MIXED QCD-EW CORRECTIONS TO DILEPTON PRODUCTION AT THE LHC IN THE HIGH INVARIANT MASS REGION

Herschel A. Chawdhry (University of Oxford)

HP2 conference 2022, Newcastle, 20/09/2022

In collaboration with F. Buccioni, F. Caola, F. Devoto, M. Heller, A. v. Manteuffel, K. Melnikov, R. Röntsch, C. Signorile-Signorile

Based on *JHEP* **06** (2022) 022 [arXiv:2203.11237]

OUTLINE

- Introduction
  - Motivation
  - Previous works
- Loop amplitudes
- Infrared subtractions
- Phenomenological results
  - Setup
  - Fiducial cross-sections
  - Behaviour at large invariant mass
  - Differential distributions
  - Forward-backward asymmetry
- Conclusion

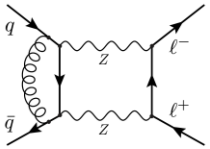
MOTIVATION

- l^+l^- production: testing SM and searching for BSM
- High m_{ll} region
  - studied by ATLAS [1707.02424] and CMS [2103.02708]
  - SMEFT operators: contributions grow like $\left(\frac{\sqrt{s}}{M}\right)^2$
  - Compare $\mu^+\mu^-$ and e^+e^- to constrain models explaining flavour anomalies
- Percent-level precision
  - Required to constrain Wilson coefficients of SMEFT operators
  - Needs QCD; EW; mixed QCDxEW
    - N3LO QCD corrections (refs: next slide) are close to 1%
    - NLO EW corrections (refs: next slide) reach tens of % at large m_{ll} due to EW Sudakov logs

PREVIOUS WORKS

- NLO EW
  - [Dittmaier, Kraemer, '02]
  - [Baur, Brein, Hollik, Schappacher, Wackeroth, '02]
  - [Baur, Wackeroth, '04]
  - [Arbuzov, Bardin, Bondarenko, Christova, Kalinovskaya, Nanava, Sadykov, '06 and '08]
  - [Zykunov, '06 and '07]
  - Carloni Calame, Montagna, Nicrosini, Vicini, '06 and '07]
  - [Dittmaier and Huber '10]
- N2LO QCD
  - [Hamberg, v. Neerven, Matsuura, '02]
  - [v. Neerven, Zijlstra, '92]
  - [Harlander, Kilgore, '02]
  - [Anastasiou, Dixon, Melnikov, Petriello, '03 and '04]
  - [Melnikov, Petriello, '06]
  - [Catani, Cieri, Ferrera, d. Florian, Grazzini, '09]
  - [Gavin, Li, Petriello, Quackenbush, '11]
- N3LO QCD
  - [Duhr, Dulat, Mistlberger, '20] x2
  - [Duhr, Mistlberger, '22]
  - [Chen, Gehrmann, Glover, Huss, Yang, Zhu, '22]
  - [Camarda, Cieri, Ferrera, '21]
  - [Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli, '22]
- Mixed QCDxEW
  - Resonant
    - [Delto, Jaquier, Melnikov, Rontsch, '20]
    - [Buccioni, Caola, Delto, Jaquier, Melnikov, Rontsch, '20]
    - [Bonciani, Buccioni, Rana, Vicini, '20 and '22]
    - [Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Rontsch, '21] x2
  - Off-shell
    - [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini, '21]

ASIDE: DIFFERENCES FROM THE ON-SHELL PROCESS

- Mixed corrections to *on-shell* W/Z production were previously calculated by several groups (see earlier refs)
  - Small: per-mille level
- Common contributions:
  - Real/virtual corrections to production or decay
    - e.g. virtual: 
- New contributions in off-shell calculation:
  - New virtual contributions
    - 2-loop 4-point massive, e.g. 

