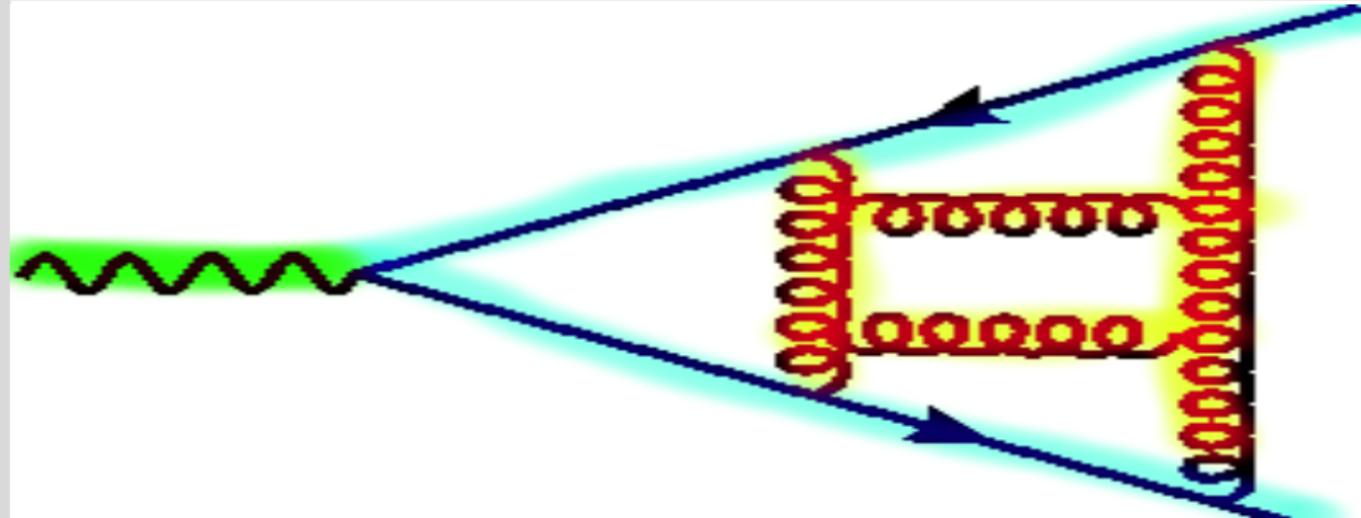


Four loop photon quark and Higgs gluon form factors

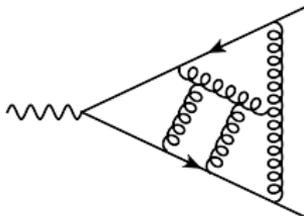
Matthias Steinhauser | High Precision for Hard Processes at the LHC, Newcastle, UK, Sep. 20-22, 2022

TTP KARLSRUHE



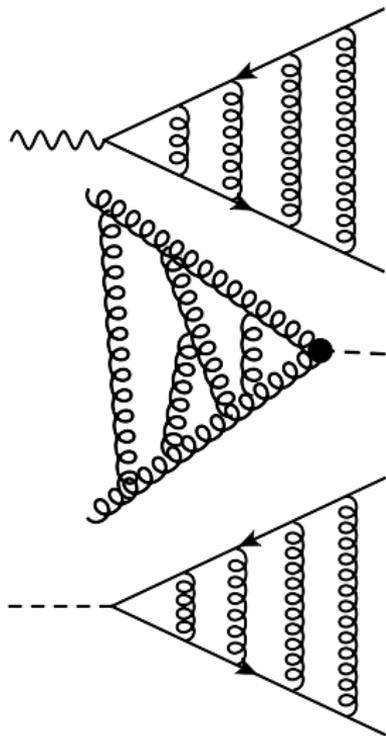
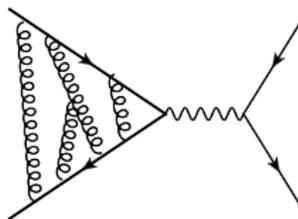
Four loop photon quark and gluon Higgs form factors

- Motivation
- History
- Technicalities
- Results
- Conclusions

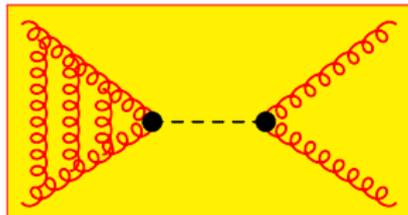


Form factors for physical processes

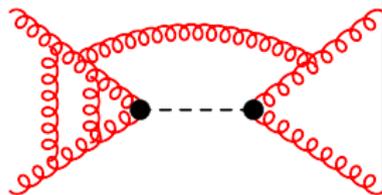
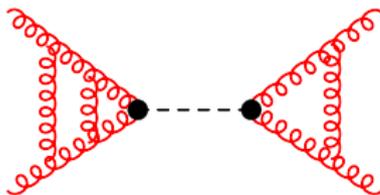
- γ_{cusp}
- $N^4\text{LO}$ virtual corrections
- $e^+e^- \rightarrow Q\bar{Q}$
- Drell-Yan
- Higgs production
- Higgs decay: $H \rightarrow Q\bar{Q}$, $A \rightarrow Q\bar{Q}$



