Fast simulations with NNLO QCD accuracy

8th International Workshop on High Precision for Hard Processes | Newcastle upon Tyne

Lucas Kunz | 20/09/2022

KARLSRUHE INSTITUTE OF TECHNOLOGY

Motivation

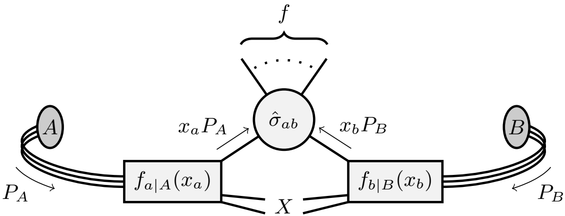

Figure by A. Huss

$$\mathrm{d}\sigma_{pp\to X} = \sum_{a,b} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \; f_a(x_a, \alpha_s(\mu_R), \mu_F)\, f_b(x_b, \alpha_s(\mu_R), \mu_F)$$

$$\times\, \mathrm{d}\hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_s(\mu_R), \mu_R, \mu_F) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)^p$$

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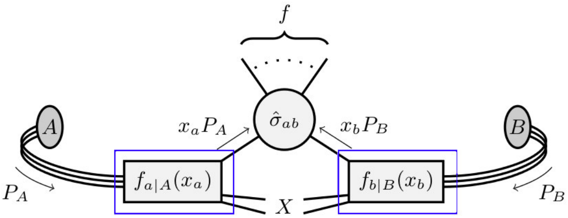

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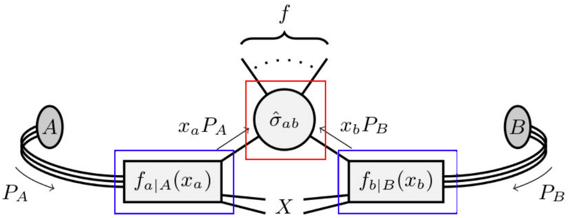

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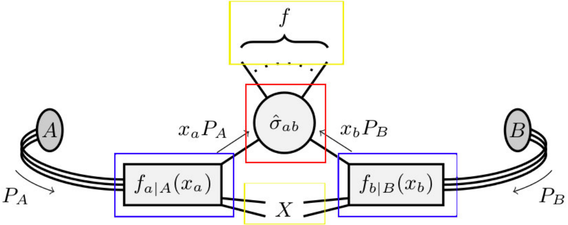

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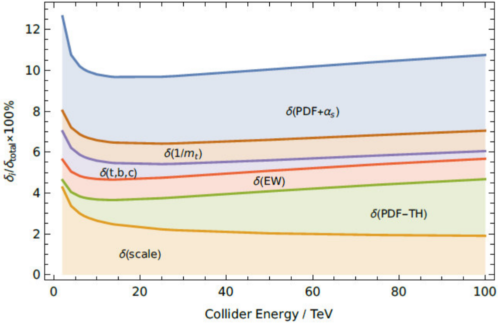
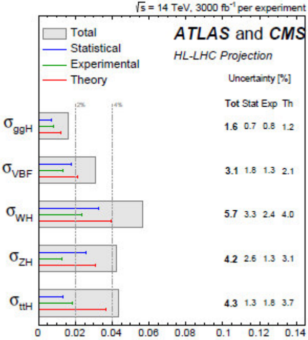
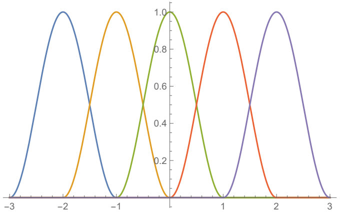


Relative uncertainty of the Higgs boson production cross section
[Dulat, Lazopoulos, Mistlberger '18]



Higgs production uncertainty estimates for the HL-LHC
[HL-LHC Working Group 2 '19]

Grid Technique - Interpolation



- Split interval $I = [a, b]$ into $N + 1$ nodes, $a = x^{[0]}$, $b = x^{[N]}$
- Partition of unity into a set of functions:
  - $1 = \sum_{i=0}^{N} E_i(x) \;\; \forall x \in I$
  - $E_i\left(x^{[i]}\right) = 1 \;\; \forall i \in \{0, \dots, N\}$
- $\Rightarrow$ Functions on the interval can be approximated: $f(x) \simeq \sum_{i=0}^{N} f^{[i]} E_i(x)$ where $f^{[i]} = f(x^{[i]})$
- $\Rightarrow$ Integrals can also be approximated: $\int_a^b f(x) g(x)\, \mathrm{d}x \simeq \sum_{i=0}^{N} f^{[i]} g_{[i]}$ with $g_{[i]} := \int_a^b E_i(x) g(x)\, \mathrm{d}x$

