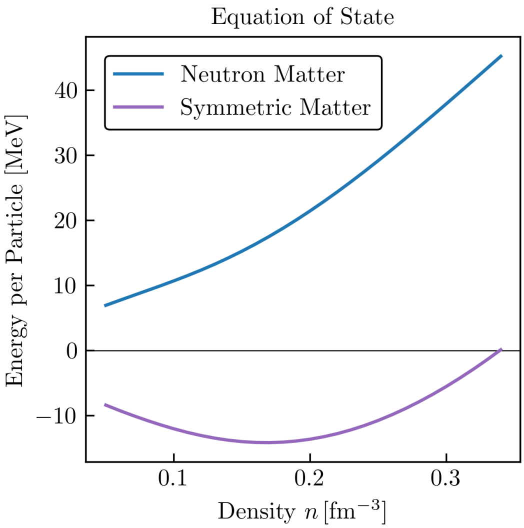
- Some key observables

E/A : Energy per particle



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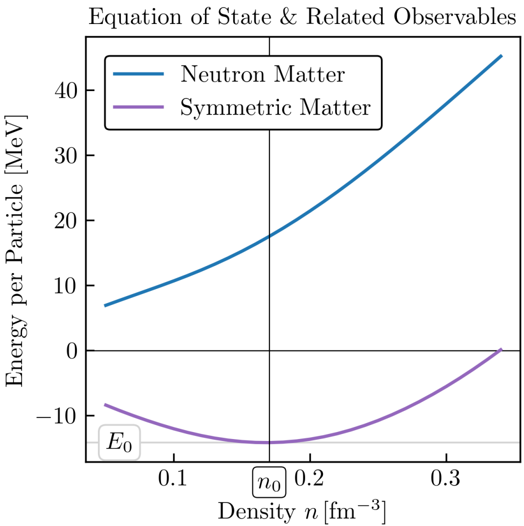

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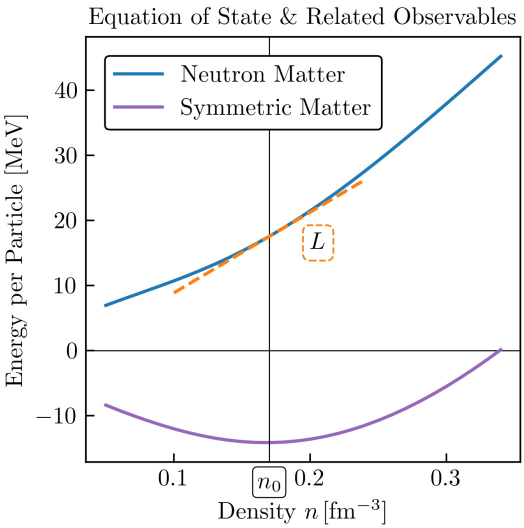

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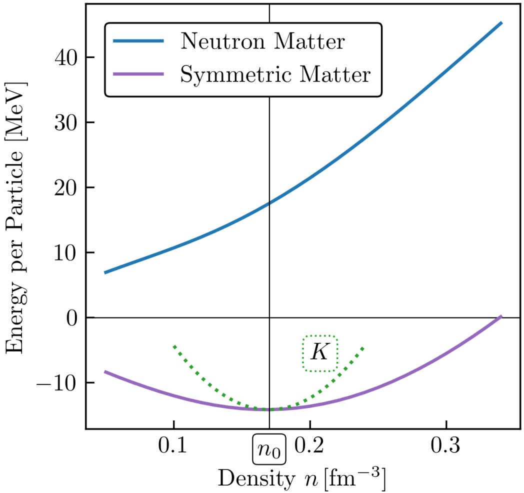
Equation of State & Related Observables



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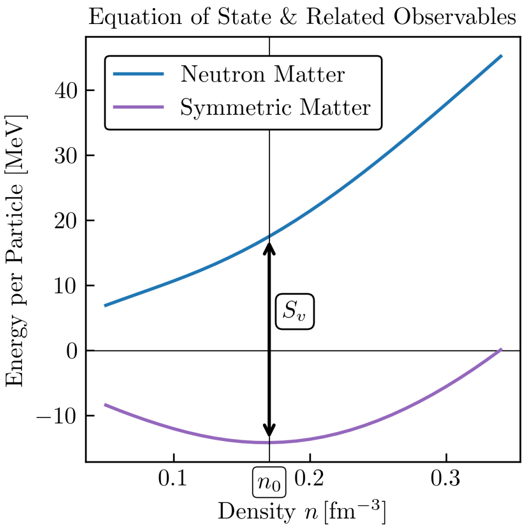
S_v : Symmetry energy; relates different systems!



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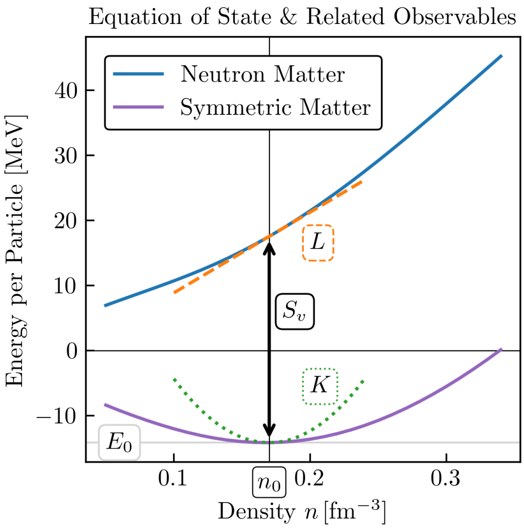
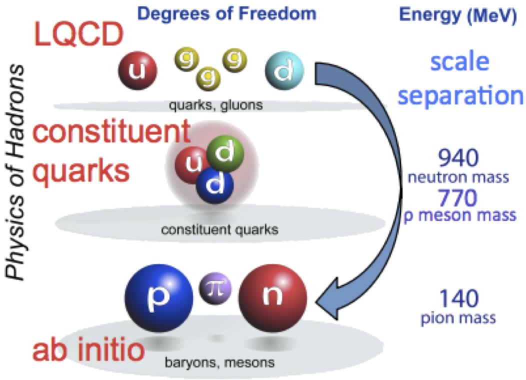
- Want consistent error propagation!



Predicting the Equation of State

Predictions in low-energy nuclear physics

- We need to know how protons/neutrons interact
- There is interesting physics at all scales



Predictions in low-energy nuclear physics

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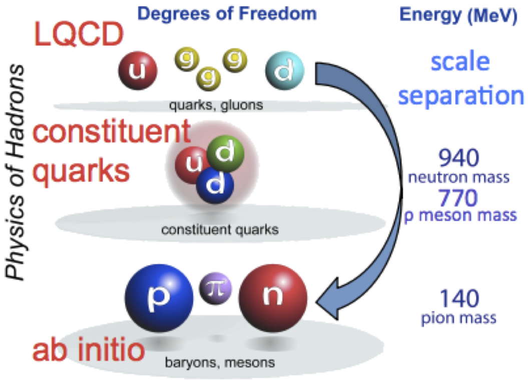

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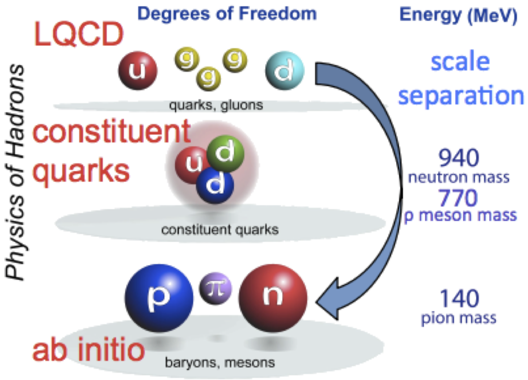

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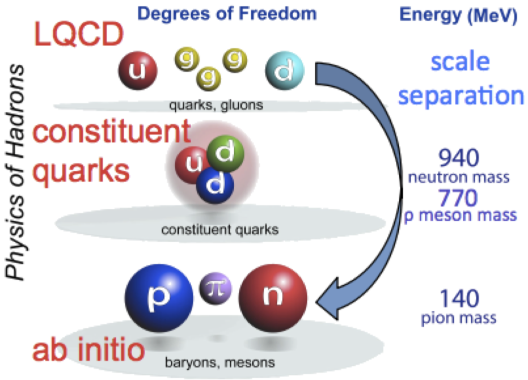
- We use chiral effective field theory here.



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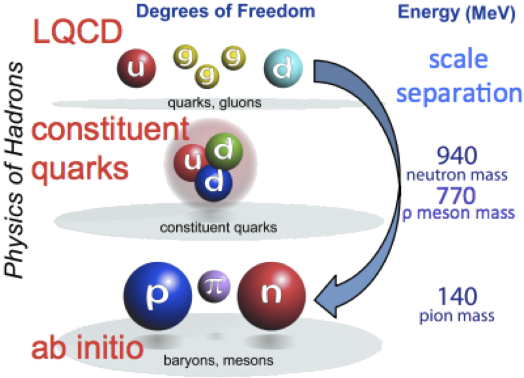
- We use chiral effective field theory here.
- Truncation error must be quantified!

