

Some developments in higher order Renormalisation

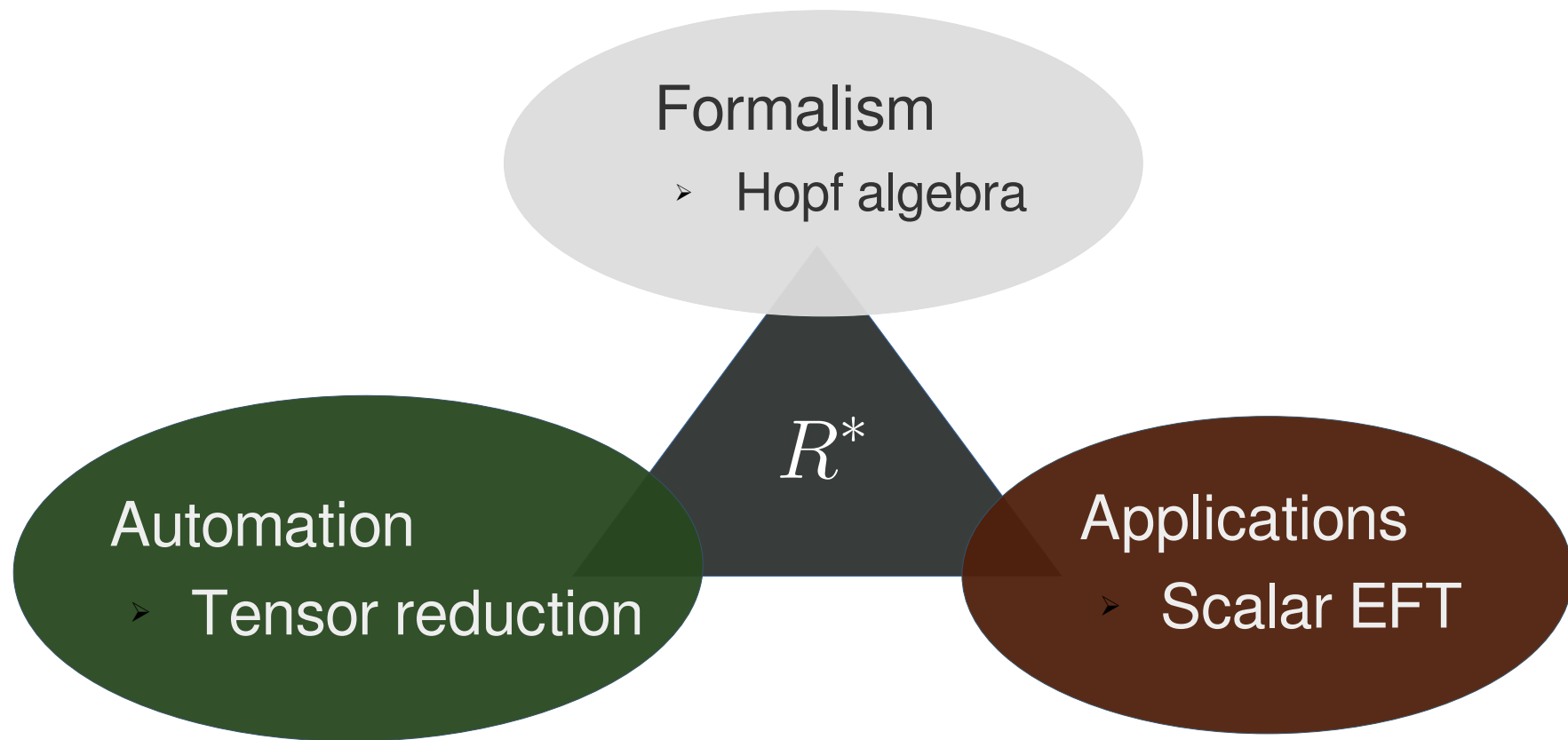
HP2- Newcastle 2022

Franz Herzog

State-of-the-art methods for multi-loop $\overline{\text{MS}}$ counterterms

R^* (local&global)	5-loop QCD,...
Massive tadpoles	5-loop QCD,...
Graphical functions	ϕ^4 up to 7 loops

This talk: focus on R^* approach



What is R^* ?

A generalisation of BPHZ to Euclidean IR divergences

[1982 Chetyrkin, Tkachov; Smirnov]

$$R^*(\Gamma) = \text{finite}$$

an offshell Feynman diagram

By adding counter-terms associated
to UV- or IR- divergent subdiagrams

2-loop example

$$R^* \left(\text{diagram 1} \right) = \text{diagram 2} + Z \left(\text{diagram 3} \right)$$

The diagram consists of a circle with a horizontal line passing through its center. There are three dots on this line: one at the left edge, one at the center, and one at the right edge. The top arc of the circle is labeled '1', the center dot is labeled '2', and the bottom arc is labeled '3'. The entire diagram is enclosed in large parentheses.

