QCD corrections to 5-point scattering with an external mass

Heribertus Bayu Hartanto

based on work with: Simon Badger, Rene Poncelet,
Andrei Popescu and Simone Zoia

arXiv:2102.02516, arXiv:2205.01687, arXiv:2209.03280

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Science & Technology Facilities Council

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Current and future LHC runs:

- increasing precision of experimental measurements
- experimental uncertainties comparable, if not, smaller than theoretical uncertainties

Perturbative QFT framework $\Rightarrow$ QCD corrections dominant

$$d\hat{\sigma} = \underbrace{d\hat{\sigma}^{(0)}}_{\text{LO}} + \underbrace{\alpha_s\, d\hat{\sigma}^{(1)}}_{\delta\text{NLO}} + \underbrace{\alpha_s^2\, d\hat{\sigma}^{(2)}}_{\delta\text{NNLO}} + \ldots$$

Theoretical error due to scale dependence: LO $> 50\%$, NLO $\sim 20 - 30\%$, NNLO $\sim 1 - 10\%$

Current precision frontier $\Rightarrow$ NNLO QCD computation for $2 \to 3$ processes

Dramatic progress for massless $2 \to 3$ scattering: $pp \to \gamma\gamma\gamma$, $pp \to \gamma\gamma j$, $pp \to jjj$ at NNLO QCD

✓ two-loop amplitudes (Feynman integrals, special function basis, finite-field technique, ...)
talks by Ryan Moodie, Giuseppe De Laurentis

✓ NNLO subtraction scheme talks by Kirill Melnikov, Rene Poncelet, Gloria Bertolotti

<u>This talk</u>: $2 \to 3$ process with an external mass

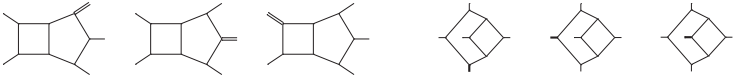
1. 5-Point 1-Mass Scattering Amplitudes
2. $Wb\bar{b}$ production at the LHC

5-Point 1-Mass Scattering Amplitudes

$2 \to 3$ scattering with an off-shell leg

$pp \to Hjj, Vjj, V\gamma j, V\gamma\gamma, Vb\bar{b}, Hb\bar{b}, \ldots$

- Increase in algebraic complexity and more intricate analytic structure compared to massless case
- All two-loop planar integrals and non-planar hexabox family are known



[Papadopoulos,Tommasini,Wever(2015)] [Papadopoulos,Wever(2019)] [Abreu,Ita,Moriello,Page,Tschernow,Zeng(2020)]
[Canko,Papadopoulos,Syrrakos(2020)] [Syrrakos(2020)][Abreu,Ita,Page,Tschernow(2021)] [Kardos,Papadopoulos,Smirnov,Syrrakos,Wever(2022)]

$\Rightarrow$ Planar special function basis[Badger,**HBH**,Zoia(2021)][Chicherin,Sotnikov,Zoia(2021)]

$\Rightarrow$ Fast numerical evaluation (PentagonFunctions++) [Chicherin,Sotnikov,Zoia(2021)]

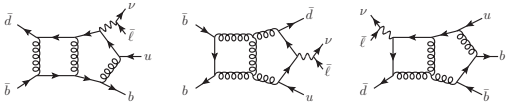
- A number of two-loop QCD amplitudes known analytically at leading colour
  - $u\bar{d} \to Wb\bar{b}$ (on-shell W, massless b) [Badger,**HBH**,Zoia(2021)]
  - $gg/q\bar{q} \to Hb\bar{b}$ (massless b) [Badger,**HBH**,Krys,Zoia(2021)]
  - $q\bar{q}' \to W^+(\to \bar{\ell}\nu)gg$ and $q\bar{q}' \to W^+(\to \bar{\ell}\nu)Q\bar{Q}$ [Abreu,Febres Cordero,Ita,Klinkert,Page(2021)]
  - $u\bar{d} \to W^+(\to \bar{\ell}\nu)\gamma g$ [Badger,**HBH**,Krys,Zoia(2022)] talk by Jakub Krys
  - $u\bar{d} \to W^+(\to \bar{\ell}\nu)b\bar{b}$ (massless b) [**HBH**,Poncelet,Popescu,Zoia(2022)] $\leftarrow$ this talk

