

# Two-loop mixed QCD-electroweak amplitudes for Z+jet production

with Caola, Chawdhry, Liu

Piotr Bargiełā

University of Oxford

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# Presentation plan

## 1 Motivation

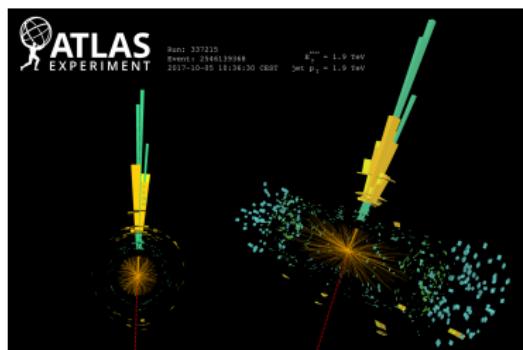
## 2 Computation

## 3 Results

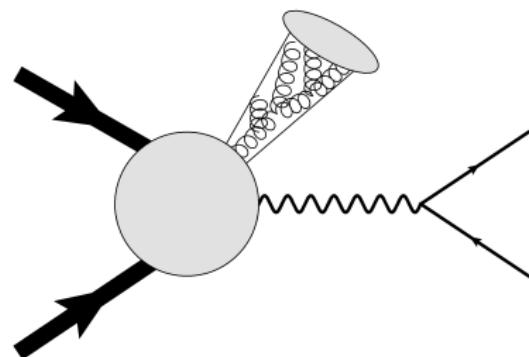
# Motivation

## Dark Matter searches at the LHC

experiment  
monojet + **missing** transverse energy



theory  
 $p p \rightarrow \text{jet} + V (\rightarrow \nu \bar{\nu} / \chi \bar{\chi})$



# Motivation

- precision measurement of the BSM signal  $p p \rightarrow \text{jet} + V (\rightarrow \chi\bar{\chi})$  requires control over the SM background  $p p \rightarrow \text{jet} + V (\rightarrow \nu\bar{\nu})$
  - $V + \text{jet}$  channels in decreasing contribution order (overall  $\sigma$  drop  $\sim \mathcal{O}(10^1)$ ) :  
 $\gamma, W \rightarrow l\nu, Z \rightarrow \nu\bar{\nu}, Z \rightarrow l\bar{l}$
  - statistical uncertainty : few % for  $p_T < 2$  TeV and 10 % above 2.5 TeV
- perturbative corrections :

$$\sigma = \sigma^0 \left( 1 + \alpha_s \delta^{(1,0)} + \alpha_s^2 \delta^{(2,0)} + \alpha \delta^{(0,1)} + \alpha_s \alpha \delta^{(\text{QCD-EWK})} + \mathcal{O}(\alpha_s^2, \alpha^2) \right)$$

$$\delta^{(QCD-EWK)} = \delta_{QCD} \times \delta_{EWK} + \delta_{NF}$$

- factorizable :  $\alpha \sim 1\%$  but Sudakov  $\log$  **enhancement**  $\frac{\alpha}{4\pi s_w^2} \log^2(\frac{s}{m_Z^2}) \sim 10\%$
- nonfactorizable** :  $\delta_{NF} \gtrsim \delta_{Z+2\text{jets}}^{(0,1)} - \delta_{Z+1\text{jet}}^{(0,1)}$  [[Lindert et al. arXiv:1705.04664](#)]

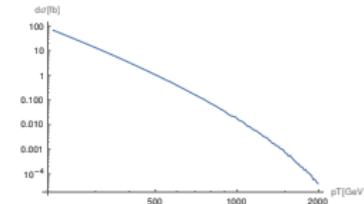
QCD-EWK dominates at high  $p_T \Rightarrow$  expect  $\delta^{(QCD-EWK)} \sim \mathcal{O}(\delta^{(2,0)}) \sim$  few %  
on the importance of QCD-EWK corrections see also [[talk by Chawdhry, Devoto](#)]

# Introduction

- for definiteness, consider the neutral  $Z$  current :

$$pp \rightarrow Z(\overset{\Gamma}{\rightarrow} l\bar{l}) + \text{jet}$$

- on-shell  $Z$  : enough to describe the decay at high  $p_T$
- high  $p_T$  range : (200, 2000) GeV at  $\sqrt{s_H} = 13$  TeV
- number of events :  $(10^7, 10^1)$  at  $\mathcal{L} = 300 \text{ fb}^{-1}$



