Higher-order QCD corrections and matching at future lepton colliders

Christian T Preuss (ETH Zürich)

High Precision for Hard Processes
Newcastle upon Tyne
21/09/2022

ETH zürich

Outline

1) Hadronic Higgs decays (at future lepton colliders)

2) Event shapes in hadronic Higgs decays
[Gehrmann-De Ridder, Coloretti, CTP 2202.07333]
[Gehrmann-De Ridder, Geissbühler, CTP, Williams 22??.?????]

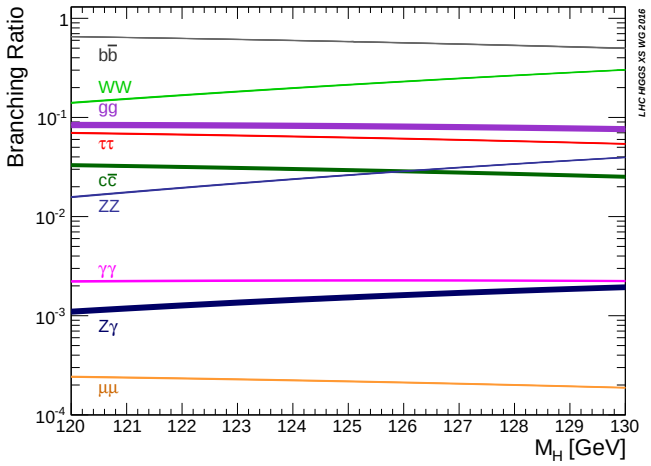
3) Towards fully-differential NNLO+PS matching
[Campbell, Höche, Li, CTP, Skands 2108.07133] + WIP

Hadronic Higgs decays (at future lepton colliders)

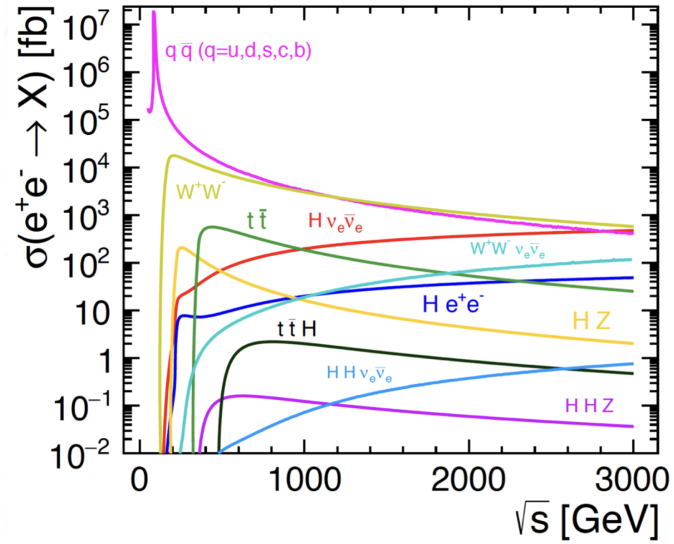
Hadronic Higgs decays

Clean Higgs production will be one of the **striking features** at a future lepton collider.

- sub-dominant Higgs couplings and decay channels will remain elusive at the LHC (e.g., $H \to gg$ decay)
- hadronic Higgs decays offer great potential for **precision QCD studies** (quark vs. gluon jets, C_{F} vs. C_{A}, ...)



[LHC Higgs WG]



Plethora of work on $e^+e^- \to$ jets related to **LEP** studies:

- N^3LO 2j inclusive: [Gorishnii et al. PLB 259 (1991) 144-150] +...
- NNLO 3j differential: [Gehrmann-De Ridder et al. 0711.4711] +...
- N^3LL resummation: [Abbate et al. 1006.3080] +...

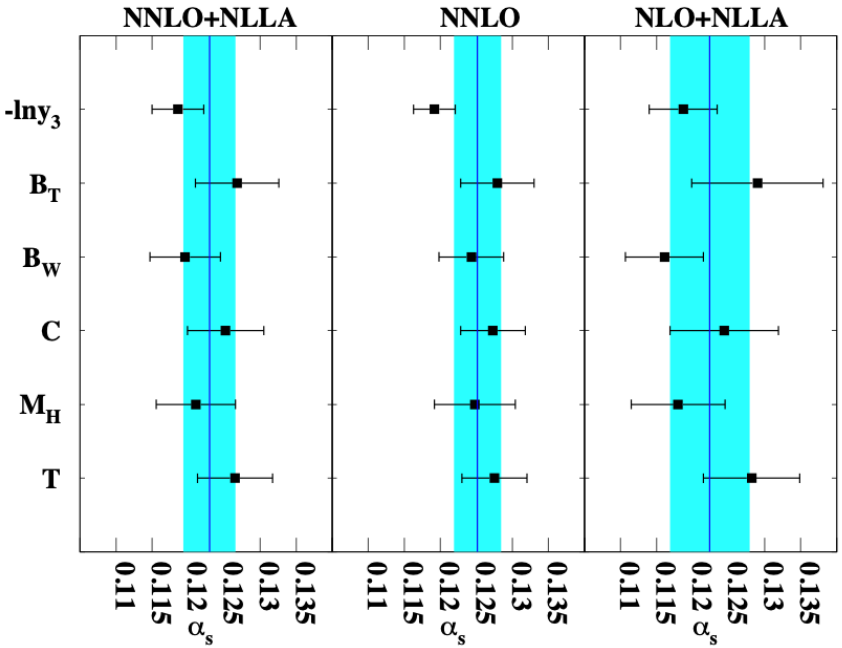
$\Rightarrow$ "Redo" and **extend** work for **hadronic Higgs decays** with focus on **global event shapes**