2-loop example

$$\begin{aligned}
 R^* \left(\text{Diagram 1} \right) &= \text{Diagram 2} + Z \left(\text{Diagram 3} \right) \\
 &+ Z \left(\text{Diagram 4} \right) * \text{Diagram 5} + \tilde{Z} \left(\text{Diagram 6} \right) * \text{Diagram 7}
 \end{aligned}$$

The diagrams are as follows:

- Diagram 1:** A circle with a horizontal line through its center. The line has two external segments extending to the left and right. There are three dots on the circle: one at the top, one at the left intersection of the line and circle, and one at the right intersection. The top dot is labeled '1', the left dot is labeled '2', and the right dot is labeled '3'.
- Diagram 2:** Identical to Diagram 1.
- Diagram 3:** Identical to Diagram 1.
- Diagram 4:** A circle with a horizontal line through its center. The line has two external segments extending to the left and right. There are two dots on the circle: one at the top and one at the bottom. The top dot is labeled '2' and the bottom dot is labeled '3'.
- Diagram 5:** A horizontal line with a small circle (loop) attached to it. The loop has a single dot at its top, labeled '1'.
- Diagram 6:** A vertical line segment with two open circles at its ends. The segment is labeled '1'.
- Diagram 7:** Identical to Diagram 4.

2-loop example

$$\begin{aligned}
 R^* \left(\text{Diagram 1} \right) &= \text{Diagram 1} + Z \left(\text{Diagram 1} \right) \\
 &+ Z \left(\text{Diagram 2} \right) * \text{Diagram 3} + \tilde{Z} \left(\text{Diagram 4} \right) * \text{Diagram 5} \\
 &+ \tilde{Z} \left(\text{Diagram 4} \right) * Z \left(\text{Diagram 2} \right) * 1
 \end{aligned}$$

Diagram 1: A circle with three external lines labeled 1, 2, and 3. Line 1 is at the top, line 2 is in the middle, and line 3 is at the bottom.

Diagram 2: A circle with two external lines labeled 2 and 3. Line 2 is at the top and line 3 is at the bottom.

Diagram 3: A horizontal line with a loop on top labeled 1.

Diagram 4: A vertical line with two open circles at the ends and a solid dot in the middle, labeled 1.

Diagram 5: A circle with two external lines labeled 2 and 3. Line 2 is at the top and line 3 is at the bottom.

General structure at L loops

$$R^*(\Gamma) = \sum_{\substack{S \subseteq \Gamma, \tilde{S} \subseteq \Gamma \\ S \cap \tilde{S} = \emptyset}} \tilde{Z}(\tilde{S}) * Z(S) * \Gamma / S \setminus \tilde{S}$$

General structure at L loops

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UV counterterm

$$Z(\Gamma) = -K \left(\sum_{\substack{S \subsetneq \Gamma, \tilde{S} \subseteq \Gamma \\ S \cap \tilde{S} = \emptyset}} \tilde{Z}(\tilde{S}) * Z(S) * \Gamma / S \setminus \tilde{S} \right)$$

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IR counterterm

$$\tilde{Z}(\Gamma_0) = -K \left(\sum_{\substack{S \subseteq \Gamma_0, \tilde{S} \subsetneq \Gamma_0 \\ S \cap \tilde{S} = \emptyset}} \tilde{Z}(\tilde{S}) * Z(S) * \Gamma_0 / S \setminus \tilde{S} \right)$$

Hopf algebraic formulation of R^*

with M Borinsky and R Beekveldt

$$\begin{array}{ccccc} & H \otimes H & \xrightarrow{S \otimes \text{id}} & H \otimes H & \\ & \uparrow \Delta & & \downarrow m & \\ H & \xrightarrow{\varepsilon} & K & \xrightarrow{\eta} & H \\ & \downarrow \Delta & & \uparrow m & \\ & H \otimes H & \xrightarrow{\text{id} \otimes S} & H \otimes H & \end{array}$$

Hopf algebraic formulation of R^*

with M Borinsky and R Beekveldt

Introduce IR and UV coactions for IR and UV divergent graphs:

$$\Delta_{\mathcal{M}}^{\text{UV}}(\Gamma) = \sum_{\gamma_{\text{UV}} \subseteq \Gamma} \gamma_{\text{UV}} \otimes \Gamma / \gamma_{\text{UV}}$$

$$\Delta_{\mathcal{M}}^{\text{IR}}(\Gamma) = \sum_{\gamma_{\text{mm}} \subseteq \Gamma} \gamma_{\text{mm}} \otimes \underbrace{\Gamma / \gamma_{\text{mm}}}_{= \gamma_{\text{IR}}}$$

