



Color singlet decays at NNLO in MCFM

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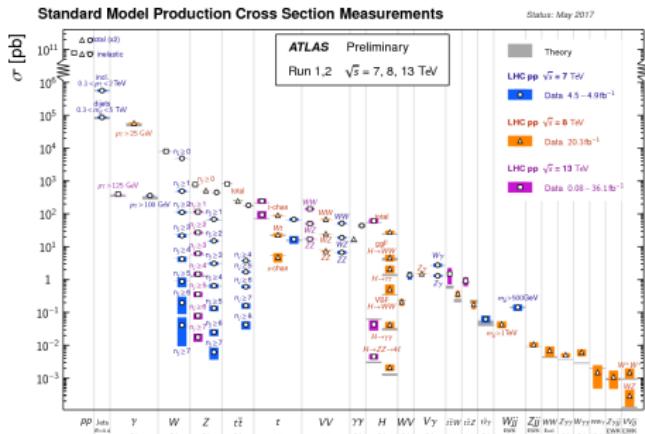
LHC physics in the precision era

So far no direct hints for BSM

- ▶ Deviations from SM small and featureless?
- ▶ Percent-level theory predictions $\Rightarrow N^x\text{LO}$ precision

NNLO parton-level MCs crucial for data analysis and phenomenology!

- ▶ (Subtracted) real emission corrections typically cause biggest numerical convergence problems
- ▶ *NLO revolution* aided by completely generic implementations of universal subtraction schemes [[Catani, Seymour '96; Frixione, Kunszt, Signer '96-'97]]
- ▶ Despite many dedicated calculations, no such implementation available yet at NNLO
- ▶ **This talk:** First steps towards including nested soft-collinear subtraction in MCFM



Methods for NNLO calculations

General aim: Extract singularities analytically, compute remainder with MC

1. Slicing:

- ▶ Find observable sensitive to IR real radiation, r_{cut}
- ▶ Slice phase space into *resolved* and *unresolved* parts:

$$\int |\mathcal{M}|^2 \mathcal{F} d\phi_d = \int_0^{r_{cut}} \left(|\mathcal{M}|^2 \mathcal{F} d\phi_d \right)_{C,S} + \int_{r_{cut}}^1 |\mathcal{M}|^2 \mathcal{F} d\phi_4 + \mathcal{O}(r_{cut})$$

- ▶ q_T [Catani,Grazzini '07] or N-jettiness [Gaunt et al '15; Boughezal et al '15]
- ▶ Implemented at NNLO in MCFM (N-jettiness, q_T) and MATRIX (q_T).
[Boughezal et al '17] [Kallweit,Grazzini,Wiesemann, '17]
- ▶ **Pros:** straightforward to implement, recycles existing NLO calculations
- ▶ **Cons:** slow convergence, large power corrections.

2. Subtraction:

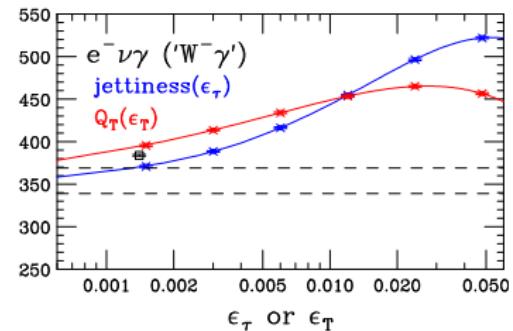
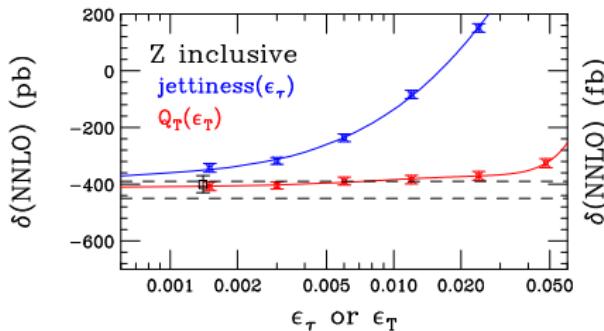
- ▶ Find set of subtraction terms, \mathcal{D} , which reproduce $|\mathcal{M}|^2$ in unresolved limits,

$$\int |\mathcal{M}|^2 \mathcal{F} d\phi_d = \int \left(|\mathcal{M}|^2 \mathcal{F} - \mathcal{D} \right) d\phi_4 + \int \mathcal{D} d\phi_4$$

- ▶ \mathcal{D} difficult to identify as it must be simple and integrable over unresolved PS.
- ▶ Integrating counter-term non-trivial if singularities are overlapping.
- ▶ *NLO fully solved* [Catani,Seymour '96], [Frixione,Kunszt,Signer '96].