  - New real contributions
    - singularities from emission off initial *and* final state

NOTE

- Independent calculation of off-shell Drell-Yan [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini, '21]
  - Found percent-level effects
  - We couldn't directly compare our results (they use bare leptons, we use dressed leptons) but we qualitatively confirmed those findings
  - Quantitative checks ongoing in private – looks OK

VIRTUAL CONTRIBUTIONS: 1-LOOP AMPLITUDES

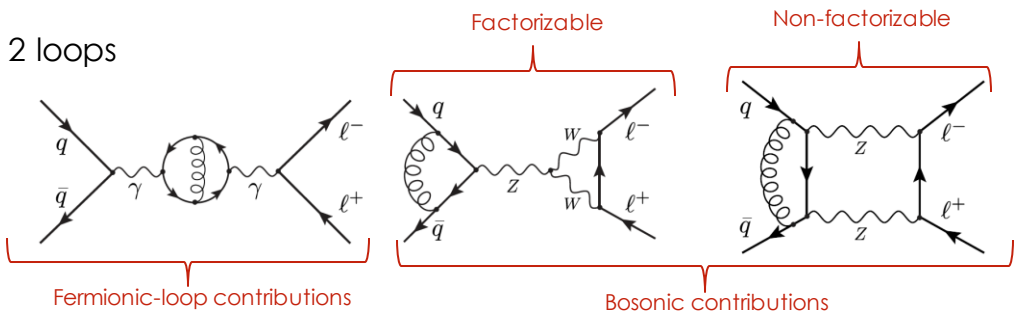
- 1-loop QCD amplitude for $q\bar{q} \to l^+l^-$:

$$\langle F_{\mathrm{LV}}^{(\mathrm{QCD}),\,\mathrm{fin}}(1_q, 2_{\bar{q}}, 3, 4)\rangle = \mathcal{C}^{\mathrm{QCD}} \langle F_{\mathrm{LM}}(1_q, 2_{\bar{q}}, 3, 4)\rangle, \quad \text{with} \quad \mathcal{C}^{\mathrm{QCD}} = -8C_F \frac{\alpha_s(\mu)}{2\pi}$$

- 1-loop QCD amplitude for $q\bar{q} \to l^+l^- + \gamma$
  - Adapting $q\bar{q} \to Z + j$ [Bern, Dixon, Kosower, '98] [MCFM: Campbell, Ellis, '99]
- 1-loop EW amplitudes for $q\bar{q} \to l^+l^-$ and $q\bar{q} \to l^+l^- + g$
  - Computed using OpenLoops2

2-LOOP AMPLITUDES

- $q\bar{q} \to l^+l^-$ at 2 loops
  - E.g.



- Fermionic-loop contributions
  - calculated independently
  - checked against [Dittmaier, Schmidt, Schwarz, 2009.02229] [Djouadi, Gambino, hep-ph/9309298]
- Bosonic contributions split (gauge-invariantly) into factorizable & non-factorizable

Difficult; slow; small

(Reminder: $m_{ll} > 200$ GeV)

Easy; fast; large

$$\mathcal{H}^{(1,1)}_{\text{non-fact}} = \mathcal{H}^{(1,1)} - \frac{\mathcal{H}^{(1,0)}\mathcal{H}^{(0,1)}}{\mathcal{H}^{(0,0)}}$$

$\mathcal{H}^{(m,n)}$: infrared subtracted finite helicity remainders at $O(\alpha^{1+m}\alpha_s^n)$, no fermionic loops

- Bosonic non-factorizable contributions
  - Using fully-analytic amplitudes from [Heller, v. Manteuffel, Schabinger, Spiesberger, 2012.05918]
  - GPLs evaluated with HandyG library
  - Optimised speed with Q-linear relations, MultivariateApart, common sub-expression elimination
    - Double-virtual evaluation time: ~0.7s per phase-space point
  - ~10 times smaller than factorizable part (Reminder: $m_{ll} > 200$ GeV)
  - Top quarks excluded