Hopf algebraic formulation of R^*

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Introduce IR and UV coactions for IR and UV divergent graphs:

$$\Delta_{\mathcal{M}}^{\text{UV}}(\Gamma) = \sum_{\gamma_{\text{UV}} \subseteq \Gamma} \gamma_{\text{UV}} \otimes \Gamma / \gamma_{\text{UV}} \qquad \Delta_{\mathcal{M}}^{\text{IR}}(\Gamma) = \sum_{\gamma_{\text{mm}} \subseteq \Gamma} \gamma_{\text{mm}} \otimes \underbrace{\Gamma / \gamma_{\text{mm}}}_{= \gamma_{\text{IR}}}$$

The coactions are *co-associative*:

$$(\Delta_{\text{UV}} \otimes \text{id}) \circ \Delta_{\text{IR}} = (\text{id} \otimes \Delta_{\text{IR}}) \circ \Delta_{\text{UV}}$$

Hopf algebraic formulation of R^*

with M Borinsky and R Beekveldt

Allows to rewrite the original R^* -operation:

$$\begin{aligned} R^*(\Gamma) &= m_{\mathcal{A}}^{(2)} \circ (Z \otimes \phi \otimes \tilde{Z}) \circ (\text{id} \otimes \Delta_{\mathcal{M}}^{IR}) \circ \Delta_{\mathcal{M}}^{UV}(\Gamma) \\ &= \sum_{\gamma_{UV} \subseteq \Gamma} \sum_{\gamma_{mm} \subseteq \Gamma / \gamma_{UV}} Z(\gamma_{UV}) \phi(\gamma_{mm}) \tilde{Z}(\Gamma / \gamma_{UV} / \gamma_{mm}) \end{aligned}$$

Hopf algebraic formulation of R^*

with M Borinsky and R Beekveldt

- UV and IR counterterms are related via the *antipode*

$$\tilde{Z} = Z \circ S$$



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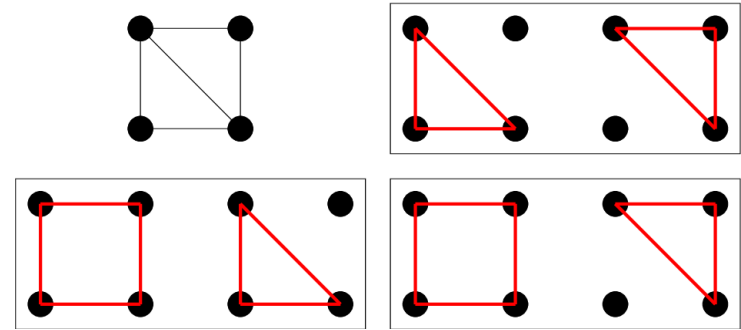
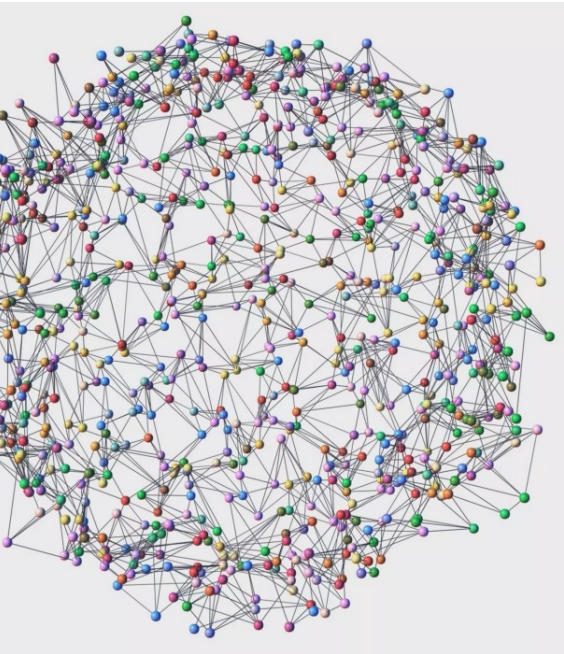
- Example:

$$\tilde{Z}\left(\text{circle with horizontal line and dots}\right) = Z \circ S\left(\text{circle with horizontal line and dots}\right) = -Z\left(\text{circle with horizontal line and dots}\right) + Z\left(\text{circle with two dots}\right)^2 + Z\left(\text{arc with two dots}\right)Z\left(\text{circle with one dot}\right)$$