Numerical performance of slicing techniques

- Comparative study with jettiness and q_T slicing method [Campbell,Ellis,Seth '22]



- Rough estimate of computing power required:

$$\frac{\epsilon_T = q_T/M_X}{t(\text{CPU days})} \quad \begin{matrix} 0.0015 \\ 200 \end{matrix} \quad \begin{matrix} 0.003 \\ 100 \end{matrix} \quad \begin{matrix} 0.006 \\ 50 \end{matrix} \quad \begin{matrix} 0.012 \\ 15 \end{matrix} \quad \begin{matrix} 0.024 \\ 3 \end{matrix} \quad \begin{matrix} 0.048 \\ 1 \end{matrix}$$

Subtraction methods

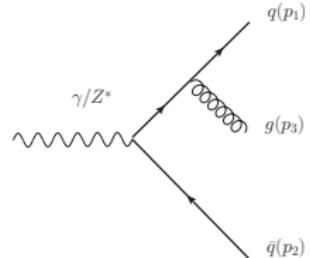
Commonly used techniques:

- ▶ Antenna subtraction [Gehrmann-DeRidder,Gehrmann,Glover '05, ...]
- ▶ Stripper [Czakon '10, ...]
- ▶ ColorfulNNLO [delDuca,Somogyi,Trócsányi, '05, ...]
- ▶ Geometric [Herzog '18]
- ▶ Local analytic sector [Magnea,Maina,Pellicoli,Signorile,Torrielli '18]
- ▶ Nested soft collinear subtraction [Czakon,Heymes '14] [Caola,Melnikov,Röntsch '17, ...]
- ▶ Projection to Born [Cacciari et al '15, ...]

Why use the nested soft collinear scheme?

- ▶ Physically transparent (collinear C / soft (S) singularities made explicit)
- ▶ Modular (singularities extracted using partial fractioning & sectorization)
- ▶ Analytic (integrals obtained by Melnikov et al)
- ▶ Numerically efficient

Toy example: Color singlet decay at NLO



Notation: At NLO consider $Z/\gamma^* \rightarrow q(p_1)\bar{q}(p_2)g(p_3)$,

Real-emission cross-section:

$$d\sigma^R = \frac{1}{2s} \int [dg_3] \mathcal{F}(1, 2, 3) \quad \text{and} \quad \int [dg_3] = \int \frac{d^{d-1}p_3}{(2\pi)^d 2E_3} \theta(E_{\max} - E_3)$$

$E_{\max} \leftrightarrow$ arbitrary parameter chosen large enough to accommodate all possible kinematic configurations

$$\mathcal{F}(1, 2, 3) = d\text{Lips}_V |\mathcal{M}(1, 2, 3)|^2 \mathcal{F}_{\text{kin}}(1, 2, 3)$$

- ▶ $d\text{Lips}_V$ is Lorentz-invariant phase space for colorless particles including $\delta^{(d)}(p_1 + p_2 + p_3)$.
- ▶ $\mathcal{F}_{\text{kin}}(1, 2, 3)$ is an *IR safe* observable that depends on kinematic variables of all particles in the process.

Toy example: Color singlet decay at NLO

The construction of subtraction terms:

$$\int [dg_3] \mathcal{F}(1, 2, 3) = \langle S_3 \mathcal{F}(1, 2, 3) \rangle + \langle (C_{12} + C_{13})(1 - S_3) \mathcal{F}(1, 2, 3) \rangle \\ + \langle \hat{O}_{\text{NLO}} \mathcal{F}(1, 2, 3) \rangle$$

with $\hat{O}_{\text{NLO}} = (I - C_{31} - C_{32})(I - S_3)$ finite.