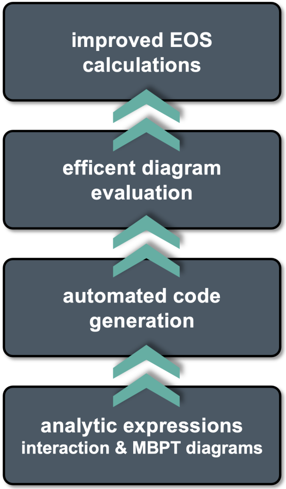


Evaluating diagrams: automatic code generation [arxiv:1710.08220]

- Chiral EFT provides forces; we need to simulate large systems → many-body perturbation theory (MBPT)
- Number of many-body diagrams increases rapidly

$$\begin{aligned} n &= 2: \; 1 \\ n &= 3: \; 3 \\ n &= 4: \; 39 \\ n &= 5: \; 840 \\ n &= 6: \; 27{,}300 \\ n &= 7: \; 1{,}232{,}280 \end{aligned}$$

- Need a systematic way to build & evaluate diagrams → automatic code generation
- Accelerated with GPU, MPI, and openMP

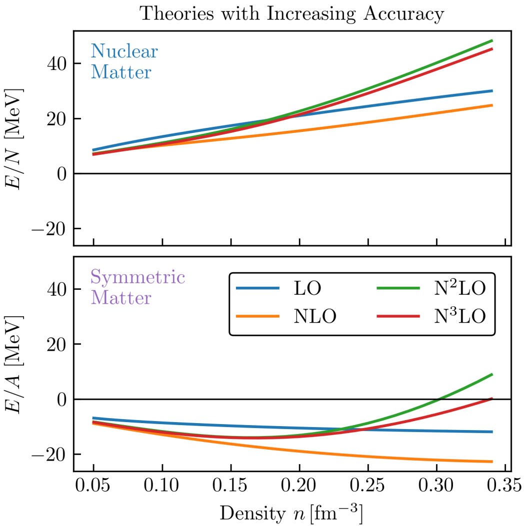


Drischler, Hebeler, and Schwenk, PRL 122, 042501
Drischler & McElvain, in prep.

A Model for Effective Field Theory Uncertainty

Takeaway points

- Theory error enabled by multiple models + physical intuition



Takeaway points

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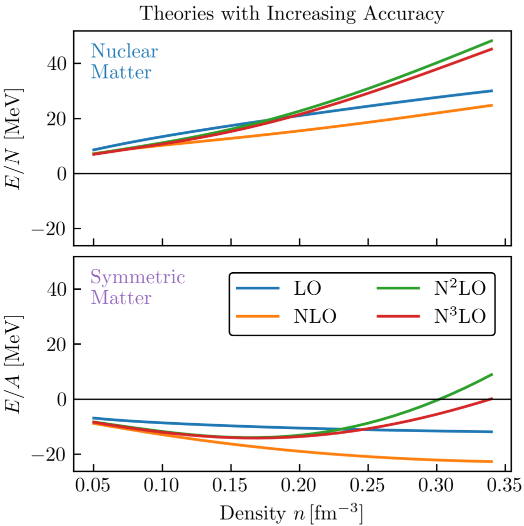
- Correlations in errors matter

Type-x: One observable at multiple locations $y(x)$ vs $y(x')$

Type-y: Multiple observables $y(x)$ vs $y'(x')$



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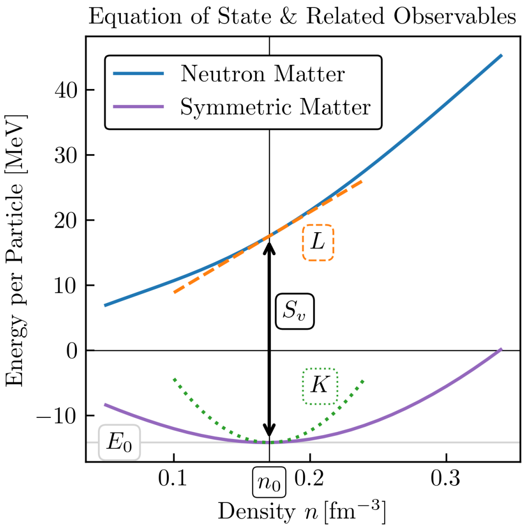

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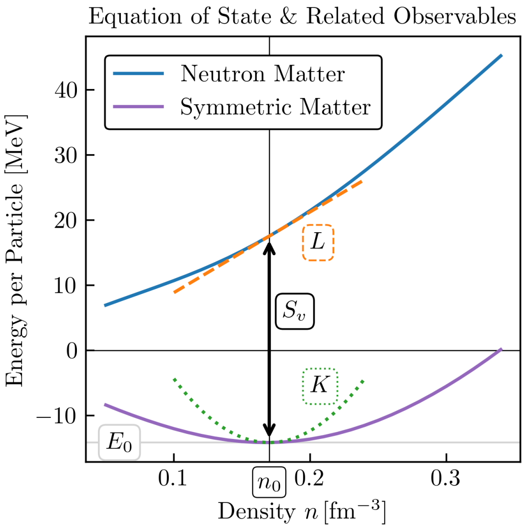
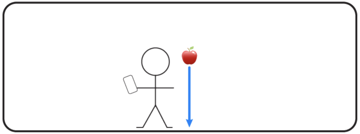
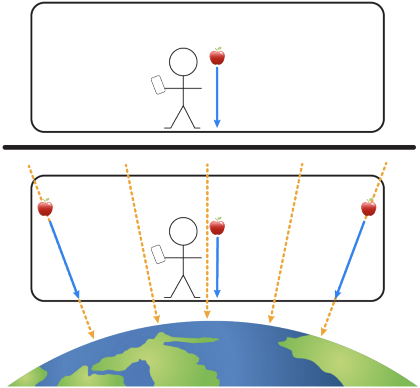
- One model connects uncertainties across multiple observables
- The truncation error model makes testable predictions





Grav. force (short distances):

$$F = -mg$$



Grav. force (short distances):

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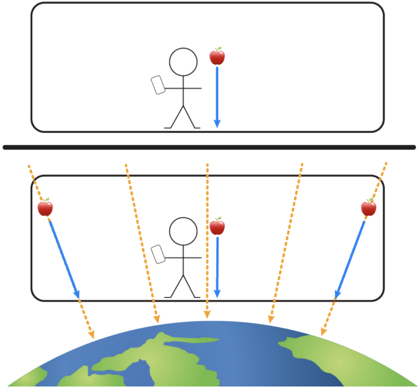
Grav. force (large distances):

$$F = -\frac{GMm}{r^2}$$

The laws look quite different!



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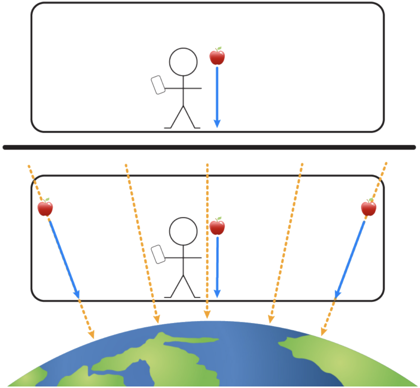
Connected via series expansion about radius of Earth R:

$$F \approx -mg + 2mg\left(\frac{r-R}{R}\right) - 3mg\left(\frac{r-R}{R}\right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R}\right)^3\right]$$

Scales in Physics



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???

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Can fit unknown parameters to data $\Rightarrow$ inverse problem!

$$F \approx a_0 + a_1 \left(\frac{r-R}{R}\right) + a_2 \left(\frac{r-R}{R}\right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R}\right)^3\right]$$

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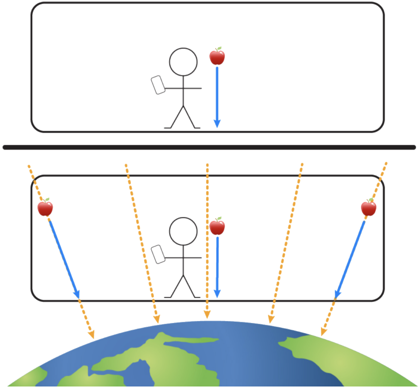

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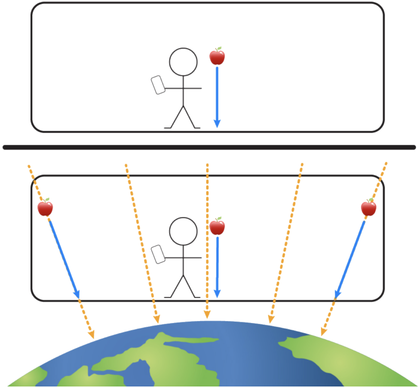
The laws look quite different!

Use prior info from physics:

$$F \approx mg \left\{ a_0' + a_1' \left(\frac{r-R}{R} \right) + a_2' \left(\frac{r-R}{R} \right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R} \right)^3 \right] \right\}$$

Scales in Physics



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The laws look quite different!

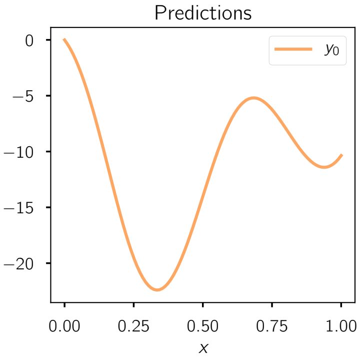
Propagate full uncertainty

$$F \approx mg \left\{ a'_0 + a'_1 \left(\frac{r-R}{R} \right) + a'_2 \left(\frac{r-R}{R} \right)^2 + \mathcal{O}\left[\left(\frac{r-R}{R} \right)^3 \right] \right\}$$

Chiral EFT in one slide

- $V_{NN}(\vec{a}) = V_{LO} + V_{NLO} + \cdots + V_{N^kLO} \Longrightarrow \boxed{\text{MBPT}} \Longrightarrow y_k(x; \vec{a})$

$$\{y_0\}$$



$y_0 \rightarrow \text{LO}$

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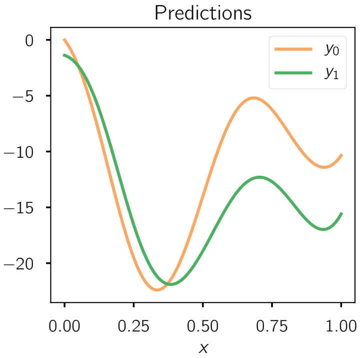
$$\{y_0, y_1\}$$