INFRARED SUBTRACTIONS

- Nested soft-collinear subtraction scheme [Caola, Melnikov, Rontsch, '17]
  - Fully analytic and local
  - Color coherence: subtract soft then collinear singularities
  - Collinear singularities: sector decomposition [Czakon, '10]
- Mixed QCD-EW: simplifications compared to NNLO QCD
  - Soft limits of photon and gluon commute
  - No singularities from photon collinear to gluon
    - but triple-collinear limits remain complicated
- Off-shell Z: more limits (sectors) compared to on-shell
  - due to emission from initial <u>and</u> final states

OUTLINE

- Introduction
  - Motivation
  - Previous works
- Loop amplitudes
- Infrared subtractions
- Phenomenological results
  - Setup
  - Fiducial cross-sections
  - Behaviour at large invariant mass
  - Differential distributions
  - Forward-backward asymmetry
- Conclusion

SETUP

- pp collisions with $\sqrt{S} = 13.6\ \text{TeV}$
- Renormalisation
  - on-shell for wavefunctions, masses, α
  - $\overline{MS}$ for α_s and PDFs
- Complex-mass scheme [hep-ph/0505042 , 2009.02229]
  - $m_Z = 91.1876\ \text{GeV}$ $\Gamma_Z = 2.4952\ \text{GeV}$
  - $m_W = 80.398\ \text{GeV}$ $\Gamma_W = 2.1054\ \text{GeV}$
  - $m_H = 125\ \text{GeV}$ $\Gamma_H = 4.165\ \text{MeV}$
- $m_t = 173.2\ \text{GeV}$
- G_μ input scheme for EW parameters, with $G_F = 1.16639 \cdot 10^{-5}\text{GeV}^{-2}$
  - $\hookrightarrow \alpha \approx 1/132.277$
- NNPDF31_nnlo_as_0118_luxqed everywhere (even for LO and NLO)
  - $\hookrightarrow \alpha_s(m_Z) = 0.118$
  - LHAPDF, HOPPET
  - We don't estimate PDF uncertainties

SETUP (CONTINUED)

- Cuts: (based on ATLAS analysis 1606.01736)

$$m_{\ell\ell} > 200\,\text{GeV}\,, \quad p_{T,\ell^\pm} > 30\,\text{GeV}\,, \quad \underbrace{\sqrt{p_{T,\ell^-} p_{T,\ell^+}}}_{\text{Salam \& Slade 2106.08329}} > 35\,\text{GeV}\,, \quad |y_{\ell^\pm}| < 2.5$$

- Dressed leptons (also for cuts)
  - Cluster leptons and photons into "lepton jets" when

$$R_{l\gamma} \equiv \sqrt{(y_l - y_\gamma)^2 + (\varphi_l - \varphi_\gamma)^2} < R_{\text{cut}}, \text{ with } R_{\text{cut}} = 0.1$$

  - Recombine in *E scheme*: $p^\mu_{\text{dressed}} = p^\mu_{\text{lepton}} + p^\mu_{\text{photon}}$
- $\mu_R = \mu_F = \frac{m_{ll}}{2}$, with 3-point variation for scale uncertainties

FIDUCIAL CROSS-SECTION

σ[fb]	LO $\sigma^{(0,0)}$	δ NLO QCD $\delta\sigma^{(1,0)}$	δ NLO EW $\delta\sigma^{(0,1)}$	δ N²LO QCD $\delta\sigma^{(2,0)}$	δ mixed $\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	−49.907	44.60	−16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

Large due to enhancement by $\ln(m_{ll}^2/p_{T,l^\pm}^2) \approx 5$

Checked against Sherpa and MoCaNLO+Recola

FIDUCIAL CROSS-SECTION (CONTINUED)