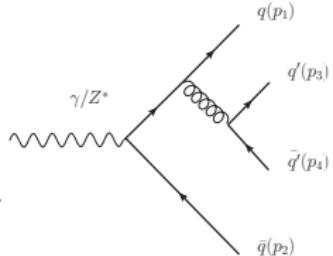
Limit operators at NLO ($2\eta_{ij} := 1 - \cos \theta_{ij}$):

- ▶ Soft: $S_i \mathcal{F} = \lim_{E_i \rightarrow 0} \mathcal{F} \rightarrow \propto \frac{1}{E_i^2} \sum_{j, k \neq i} \langle \mathcal{M} | \mathbf{T}_j \mathbf{T}_k | \mathcal{M} \rangle \frac{\eta_{jk}}{\eta_{ji} \eta_{ik}}$
- ▶ Collinear: $C_{ij} \mathcal{F} = \lim_{\eta_{ij} \rightarrow 0} \mathcal{F} \rightarrow \propto \frac{1}{s_{ij}} |\mathcal{M}|^2 \otimes P_{(ij) \rightarrow i}(z)$

N.B. Subtracted terms indeed *Analytic* e.g. soft term

$$\langle S_3 \mathcal{F}(1, 2, 3) \rangle = -2^{1-2\varepsilon} \frac{[\alpha_s]}{\varepsilon} \left[\frac{(4\pi)^\varepsilon}{8\pi^2} \right] \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} {}_2F_1(1, 1, ;, 1-\varepsilon, 1-\eta_{12}) \eta_{12}$$

First non-trivial case: Color singlet decay at NNLO



Singularity structure of $Z/\gamma^* \rightarrow q(p_1)\bar{q}(p_2)q'(p_3)\bar{q}'(p_4)$

[Gehrmann, Gehrmann-DeRidder, Glover '03]

$$\int [dg_3][dg_4]|\mathcal{M}|^2 = -\frac{1}{12\varepsilon^3} - \frac{7}{18\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{11\pi^2}{72} - \frac{407}{216}\right) + \dots$$

Singular limits: [Campbell, Glover '98], [Catani, Grazzini '99]

- ▶ Both $q'_{3,4}$ soft: $\int \$ = \int [dg_3][dg_4] \sum_{i,j=1,2} \mathcal{I}_{ij}(3,4)$
- ▶ Double collinear $q'_{3,4}$: $\int C_{34} = \int [dg_3][dg_4] P_{q_3 q'_4}$
- ▶ Triple collinear $q'_{3,4}, q_i$: $\int \mathcal{C}_i = \int [dg_3][dg_4] P_{\bar{q}'_3 q'_4 q_i}$
- ▶ Iterated double collinear: $q'_{3,4}, q_i$: $\int C_{34} \mathcal{C}_i = \int [dg_3][dg_4] P_{qg} \otimes P_{gq'}$
- ▶ iterated soft-collinear ($\$ C_{34}, \$ \mathcal{C}_i, \$ \mathcal{C}_i C_{34}$)

Finite remainder: Subtracted $Z/\gamma^* \rightarrow q(p_1)\bar{q}(p_2)q'(p_3)\bar{q}'(p_4)$ contribution

$$d\Gamma_{Z/\gamma^* \rightarrow q\bar{q}q'\bar{q}'}^{\text{NNLO}} = \sum_{i=1,2} \left\langle [I - C_{34}] [I - \mathcal{C}_i] [I - \$] [dg_3][dg_4] \mathcal{F}(1_q, 2_{\bar{q}}, 3_{q'}, 4_{\bar{q}'}) \right\rangle$$

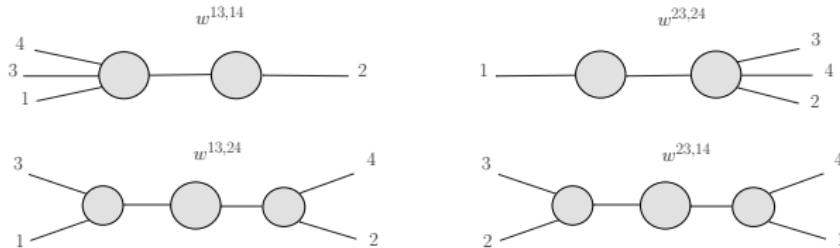
Color singlet decay (NNLO)

What remains?