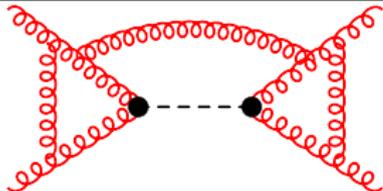
Example: Higgs production at the LHC



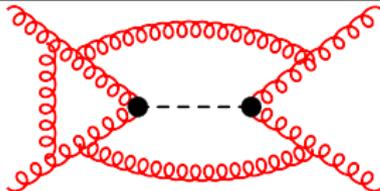
[Baikov,Chetyrkin,Smirnov,Smirnov,
Steinhauser'09],
[Gehrmann,Glover,Huber,Ikizlerli,
Studerus'10]; [Lee,Smirnov'10]



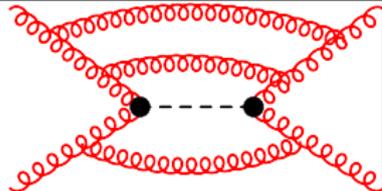
[Duhr,Gehrmann'13], [Li,Zhu'13],
[Dulat,Mistlberger'14],
[Duhr,Gehrmann,Jaquier'14]



[Anastasiou,Duhr,Dulat,Herzog,
Mistlberger'13], [Kilgore'13]



[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,
Herzog,Mistlberger'14],
[Li,von Manteuffel,Schabinger,Zhu'14]



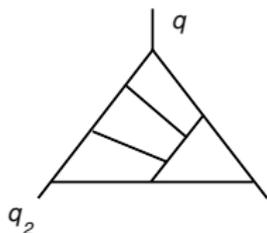
[Anastasiou,Duhr,Dulat,Mistlberger'13]

$N^3\text{LO}$: [Anastasiou,Duhr,Dulat,Herzog,Mistlberger'15]

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger'16; Mistlberger'18]

- 4 loops: [this talk]

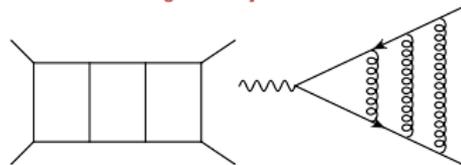
- 3-point, 1 scale ($q^2 = s \neq 0; q_1^2 = q_2^2 = 0$)
- 3-point, 2 scales ($q^2 = s \neq 0; q_1^2 = 0; q_2^2 \neq 0$)



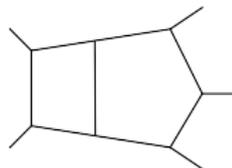
- 3 loops: [Henn,Mistlberger,Smirnov,Wasser'20; Bargiela,Caola,von Manteuffel,Tancredi'21;

Caola,Chakraborty,Gambuti,von Manteuffel,Tancredi'22; ... ; Giulio Gambuti's and Fabian Lange's talks]

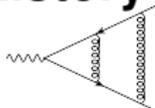
- 4-point, 2 scales (s, t)
- 3-point, 2 scales (s, m^2)



- 2 loops: 5-point [Abreu,Agarwal,Badger,Bronnum-Hansen,...,Zoia]
- 2 loops, $2 \rightarrow 2$ with masses, ...



History of massless form factors



[Kramer,Lampe'87; Matsuura,van der Marck,van Neerven'88;

Harlander'00 (ggh); Ravindran,Smith,van Neerven'05; Gehrmann,Huber,Maitre'05]

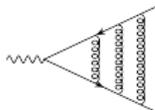
pole part: [Moch,Vermaseren,Vogt'05]

fermionic part: [Moch,Vermaseren,Vogt'05]

full: [Baikov,Chetyrkin,Smirnov,Smirnov,Steinhauser'09]

[Gehrmann,Glover,Huber,Ikizlerli,Studerus'10]; [Lee,Smirnov'10]

MIs: [Heinrich,Huber,Kosower,Smirnov'09]



all n_f terms, large- N_c : [Henn,Smirnov,Smirnov,Steinhauser'16]

n_f^3 terms Higgs-gluon and photon-quark FF: [von Manteuffel,Schabinger'16]

full large- N_c : [Henn,Lee,Smirnov,Smirnov,Steinhauser'16]

complete n_f^2 terms: [Lee,Smirnov,Smirnov,Steinhauser'17]