Why (global) event shapes?

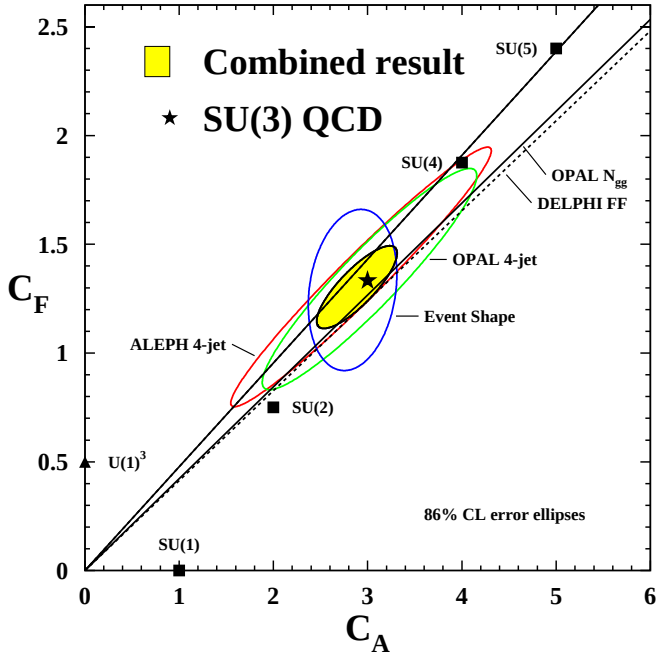
- can be regarded as a class of "**good observables**"
- directly access **geometrical event properties** (non-identified particles)
- can be **calculated reliably** order-by-order in perturbative QCD

Studied extensively at LEP in Z/γ^* decays for **QCD precision measurements**

e.g. α_s extraction [Dissertori et al. 0906.3436]



e.g. C_A, C_F determination [Kluth hep-ex/0309070]



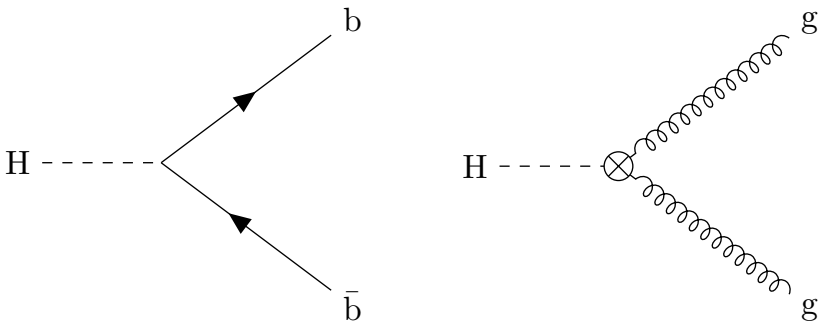
Classical set of 3-jet observables:
T, C, B_{T}, B_{W}, M_{H}, y_{23}

Additional set of 4-jet observables:
T_{Minor}, D, A, B_{Min}, M_{L}, y_{34}

Event shapes in hadronic Higgs decays

Hadronic Higgs decays

Hadronic Higgs decays assumed to proceed in **two categories***:



- Decays to b-quark pair mediated by $Hb\bar{b}$ **Yukawa coupling**: $H \to b\bar{b}$
- Decays to two gluons mediated by Hgg **effective coupling**: $H \to gg$

Event shapes **vanish** in **2-jet limit**.

LO contribution from $H \to 3j$ decays.

At **NLO** in QCD, include
virtual contributions from quark- and gluon loops;
real contributions from gluon emissions and gluon splittings.

	$H \to b\bar{b}$ type	$H \to gg$ type	
LO	$H \to b\bar{b}g$	$H \to ggg$	tree-level
		$H \to gq\bar{q}$	tree-level
NLO	$H \to bbg$	$H \to ggg$	one-loop
		$H \to gq\bar{q}$	one-loop
	$H \to b\bar{b}gg$	$H \to gggg$	tree-level
	$H \to b\bar{b}q\bar{q}$	$H \to ggq\bar{q}$	tree-level
	$H \to b\bar{b}b\bar{b}$	$H \to q\bar{q}q'\bar{q}'$	tree-level

Can event shapes help to discriminate between Higgs decay channels?