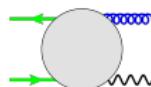
timeline of lower orders

- ↓
- ⌚ NLO QCD [[Giele et al. arXiv:9302225](#)]
  - ⌚ NLO EWK [[Denner et al. arXiv:1103.0914](#)]
  - ⌚ NNLO QCD [[Gehrmann-De Ridder arXiv:1507.02850](#)]

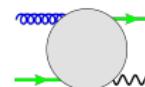
challenge = **amplitude** + subtraction

4 partonic amplitude channels :

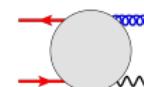
$$u\bar{u} \rightarrow g Z$$



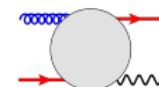
$$u g \rightarrow u Z$$



$$d\bar{d} \rightarrow g Z$$



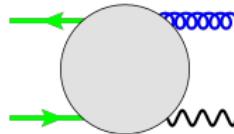
$$d g \rightarrow d Z$$



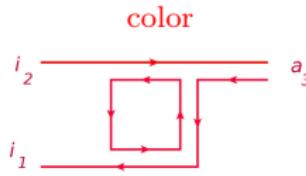
# Amplitude structure

for definiteness, consider

$$u(p_1) + \bar{u}(p_2) \rightarrow g(-p_3) + Z(-p_4)$$

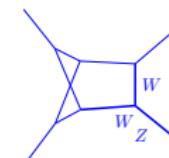


$$A = g_s^2 e^2 g_{L/R} T_{i_1 i_2}^{a_3} \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{\bar{u}(p_2)(k_1 \not{p}_3 \not{p}_3 \not{p}_4 k_2 + \dots) u(p_1)}{\mathcal{D}_1 \dots \mathcal{D}_7}$$

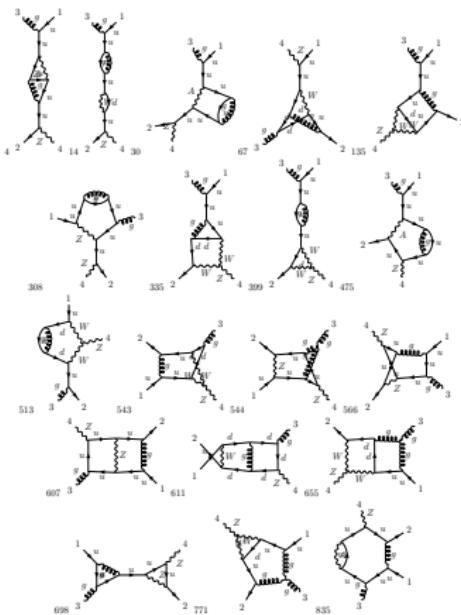


tensors

integrals



# Complexity


 $u\bar{u} \rightarrow gZ$ 

	0L	1L QCD	1L EWK	2L
# diagrams	2	13	35	900
# families	0	1	4	18
# integrals	0	105	275	60968
# Master Integrals	0	7	26	1269

complexity summary

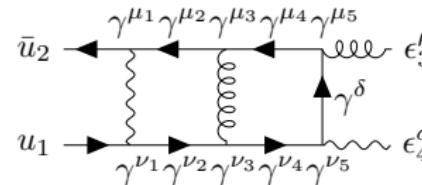
$\times 47$  pages

# Tensors in $d=4-2\epsilon$ dimensions

for further steps, scalar integrals required

$$A = g_s^2 e^2 g_{L/R} T_{i_1 i_2}^{a_3} \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{\bar{u}(p_2)(k_1^\mu p_3^\nu k_2^\rho + \dots) u(p_1)}{\mathcal{D}_1 \dots \mathcal{D}_7} = \sum_{i=1}^? \mathcal{F}_i T_i$$

for example, consider vector current



$\Sigma(\text{diagrams}) \# \text{Lorentz indices} > \# \text{all invariant structures}$

$$\begin{aligned} \#T_i &= 17 \text{ (Lorentz invariant tensors)} - 4 \text{ (by transversality } \epsilon_3 \cdot p_3 = 0) \\ &\quad - 6 \text{ (by gauge fixing } \epsilon_i \cdot p_{i-1} = 0) \end{aligned}$$

$= 7$  (independent in  $d$  dimensions)