$(d_F^{abcd})^2$: [Lee,Smirnov,Smirnov,Steinhauser'19; von Manteuffel,Panzer,Schabinger'20]

full $1/\epsilon^2$ (γ_{cusp}): [Brüser et al'19; Mistlberger, Korchemsky'19; Lee et al.'19;

von Manteuffel,Panzer,Schabinger'20] [numerically [Moch,Ruijl,Ueda,Vermaseren,Vogt'17'18]]

full $1/\epsilon$ (γ_{coll}): [Henn,Smirnov,Smirnov,Steinhauser'16; Lee et al.'17'19;

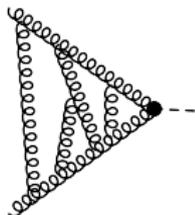
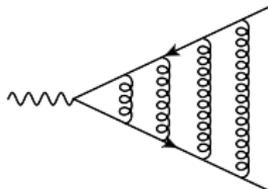
von Manteuffel,Panzer,Schabinger'20; Agarwal,von Manteuffel,Panzer,Schabinger'21]

full result: combine 2 groups with complementary techniques

all- n_f , $N = 4$ SYM, $q\bar{q}\gamma$, ggh: [Lee,von Manteuffel,Schabinger,Smirnov,Smirnov,Steinhauser'21'22]

$b\bar{b}h$: [Chakraborty,Huber,Lee,von Manteuffel,Schabinger,Smirnov,Smirnov,Steinhauser'22]

- Generation of amplitudes, projection to scalar integrals, traces, ...



$$F_q(q^2) = -\frac{1}{4(1-\epsilon)q^2} \text{Tr}(\not{q}_2 \Gamma_q^\mu \not{q}_1 \gamma_\mu)$$

$$F_g(q^2) = \frac{(q_1 \cdot q_2 g_{\mu\nu} - q_{1,\mu} q_{2,\nu} - q_{1,\nu} q_{2,\mu})}{2(1-\epsilon)} \Gamma_g^{\mu\nu}$$

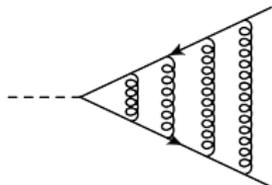
$$F_b(q^2) = -\frac{1}{2q^2} \text{Tr}(\not{q}_2 \Gamma_b \not{q}_1)$$

- Reduction to MIs

- FIRE [Smirnov, Chukharev'19]
- Finred [von Manteuffel]

- Computation of MIs

- A. finite integrals
- B. differential equations



- Construct finite integrals in $d = d_0 - 2\epsilon$ dimensions; $d_0 = 4, 6, \dots$

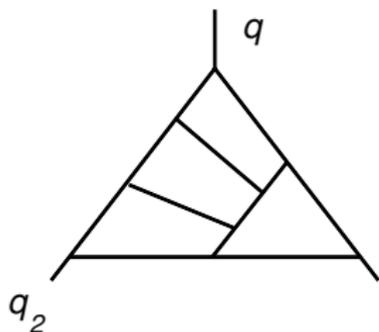
[Panzer'14; von Manteuffel, Panzer, Schabinger'14]

- Expand for $\epsilon \rightarrow 0$
- If linearly reducible: integrate analytically with HyperInt [Panzer'14]
- Compute individual integrals; up to weight 8
- Example: 12-line MI:

$$\begin{aligned}
 \begin{array}{c} (6-2\epsilon) \\ \text{Diagram} \end{array} &= \left(-\frac{119}{48} \zeta_7 - \frac{5}{6} \zeta_5 \zeta_2 - \frac{53}{10} \zeta_3 \zeta_2^2 + 3 \zeta_3^2 + \frac{79}{42} \zeta_2^3 + \frac{25}{6} \zeta_5 - \frac{5}{3} \zeta_3 \zeta_2 + \frac{1}{15} \zeta_2^2 + 2 \zeta_3 \right) \\
 &+ \epsilon \left(-\frac{991}{30} \zeta_{5,3} - \frac{323}{2} \zeta_5 \zeta_3 - \frac{81}{2} \zeta_3^2 \zeta_2 + \frac{127223}{31500} \zeta_2^4 - \frac{2827}{24} \zeta_7 + \frac{73}{6} \zeta_5 \zeta_2 - 14 \zeta_3 \zeta_2^2 \right. \\
 &\left. + \frac{41}{3} \zeta_3^2 + \frac{1696}{315} \zeta_2^3 + \frac{401}{3} \zeta_5 + \frac{206}{3} \zeta_3 \zeta_2 + \frac{23}{15} \zeta_2^2 + 14 \zeta_3 \right) + \mathcal{O}(\epsilon^2)
 \end{aligned}$$

- F_q and F_g up to $1/\epsilon$ [von Manteuffel, Panzer, Schabinger'20]

Computation of MIs: B. differential equations



- Physics: $q^2 \neq 0$ and $q_2^2 = (q_2 + q)^2 = 0$
- However: integrals are **simple** for $q_2^2 = q^2$

[Baikov,Chetyrkin,Kühn'05'08;...;Lee,Smirnov,Smirnov'11]