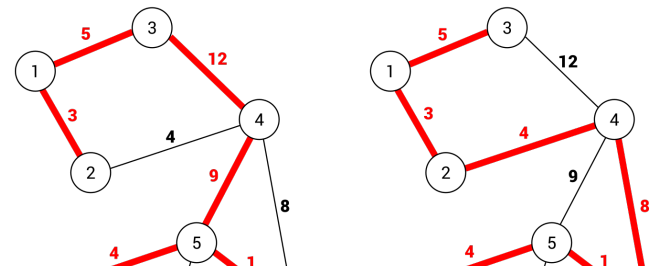
$$S\left(\text{circle with horizontal line and dots}\right) = -\text{circle with horizontal line and dots} + \left(\text{circle with two dots}\right)^2 + \text{arc with two dots} \cdot \text{circle with one dot}$$

R* Automation

Its mostly just recursion and
basic graph theory algorithms



$$Z(\Gamma) = -K \left(\sum_{\substack{S \subsetneq \Gamma, \tilde{S} \subseteq \Gamma \\ S \cap \tilde{S} = \emptyset}} \tilde{Z}(\tilde{S}) * Z(S) * \Gamma / S \setminus \tilde{S} \right)$$



R* Automation

And Taylor expansion

$$Z(\Gamma) = \frac{1}{n!} \sum_i \left(\prod_{j=1}^{\omega} Q_{i(j)}^{\mu_j} \right) Z\left(\prod_{j=1}^{\omega} \partial_{i(j)}^{\mu_j} \Gamma\right)$$

R* Automation

And Taylor expansion

Pros:

$$Z(\Gamma) = \frac{1}{n!} \sum_i \left(\prod_{j=1}^{\omega} Q_{i(j)}^{\mu_j} \right) Z\left(\prod_{j=1}^{\omega} \partial_{i(j)}^{\mu_j} \Gamma \right)$$

- Reduce higher degree divergences to logarithmic ones
- log counterterms are independent of external kinematics, allows for arbitrary IRR

$$Q \text{ --- } \text{[shaded circle]} \text{ --- } -Q = \int \frac{d^D k}{\pi^{D/2}} \left(\frac{1}{(k+Q)^2} \text{[shaded circle]} \text{ --- } k \text{ --- } -k \right) = \left[\text{[shaded circle]} \text{ --- } k \text{ --- } -k \right]_{k^2=1} \int \frac{d^D k}{\pi^{D/2}} \frac{1}{(k+Q)^2 (k^2)^{1+(L-1)\epsilon}}$$

R* Automation

And Taylor expansion

Cons:
$$Z(\Gamma) = \frac{1}{n!} \sum_i \left(\prod_{j=1}^{\omega} Q_{i(j)}^{\mu_j} \right) Z\left(\prod_{j=1}^{\omega} \partial_{i(j)}^{\mu_j} \Gamma \right)$$

- Many terms+and many indices are created in the process.
- Tough tensor reductions of vacuum integrals!
- For the 5-loop beta function required up to rank ~18

Tensor Reduction: basic problem

$$I^{\mu_1 \dots \mu_n} = \sum_{\sigma \in S_2^n} g^{\mu_{\sigma(1)} \mu_{\sigma(2)}} \dots g^{\mu_{\sigma(n-1)} \mu_{\sigma(n)}} I_{\sigma(1) \dots \sigma(n)}$$

➤ Number of scalar coefficients:

$$|S_2^n| = \frac{n!}{(n/2)! 2^{n/2}} = (n-1)!!$$

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- Number of scalar coefficients:

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n	2	4	6	8	10	12	14	16	18	20
$ S_2^n $	1	3	15	105	945	10,395	135,135	2,027,025	34,459,425	654,729,075

- i.e. Passarino-Veltman reduction would lead to solving a *dense* 135,135x135,135 linear system at rank 14!

Tensor reduction: Projectors

- Brute force approach is unfeasible for large n !
(where !=exclamation mark not factorial)

Use shorthand

$$I = \sum_{\sigma \in S_2^n} g(\sigma) I(\sigma)$$

Tensor reduction: Projectors

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Use shorthand

$$I = \sum_{\sigma \in S_2^n} g(\sigma) I(\sigma)$$

Define projector $P(\sigma) = P^{\mu_{\sigma(1)} \cdots \mu_{\sigma(n)}}$ such that $P(\sigma) \cdot g(\sigma') = \delta_{\sigma\sigma'}$

then

$$I(\sigma) = P(\sigma) \cdot I$$

Tensor reduction: Projectors

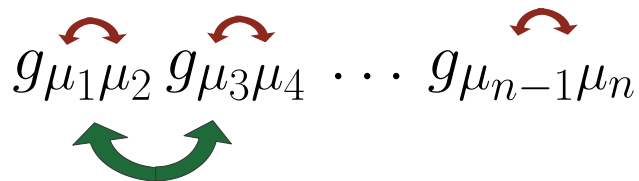
General Ansatz $P(\sigma) = \sum_{\sigma'} c(\sigma, \sigma') g(\sigma')$