$y_0 \rightarrow$ LO

$y_1 \rightarrow$ NLO

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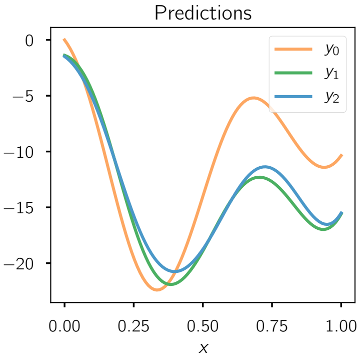
$$\{y_0, y_1, y_2\}$$



$y_0 \to$ LO

$y_1 \to$ NLO

$y_2 \to$ N^{2}LO

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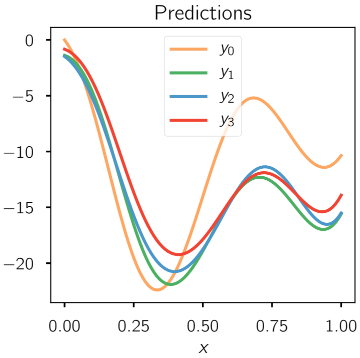
$$\{y_0, y_1, y_2, y_3\}$$



$$y_0 \to \text{LO}$$
$$y_1 \to \text{NLO}$$
$$y_2 \to \text{N}^2\text{LO}$$
$$\vdots$$
$$y_k \to \text{N}^k\text{LO}$$

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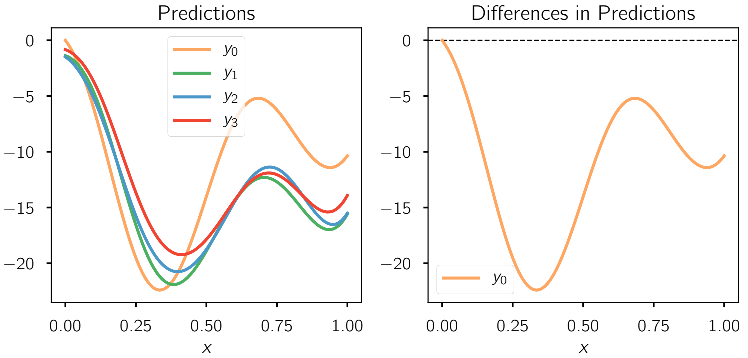
- $V_{NN}(\vec{a}) = V_{LO} + V_{NLO} + \cdots + V_{N^kLO} \Longrightarrow \boxed{\text{MBPT}} \Longrightarrow y_k(x; \vec{a})$
- One can change variables for convenience/insight.

$$y_0 = y_0$$



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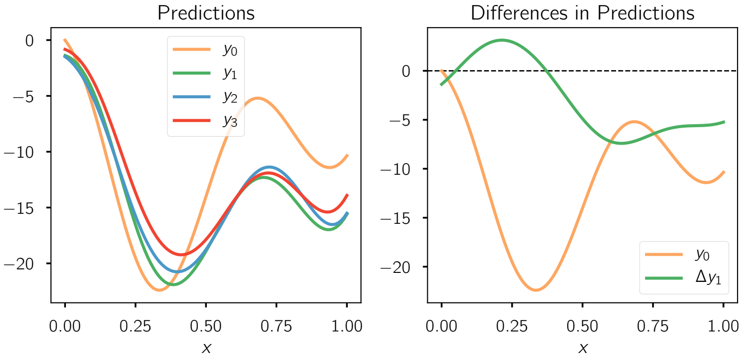
$$y_1 = y_0 + \Delta y_1$$



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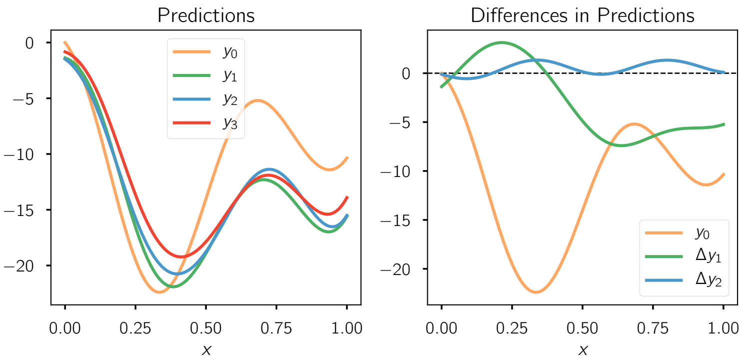
$$y_2 = y_0 + \Delta y_1 + \Delta y_2$$



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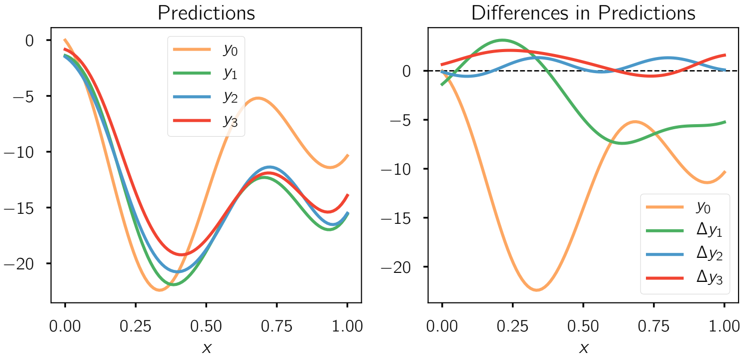
- One can change variables for convenience/insight.
- $\Delta y_n = y_{\text{ref}} c_n Q^n$

$$y_3 = y_0 + \Delta y_1 + \Delta y_2 + \Delta y_3$$



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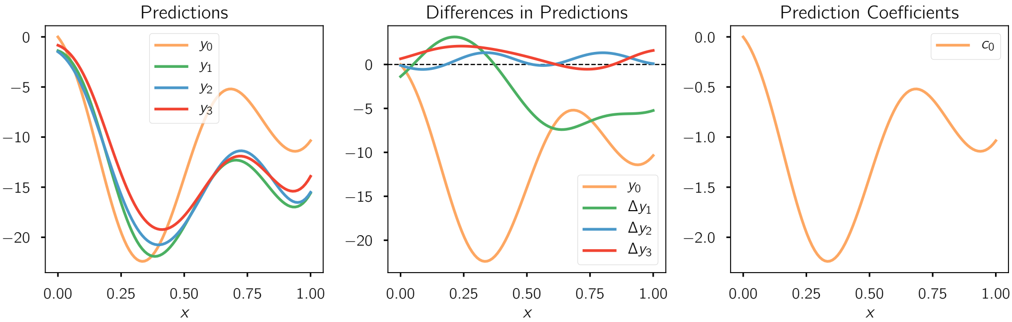
$$y_0 = y_{\text{ref}}\left[c_0 Q^0\right]$$



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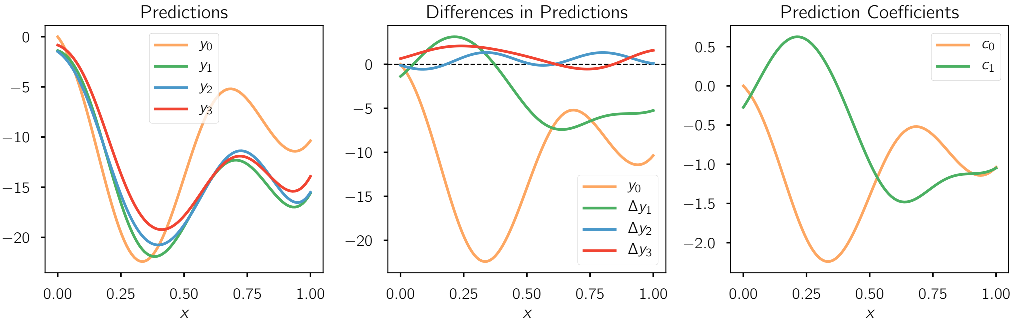

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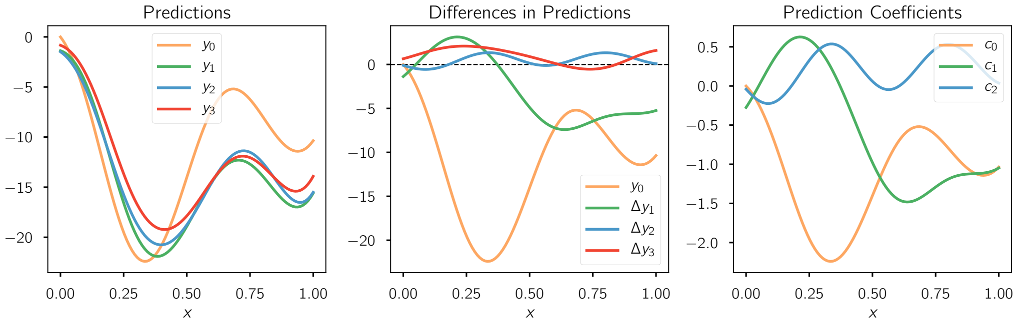