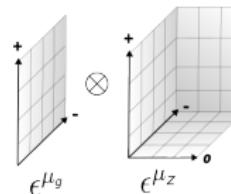
$= ?$  (independent in 4 dimensions)

$$\begin{aligned} T_i = \epsilon_{3,\mu}(p_3) \epsilon_{4,\nu}(p_4) \bar{u}(p_2) \quad &(p_1^\nu \gamma^\mu, p_1^\mu p_1^\nu p_3^\rho, \\ &p_2^\nu \gamma^\mu, p_1^\mu \gamma^\nu, \\ &p_1^\mu p_2^\nu p_3^\rho, g^{\mu\nu} p_3^\rho, \gamma^\mu p_3^\rho \gamma^\nu) \quad u(p_1) \end{aligned}$$

## Tensors in 4 dimensions

recent loop-universal **claim** in the 'tHV scheme [Peraro, Tancredi [arXiv:2012.00820](#)] :  
 # tensors indpt in 4-dim = # indpt helicity states (here =  $3 \times 2^2/2 = 6$ )

$$A = \sum_{i=1}^7 \mathcal{F}_i T_i = \sum_{i=1}^6 \overline{\mathcal{F}}_i \overline{T}_i$$



orthogonalization : projects out  $\overline{T}_7$  from the **physical** 4-dim subspace

$$\sum_{pol} \overline{T}_i^\dagger \overline{T}_j = \left( \begin{array}{c|c} 6 \times 6 \text{ (4-dim)} & 0 \\ \hline 0 & 1 \times 1 \text{ (-2epsilon-dim)} \end{array} \right)$$

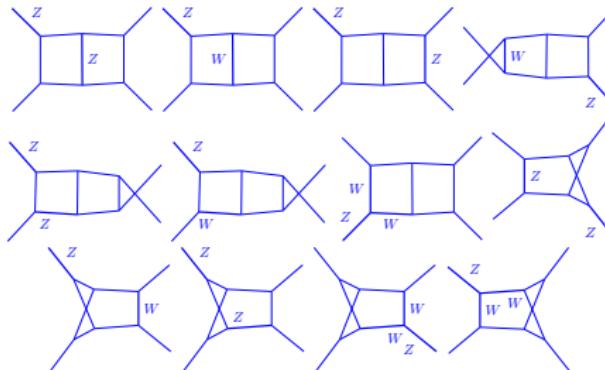
gain : 1-1 **correspondence** between form factors and helicity amplitudes

$$\overline{\mathcal{F}}_i \iff A_{\vec{\lambda}_i}$$

⇒ unphysical information removed

## Feynman integrals

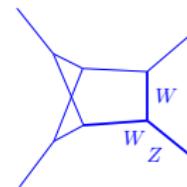
$$A = \sum_{f \in \text{fam}} \sum_{\vec{n} \in \text{int}} \int \frac{d^d k_1}{(2\pi)^d} \frac{d^d k_2}{(2\pi)^d} \frac{c_{f,\vec{n}}(d, m_k, s_{ij})}{\mathcal{D}_{f,1}^{n_1} \dots \mathcal{D}_{f,9}^{n_9}}$$

example integral **families**

- multiple **scales** :  $\{s_{23}, s_{13}, m_Z, m_W, (m_t, m_H)\}$
- usual approach : Integration By Parts reduction ( $6 \times 10^4 \rightarrow 1 \times 10^3$ )

## IBP ineffective

- reduction with **kira-2.2**
- most involved topology : 2-loop non-planar with the  $W^+W^-Z$  vertex
- number of integrals to reduce : 1181 ,  $(ISP)^4$
- number of master integrals : 95



amplitude<sub>NPL,WWZ</sub> = integrand (5.4MB) /. IBPs (640MB) = simpler ?  
physical pole motivation  $\Rightarrow$  **partial fraction** coefficients of Master Integrals

- algebraic geometry  $\Rightarrow$  **Groebner basis**
- number of denominator factors  $\mathcal{P}(d, m_k, s_{ij})$  : 131
- **Singular** ineffective



either choose better Master basis [[Bonetti et al. arXiv:2203.17202](#)]

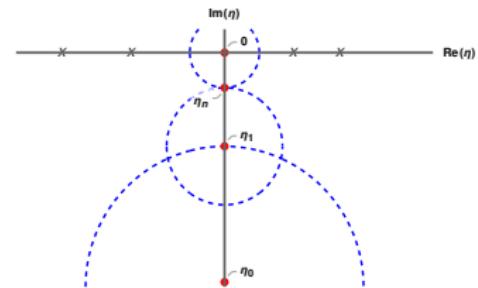
or evaluate integrals numerically without IBP

our strategy : **numerical** because  $2 \rightarrow 2$  process easy to grid for phenomenology