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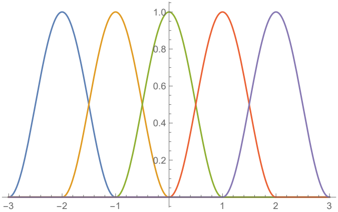

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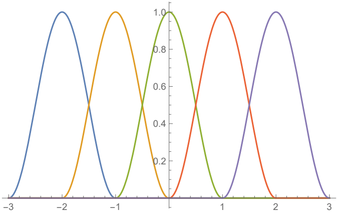

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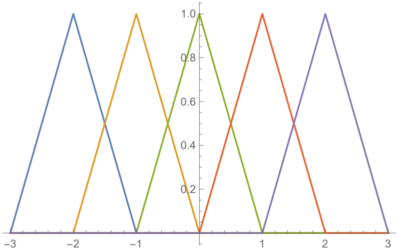
Grid Technique - Interpolation

Example: $f(x) = \frac{1}{3}x^3 + x^2 - 1$ on the Interval $I = [-2, 2]$

Five nodes $\{-2, -1, 0, 1, 2\}$



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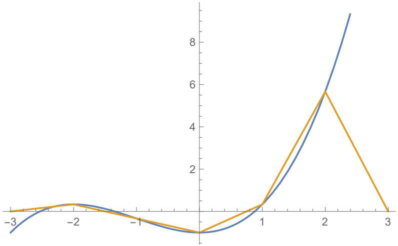

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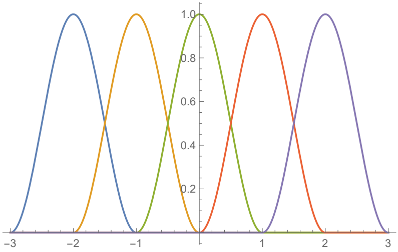

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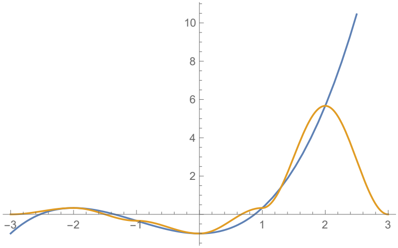

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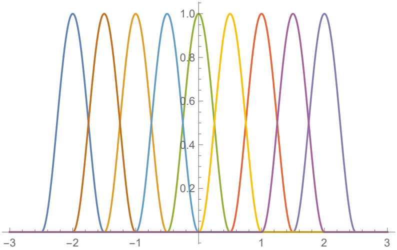
Example: $f(x) = \frac{1}{3}x^3 + x^2 - 1$ on the Interval $I = [-2, 2]$

Nine nodes $\{-2, -1.5, \ldots, 1.5, 2\}$



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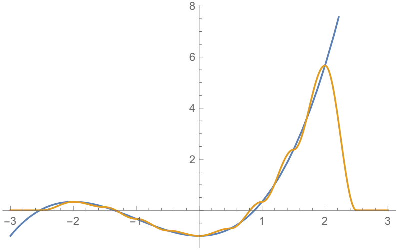

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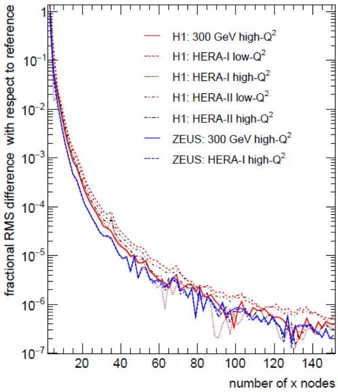


Fractional root mean square difference between interpolation and reference [Britzger, Gehrmann, Huss, Rabbertz, et al. '19]

Grid Technique - Formulae

$$\mathrm{d}\hat{\sigma}_{ab\to X}(x, \alpha_s, \mu) = \sum_k \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^{k+r} \mathrm{d}\hat{\sigma}^{(k)}_{ab\to X}(x, \alpha_s, \mu)$$

Evaluation with Monte Carlo event generator:

- Fixed-order ($k = 0, 1, ...$) parton level calculations
- Phase-space samples (x_m, Φ_m) with weights $w^{(k)}_{ab\to X,\, m}$

$$\Rightarrow \sigma_{pp\to X}(x, \alpha_s, \mu) = \sum_{a,b} \sum_k \sum_{m=1}^{M_p} \left(\frac{\alpha_s(\mu_{R,m})}{2\pi}\right)^{k+r} \hat{\sigma}^{(k)}_{ab\to X,\, m} \times w^{(k)}_{ab\to X,\, m}\, f_a(x_{a,m}, \mu_{F,m})\, f_b(x_{b,m}, \mu_{F,m})$$