- ▶ Triple collinear limits are separated using phase-space partition functions:

$$1 = w^{13,14} + w^{23,24} + w^{13,24} + w^{14,23}$$

- ▶ $w^{13,24}, w^{14,23}$ and $w^{13,14}, w^{23,24}$ project onto the relevant double and triple collinear splittings.

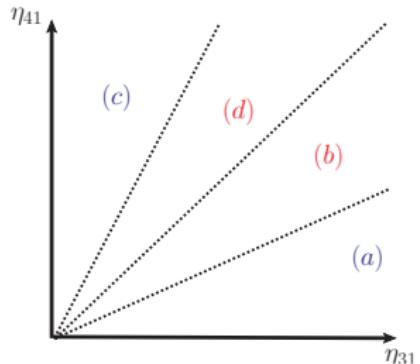


- ▶ The partitions needed to cancel all singularities which do not belong in desired sector.

Color singlet decay (NNLO)

Sectors: angular decomposition to separate singular regions:

$$\begin{aligned} 1 &= \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)} \\ &= \theta(\eta_{41} < \frac{\eta_{31}}{2}) + \theta(\frac{\eta_{31}}{2} < \eta_{41} < \eta_{31}) \\ &\quad + \theta(\eta_{31} < \frac{\eta_{41}}{2}) + \theta(\frac{\eta_{41}}{2} < \eta_{31} < \eta_{41}) \end{aligned}$$



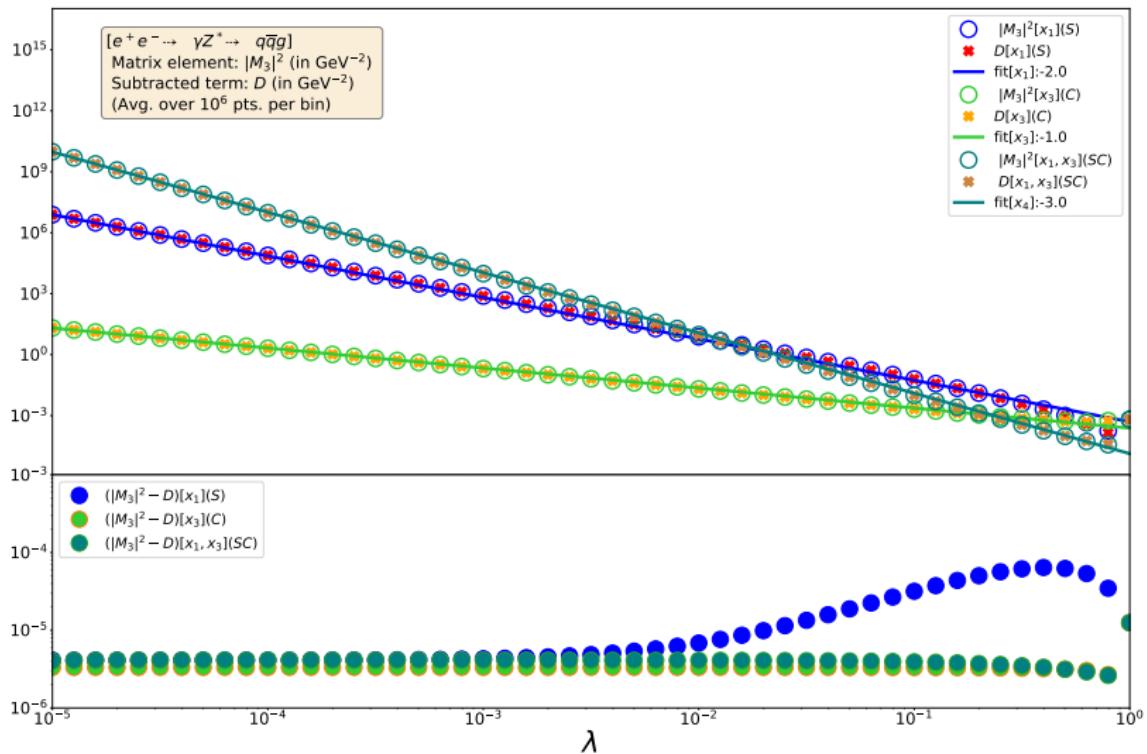
Sector-specific phase-space mapping [Czakon '10]