- Reduction: FIRE [Smirnov,Chukharev'19]
 \oplus LiteRed [Lee]

- Introduce arbitrary $q_2^2 \leftrightarrow$ differential equations

- Boundary conditions: $q_2^2 = q^2$

- Transport information from $q_2^2 = q^2$ to $q_2^2 = 0$

[Henn,Smirnov,Smirnov'14]

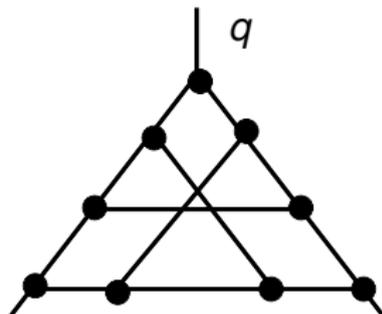
- **All MIs** of a given family are computed simultaneously

- Use **canonical basis**

[Henn'13'14] [Lee'14] **Libra** [Lee'20]

solution: iterated integrals

- $q_2^2 \rightarrow 0$: $(q_2^2/q^2)^{a\epsilon}$ extract term with $a = 0$



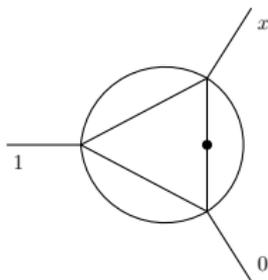
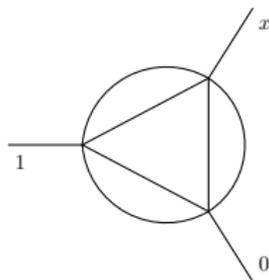
- $x = \frac{q_2^2}{q^2}$
- alphabet: $\frac{1}{x}, \frac{1}{x+1}, \frac{1}{x-1}, \frac{1}{x-4}, \frac{1}{x-1/4}, \frac{1}{(1-x)x_1}, \frac{1}{xx_2}, \frac{1}{xx_3}$
with
 $x_1 = \sqrt{x}, x_2 = \sqrt{x - 1/4}, x_3 = \sqrt{1/x - 1/4}$
- each iterated integral in the results for master integrals contains at most one of x_1, x_2, x_3
- construct uniformly transcendental (UT) basis (of 1-scale integrals)

Example

$$c_0 = L_0^{-1} U_{01} L_1 c_1 = L_0^{-1} U_{01} C_1$$

 \Leftrightarrow

$$C_0 = U_{01} C_1$$



$$\frac{d}{dx} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix} = \begin{pmatrix} -\frac{2(2\epsilon-1)}{x} & \frac{\epsilon(3\epsilon-1)}{x(2\epsilon-1)(5\epsilon-3)} \\ \frac{2(2\epsilon-1)^2(5\epsilon-3)}{(x-1)x(3\epsilon-1)} & -\frac{(x+1)\epsilon}{(x-1)x} \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}$$

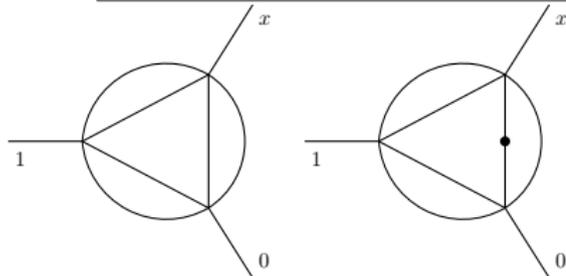
$$x = q_2^2/q^2$$

Example

$$c_0 = L_0^{-1} U_{01} L_1 c_1 = L_0^{-1} U_{01} C_1$$

 \Leftrightarrow

$$C_0 = U_{01} C_1$$



$$\frac{d}{dx} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix} = \begin{pmatrix} -\frac{2(2\epsilon-1)}{x} & \frac{\epsilon(3\epsilon-1)}{x(2\epsilon-1)(5\epsilon-3)} \\ \frac{2(2\epsilon-1)^2(5\epsilon-3)}{(x-1)x(3\epsilon-1)} & -\frac{(x+1)\epsilon}{(x-1)x} \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}$$

■ **Libra** [Lee'20] $\Leftrightarrow \epsilon$ form: $j = TJ$

$$\frac{d}{dx} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \epsilon S(x) \begin{pmatrix} J_1 \\ J_2 \end{pmatrix}$$

$$S(x) = \frac{S_0}{x} + \frac{S_1}{x-1} \quad S_0 = \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix} \quad S_1 = \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}$$

Example

■ **Libra** [Lee'20] $\Leftrightarrow \epsilon$ form: $j = Tj$

$$C_0 = U_{01} C_1$$

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■ **Libra** [Lee'20] $\Leftrightarrow \begin{pmatrix} [j_1]_{x^0} \\ [j_1]_{x^2-3\epsilon} \end{pmatrix} = L_0^{-1} U_{01} L_1 \begin{pmatrix} [j_1]_{(1-x)^0} \\ [j_2]_{(1-x)^{-2\epsilon}} \end{pmatrix}$