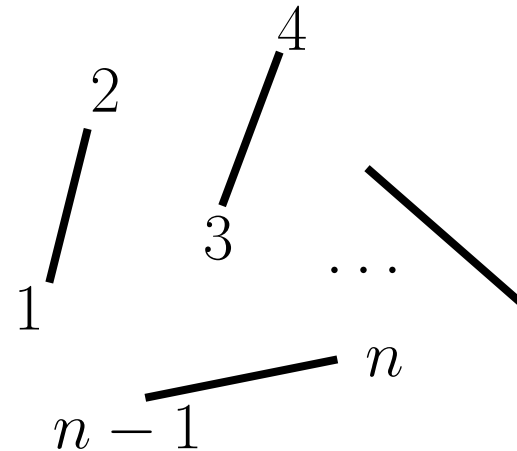
Tensor reduction: Projectors

General Ansatz
$$P(\sigma) = \sum_{\sigma'} c(\sigma, \sigma') g(\sigma')$$

We constrain the coefficients using symmetry (stabilizer) group of $g(\sigma)$:

$$S_2 \times \cdots \times S_2 \times S_{n/2}$$

$$g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} \cdots g_{\mu_{n-1} \mu_n}$$




Tensor reduction: Projectors

The orbit partition formula:

$$P(\sigma) = \sum_k c_k \sum_{\tau \in C_k(\sigma)} g(\tau)$$

k sums over orbits:

C_k : sets of permutations
which are related by internal
symmetry group.

Tensor reduction: Projectors

The orbit partition formula:

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Example: Projector of $g^{\mu_1\mu_2} g^{\mu_3\mu_4}$ (G: $1 \leftrightarrow 2, 3 \leftrightarrow 4, 12 \leftrightarrow 34$)

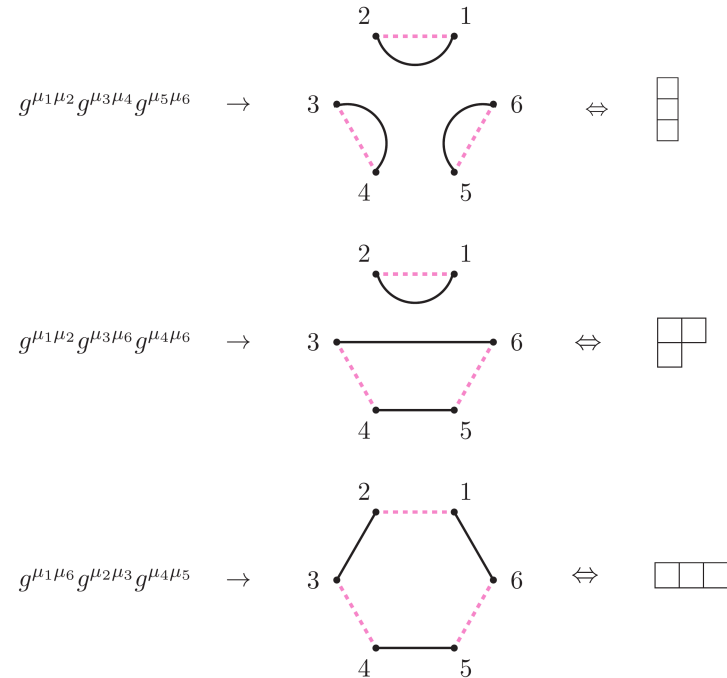
$$P_{\mu_1\mu_2\mu_3\mu_4} = c_{1,1} g_{\mu_1\mu_2} g_{\mu_3\mu_4} + c_2 (g_{\mu_1\mu_3} g_{\mu_2\mu_4} + g_{\mu_1\mu_4} g_{\mu_2\mu_3})$$



Tensor reduction: Projectors

in preparation with J. Vermaseren, J Goode and Sam Teale

- Can label *orbits* (independent c_k s) by *integer partitions* of $n/2$
- Integer partitions $\leftrightarrow$ *cycle structure* of bi-chord graphs