# Auxiliary Mass Flow method

numerical evaluation

- auxiliary mass :  $\frac{1}{\mathcal{D}_k + i0^+} \rightarrow \frac{1}{\mathcal{D}_k - \eta}$   
[\[Xiao Liu et al. arXiv:1711.09572\]](#)
- differential equations :  $\frac{\partial}{\partial \eta} \mathcal{I}(\eta) = A(\eta) \mathcal{I}(\eta)$   
 easy to solve
- boundary conditions at  $\eta = \infty$  :  
 expansion by regions



$$\frac{1}{((l+p)^2 - m^2 - \eta)^\nu} \simeq \frac{1}{(l^2 - \eta)^\nu} \sum_{i=0}^N \frac{\Gamma(\nu + i)}{i! \Gamma(\nu)} \left( -\frac{2l.p + p^2 - m^2}{l^2 - \eta} \right)^i$$

- iterative strategy : reduction to **vacuum** bubbles
- analytic continuation** : path  $\{i\infty, \eta_0, \eta_1, \dots, \eta_n, 0\}$

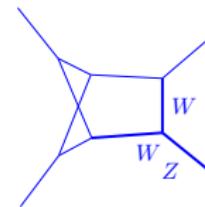


full control on precision with  $N$  and  $n$

# AMFlow example

- implemented in **AMFlow.m**
- analytic IBP reduction at fixed mass values vastly reduces evaluation time
- example evaluation with at least 27 digits **precision** (at  $\frac{m_W^2}{m_Z^2} = \frac{7}{9}$ )

$$\begin{aligned}
 & \frac{0.011955742709286810865609312312731412704773730415683}{\epsilon^4} \\
 & - \frac{0.51400897331764447006253493086301699478672740697984 - 0.07512062155069217213862554549556338060978694623307i}{\epsilon^3} \\
 & - \frac{11.1869669024863939355248164495030395570523614569652 - 3.2296095711745512165046978207303473280648508875869i}{\epsilon^2} \\
 & - \frac{163.88156956771300428552763557770868752429557895669 - 71.27829180648333783257397027808244347742401990587i}{\epsilon} \\
 & + (-1813.1703568796044594193394262418828425021433496769 + 1072.1983766480299427459157788856947923510195402455i) =
 \end{aligned}$$



**advantage** : evaluate numerically the whole amplitude  
 $\Rightarrow$  no large cancellations in a family

$$A = \sum_{f \in \text{fam}} \#_f$$

# UV and IR structure

$$A_b^{(2)} = \frac{c_{4,\text{IR}}}{\epsilon^4} + \frac{c_{3,\text{IR}}}{\epsilon^3} + \frac{c_{2,\text{IR}} + c_{2,\text{UV}}}{\epsilon^2} + \frac{c_{1,\text{IR}} + c_{1,\text{UV}}}{\epsilon^1} + \mathbf{fin}$$

extract physical finite part and check universal pole structure

**UV**

QCD :  $\overline{\text{MS}}$  scheme

$$A^{(1,QCD)} = (1 + g_s^2 \beta_0) A_b^{(1,QCD)}$$

EWK : on-shell  $G_\mu$  scheme [[Denner arXiv:0709.1075](#)]

$$A^{(1,EWK)} = (1 + e^2 \delta_Z^{(1)}) A_b^{(1,EWK)}$$

mixed :

$$A = (1 + g_s^2 \beta_0 + e^2 \delta_Z^{(1)} + e^2 g_s^2 \delta_Z^{(2)}) A_b$$

**IR**

$$A^{(2)} = \mathcal{I}_2 A^{(0)} + \mathcal{I}_{1,QCD} A^{(1,EWK,fin)} + \mathcal{I}_{1,EWK} A^{(1,QCD,fin)} + A^{(2,fin)}$$

[[Buccioni et al. arXiv:2203.11237](#)]