- Estimate uncertainties with envelope of:
  - $\mu\,(=\mu_R=\mu_F)=\frac{m_{ll}}{4}$ or $\frac{m_{ll}}{2}$ or m_{ll}
  - $G_\mu = 1.16639\cdot 10^{-5}\mathrm{GeV}^{-2}$ or $\alpha(m_Z)=1/128$
- Reminder: not estimating PDF uncertainties
  - Note that $q\bar{q}$ luminosity uncertainty is ~2% for $m_{ll} < 1\ \mathrm{TeV}$ and ~5% for m_{ll}~$2\ \mathrm{TeV}$

Uncertainties: +12% / -6%

$$\underbrace{\sigma^{(0,0)}}_{\text{LO}} + \underbrace{\delta\sigma^{(1,0)}}_{\text{NLO QCD}} + \underbrace{\delta\sigma^{(0,1)}}_{\text{NLO EW}} + \underbrace{\delta\sigma^{(2,0)}}_{\text{N2LO QCD}} = 1928.3^{+1.8\%}_{-0.15\%}\ \mathrm{fb}$$

$$\sigma_{\mathrm{QCD}\times\mathrm{EW}} \equiv \underbrace{\sigma^{(0,0)}}_{\text{LO}} + \underbrace{\delta\sigma^{(1,0)}}_{\text{NLO QCD}} + \underbrace{\delta\sigma^{(0,1)}}_{\text{NLO EW}} + \underbrace{\delta\sigma^{(2,0)}}_{\text{N2LO QCD}} + \underbrace{\delta\sigma^{(1,1)}}_{\text{mixed}} = 1912.6^{+0.65\%}_{-0\%}\ \mathrm{fb}$$

BEHAVIOUR AT LARGE m_{ll}

- At high m_{ll}, EW corrections dominated by universal Sudakov logs
  - So expect mixed QCD-EW corrections to be well-described by product of QCD and EW corrections
  - Define factorised approximation:

$$\delta\sigma^{(1,1)}_{\text{fact}} = \delta^{(1,0)}_{\text{NLO}}\,\delta^{(0,1)}_{\text{NLO}}\,\sigma^{(0,0)}$$

!! Do not confuse with earlier discussion about treatment of 2-loop amplitudes !!

where

$$\delta^{(1,0)}_{\text{NLO}} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}}, \qquad \delta^{(0,1)}_{\text{NLO}} = \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}}$$

and compare it against exact result

  - Consider 4 windows:

$$\begin{aligned}
\Phi^{(1)}: &\quad 200\,\text{GeV} < m_{\ell\ell} < 300\,\text{GeV},\\
\Phi^{(2)}: &\quad 300\,\text{GeV} < m_{\ell\ell} < 500\,\text{GeV},\\
\Phi^{(3)}: &\quad 500\,\text{GeV} < m_{\ell\ell} < 1.5\,\text{TeV},\\
\Phi^{(4)}: &\quad 1.5\,\text{TeV} < m_{\ell\ell} < \infty.
\end{aligned}$$

BEHAVIOUR AT LARGE m_{ll} (CONTINUED)

sum

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$	$\sigma_{\text{QCD}\times\text{EW}}$
$\Phi^{(1)}$	1169.8	254.3	−30.98	10.18	−10.74	−6.734	$1392.6^{+0.75\%}_{-0\%}$
$\Phi^{(2)}$	368.29	71.91	−11.891	2.85	−4.05	−2.321	$427.1^{+0.41\%}_{-0.02\%}$
$\Phi^{(3)}$	82.08	14.31	−4.094	0.691	−1.01	−0.7137	$91.98^{+0.22\%}_{-0.14\%}$
$\Phi^{(4)} \times 10$	9.107	1.577	−1.124	0.146	−0.206	−0.1946	$9.500^{+0\%}_{-0.97\%}$

Exact result

Factorised approximation

As before, estimating uncertainties by varying scales and input scheme

Recall:

$\Phi^{(1)}$: $200\,\text{GeV} < m_{\ell\ell} < 300\,\text{GeV}$,

$\Phi^{(2)}$: $300\,\text{GeV} < m_{\ell\ell} < 500\,\text{GeV}$,

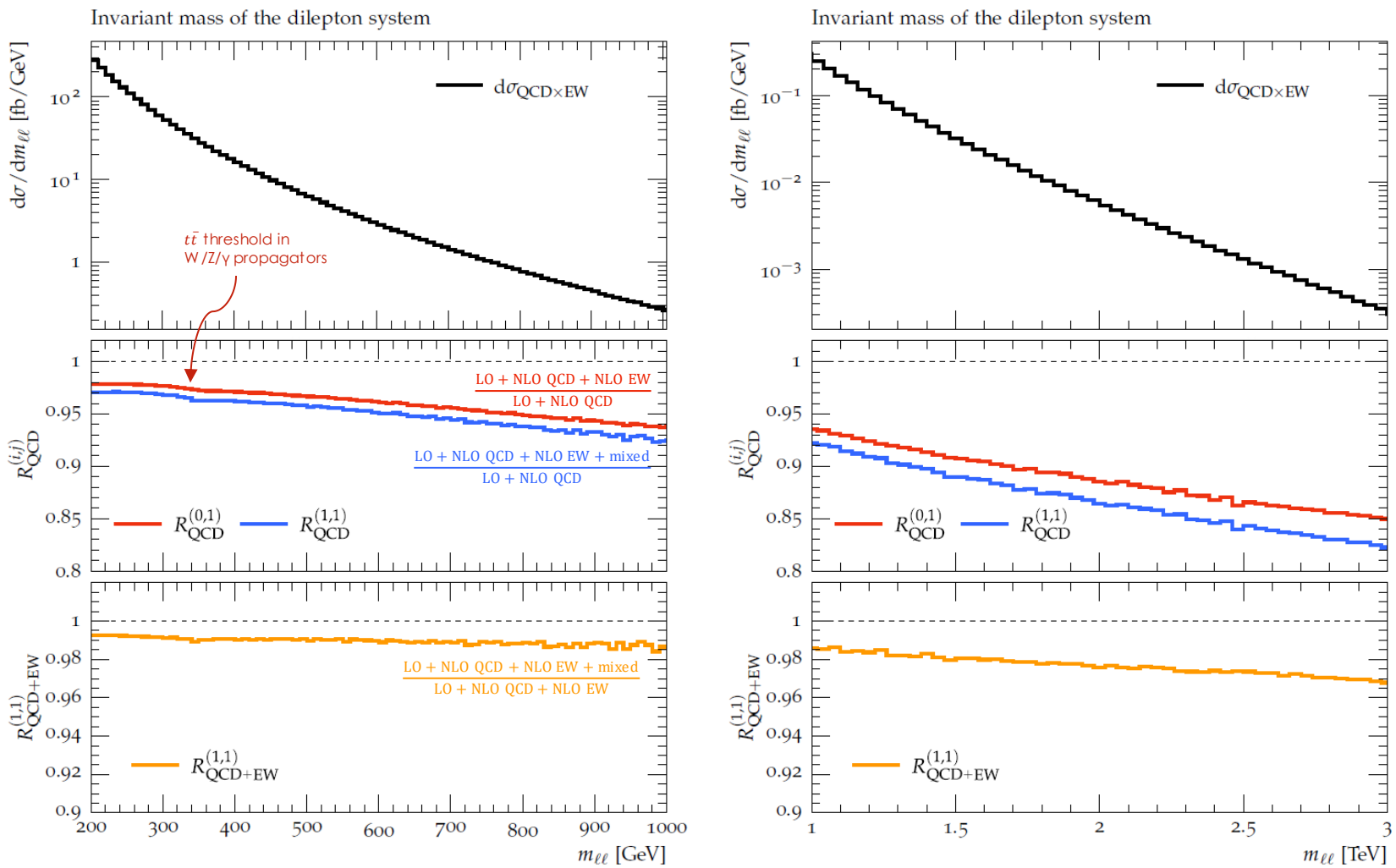
$\Phi^{(3)}$: $500\,\text{GeV} < m_{\ell\ell} < 1.5\,\text{TeV}$,

$\Phi^{(4)}$: $1.5\,\text{TeV} < m_{\ell\ell} < \infty$.