Corresponding limits: $\theta^{(a)} \leftrightarrow C_{41}$, $\theta^{(b)} \leftrightarrow C_{34}$, $\theta^{(c)} \leftrightarrow C_{31}$, $\theta^{(d)} \leftrightarrow C_{34}$

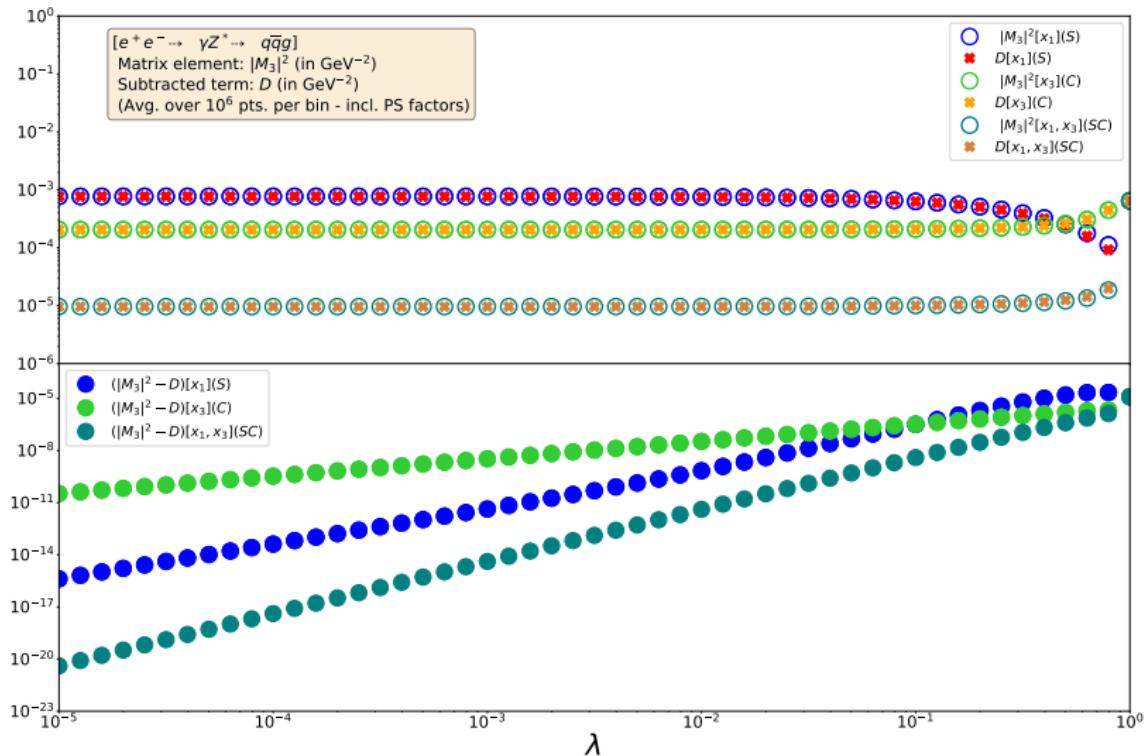
Fully-regulated double-real emission contribution in 4 dimensions

$$\begin{aligned} d\sigma^{RR} = & \sum_{i,j=1,2,i \neq j} \left\langle [1 - \$][dg_4][dg_3]w^{i3,j4} \mathcal{F}(1,2,3,4) \right\rangle \\ & + \sum_{i=1,2,i \neq j} \left\langle \left[(1 - \$)(1 - \mathcal{C}_i) \left(1 - (\theta^{(a)} + \theta^{(b)}) C_{34} \right) \right] \right. \\ & \quad \left. [dg_4][dg_3]w^{i3,i4} \mathcal{F}(1,2,3,4) \right\rangle \end{aligned}$$