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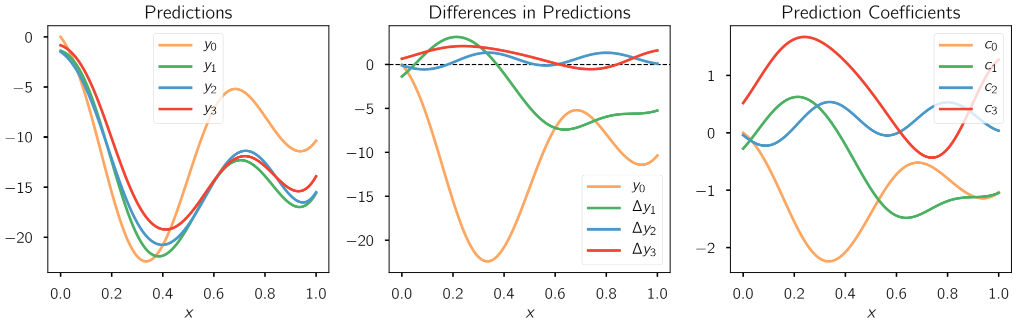
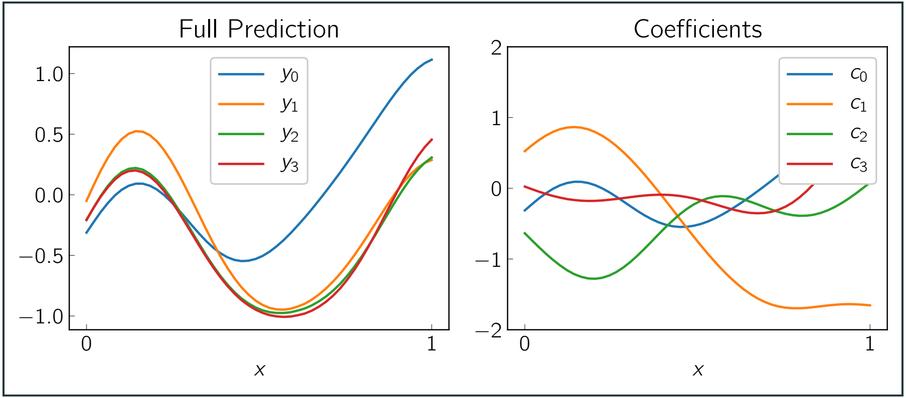


Model building

Main equation

$$y_k = y_{\text{ref}} \sum_{n=0}^{k} c_n Q^n$$

$$c_n \equiv \frac{y_n - y_{n-1}}{y_{\text{ref}} Q^n}$$



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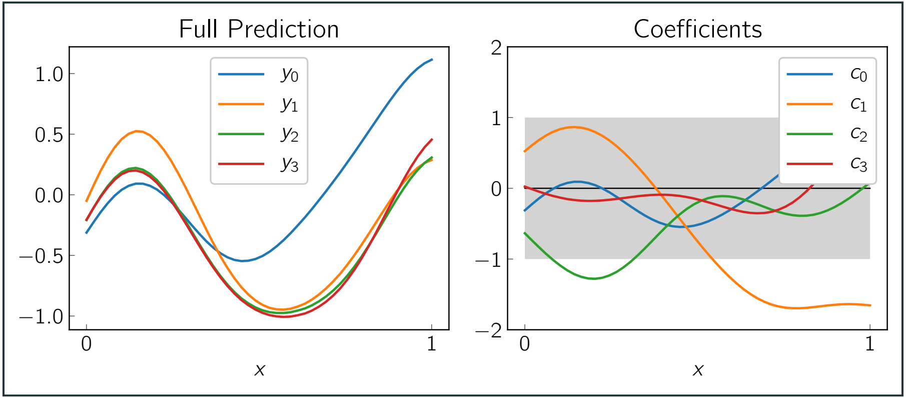
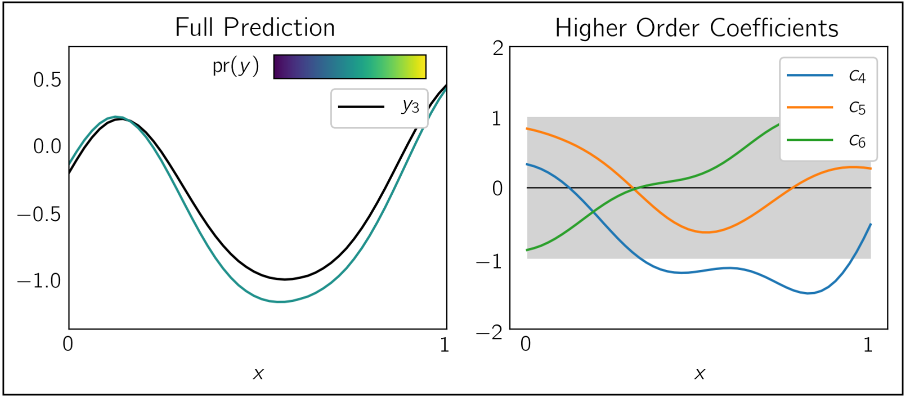


Model building

Main equation

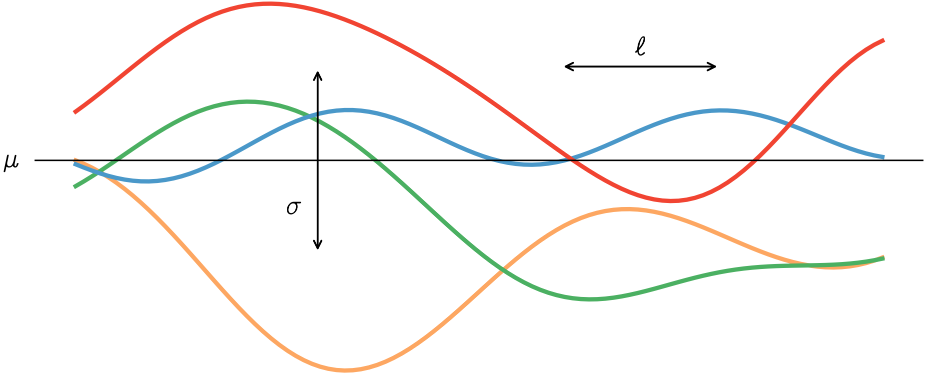
$$y = y_{\text{ref}} \sum_{n=0}^{\infty} c_n Q^n$$

$$c_n \equiv \frac{y_n - y_{n-1}}{y_{\text{ref}} Q^n}$$



Gaussian processes: how we induct on the c_n

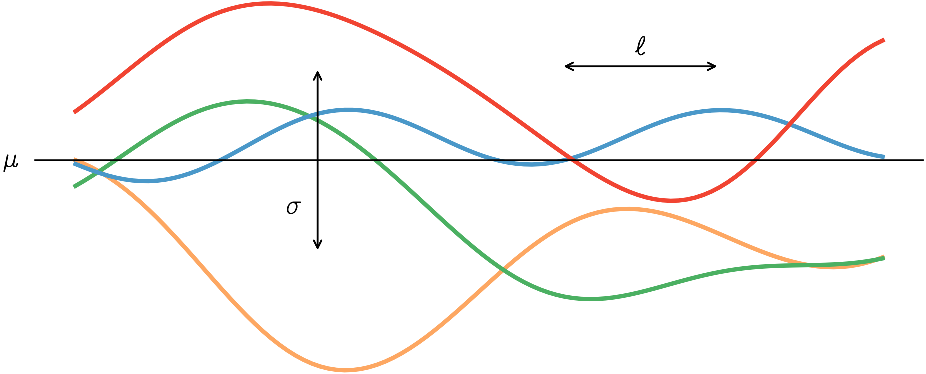
What are Gaussian processes?



Gaussian processes: how we induct on the c_n

What are Gaussian processes?

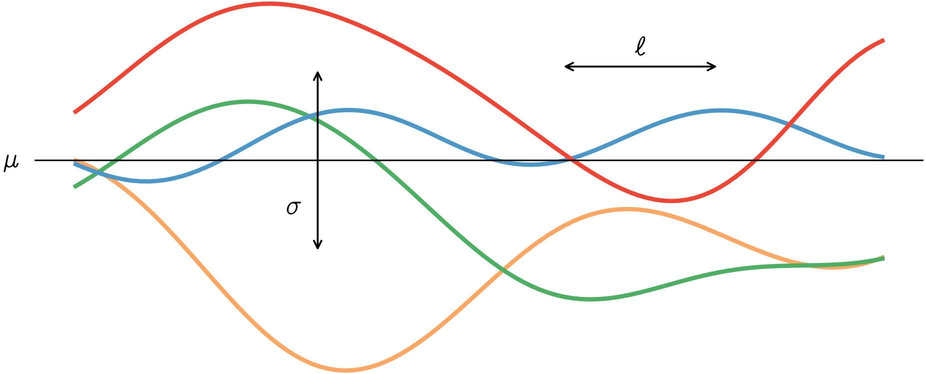
- An infinite dimensional generalization of the Gaussian distribution



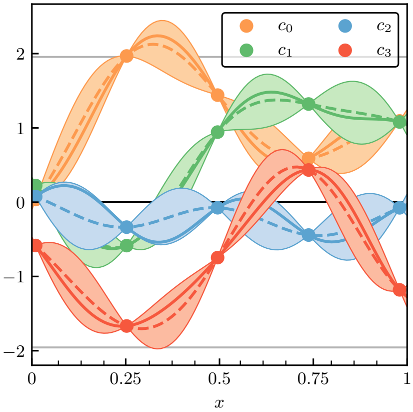
Gaussian processes: how we induct on the c_n

What are Gaussian processes?