$[j_1]_{(1-x)^0}$ simple
 $[j_2]_{(1-x)^{-2\epsilon}} = 0$
 $[j_1]_{x^0}$ wanted

$$L_0 = f(\epsilon)^{-1} \begin{pmatrix} \frac{2(3-5\epsilon)(1-2\epsilon)^2}{3(1-3\epsilon)} & \frac{(3-5\epsilon)(1-3\epsilon)(2-3\epsilon)}{2(1-4\epsilon)} \\ \frac{4(3-5\epsilon)(1-2\epsilon)^2}{3(1-3\epsilon)} & -\frac{(3-5\epsilon)(1-3\epsilon)(2-3\epsilon)}{2(1-4\epsilon)} \end{pmatrix}$$

$$L_1 = f(\epsilon)^{-1} \begin{pmatrix} (3-5\epsilon)(1-2\epsilon) & 0 \\ 0 & \frac{(1-3\epsilon)\epsilon}{1-4\epsilon} \end{pmatrix}$$

$$U_{01} = \begin{pmatrix} 1 + 8\zeta_3 \epsilon^3 - \frac{\pi^4 \epsilon^4}{9} + \dots & -\frac{\pi^2 \epsilon^2}{3} - 2\zeta_3 \epsilon^3 - \frac{5\pi^4 \epsilon^4}{18} + \dots \\ \frac{2\pi^2 \epsilon^2}{3} - 4\zeta_3 \epsilon^3 + \frac{5\pi^4 \epsilon^4}{9} + \dots & 1 - 8\zeta_3 \epsilon^3 - \frac{\pi^4 \epsilon^4}{9} + \dots \end{pmatrix}$$

Example

$$C_0 = U_{01} C_1$$

$$\begin{aligned}
 [j_1]_{(1-x)^0} & \text{ simple} \\
 [j_2]_{(1-x)^{-2\epsilon}} & = 0 \\
 [j_1]_{x^0} & \text{ wanted}
 \end{aligned}$$

■ **Libra** [Lee'20] $\Leftrightarrow \begin{pmatrix} [j_1]_{x^0} \\ [j_1]_{x^{2-3\epsilon}} \end{pmatrix} = L_0^{-1} U_{01} L_1 \begin{pmatrix} [j_1]_{(1-x)^0} \\ [j_2]_{(1-x)^{-2\epsilon}} \end{pmatrix}$

$$L_0 = f(\epsilon)^{-1} \begin{pmatrix} \frac{2(3-5\epsilon)(1-2\epsilon)^2}{3(1-3\epsilon)} & \frac{(3-5\epsilon)(1-3\epsilon)(2-3\epsilon)}{2(1-4\epsilon)} \\ \frac{4(3-5\epsilon)(1-2\epsilon)^2}{3(1-3\epsilon)} & -\frac{(3-5\epsilon)(1-3\epsilon)(2-3\epsilon)}{2(1-4\epsilon)} \end{pmatrix}$$

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$$U_{01} = \begin{pmatrix} 1 + 8\zeta_3\epsilon^3 - \frac{\pi^4\epsilon^4}{9} + \dots & -\frac{\pi^2\epsilon^2}{3} - 2\zeta_3\epsilon^3 - \frac{5\pi^4\epsilon^4}{9 \cdot 18} + \dots \\ \frac{2\pi^2\epsilon^2}{3} - 4\zeta_3\epsilon^3 + \frac{5\pi^4\epsilon^4}{9} + \dots & 1 - 8\zeta_3\epsilon^3 - \frac{\pi^4\epsilon^4}{9} + \dots \end{pmatrix}$$

■ Use $f(\epsilon)$ for UT $\Leftrightarrow C_1 = L_1 \begin{pmatrix} [j_1]_{(1-x)^0} \\ 0 \end{pmatrix}$ and $C_0 = U_{01} C_1$ are UT

- Use $f(\epsilon)$ for UT $\Leftrightarrow C_1 = L_1 \begin{pmatrix} [j_1]_{(1-x)^0} \\ 0 \end{pmatrix}$ and $C_0 = U_{01} C_1$ are UT
- Compare

$$C_0 = U_{01} C_1 = \begin{pmatrix} \frac{1}{4} - \frac{\pi^2}{12} \epsilon^2 - \frac{71}{6} \zeta_3 \epsilon^3 + \dots \\ \frac{\pi^2}{6} \epsilon^2 - \zeta_3 \epsilon^3 + \dots \end{pmatrix}$$

to

$$C_0 = L_0 C_0 = f(\epsilon)^{-1} \begin{pmatrix} \frac{2[j_1]_{x^0} (5\epsilon-3)(2\epsilon-1)^2}{3(3\epsilon-1)} + \frac{[j_1]_{x^{2-3\epsilon}} (3\epsilon-2)(3\epsilon-1)(5\epsilon-3)}{2(4\epsilon-1)} \\ \frac{4[j_1]_{x^0} (2\epsilon-1)^2 (5\epsilon-3)}{3(3\epsilon-1)} - \frac{[j_1]_{x^{2-3\epsilon}} (3\epsilon-2)(3\epsilon-1)(5\epsilon-3)}{2(4\epsilon-1)} \end{pmatrix}$$