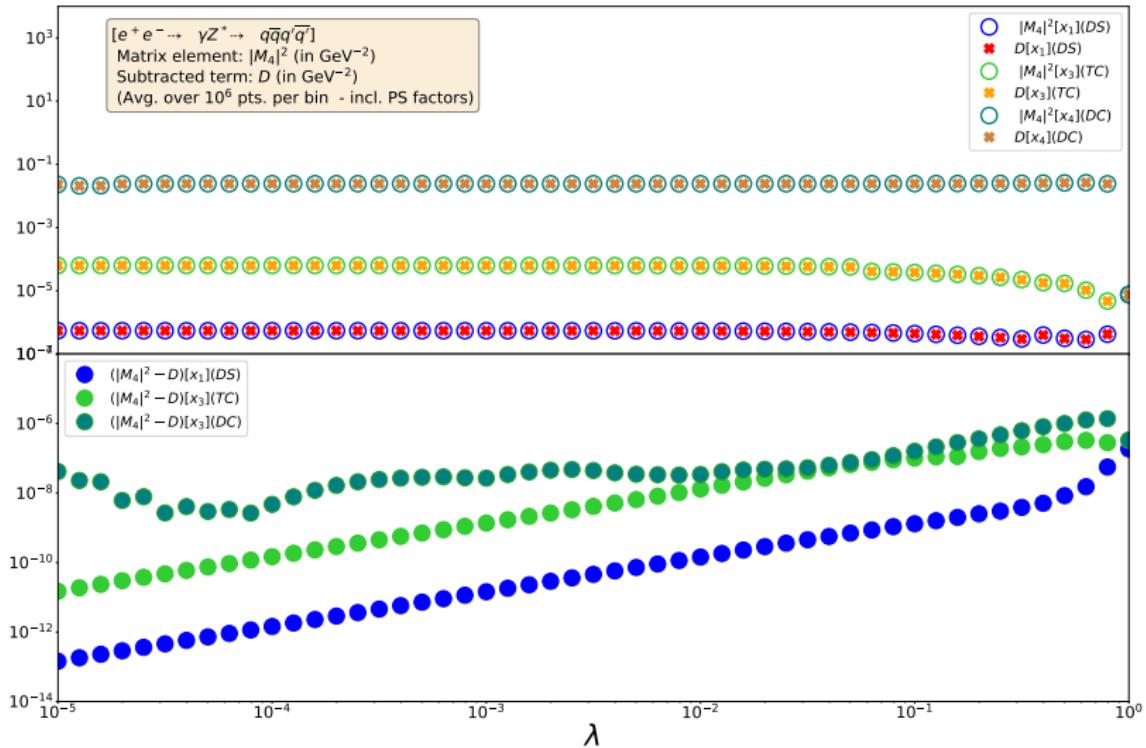
Scaling (NLO)



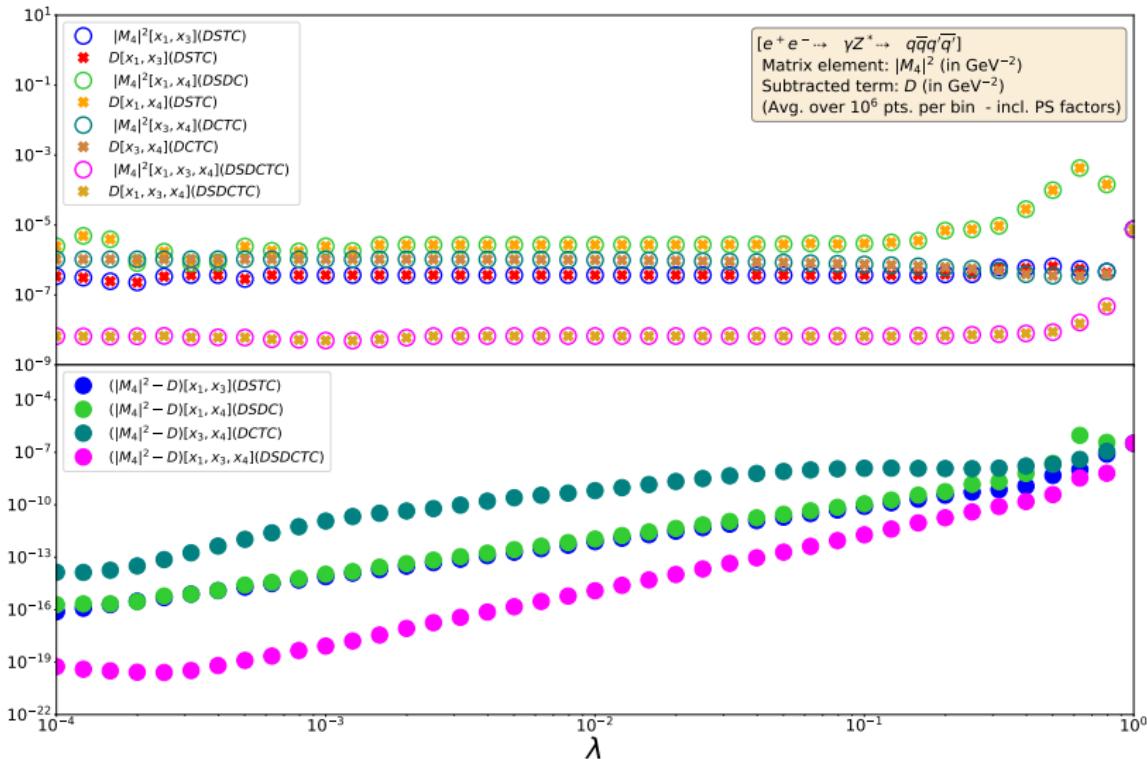
Scaling (NLO)