## Preliminary results

- in our framework, EWK treated in the same way as QED  
⇒ the simplest consistency check is the mixed QCD-QED correction
- input parameters :  $E = 500, \cos \theta = 0.123, \mu^2 = s$   
 $m_Z = 91.1876, m_W = 80.377, m_t = 172.69, m_H = 125.25$  [[Workman et al. PDG](#)]
- numerical value :

$$\begin{aligned} A^{(2,\text{fin})} = & e^2 g_s^2 g_{L/R} Q_{up}^2 T_{i,\bar{i}}^{ag} \bar{u}(p_2) p_1 \cdot \epsilon_4 \not{\epsilon}_3 u(p_1) \\ & \left( \frac{-5.24754 \cdot 10^{-17}}{\epsilon^4} \right. \\ & + \frac{-8.96438 \cdot 10^{-16} - 4.260140 \cdot 10^{-16} i}{\epsilon^3} \\ & + \frac{-8.80956 \cdot 10^{-15} - 7.52783 \cdot 10^{-15} i}{\epsilon^2} \\ & + \frac{-4.76398 \cdot 10^{-14} - 7.3129 \cdot 10^{-14} i}{\epsilon^1} \\ & \left. -0.00578014 - 0.00117727i \right) \end{aligned}$$

stay tuned for the full QCD-EWK

# Outlook

- discussed methods generalizable for the **top** loop
- careful treatment of anomalies
- **hadronic cross section**

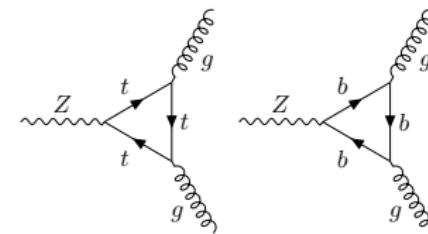
THANK YOU

## The top loop

- closed chiral fermion loops
- new tensors  $\bar{T}_i$  with  $\gamma_\mu \gamma_5$
- Larin's prescription

[[Larin arXiv:9302240](#)]

$$\gamma_\mu \gamma_5 = \frac{i}{3!} \epsilon_{\mu\mu_1\mu_2\mu_3} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3}$$



- massive top quark

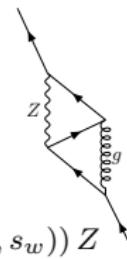
⇒ "anomaly" noncancellation :

$$\sim f(m_t) \mathcal{O}(\epsilon^0)$$

Details of the  $G_\mu$  scheme

- input parameters :  $\{G_\mu, m_Z, m_W\}$
- $\alpha(0)(1 + \Delta r) = \frac{\sqrt{2}G_\mu m_W^2}{\pi} \left(1 - \frac{m_W^2}{m_Z^2}\right)$
- amplitude renormalization

$$A(g_{s,0}, g_{L/R,0}(e_0, c_{w,0}, s_{w,0})) \sqrt{Z_u Z_{\bar{u}} Z_g Z_Z} = A(g_s, g_{L/R}(e, c_w, s_w)) Z$$



$$Z = 1 + \delta_Z = 1 + \frac{1}{2}(\delta_{Z_g} + \delta_{Z_u} + \delta_{Z_{\bar{u}}} + \delta_{Z_{ZZ}} - \frac{Q_f}{g_{L/R}} \delta_{Z_{AZ}}) + \delta_{g_{L/R}} + \delta_{g_s}$$

all SM particles contribute  $\Rightarrow$  much more involved than  $\overline{\text{MS}}$

# Catani operator

$$\frac{2\Gamma(1-\epsilon)}{e^{\gamma_E \epsilon}} \mathcal{I}_{1,QCD}(\epsilon) = \left(-\frac{\mu^2}{s}\right)^\epsilon (C_A - 2C_F) \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right) - \left(\left(-\frac{\mu^2}{u}\right)^\epsilon + \left(-\frac{\mu^2}{t}\right)^\epsilon\right) \left(C_A \left(\frac{1}{\epsilon^2} + \frac{3}{4\epsilon}\right) + \frac{\beta_0}{4\epsilon}\right),$$

$$-\frac{\Gamma(1-\epsilon)}{e^{\gamma_E \epsilon}} \mathcal{I}_{1,EWK}(\epsilon) = \left(-\frac{\mu^2}{s}\right)^\epsilon S_\epsilon Q_{up}^2 \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right),$$

$$\mathcal{I}_2(\epsilon) - \mathcal{I}_{1,QCD}(\epsilon) \mathcal{I}_{1,EWK}(\epsilon) = \frac{e^{\gamma_E \epsilon}}{\Gamma(1-\epsilon)} \left(-\frac{\mu^2}{s}\right)^{2\epsilon} \frac{1}{\epsilon} S_\epsilon C_F Q_{up}^2 \left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8}\right)$$