$u\bar{d} \to W^+(\to \bar{\ell}\nu)b\bar{b}$ at two loops [HBH,Poncelet,Popescu,Zoia(2022)]

⇒ leading colour approximation and massless b quark

⇒ closely follow on-shell $u\bar{d} \to Wb\bar{b}$ calculation [Badger,HBH,Zoia(2021)]

⇒ work with $\sum \mathcal{M}^{(0)*}\mathcal{M}^{(2)}$ using CDR+Larin scheme



Incorporate $W \to \ell\nu$ decay:

$$A_6^{(L)} = A_5^{(L)\mu}\, D_\mu\, P(s_{56}), \qquad M_6^{(L)} = \sum_{\text{spin}} A_6^{(0)\dagger} A_6^{(L)} = M_5^{(L)\mu\nu}\, \mathcal{D}_{\mu\nu}\, |P(s_{56})|^2$$

$$M_5^{(L)\mu\nu} = \sum_{i=1}^{16} a_i^{(L)} v_i^{\mu\nu}, \qquad a_i^{(L)} = \sum_j \Delta_{ij}^{-1}\, \tilde{M}_{5,j}^{(L)}, \qquad \Delta_{ij} = v_{i\mu\nu} v_j^{\mu\nu}, \qquad v_i^{\mu\nu} \in \{p_1^\mu, p_2^\mu, p_3^\mu, p_W^\mu\}$$

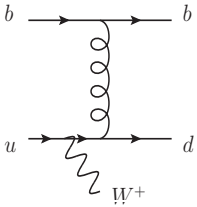
⇒ Derive analytic expressions using finite-field reconstruction methods talks by Ryan Moodie, Jakub Krys (QGRAF[Nogueira], FORM[Vermaseren], LiteRed[Lee], FiniteFlow[Peraro], Mathematica)

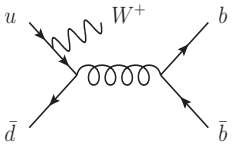
⇒ Evaluate master integrals (special functions) using PentagonFunctions++[Chicherin,Sotnikov,Zoia(2021)]

⇒ Cross check against $q\bar{q}' \to W^+(\to \bar{\ell}\nu)Q\bar{Q}$ amplitude [Abreu,Febres Cordero,Ita,Klinkert,Page(2021)]

$Wb\bar{b}$ production at the LHC

$W + b$ jets





$\Rightarrow$ testing perturbative QCD

$\Rightarrow$ modelling of flavoured jets

$\Rightarrow$ theoretical approach: 4FS vs 5FS

$W + 1b$ jet: probe b quark PDFs

$W + 2b$ jets: backgrounds for

- Higgs-strahlung $pp \to WH(H \to b\bar{b})$
- single top $pp \to bt(t \to bW)$

Measured at Tevatron[hep-ex/0410062][arXiv:1210.0627] and LHC[arXiv:1109.1470][arXiv:1302.2929][arXiv:1312.6608][arXiv:1608.07561]

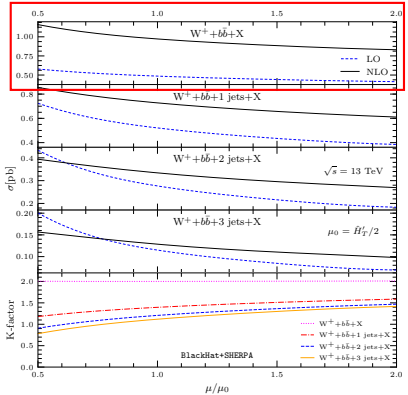
Theoretical predictions available at NLO:

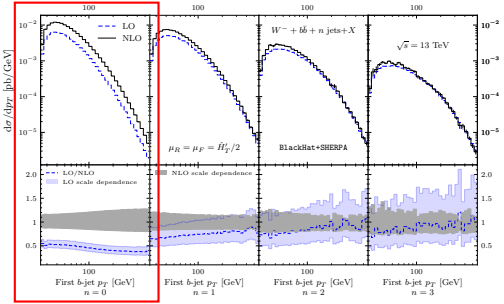
$W + 1b$: [Campbell,Ellis,Maltoni,Willenbrock(2006)][Campbell,Ellis,Febres Cordero,Maltoni,Reina,Wackeroth,Willenbrock(2008)] [Caola,Campbell,Febres Cordero,Reina,Wackeroth(2011)]