- An infinite dimensional generalization of the Gaussian distribution
- A popular machine learning tool for non-parametric regression

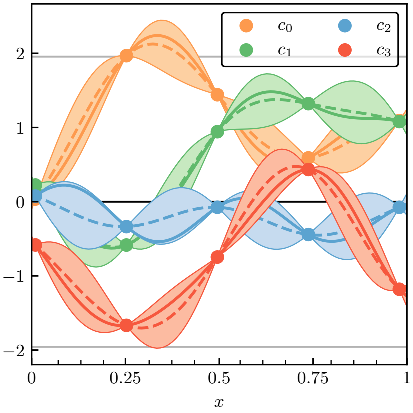


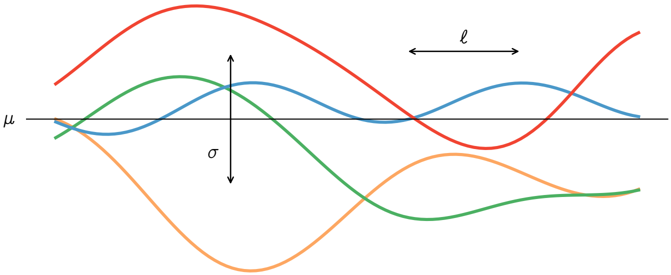
Learning parameters



Learning parameters

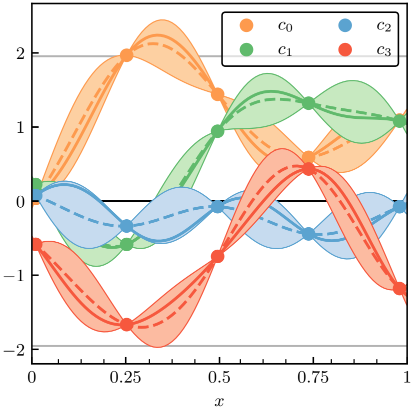
$$y_3 = y_{\text{ref}}\left[c_0 Q^0 + c_1 Q^1 + c_2 Q^2 + c_3 Q^3\right]$$

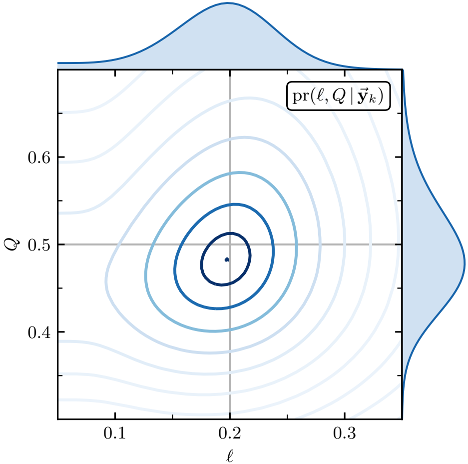




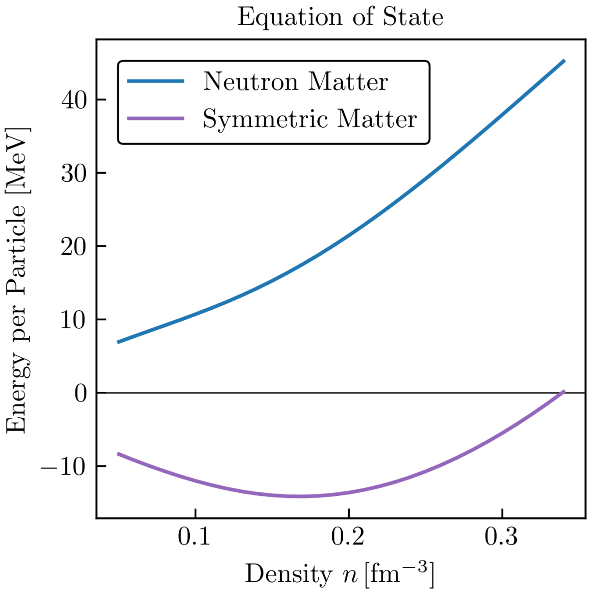
Learning parameters

$$y_3 = y_{\mathrm{ref}}\left[c_0 Q^0 + c_1 Q^1 + c_2 Q^2 + c_3 Q^3\right]$$



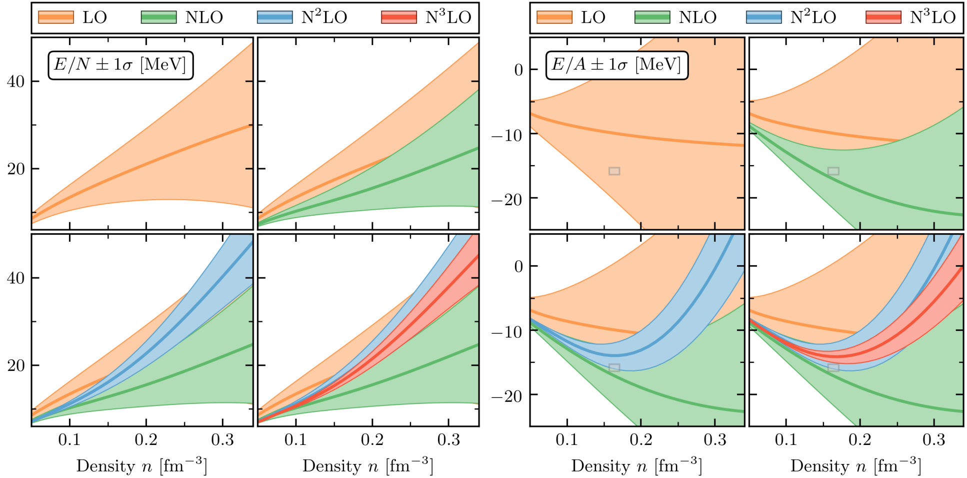


Uncertainty quantification: the equation of state

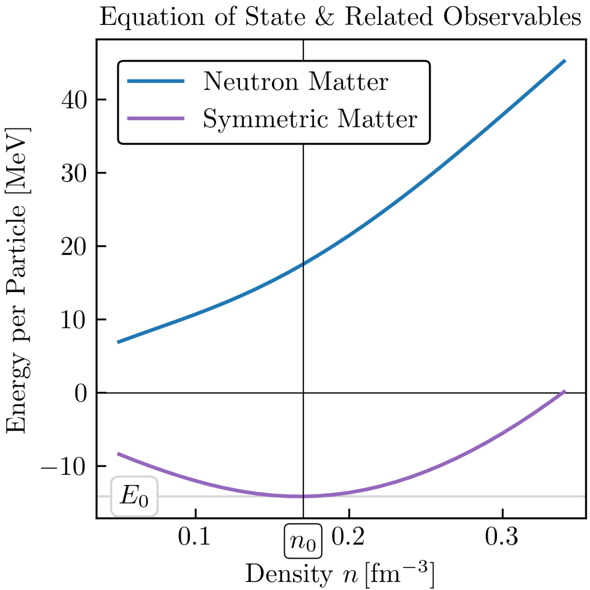


The equation of state

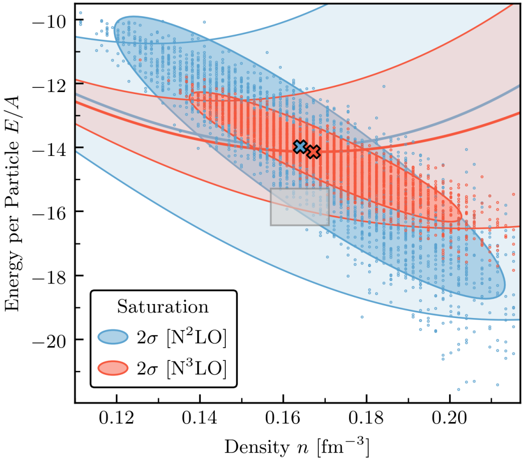
Note: correlations not shown!



Uncertainty quantification: the saturation point



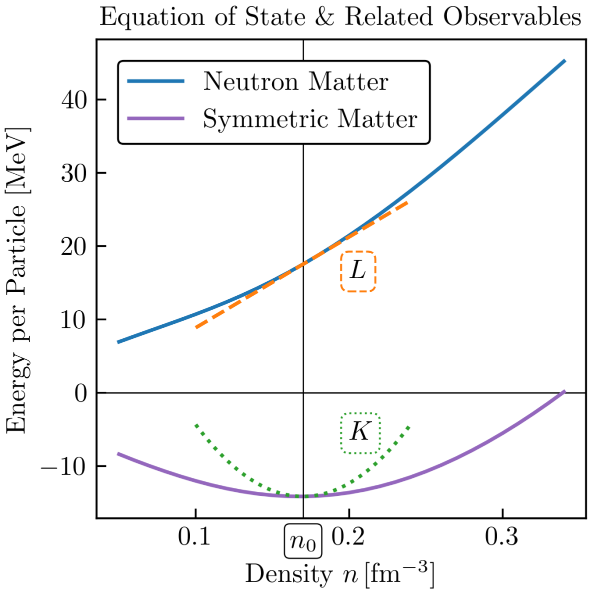
Saturation properties



$$\mathrm{pr}(E_0, n_0 \mid \mathcal{D})$$

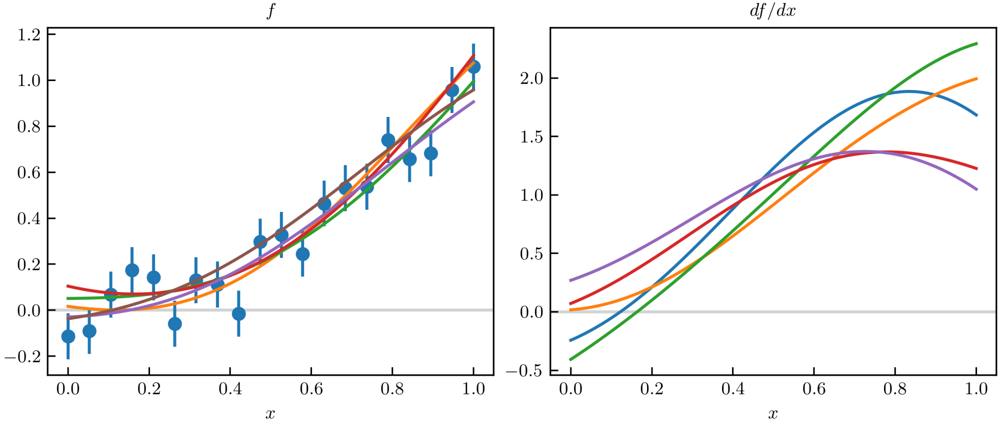
- Crosses denote minimum of theory curve
- Blue: our 2nd-best theory; Red: our best theory
- Create distribution by sampling: draw curve, get minimum, repeat
- Approximate by ellipse
- (Little grey box shows constraints from density function theory)

Uncertainty quantification: gradient properties

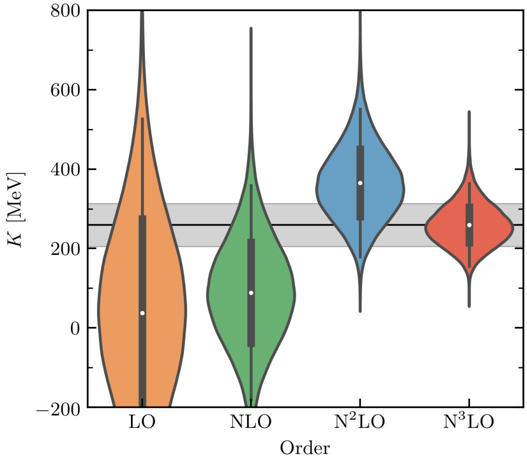


Obtaining derivatives

- Gaussian processes are closed under differentiation
- Can draw random functions and its corresponding derivative
- Correlations between f and f' come free!