Scaling (NNLO)



Scaling (NNLO)



Integration (NNLO)

- ▶ **LEP setup:** $e^+e^- \rightarrow \gamma^* \rightarrow 2$ jets ($E_{\text{cms}}=91.2$ GeV)
- ▶ Numerical safety cutoff at $k_{T,3 \rightarrow 4} = Q\sqrt{y_{3 \rightarrow 4}}$
(Durham jet algorithm: $y_{ij} = 2(1 - \cos \theta_{ij})\min(E_i^2, E_j^2)/s$)
- ▶ Subtracted double-real contribution from $(d\bar{d}u\bar{u})$ final state

$k_{T,3 \rightarrow 4}$	σ	time
5 MeV	3.66(19) fb	96 s
5 keV	3.96(17) fb	100 s
5 eV	3.95(11) fb	100 s

- ▶ Diagrammatic calculation can lead to minor instabilities in double-soft limit:

$$|\mathcal{M}(1, 2, 3, 4)|^2 = \frac{2s_{13}s_{14}}{s_{134}^2 s_{34}^2} - \frac{s_{13}s_{24} + s_{14}s_{23} - s_{12}s_{34}}{s_{134}s_{234}s_{34}^2} + \left(\begin{array}{c} 1 \leftrightarrow 2 \\ 3 \leftrightarrow 4 \end{array} \right) + \dots \quad \text{vs}$$

$$\sum_{i,j=1,2} \mathcal{I}_{ij}(3, 4) = \frac{2s_{12}}{(s_{13} + s_{14})(s_{23} + s_{24})s_{34}} \left(1 - \frac{s_{12}s_{34}(\textcolor{red}{s_{13}s_{24} - s_{14}s_{23}})^2}{(s_{13} + s_{14})(s_{23} + s_{24})} \right)$$

Scaling tests affected below $\lambda \approx 10^{-3}$ but integration results unchanged

Summary and outlook

So far:

- ▶ Implemented technique for simplest double-real test case
- ▶ Used all building blocks except single soft subtractions
- ▶ Encouraging results when compared to other methods

subtraction			slicing		
$k_{T,3 \rightarrow 4}$	$\Delta\sigma$	time	ϵ_τ	$\Delta\sigma$	time
5 MeV	0.38 fb	96 s	0.001	1.15 fb	110 s
5 keV	0.34 fb	100 s	0.0001	3.05 fb	110 s

- ▶ If pattern unchanged by single-soft subtractions, expect significant speedup for practical calculations

Next steps:

- ▶ Include integrated counterterms and assemble complete calculation
- ▶ Implement into MCFM in generic form
- ▶ Provide interfaces to other codes (e.g. for PS matching)

n_f -contribution to double-real correction (slicing method)