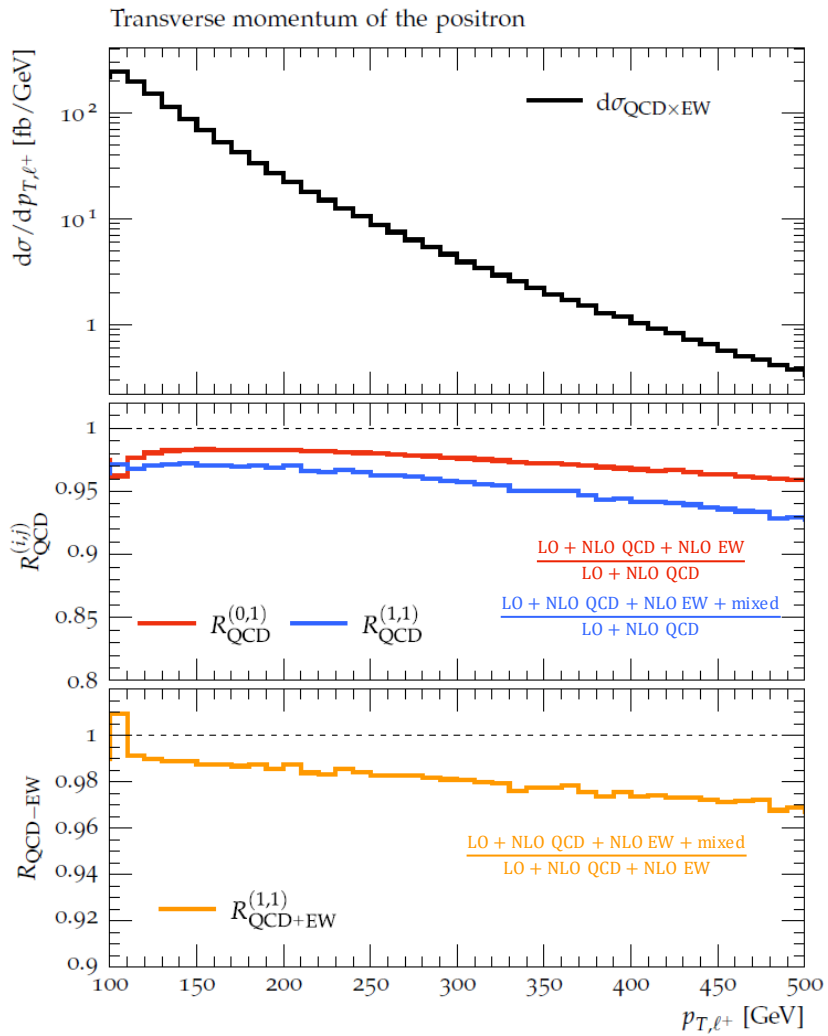
Recall: $\delta\sigma^{(1,1)}_{\text{fact}} = \delta^{(1,0)}_{\text{NLO}}\,\delta^{(0,1)}_{\text{NLO}}\,\sigma^{(0,0)}$

$$\delta^{(1,0)}_{\text{NLO}} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}}, \qquad \delta^{(0,1)}_{\text{NLO}} = \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}}$$