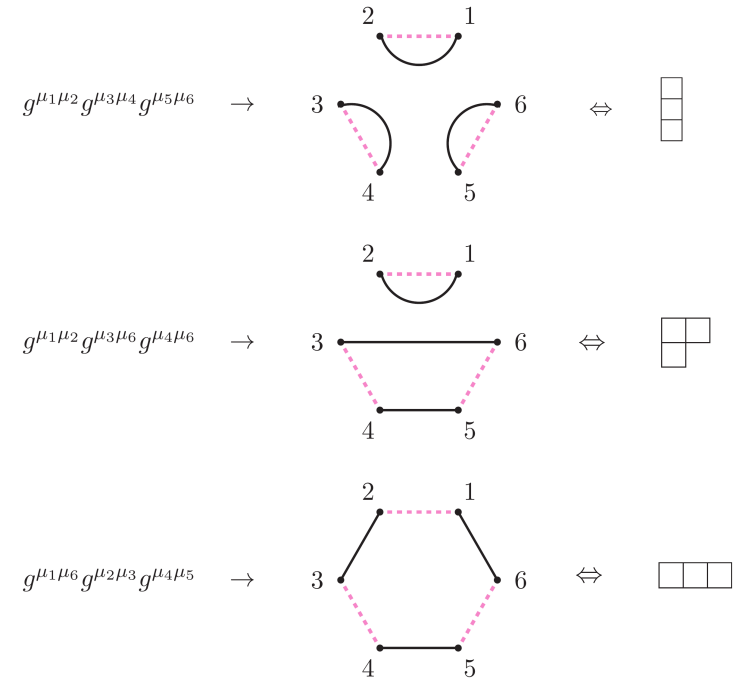


Tensor reduction: Projectors

in preparation with J. Vermaseren, J Goode and Sam Teale

- Can label *orbits* (independent c_k s) by *integer partitions* of $n/2$
- Integer partitions $\leftrightarrow$ *cycle structure* of bi-chord graphs
- Orbit partition tames growth of system size:

n	2	4	6	8	10	12	14	16	18	20
$ S_2^n $	1	3	15	105	945	10,395	135,135	2,027,025	34,459,425	654,729,075
$p(\frac{n}{2})$	1	2	3	5	7	11	15	22	30	42



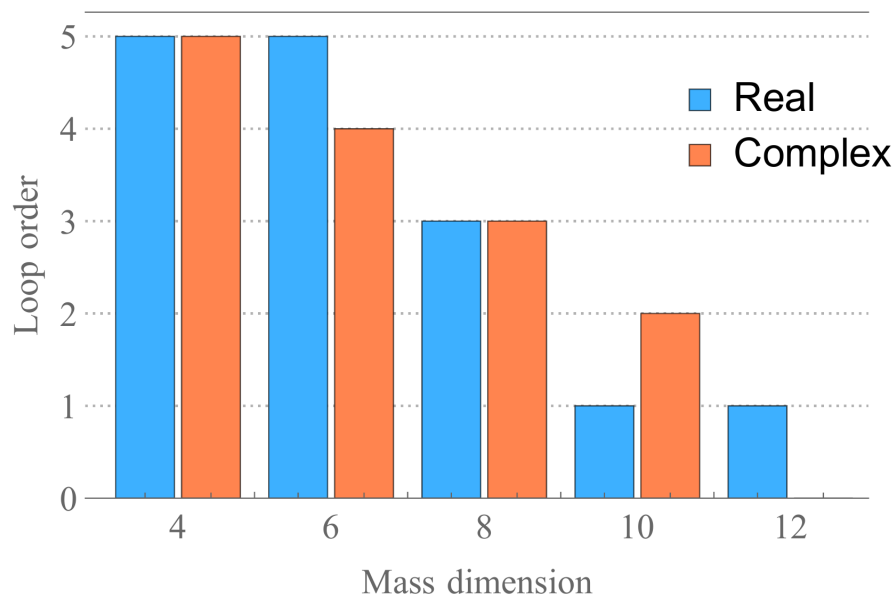
Extension to spinors in progress

Application: Scalar EFT

arxiv:2105.12742 with W Cao, T Melia and J Nepveu

Compute higher orders both in loop and EFT expansion with R^* :

- Great testing ground for method
- Investigate non-renormalisation theorems



Scalar EFT Basics

- Lagrangian

$$\mathcal{L} = \mathcal{L}^{(4)}(\phi, \partial_\mu \phi) + \sum_{n>4} \frac{1}{\Lambda^{n-4}} \sum_i \tilde{c}_i^{b(n)} \tilde{\mathcal{O}}_i^{b(n)}$$

- Basis of operators not unique
 - Redundancies IBP and Field redefinitions
- Couplings mix under renormalisation

$$\mu \frac{d}{d\mu} c_i^{(n)} = \sum_j \gamma_{ij}^{(n)} c_j^{(n)}$$

Complex scalar operators

at mass dimension 8

$$\mathcal{O}_8^{(8)} = \frac{1}{4! 4!} \begin{array}{c} \bullet \circ \\ \bullet \circ \\ \bullet \circ \\ \bullet \circ \end{array} = \frac{1}{4!4!} (\phi^* \phi)^4$$

$$\mathcal{O}_{6,i}^{(8)} = -\frac{1}{4} \begin{array}{c} \bullet \circ \\ \bullet \circ \\ \bullet \circ \end{array}, -\frac{1}{12} \begin{array}{c} \bullet \circ \\ \bullet \circ \\ \bullet \circ \end{array}, -\frac{1}{12} \begin{array}{c} \bullet \circ \\ \bullet \circ \\ \bullet \circ \end{array}$$