$W + 2b$: $m_b = 0$[Ellis,Veseli(1999)], onshell W[Febres Cordero,Reina,Wackeroth(2006,2009)], $W(\ell\nu)b\bar{b}$[Badger,Campbell,Ellis(2010)] NLO+PS[Oleari,Reina(2011)][Frederix etal(2011)], $W(\ell\nu)b\bar{b}j$ [Luisoni,Oleari,Tramontano(2015)] $W(\ell\nu)b\bar{b}+ \leq 3$j[Anger,Febres Corder,Ita,Sotnikov(2018]

$Wb\bar{b}$+jets production at NLO QCD

[Anger,Febres Cordero,Ita,Sotnikov(2017)]





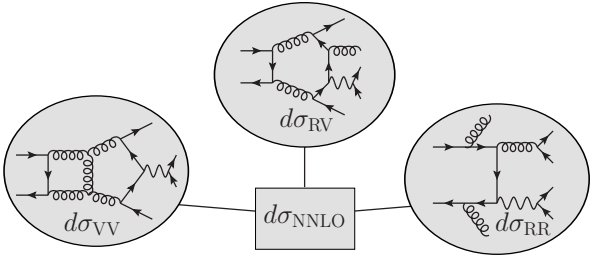
⇒ $n = 0$: large NLO corrections, large NLO scale dependence

⇒ opening of qg channel at NLO

Our work: compute NNLO QCD corrections to $Wb\bar{b}$ production

NNLO QCD corrections to $W(\to \ell\nu)b\bar{b}$ production

[**HBH**,Poncelet,Popescu,Zoia;arXiv:2205.01687,arXiv:2209.03280]



- Amplitudes:
  - Tree-level $pp \to W(\to \ell\nu)b\bar{b}jj$: AvH[Bury,van Hameren(2015)]
  - 1-loop $pp \to W(\to \ell\nu)b\bar{b}j$: OpenLoops[Bucionni,Lang,Lindert,Maierhoefer,Pozzorini,Zhang,Zoller(2018,2019)]
  - 2-loop $u\bar{d} \to W(\to \ell\nu)b\bar{b}$: [**HBH**,Poncelet,Popescu,Zoia(2022)]
- NNLO subtraction scheme: Sector Improved Residue Subtraction Scheme (Stripper) [Czakon(2010)][Czakon,Heymes(2014)]
- First NNLO QCD calculation for $2 \to 3$ process involving external mass

Setup (follows CMS measurement[arXiv:1608.07561])

⇒ **5FS**, LHC 8 TeV, PDFs: NNPDF31, cuts: $p_{T,\ell} > 30$ GeV, $|\eta_\ell| < 2.1$, $p_{T,j} > 25$ GeV, $|\eta_j| < 2.4$

⇒ jet algorithm: flavour-k_T[Banfi,Salam,Zanderighi(2006)] and flavour anti-k_T[Czakon,Mitov,Poncelet(2022)] with $R = 0.5$

⇒ central scale: $\mu_R = \mu_F = \mu_0$, where $H_T = E_T(\ell\nu) + p_T(b_1) + p_T(b_2)$.

⇒ final states: **inclusive** (at least 2 b jets) and **exclusive** (exactly two b jets and no other jets).

⇒ scale uncertainties: inclusive → 7-pt variation $1/2 \leq \mu_R/\mu_F \leq 2$

exclusive → 7-pt variation and uncorrelated prescription[Stewart,Tackmann(2012)]

Uncorrelated scale variation $\sigma_{Wb\bar{b},\mathrm{exc}} = \sigma_{Wb\bar{b},\mathrm{inc}} - \sigma_{Wbbj,inc}$ $\Delta\sigma_{Wb\bar{b},\mathrm{exc}} = \sqrt{\left(\Delta\sigma_{Wb\bar{b},\mathrm{inc}}\right)^2 + \left(\Delta\sigma_{Wbbj,inc}\right)^2}$