Grid Technique - Formulae

For pp collisions: 4 functions $E_i(x_a)$, $E_j(x_b)$, $E_v(\mu_R)$, $E_w(\mu_F)$

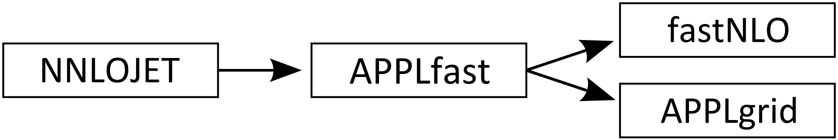
$$\Rightarrow \sigma_{pp\to X}(x, \alpha_s, \mu) = \sum_{i,j,v,w=0}^{N} \sum_{a,b} \sum_{k} \left(\frac{\alpha_s^{[v]}}{2\pi}\right)^{k+r} f_a^{[i,w]} f_b^{[j,w]} \times \hat{\sigma}^{(k)}_{ab\to X\;[i,j,v,w]}$$

with

$$\hat{\sigma}^{(k)}_{ab\to X\;[i,j,v,w]} := \sum_{m=1}^{M_p} E_i(x_{a,m})\, E_j(x_{b,m})\, E_v(\mu_{R,m})\, E_w(\mu_{F,m}) \times w^{(k)}_{ab\to X,\,m}\, \hat{\sigma}^{(k)}_{ab\to X,\,m}$$

Grid Technique - Workflow

- NNLOJET: fixed order Monte Carlo calculations
- fastNLO/APPLgrid: grid libraries
- APPLfast: interface connecting the two sides



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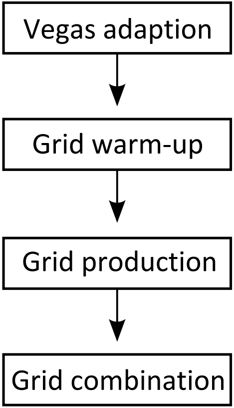

- generate optimised Vegas phase-space grid
- optimise the limits for x and μ (experimental fiducial cuts)
- grids filled with the weights generated from a full NNLOJET run
- grids from individual jobs are combined into final master grid

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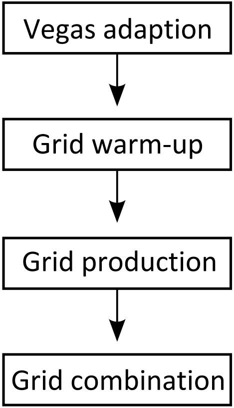

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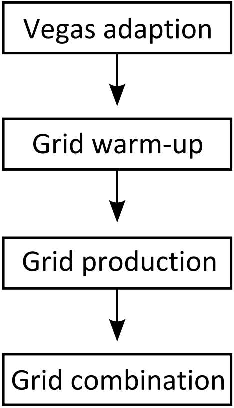

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Grid Technique - Workflow

Vegas adaption ↓ Grid warm-up ↓ Grid production ↓ Grid combination

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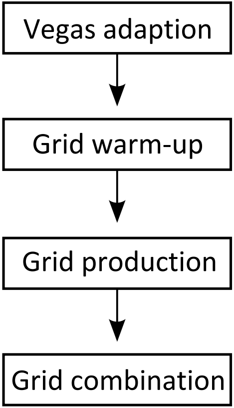

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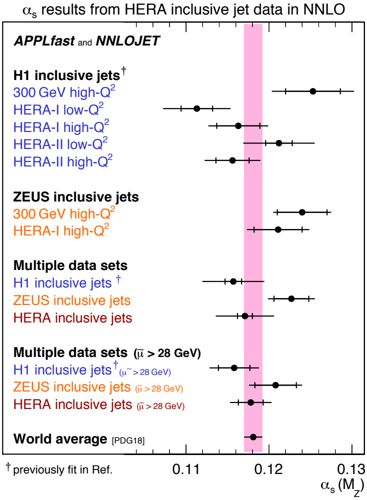

Use cases:

- observable calculations
- PDF fitting
- determination of the strong coupling constant [H1 collaboration '17] [Britzger, Gehrmann, Huss, Rabbertz, et al. '19] [Britzger, Gehrmann, Huss, Rabbertz, et al. '22]

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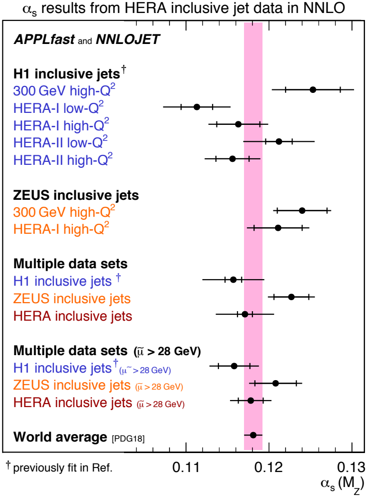