\Leftrightarrow extraction of $[j_1]_{x^0}$ NOT possible

Example

- Use $f(\epsilon)$ for UT $\Leftrightarrow C_1 = L_1 \begin{pmatrix} [j_1]_{(1-x)^0} \\ 0 \end{pmatrix}$ and $C_0 = U_{01} C_1$ are UT

- Compare

$$C_0 = U_{01} C_1 = \begin{pmatrix} \frac{1}{4} - \frac{\pi^2}{12} \epsilon^2 - \frac{71}{6} \zeta_3 \epsilon^3 + \dots \\ \frac{\pi^2}{6} \epsilon^2 - \zeta_3 \epsilon^3 + \dots \end{pmatrix}$$

to

$$C_0 = L_0 C_0 = f(\epsilon)^{-1} \begin{pmatrix} \frac{2[j_1]_{x^0} (5\epsilon-3)(2\epsilon-1)^2}{3(3\epsilon-1)} + \frac{[j_1]_{x^{2-3\epsilon}} (3\epsilon-2)(3\epsilon-1)(5\epsilon-3)}{2(4\epsilon-1)} \\ \frac{4[j_1]_{x^0} (2\epsilon-1)^2 (5\epsilon-3)}{3(3\epsilon-1)} - \frac{[j_1]_{x^{2-3\epsilon}} (3\epsilon-2)(3\epsilon-1)(5\epsilon-3)}{2(4\epsilon-1)} \end{pmatrix}$$

\Leftrightarrow extraction of $[j_1]_{x^0}$ NOT possible

- Introduce $\tilde{C}_0 = Q^{-1} C_0$ with $Q = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$ (Q diagonalizes S_0 , conserves UT property)

$$\Leftrightarrow \tilde{C}_0 = \begin{pmatrix} \frac{2(2\epsilon-1)^2(5\epsilon-3)}{3(3\epsilon-1)} [j_1]_{x^0} \\ -\frac{(3\epsilon-2)(3\epsilon-1)(5\epsilon-3)}{2(4\epsilon-1)} [j_1]_{x^{2-3\epsilon}} \end{pmatrix} \stackrel{!}{=} Q^{-1} U_{01} C_1$$

$\Leftrightarrow [j_1]_{x^0}$

- Many MIs computed with both methods,
“A. finite integrals” and “B. differential equations”
- FIESTA [Smirnov,Shapurov,Vysotsky]; 10^{-4} relative uncertainty
- Correct IR behaviour
- $1/\epsilon^2$ poles $\Leftrightarrow \gamma_{\text{cusp}}$

[Brüser et al'19; Mistlberger, Korchemsky'19; Lee et al.'19; von Manteuffel, Panzer, Schabinger'20]

$1/\epsilon$ poles $\Leftrightarrow \gamma_{\text{coll}}$ [Henn, Smirnov, Smirnov, Steinhauser'16; Lee et al.'17'19;

von Manteuffel, Panzer, Schabinger'20; Agarwal, von Manteuffel, Panzer, Schabinger'21]

- weight-8 coefficients of $F_q^{(4)}$ and $F_g^{(4)}$ agree after
 $C_F \rightarrow C_A, N_F \rightarrow N_A, d_A^{abcd} d_F^{abcd} \rightarrow d_A^{abcd} d_A^{abcd}$

IR structure

$1/\epsilon$ poles after UV renormalization

↪ Z factors to subtract IR poles

$$\begin{aligned} F_b^{\text{fin}} &= Z_q^{-1} F_b^{\text{ren}} \\ F_q^{\text{fin}} &= Z_q^{-1} F_q^{\text{ren}} \\ F_g^{\text{fin}} &= Z_g^{-1} F_g^{\text{ren}} \end{aligned}$$