THE m_{ll} DISTRIBUTION



p_T DISTRIBUTION OF l^+



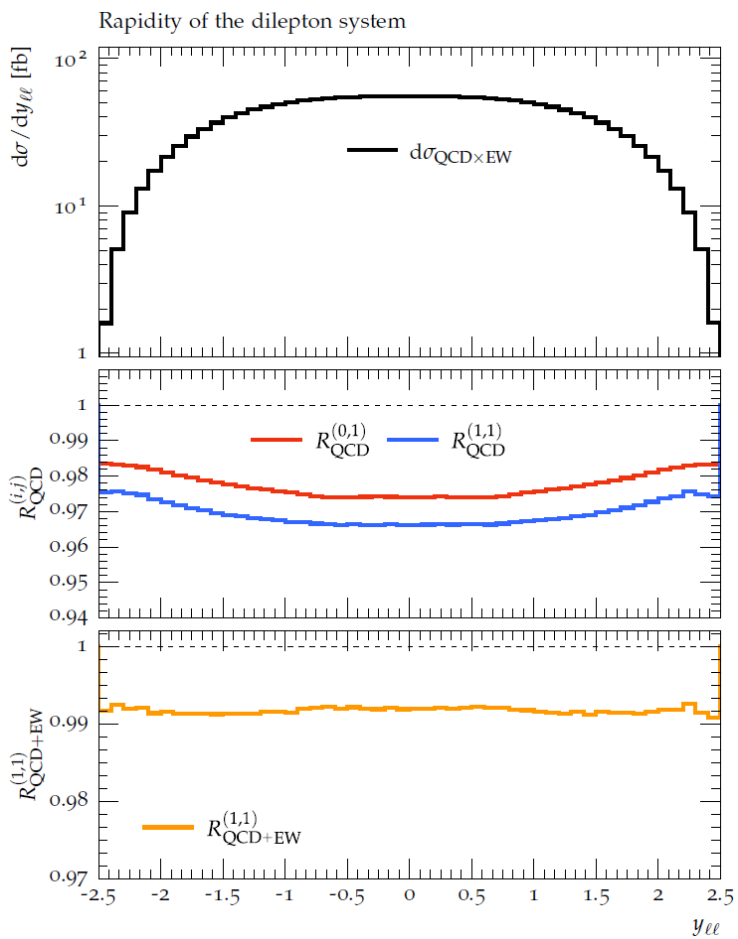
- Related to mll distribution
- NLO EW large and negative
- QCDxEW corrections unusually large at low pT (largest contribution to cross-section)
- Mixed QCDxEW follows NLO EW but grows to -3% at pT=500 GeV

THE y_{ll} DISTRIBUTION

$$\frac{\text{LO + NLO QCD + NLO EW}}{\text{LO + NLO QCD}}$$

$$\frac{\text{LO + NLO QCD + NLO EW + mixed}}{\text{LO + NLO QCD}}$$

$$\frac{\text{LO + NLO QCD + NLO EW + mixed}}{\text{LO + NLO QCD + NLO EW}}$$



NLO EW corrections are fairly flat and $\sim -3\%$

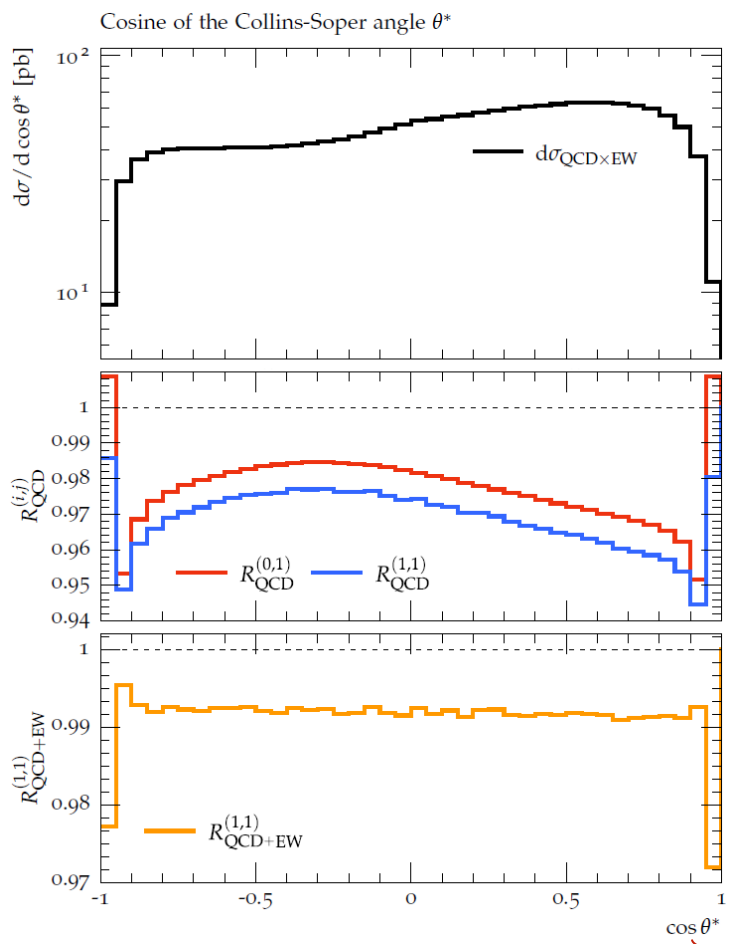
Mixed corrections are fairly flat and $\sim -1\%$