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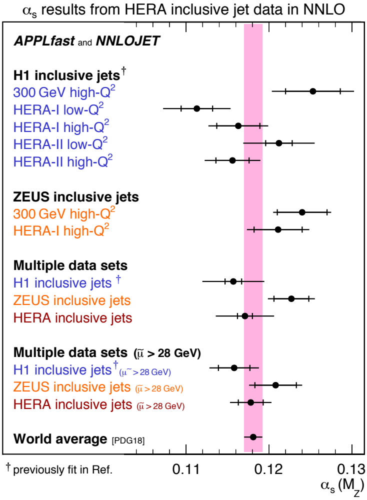

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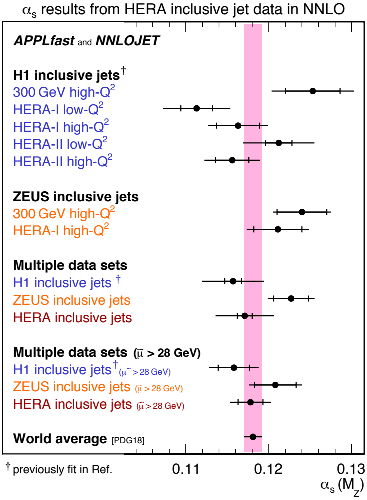

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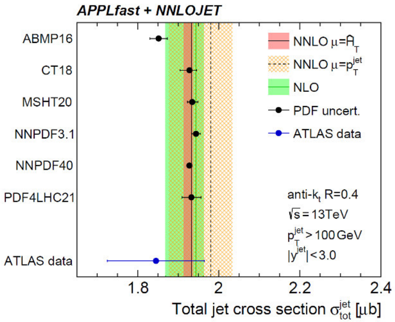

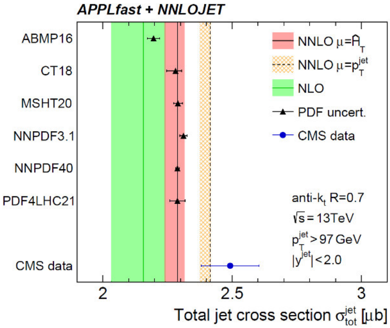

Comparison of the total jet cross section using different PDFs
[Britzger, Gehrmann, Huss, Rabbertz, et al. '22]

Current status

- Interface adapted to use modules 2 of NNLOJET
  - better colour sampling
  - full colour dijet code
  - more flexible decomposition of logarithmic scale coefficients
- Validation at NNLO in progress
- Runtime comparison at NLO (in minutes, Drell-Yan, 1000000 events):

setup	warmup (LO/R/V)			production (LO/R/V)		
modules 1	18.75	20.32	35.53	2.41	8.99	10.62
modules 2	3.48	5.59	17.49	0.776	1.70	6.85

Next steps

- finalize validation of NNLO code
  - produce grids for jet+X and di-jet processes at the LHC
  - extract value of $\alpha_s(\mathrm{M_Z})$ from LHC data
  - reproduce results in [Britzger, Gehrmann, Huss, Rabbertz, et al. '22]
- optimize workflow and runtime
- calculate di-jet differential distributions at full colour
- provide setup for further developments and calculations

Thank you for your attention!

Backup - Logarithm decomposition

Karlsruhe Institute of Technology

$$\sigma_{pp\to X}(x,\alpha_s,\mu) = \sum_{i,j,v,w=0}^{N} \sum_{a,b} \sum_{k} \left(\frac{\alpha_s^{[v]}}{2\pi}\right)^{k+r} f_a^{[i,w]}\, f_b^{[j,w]}\, \hat{\sigma}^{(k)}_{ab\to X\;[i,j,v,w]}$$

$$\mathrm{d}\hat{\sigma}^{(k)}_{ab\to X\;[i,j,v,w]}\left(\mu_R^2,\mu_F^2\right) = \sum_{\alpha+\beta\le k} \mathrm{d}\hat{\sigma}^{(k|\alpha,\beta)}_{ab\to X\;[i,j,v,w]} \ln^{\alpha}\left(\frac{\mu_R^2}{\mu_0^2}\right) \ln^{\beta}\left(\frac{\mu_F^2}{\mu_0^2}\right)$$

$$\hat{\sigma}^{(k|\alpha,\beta)}_{ab\to X\;[i,j,v,w]} = \sum_{m=1}^{M_p} E_i(x_{a,m})\, E_j(x_{b,m})\, E_v(\mu_{R,m})\, E_w(\mu_{F,m})\, w^{(k)}_{ab\to X,\,m}\, \hat{\sigma}^{(k|\alpha,\beta)}_{ab\to X,\,m}$$