$$\begin{aligned} \log Z_r &= a \left(\frac{\Gamma'_1}{4\epsilon^2} + \frac{\Gamma_1}{2\epsilon} \right) + a^2 \left(-\frac{3\beta_0\Gamma'_1}{16\epsilon^3} + \frac{\Gamma'_2 - 4\beta_0\Gamma_1}{16\epsilon^2} + \frac{\Gamma_2}{4\epsilon} \right) + a^3 \left(\frac{11\beta_0^2\Gamma'_1}{72\epsilon^4} + \frac{12\beta_0^2\Gamma_1 - 8\beta_1\Gamma'_1 - 5\beta_0\Gamma'_2}{72\epsilon^3} \right. \\ &+ \frac{\Gamma'_3 - 6\beta_0\Gamma_2 - 6\beta_1\Gamma_1}{36\epsilon^2} + \frac{\Gamma_3}{6\epsilon} \left. \right) + a^4 \left(-\frac{25\beta_0^3\Gamma'_1}{192\epsilon^5} + \frac{\beta_0(13\beta_0\Gamma'_2 + 40\beta_1\Gamma'_1 - 24\beta_0^2\Gamma_1)}{192\epsilon^4} \right. \\ &+ \frac{-7\beta_0\Gamma'_3 - 9\beta_1\Gamma'_2 + 24\beta_0^2\Gamma_2 - 15\beta_2\Gamma'_1 + 48\beta_0\beta_1\Gamma_1}{192\epsilon^3} + \frac{\Gamma'_4 - 8\beta_0\Gamma_3 - 8\beta_1\Gamma_2 - 8\beta_2\Gamma_1}{64\epsilon^2} + \frac{\Gamma_4}{8\epsilon} \left. \right) + \mathcal{O}(a^5) \end{aligned}$$

[Catani'98, ..., Becher,Neubert'09; Gardi,Magnea'09; ... Agarwal,Magnea,Signorile-Signorile,Tripathi'21; ...]

Γ_n and Γ'_n only depend on the type of external state: $r = q, g$

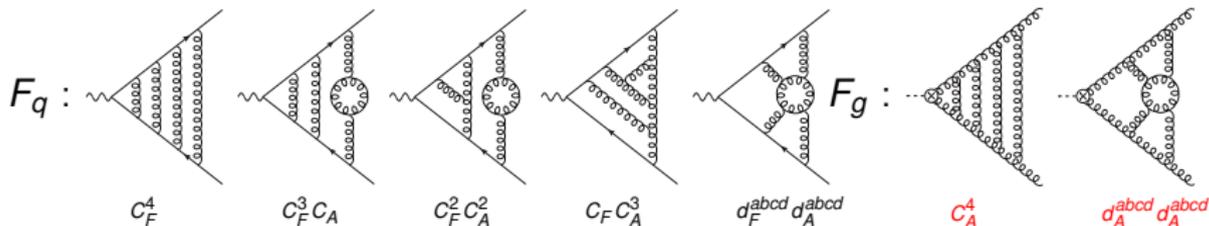
$$\Gamma_n = -\gamma_{\text{cusp},r}^{(n)} \log \left(\frac{\mu^2}{-q^2 - i0} \right) - \gamma_{\text{coll},r}^{(n)}$$

$$a = \frac{\alpha_s}{4\pi}$$

$$\Gamma'_n = -2\gamma_{\text{cusp},r}^{(n)}$$

↪ All requirements fulfilled by our explicit calculations!