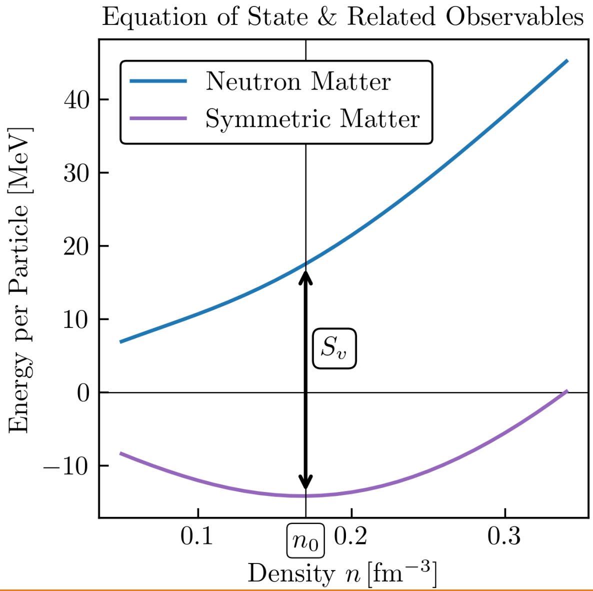
Gradient examples



$$\mathrm{pr}(K \mid \mathcal{D}) = \int dn_0 \, \mathrm{pr}(K \mid n_0, \mathcal{D}) \, \mathrm{pr}(n_0 \mid \mathcal{D})$$

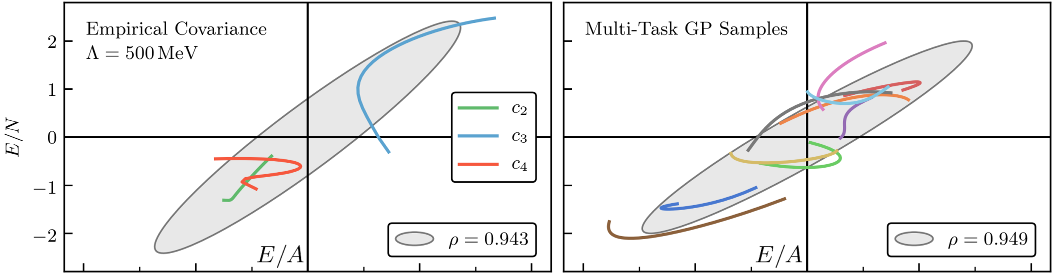
- Incompressibility is defined at satuation density n_0
- We don't know what n_0 is, but can marginalize over it (see previous slides)

Uncertainty quantification: symmetry energy



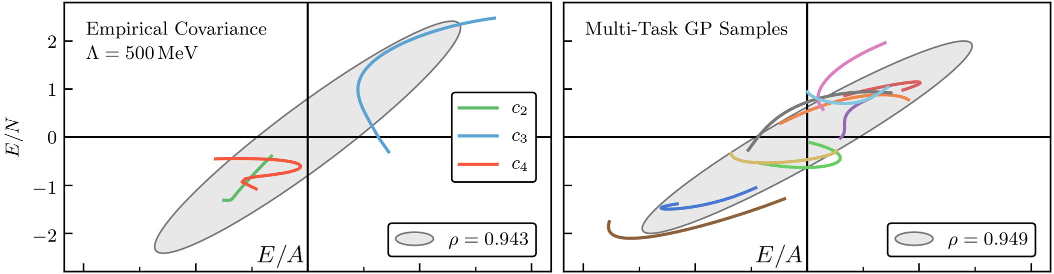
Correlated observables: estimating symmetry energy

- The convergence pattern of E/N is correlated with that of E/A.



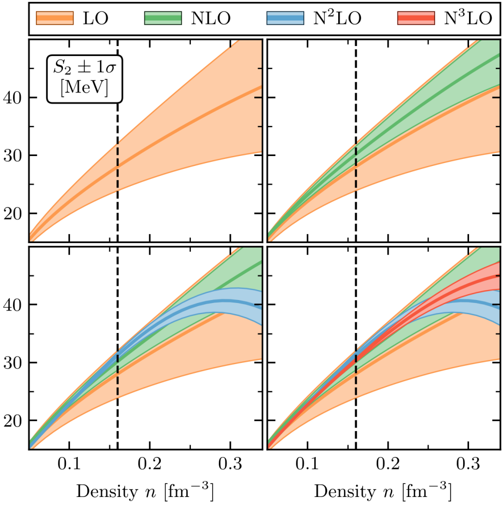
Correlated observables: estimating symmetry energy

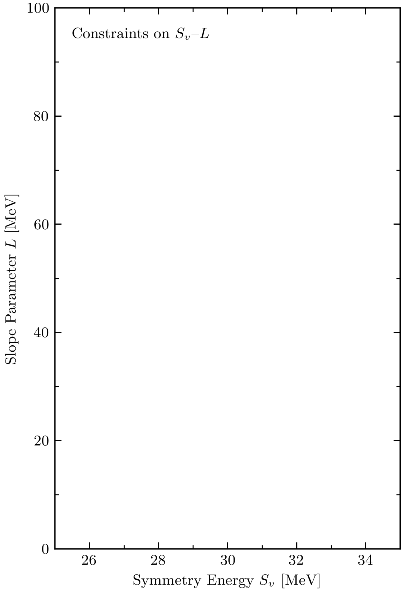
- The convergence pattern of E/N is correlated with that of E/A.
- Model with multi-task Gaussian process



Correlated observables: estimating symmetry energy

- The convergence pattern of E/N is correlated with that of E/A.
- Model with multi-task Gaussian process
- So the uncertainty in $S_2 = E/N - E/A$ is smaller than naively expected




Constraints on S_v–L
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Slope Parameter L [MeV]
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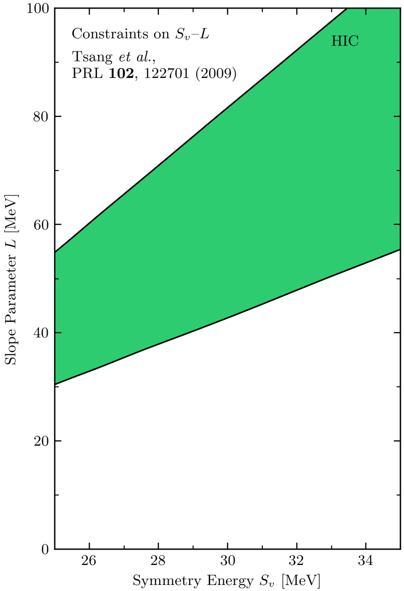
Symmetry Energy S_v [MeV]



Constraints on S_v–L
Tsang *et al.*,
PRL **102**, 122701 (2009)
HIC
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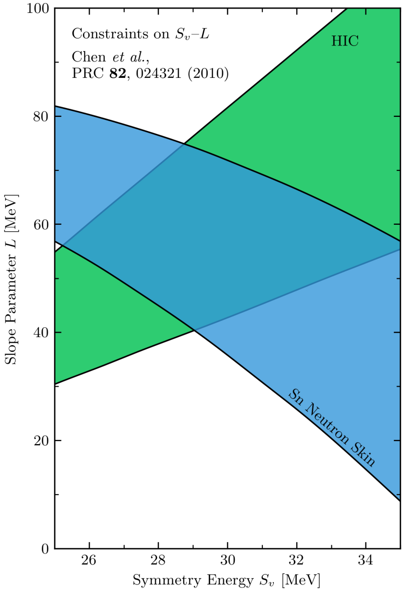
Constraints on S_v–L
Chen *et al.*,
PRC **82**, 024321 (2010)
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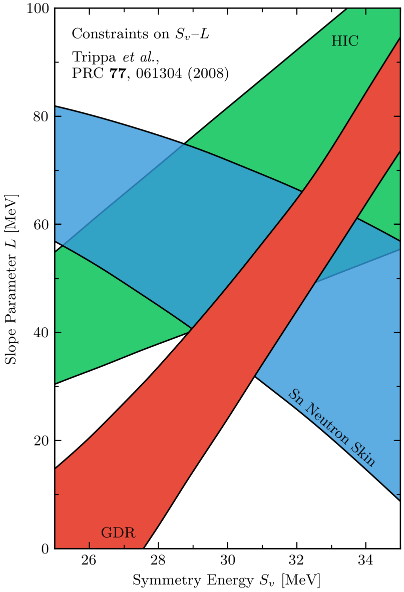
Constraints on S_v–L
Trippa *et al.*,
PRC **77**, 061304 (2008)
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Symmetry Energy S_v [MeV]
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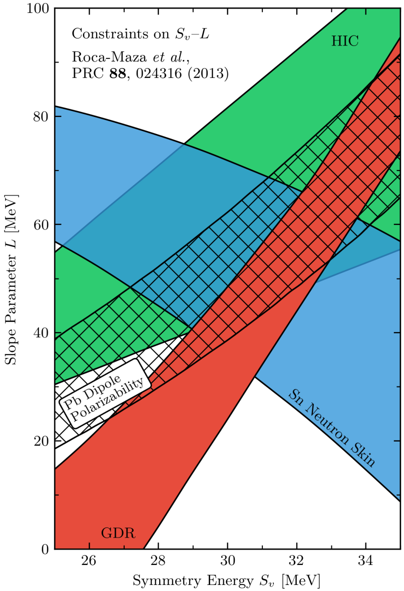
Constraints on S_v–L
Roca-Maza *et al.*,
PRC **88**, 024316 (2013)
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Symmetry Energy S_v [MeV]
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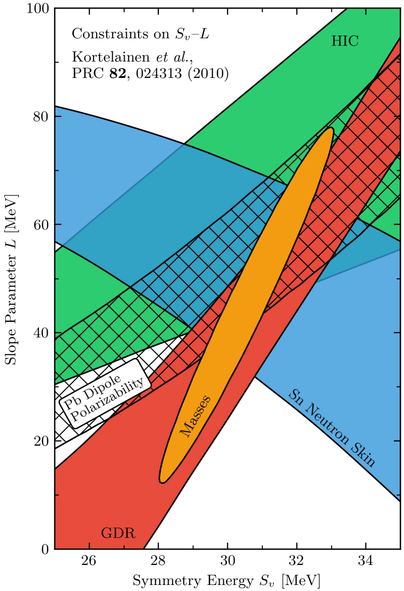
Constraints on S_v–L
Kortelainen *et al.*,
PRC **82**, 024313 (2010)
HIC
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Symmetry Energy S_v [MeV]
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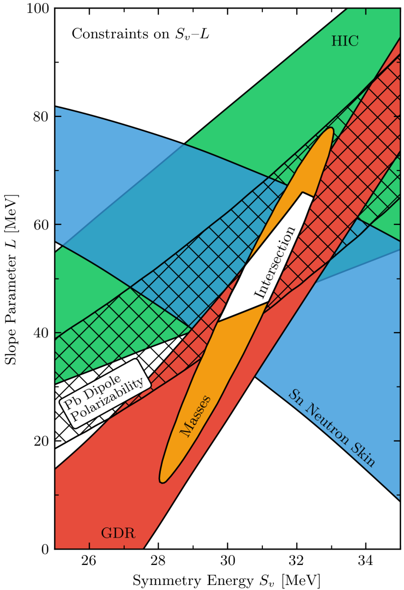