$$\mathcal{O}_{4,i}^{(8)} = \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}, \frac{1}{4} \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}, \frac{1}{4} \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array},$$

$$\begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}, \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}, \frac{1}{2} \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array},$$

$$\frac{1}{2} \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}, \frac{1}{4} \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}, \frac{1}{2} \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array}$$

$$\mathcal{O}_2^{(8)} = - \begin{array}{c} \bullet \circ \\ \bullet \circ \end{array} = -(\partial^\alpha \partial^\beta \partial^\gamma \phi^*)(\partial_\alpha \partial_\beta \partial_\gamma \phi)$$

- dots = complex fields
- Circles= complex conjugate fields
- Lines= contracted pair of derivatives
- Conformal Primaries:

$$\mathcal{O}_8^{(8)c} = \mathcal{O}_8^{(8)}$$

$$\mathcal{O}_6^{(8)c} = -\frac{2}{3} \mathcal{O}_{6,1}^{(8)} + (\mathcal{O}_{6,2}^{(8)} + \mathcal{O}_{6,3}^{(8)})$$

$$\begin{aligned} \mathcal{O}_4^{(8)c}(x, y) = & (5x - y) \mathcal{O}_{4,1}^{(8)} + 2x \mathcal{O}_{4,2}^{(8)} + 2x \mathcal{O}_{4,3}^{(8)} - 4x \mathcal{O}_{4,4}^{(8)} - 4x \mathcal{O}_{4,5}^{(8)} \\ & + (4y - 16x) \mathcal{O}_{4,6}^{(8)} + (4y - 16x) \mathcal{O}_{4,7}^{(8)} + 4y \mathcal{O}_{4,8}^{(8)} + (36x - 8y) \mathcal{O}_{4,9}^{(8)} \end{aligned}$$

Anomalous dimension matrix at mass dimension 8

- Most zeroes were expected by a selection rule by Bern et al.
- Two unexpected zeroes were found!

$$\gamma_c^{(8)} = \begin{pmatrix} g^3 \mathcal{O}_8^{(8)c} & g^2 \mathcal{O}_6^{(8)c} & g \mathcal{O}_4^{(8)c}{}_{(1,0)} & g \mathcal{O}_4^{(8)c}{}_{(0,1)} \\ \begin{pmatrix} 29g - 409g^2 \\ + \left(2352\zeta_3 + \frac{57765}{8}\right)g^3 \end{pmatrix} & \boxed{0g} - 240g^2 + \left(2304\zeta_3 + 5882\right)g^3 & \frac{54216g}{5} - \frac{958314g^2}{5} + \left(\frac{4083264\zeta_3}{5} + \frac{55731313}{15}\right)g^3 & -\frac{8856g}{5} + \frac{159894g^2}{5} - \left(\frac{713664\zeta_3}{5} + \frac{9281093}{15}\right)g^3 \\ 0g + 0g^2 + \boxed{0g^3} & 4g - \frac{122g^2}{3} + \left(216\zeta_3 + \frac{4559}{12}\right)g^3 & -\frac{679g}{5} + \frac{14209g^2}{12} - \left(\frac{29216\zeta_3}{5} + \frac{3248605}{324}\right)g^3 & \frac{164g}{5} - \frac{9233g^2}{36} + \left(\frac{6056\zeta_3}{5} + \frac{2740291}{1296}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 0g + 0g^2 + \frac{5g^3}{108} & \frac{11g}{3} - \frac{29g^2}{180} - \left(\frac{32\zeta_3}{3} + \frac{97091}{9720}\right)g^3 & -\frac{4g}{3} + \frac{1057g^2}{540} - \left(\frac{16\zeta_3}{3} + \frac{10868}{1215}\right)g^3 \\ 0g + 0g^2 + 0g^3 & 0g + 0g^2 + \frac{115g^3}{324} & \frac{46g}{3} - \frac{14557g^2}{540} + \left(96\zeta_3 + \frac{198001}{2430}\right)g^3 & -\frac{19g}{3} + \frac{2809g^2}{180} - \left(\frac{176\zeta_3}{3} + \frac{718739}{9720}\right)g^3 \end{pmatrix}$$

which multiplies the couplings of

$$\left\{ \mathcal{O}_8^{(8)c}, \mathcal{O}_6^{(8)c}, \mathcal{O}_4^{(8)c}{}_{(1,0)}, \mathcal{O}_4^{(8)c}{}_{(0,1)} \right\}.$$