Leading colour approximation is only applied to *scale independent* double virtual finite remainder

$$\mathcal{V}^{(2)}(\mu_R^2) = \mathcal{V}^{(2)}_{\mathrm{LC}}(s_{12}) + \sum_{i=1}^{4} c_i \ln^i\left(\frac{\mu_R^2}{s_{12}}\right)$$

Study tunable *softness* parameter in the flavour anti-k_T algorithm ($a = 0.2, 0.1, 0.05$):

$$d_{ij}^{(F)} \equiv d_{ij} \times \begin{cases} \mathcal{S}_{ij}\,, & fl(i) = -fl(j) \neq 0, \\ 1\,, & \text{otherwise.} \end{cases} \qquad \mathcal{S}_{ij} = 1 - \Theta(1-x)\cdot\cos\left(\frac{\pi}{2}x\right), x \equiv \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}$$

Numerical results: cross sections

$$K_{\rm NLO} = \frac{\sigma_{\rm NLO}}{\sigma_{\rm LO}} \qquad K_{\rm NNLO} = \frac{\sigma_{\rm NNLO}}{\sigma_{\rm NLO}}$$

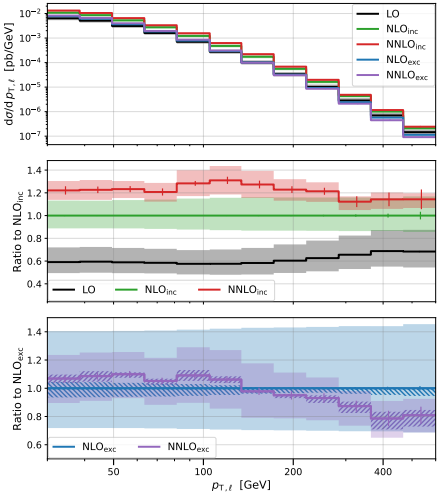
Inclusive $W^+(\to \ell^+\nu)b\bar{b}$ cross sections

Jet algorithm	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}$ [fb]	$K_{\rm NLO}$	$\sigma_{\rm NNLO}$ [fb]	$K_{\rm NNLO}$
flavour-k_T	$213.24(8)^{+21.4\%}_{-16.1\%}$	$362.0(6)^{+13.7\%}_{-11.4\%}$	1.70	$445(5)^{+6.7\%}_{-7.0\%}$	1.23
flavour anti-k_T ($a = 0.05$)	$262.52(10)^{+21.4\%}_{-16.1\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	1.91	$690(7)^{+10.9\%}_{-9.7\%}$	1.38
flavour anti-k_T ($a = 0.1$)	$262.47(10)^{+21.4\%}_{-16.1\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	1.90	$677(7)^{+10.4\%}_{-9.4\%}$	1.36
flavour anti-k_T ($a = 0.2$)	$261.71(10)^{+21.4\%}_{-16.1\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$	1.86	$647(7)^{+9.5\%}_{-8.9\%}$	1.33

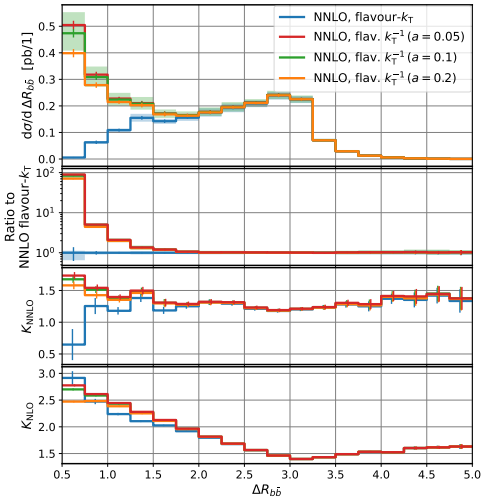
Exclusive $W^\pm(\to \ell^\pm\nu)b\bar{b}$ cross sections