Constraints on S_v–L
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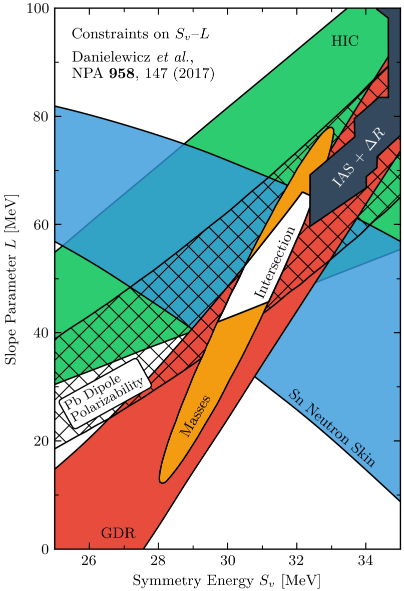

Constraints on S_v–L
Danielewicz *et al.*,
NPA **958**, 147 (2017)
HIC
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Slope Parameter L [MeV]
Symmetry Energy S_v [MeV]
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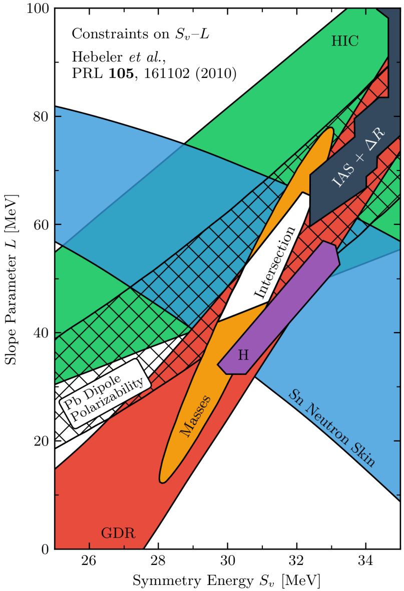
Constraints on S_v–L
Hebeler *et al.*,
PRL **105**, 161102 (2010)
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Sn Neutron Skin
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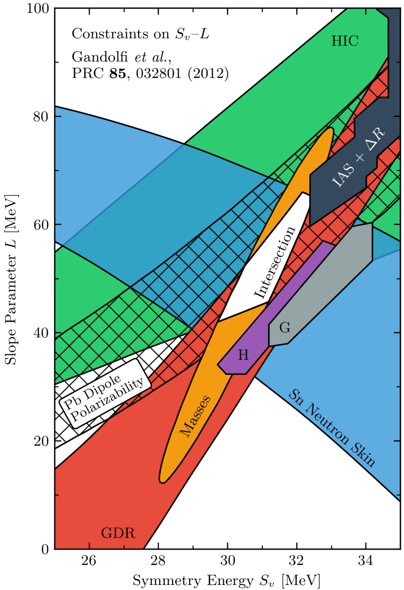

Constraints on S_v–L
Gandolfi *et al.*,
PRC **85**, 032801 (2012)
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Symmetry Energy S_v [MeV]
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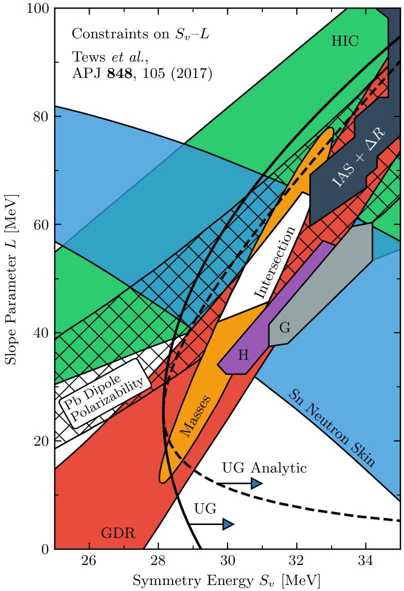
Constraints on S_v–L
Tews *et al.*, APJ **848**, 105 (2017)
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IAS + ΔR
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Slope Parameter L [MeV]
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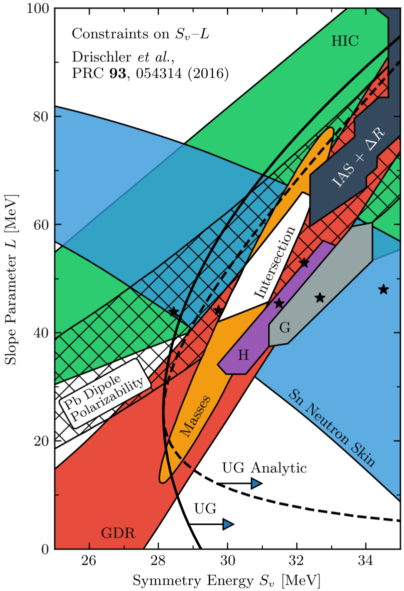
Constraints on S_v–L
Drischler *et al.*,
PRC **93**, 054314 (2016)
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Intersection
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Masses
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Sn Neutron Skin
UG Analytic
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Symmetry Energy S_v [MeV]
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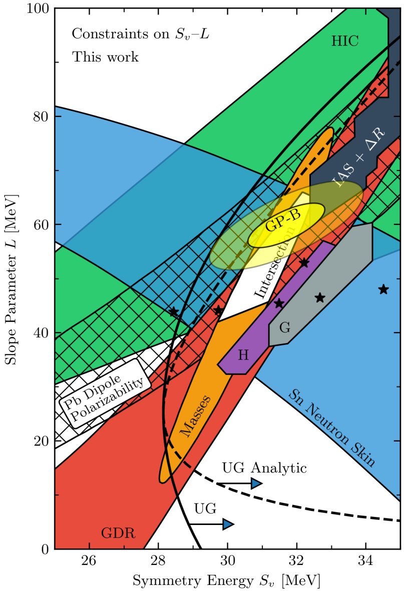

Constraints on S_v–L
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HIC
IAS + ΔR
GP-B
Intersection
Pb Dipole Polarizability
Masses
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Slope Parameter L [MeV]
Symmetry Energy S_v [MeV]
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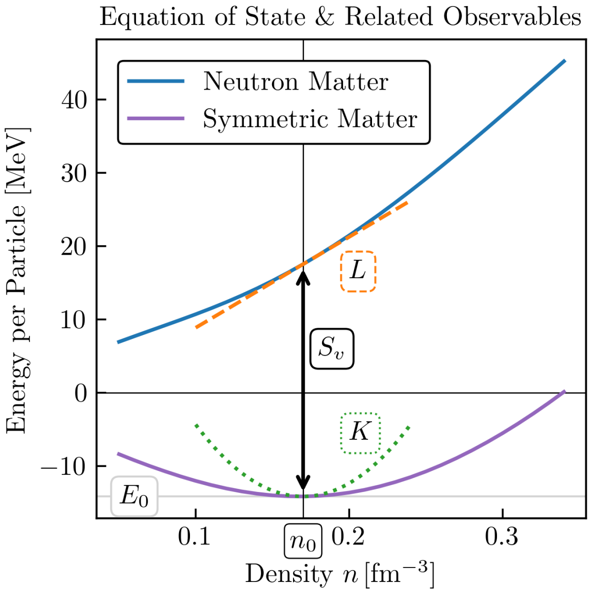
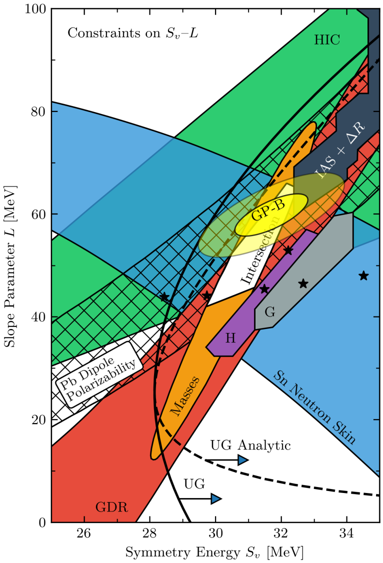
Uncertainty quantification: putting it all together