Similar at mass dimension 6

$$\gamma_c^{(6)} = \begin{pmatrix} g^2 \mathcal{O}_6^{(6)c} & g \mathcal{O}_4^{(6)c} \\ \left(\begin{array}{l} 14g - \frac{297g^2}{2} + \left(816\zeta_3 + \frac{14981}{8}\right)g^3 \\ -\left(\frac{32087\zeta_3}{2} - 2892\zeta_4 + 23320\zeta_5 + \frac{888983}{32}\right)g^4 \\ 0g + 0g^2 + \boxed{0g^3} + \frac{5g^4}{6} \end{array} \right) & \left(\begin{array}{l} \boxed{0g} - \frac{45g^2}{2} + \left(216\zeta_3 + \frac{6153}{16}\right)g^3 \\ -\left(783\zeta_3 + 8100\zeta_5 + \frac{74079}{16}\right)g^4 \\ -g + \frac{13g^2}{2} - \left(36\zeta_3 + \frac{383}{12}\right)g^3 \\ +\left(\frac{769\zeta_3}{2} - 123\zeta_4 + 560\zeta_5 + \frac{7893}{32}\right)g^4 \end{array} \right) \end{pmatrix}$$

- The existence of upper right zero appears only in the conformal primary basis – we have little understanding why
- The lower one seems basis independent and we have no idea why it appears.

and mass dimension 10

$$\gamma_c^{(10)} = \begin{pmatrix} g^4 \mathcal{O}_{10}^{(10)c} & g^3 \mathcal{O}_8^{(10)c} & g^2 \mathcal{O}_6^{(10)c}{}_{(1,0,0,0)} & g^2 \mathcal{O}_6^{(10)c}{}_{(0,1,0,0)} & g^2 \mathcal{O}_6^{(10)c}{}_{(0,0,1,0)} & g^2 \mathcal{O}_6^{(10)c}{}_{(0,0,0,i)} & g \mathcal{O}_4^{(10)c}{}_{(1,0)} & g \mathcal{O}_4^{(10)c}{}_{(0,1)} \\ 50g - \frac{1725g^2}{2} & \boxed{0g} - 1125g^2 & \frac{412500g}{7} - \frac{61613665g^2}{42} & \frac{3300g}{7} - \frac{19655g^2}{6} & \frac{141900g}{7} - \frac{6734025g^2}{14} & 0 & -\frac{500460g}{7} + \frac{31376515g^2}{21} & -64740g + \frac{16819700g^2}{21} \\ 0g + \boxed{0g^2} & 15g - \frac{385g^2}{2} & -\frac{1474g}{7} + \frac{83689g^2}{45} & -\frac{234g}{7} + \frac{386467g^2}{315} & \frac{326g}{7} - \frac{4577g^2}{7} & 0 & -\frac{43732g}{35} + \frac{5863532g^2}{315} & \frac{14272g}{5} - \frac{15354406g^2}{315} \\ 0g + 0g^2 & 0g + 0g^2 & \frac{35g}{6} - \frac{678389g^2}{15120} & -\frac{g}{6} + \frac{24379g^2}{15120} & \frac{7g}{6} - \frac{28265g^2}{3024} & 0 & \frac{1006g}{105} + \frac{1249571g^2}{15120} & -\frac{967g}{30} + \frac{1903001g^2}{3780} \\ 0g + 0g^2 & 0g + 0g^2 & \frac{4g}{3} - \frac{331g^2}{630} & -\frac{2g}{3} - \frac{5401g^2}{945} & 2g - \frac{14965g^2}{756} & 0 & -\frac{4159g}{105} + \frac{3146033g^2}{7560} & \frac{914g}{15} - \frac{103037g^2}{315} \\ 0g + 0g^2 & 0g + 0g^2 & \frac{15g}{2} - \frac{247109g^2}{5040} & -\frac{g}{2} - \frac{4127g^2}{1680} & \frac{7g}{2} - \frac{31333g^2}{1008} & 0 & -\frac{1769g}{35} + \frac{243011g^2}{336} & \frac{573g}{10} + \frac{37049g^2}{1260} \\ 0 & 0 & 0 & 0 & 0 & \frac{10g}{3} - \frac{770g^2}{27} & 0 & 0 \\ 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0 & -2g + \frac{223g^2}{24} & -g + \frac{7g^2}{24} \\ 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0g + 0g^2 & 0 & -g + \frac{7g^2}{12} & -3g + \frac{251g^2}{24} \end{pmatrix}$$

Summary & Outlook

- R^* formalism
 - Discussed Hopf algebra framework to streamline operations, e.g. IR counterterm via antipode relation
- Discussed high rank tensor problem with R^*
 - Presented an orbit partition formula for the projector
 - In progress: extend to spinor indices
- Finally discussed application in Scalar EFT
 - Two new zeros were found in mixing matrix
 - In progress: Extending non-renormalisation thms to multi-linear mixing