Results: F_q and F_g

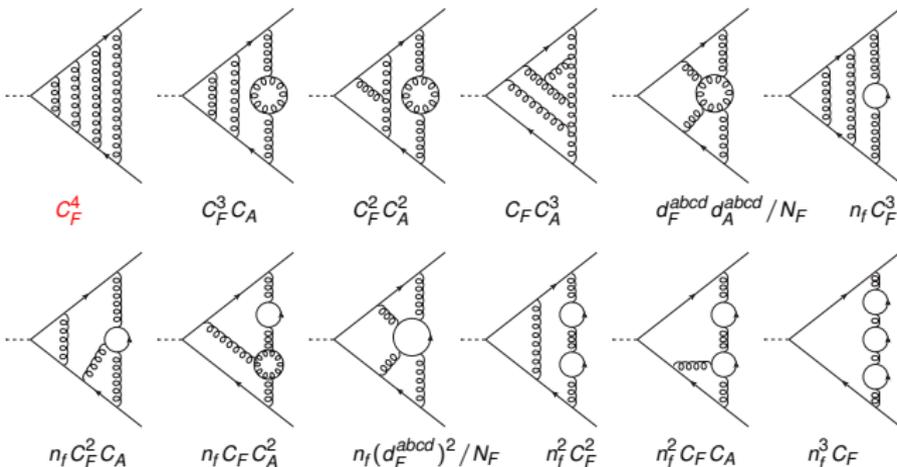


$$\begin{aligned}
 F_g^{(4)} \Big|_{\epsilon^0} = & C_A^4 \left(-\frac{2591}{90} \zeta_{5,3} + \frac{1018949}{90} \zeta_5 \zeta_3 - \frac{35689}{27} \zeta_3^2 \zeta_2 + \frac{18282694}{7875} \zeta_2^4 - \frac{27705161}{504} \zeta_7 + \frac{1160731}{270} \zeta_5 \zeta_2 - \frac{1928564}{405} \zeta_3 \zeta_2^2 \right. \\
 & - \frac{1296845}{1458} \zeta_3^2 - \frac{727183}{1134} \zeta_3^3 + \frac{6161623}{243} \zeta_5 - \frac{3233651}{729} \zeta_3 \zeta_2 + \frac{54443689}{14580} \zeta_2^2 + \frac{839716507}{104976} \zeta_2 - \frac{84995881}{52488} \zeta_3 + \frac{96887974603}{3779136} \Big) \\
 & + \frac{d_A^{abcd} d_A^{abcd}}{N_A} \left(260 \zeta_{5,3} - 5092 \zeta_5 \zeta_3 - 16 \zeta_3^2 \zeta_2 - \frac{496766}{525} \zeta_2^4 - \frac{6776}{3} \zeta_7 - 5016 \zeta_5 \zeta_2 + \frac{2992}{3} \zeta_3 \zeta_2^2 + \frac{31588}{3} \zeta_3^2 \right. \\
 & \left. + \frac{1073972}{945} \zeta_3^3 - 6460 \zeta_5 + \frac{6752}{9} \zeta_3 \zeta_2 + \frac{24616}{45} \zeta_2^2 - \frac{4682}{27} \zeta_2 - \frac{1310}{9} + \frac{68410}{9} \zeta_3 \right) \\
 & + \text{contributions with closed fermion loop}
 \end{aligned}$$

$$\zeta_{5,3} = \sum_{m=1}^{\infty} \sum_{n=1}^{m-1} \frac{1}{m^2 n^3} \approx 0.0377076729848$$

[Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser 22]

F_b



$$\begin{aligned}
 F_b = & C_F^4 \left[\frac{1}{\epsilon^8} \left(\frac{2}{3} \right) + \frac{1}{\epsilon^6} \left(-\frac{4}{3} \zeta_2 + \frac{8}{3} \right) + \frac{1}{\epsilon^5} \left(-\frac{272}{9} \zeta_3 + 12 \zeta_2 + \frac{16}{3} \right) + \frac{1}{\epsilon^4} \left(-\frac{296}{15} \zeta_2^2 - 60 \zeta_3 + \frac{80}{3} \zeta_2 + \frac{68}{3} \right) \right. \\
 & + \frac{1}{\epsilon^3} \left(-\frac{3008}{15} \zeta_5 + \frac{640}{9} \zeta_3 \zeta_2 - 12 \zeta_2^2 - \frac{2336}{9} \zeta_3 + \frac{340}{3} \zeta_2 + 52 \right) + \frac{1}{\epsilon^2} \left(\frac{19360}{27} \zeta_3^2 - \frac{6784}{315} \zeta_2^3 - 1100 \zeta_5 - 480 \zeta_3 \zeta_2 + \frac{118}{15} \zeta_2^2 \right. \\
 & + \frac{668}{9} \zeta_3 + 506 \zeta_2 - \frac{254}{3} \left. \right) + \frac{1}{\epsilon} \left(-\frac{14162}{21} \zeta_7 + \frac{5792}{5} \zeta_5 \zeta_2 + \frac{6208}{9} \zeta_3 \zeta_2^2 - 1180 \zeta_3^2 - \frac{17326}{21} \zeta_2^3 - \frac{113542}{15} \zeta_5 - \frac{21398}{9} \zeta_3 \zeta_2 \right. \\
 & + \frac{4867}{5} \zeta_2^2 + \frac{69733}{9} \zeta_3 + \frac{5159}{2} \zeta_2 - \frac{12707}{6} \left. \right) + \left(-\frac{32384}{15} \zeta_{5,3} + \frac{739328}{45} \zeta_5 \zeta_3 - \frac{66392}{27} \zeta_3^2 \zeta_2 + \frac{7486576}{7875} \zeta_2^4 - \frac{47217}{2} \zeta_7 \right. \\
 & - \frac{31928}{5} \zeta_5 \zeta_2 - \frac{11092}{5} \zeta_3 \zeta_2^2 - \frac{284228}{27} \zeta_3^2 - \frac{250138}{45} \zeta_2^3 - \frac{392059}{15} \zeta_5 - \frac{121270}{9} \zeta_3 \zeta_2 + \frac{28514}{3} \zeta_2^2 + \frac{212006}{3} \zeta_3 + \frac{53859}{4} \zeta_2 \\
 & \left. - \frac{71295}{4} \right) \Big] + \text{other colour factors}
 \end{aligned}$$

[Chakraborty, Huber, Lee, von Manteuffel, Schabinger, Smirnov, Smirnov, Steinhauser '22]

- Massless 4-loop form factors in QCD: F_q, F_g, F_b
- 4-loop $N = 4$ SYM form factor
- Analytic results for all MIs
- > 40.000 Feynman diagrams; $\mathcal{O}(10^9)$ integrals; 294 MIs
⇒ compact final results
- Virtual N⁴LO corrections for $gg \rightarrow H, DY, \dots$ available