Key results



Constraints on S_v–L
HIC
IAS + ΔR
GP-B
Intersection
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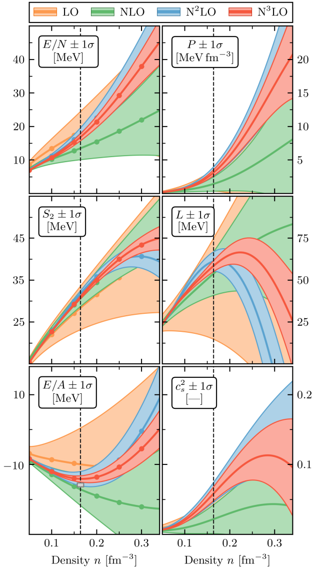
Slope Parameter L [MeV]
Symmetry Energy S_v [MeV]



LO
NLO
N^{2}LO
N^{3}LO
$E/N \pm 1\sigma$ [MeV]
$P \pm 1\sigma$ [MeV fm^{-3}]
$S_2 \pm 1\sigma$ [MeV]
$L \pm 1\sigma$ [MeV]
$E/A \pm 1\sigma$ [MeV]
$c_s^2 \pm 1\sigma$ [—]
Density n [fm^{-3}]

Takeaway points

Truncation Model

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Truncation Model

- Truncation and interpolation error informed by convergence pattern

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- Full error can be propagated, using physics insight

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- To progress, science must reason under uncertainty

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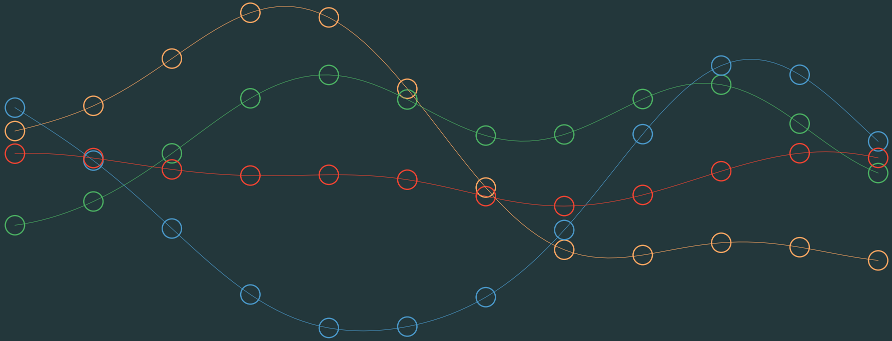
Implications

- To progress, science must reason under uncertainty
- This work is the first rigorous UQ accounting for nuclear matter
- All code is publicly available
- This promotes reproducibility and extendability

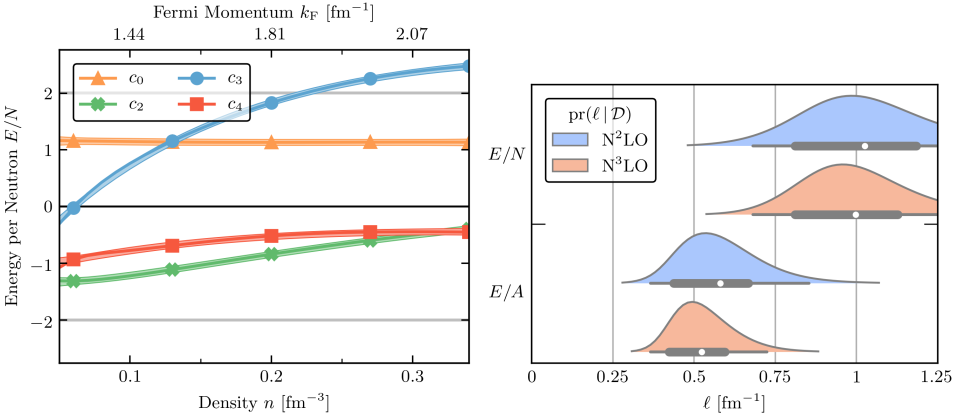
Thank you!

buqeye.github.io



Model Checking Diagnostics

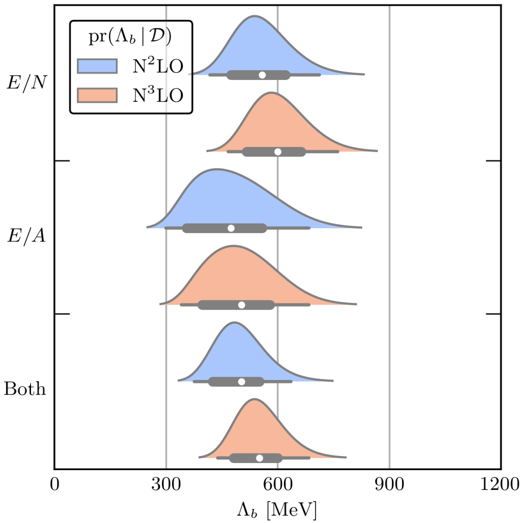
Observable coefficients



Breakdown scale estimation

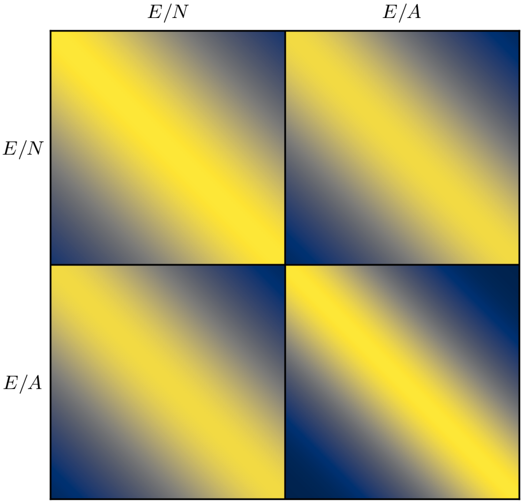
$$Q(k_F) = \frac{k_F}{\Lambda_b}$$

- k_F is the fermi momentum
- in 1-1 correspondence with the density n
- $\Lambda_b \approx 600$ MeV agrees with nucleon-nucleon scattering predictions



Multi-task correlations

- Each diagonal quadrant shows correlations within either E/N or E/A
- Diagonals are densities that are close together; off-diagonals are densities that are far apart
- Off-diagonal quadrants show correlations between E/N and E/A
- Yellow means highly correlated, blue means less correlated



Model checking diagnostics

Model checking diagnostics

Does our model refer to reality? How can we check?

Model checking diagnostics

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Assumptions

1. c_n are iid stationary GPs

Tests

1. Define a metric to measure GP-ness

Model checking diagnostics

Does our model refer to reality? How can we check?

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2. Error bands have statistical meaning

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2. Credible interval diagnostic

Model checking diagnostics

Does our model refer to reality? How can we check?

Assumptions

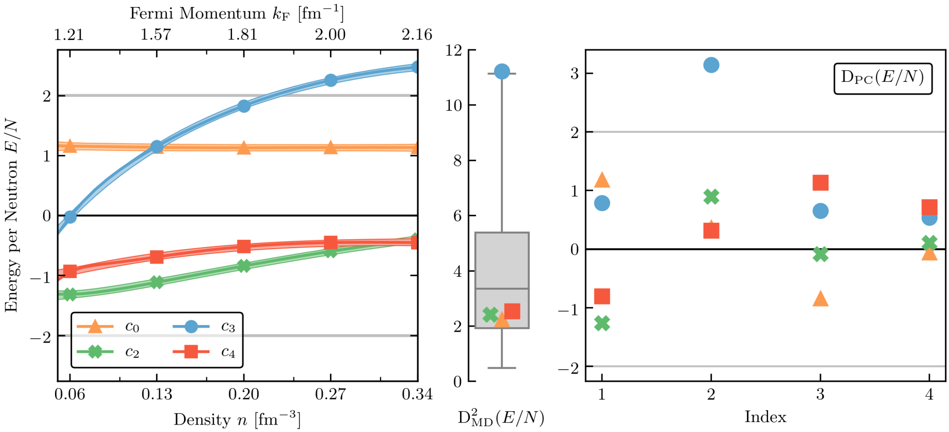
1. c_n are iid stationary GPs
2. Error bands have statistical meaning

Tests

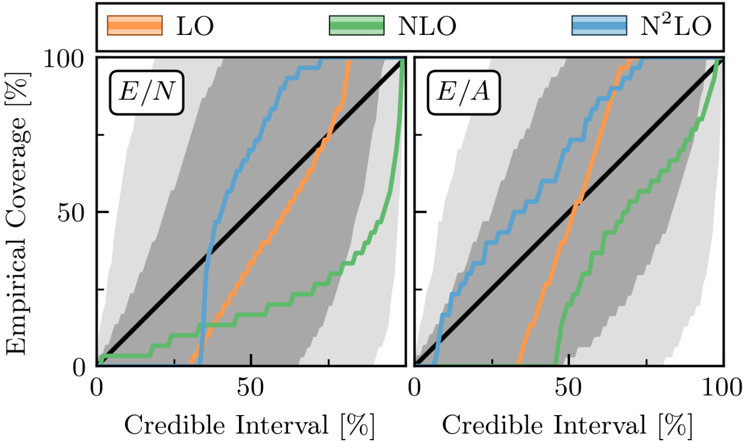
1. Define a metric to measure GP-ness
2. Credible interval diagnostic

- See Bastos & O'Hagan (2009) "Diagnostics for Gaussian Process Emulators"
- But we have multiple curves on which to test

Neutron matter diagnostics



Neutron matter diagnostics



PREX-2 [arxiv:2101.03193]

- PREx uses parity violating electron scattering to probe the size of neutrons inside a 208-Pb nucleus.
- The error bars are only 1σ
- At 2σ, it agrees with everything