DISTRIBUTION IN COLLINS-SOPER ANGLE

$$\frac{\text{LO + NLO QCD + NLO EW}}{\text{LO + NLO QCD}}$$

$$\frac{\text{LO + NLO QCD + NLO EW + mixed}}{\text{LO + NLO QCD}}$$

$$\frac{\text{LO + NLO QCD + NLO EW + mixed}}{\text{LO + NLO QCD + NLO EW}}$$



$$\cos\theta^* = \frac{p^+_{\ell^-}p^-_{\ell^+} - p^-_{\ell^-}p^+_{\ell^+}}{m_{\ell\ell}\sqrt{m^2_{\ell\ell} + p^2_{\ell\ell,\perp}}} \times \mathrm{sgn}(p_{\ell\ell,z})$$

FORWARDS-BACKWARDS ASYMMETRY

- From Collins-Soper angular distribution, calculate forwards-backwards asymmetry

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = \int_0^1 d\cos\theta^* \, \frac{d\sigma(pp \to \ell^-\ell^+)}{d\cos\theta^*}, \qquad \sigma_B = \int_{-1}^0 d\cos\theta^* \, \frac{d\sigma(pp \to \ell^-\ell^+)}{d\cos\theta^*}$$

- We find: $A_{\rm FB} = 0.1580^{+0.15\%}_{-0.07\%}$

$\Phi^{(1)}$: $200\,\text{GeV} < m_{\ell\ell} < 300\,\text{GeV}$,

$\Phi^{(2)}$: $300\,\text{GeV} < m_{\ell\ell} < 500\,\text{GeV}$,

$\Phi^{(3)}$: $500\,\text{GeV} < m_{\ell\ell} < 1.5\,\text{TeV}$,

$\Phi^{(4)}$: $1.5\,\text{TeV} < m_{\ell\ell} < \infty$.

	without mixed	with mixed
	$\tilde{A}_{\rm FB}$	$A_{\rm FB}$
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
$\Phi^{(2)}$	$0.1852^{+0.08\%}_{-0.40\%}$	$0.1847^{+0.10\%}_{-0.19\%}$
$\Phi^{(3)}$	$0.2401^{+0.13\%}_{-0.64\%}$	$0.2388^{+0.06\%}_{-0.47\%}$
$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$

As before, estimating uncertainties by varying scales and input scheme

CONCLUSION

- Mixed QCD-EW corrections to pp -> l+l-
  - Fully differential in final-state resolved particles
  - Nested soft-collinear subtraction scheme for IR singularities
  - Fully analytic 2-loop amplitudes
- At mll ~ 200 GeV, mixed corrections are ~ -1%
  - larger than NNLO QCD (in this setup)
  - Input-scheme uncertainties reduced to sub percent level
- At mll > 1 TeV, mixed corrections are larger
  - explainable by Sudakov logarithms
  - can be approximated to within 30% by product of NLO QCD and EW
  - percent-level effect on forwards-backwards asymmetry