Jet algorithm	$\sigma_{\rm LO}$ [fb]	$\sigma_{\rm NLO}$ [fb]	$K_{\rm NLO}$	$\sigma_{\rm NNLO}$ [fb]	$K_{\rm NNLO}$
flavour-k_T	$345.97(9)^{+21.4\%}_{-16.2\%}$	$408.4(5)^{+4.2\%\ (41\%)}_{-6.2\%\ (28\%)}$	1.18	$434(8)^{+1.7\%\ (16\%)}_{-2.5\%\ (16\%)}$	1.06
flavour anti-k_T ($a = 0.05$)	$425.71(12)^{+21.5\%}_{-16.2\%}$	$540.3(7)^{+6.2\%\ (42\%)}_{-7.4\%\ (29\%)}$	1.27	$636(11)^{+5.4\%\ (23\%)}_{-5.0\%\ (20\%)}$	1.18
flavour anti-k_T ($a = 0.1$)	$425.63(12)^{+21.5\%}_{-16.2\%}$	$538.7(7)^{+6.1\%\ (42\%)}_{-7.4\%\ (29\%)}$	1.27	$630(10)^{+5.0\%\ (22\%)}_{-4.8\%\ (20\%)}$	1.17
flavour anti-k_T ($a = 0.2$)	$424.37(12)^{+21.5\%}_{-16.2\%}$	$530.6(7)^{+5.8\%\ (42\%)}_{-7.2\%\ (29\%)}$	1.25	$606(10)^{+4.2\%\ (21\%)}_{-4.2\%\ (19\%)}$	1.14

Numerical results: differential distributions



$W^+(\to \ell^+\nu)b\bar{b}$ with flavour-k_T



inclusive $W^+(\to \ell^+\nu)b\bar{b}$

NNLO vs NLO-merged calculation

$$K_{\mathrm{NNLO}} = \frac{\sigma_{\mathrm{NNLO}}}{\sigma_{\mathrm{NLO}}} \qquad K_{\mathrm{NLO+}} = \frac{\sigma_{\mathrm{NLO+}}}{\sigma_{\mathrm{NLO}}}$$

Inclusive $W^+(\to \ell^+\nu)b\bar{b}$ cross sections

Jet algorithm	σ_{NLO} [fb]	σ_{NNLO} [fb]	K_{NNLO}	$\sigma_{\mathrm{NLO+}}$ [fb]	$K_{\mathrm{NLO+}}$
flavour-k_T	$362.0(6)^{+13.7\%}_{-11.4\%}$	$445(5)^{+6.7\%}_{-7.0\%}$	1.23	$426(5)^{+7.6\%}_{-8.9\%}$	1.18
flavour anti-k_T ($a = 0.05$)	$500.9(8)^{+16.1\%}_{-12.8\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	1.38	$635(6)^{+11.2\%}_{-11.1\%}$	1.27
flavour anti-k_T ($a = 0.1$)	$497.8(8)^{+16.0\%}_{-12.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	1.36	$626(6)^{+10.8\%}_{-10.9\%}$	1.26
flavour anti-k_T ($a = 0.2$)	$486.3(8)^{+15.5\%}_{-12.5\%}$	$647(7)^{+9.5\%}_{-8.9\%}$	1.33	$602(6)^{+10.2\%}_{-10.5\%}$	1.24

Large corrections and scale dep at NLO

$\Rightarrow$ $qg(\bar{q}g) \to Wb\bar{b}q(\bar{q})$ subprocess

$\Rightarrow$ upgrade $pp \to Wb\bar{b}j$ to NLO

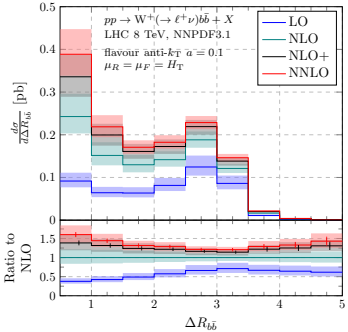
[Luisoni,Oleari,Tramontano(2015)]

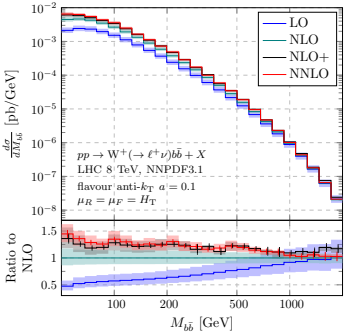
Combine $Wb\bar{b}$ and $Wb\bar{b}j$ NLO cross sections using *exclusive sums* method

[The SM and NLO Multileg and SM MC Working Groups: Summary Report(2012)]

[Anger,Febres Cordero,Ita,Sotnikov(2017)]

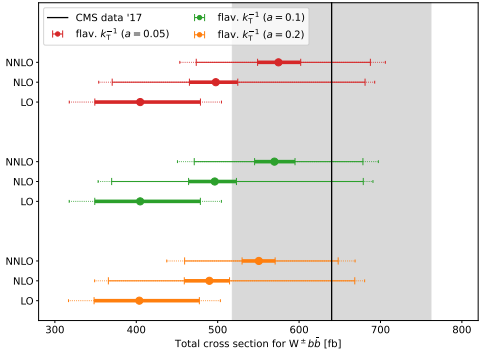
$$\sigma_{\mathrm{NLO+},Wb\bar{b},\mathrm{inc}} = \sigma_{\mathrm{NLO},Wb\bar{b},\mathrm{exc}} + \sigma_{\mathrm{NLO},Wb\bar{b}j,\mathrm{inc}}$$





Comparison to CMS data [arXiv:1608.07561]

- CMS measurement is done for exclusive final state configuration
- The use of flavour anti-k_T algorithm in theoretical calculation allows for close comparison with CMS data (standard anti-k_T)
- Correction factors included:
  hadronisation (multiplicative) $= 0.81 \pm 0.07$
  DPI (additive) $= 0.06 \pm 0.06$ pb



[**HBH**,Poncelet,Popescu,Zoia;arXiv:2209.03280]

- **Thick** bands: 7-pt scale variation (without DPI error)
  thin bands: uncorrelated prescription (without DPI error),
  dotted bands: uncorrelated prescription (with DPI error)

Conclusions

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- ✓ Progress on master integral and scattering amplitude computations for 2-loop 5-point scattering process with an external mass
- ✓ NNLO QCD predictions for $W(\to \ell\nu)b\bar{b}$ production at the LHC
  $\Rightarrow$ improved perturbative convergence, better agreement with data
- ✗ Non-planar contributions to the two-loop amplitude
- ✗ $W + 1b$ jet, $Z + b\bar{b}$,$W/Z + 2$ jets, $W\gamma j$ at NNLO QCD
- ✗ Processes with more scale(s)

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THANK YOU!!!

Back-up Slides

Flavoured jet algorithm study

Massless b beyond NLO: IR unsafe with standard jet algorithm due to soft wide angle $q\bar{q}$ pair

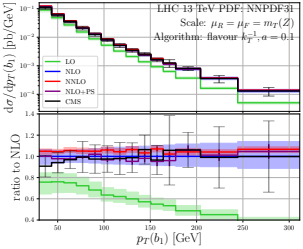
Flavour k_T [Banfi,Salam,Zanderighi(2006)] jet algorithm widely used in theoretical calculations

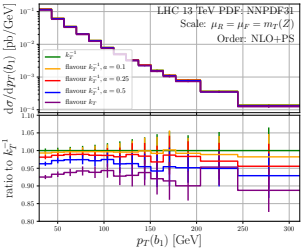
Experimental measurements use anti-k_T jet algorithm
⇒ unfold data to match theory definition by comparing NLO+PS employing both jet definitions
[Gauld,Gehrmann-De Ridder,Glover,Huss,Majer(2020)]

Flavour anti-k_T [Czakon,Mitov,Poncelet(2022)]
⇒ minimize the effect of unfolding
⇒ introduce damping function to the standard anti-k_T

$$\mathcal{S}_{ij} = 1 - \Theta(1-x)\cdot\cos\left(\frac{\pi}{2}x\right) \qquad x \equiv \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}$$

if i, j have the same non-zero flavour of opposite sign





$pp \to Z + b$ [Czakon,Mitov,Poncelet;arXiv:2205.11879]

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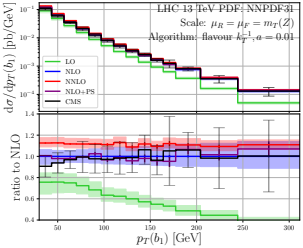

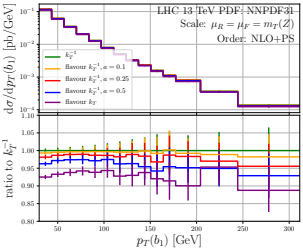

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