*Decay $H \to c\bar{c}$ in principle trivial to include, but omitted here.

Computational setup

Implemented in (publicly-available[†]) **parton-level NNLO MC** EERAD3 [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 1402.4140].

- **On-shell** Higgs decays with $\sqrt{s} = M_H \equiv 125.0\ \mathrm{GeV}$
- **vanishing** kinematical b-mass
- **large** t-mass (HEFT)

$\rightarrow$ straightforward extension to NNLO

Calculate **observable-weighted** distributions of the form

$$\frac{\mathrm{Br}^n_{H\to X}(s,\mu_\mathrm{R})}{\Gamma^n_{H\to X}(s,\mu_\mathrm{R})}\, O\, \frac{\mathrm{d}\Gamma(s,\mu_\mathrm{R},O)}{\mathrm{d}O}$$

with branching ratios defined as

$$\mathrm{Br}^n_{H\to X}(s,\mu_\mathrm{R}) = \frac{\Gamma^n_{H\to X}(s,\mu_\mathrm{R})}{\Gamma^n_{H\to b\bar{b}}(s,\mu_\mathrm{R}) + \Gamma^n_{H\to gg}(s,\mu_\mathrm{R})}$$

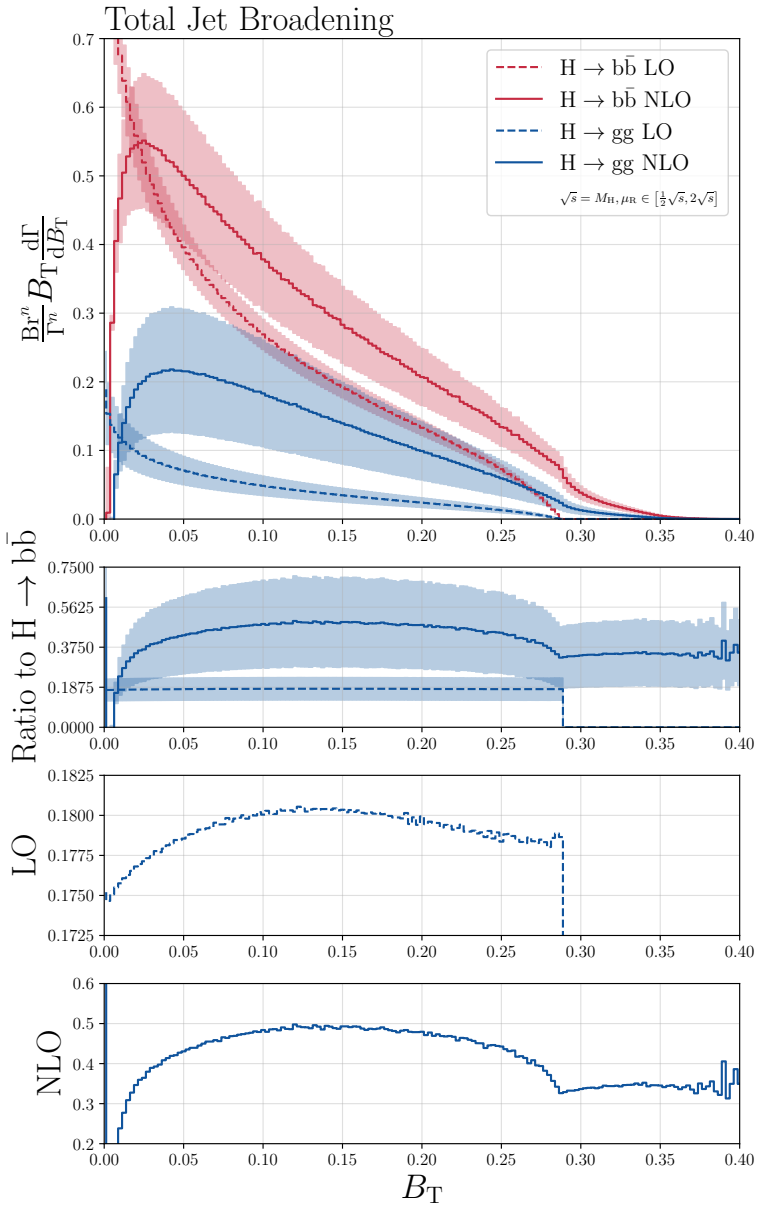
[†] Extension to Higgs decays not public yet.

Results – total jet broadening

$$B_\mathrm{T} = B_1 + B_2 \quad \text{with} \quad B_i = \frac{\sum_{j \in H_i} |\vec{p}_j \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}$$

$B_\mathrm{T} \to 0$: 2-jet event
$B_\mathrm{T} \leq \frac{1}{2\sqrt{3}}$ for 3-particle events

Broad but **peaked** ratio between decay channels.

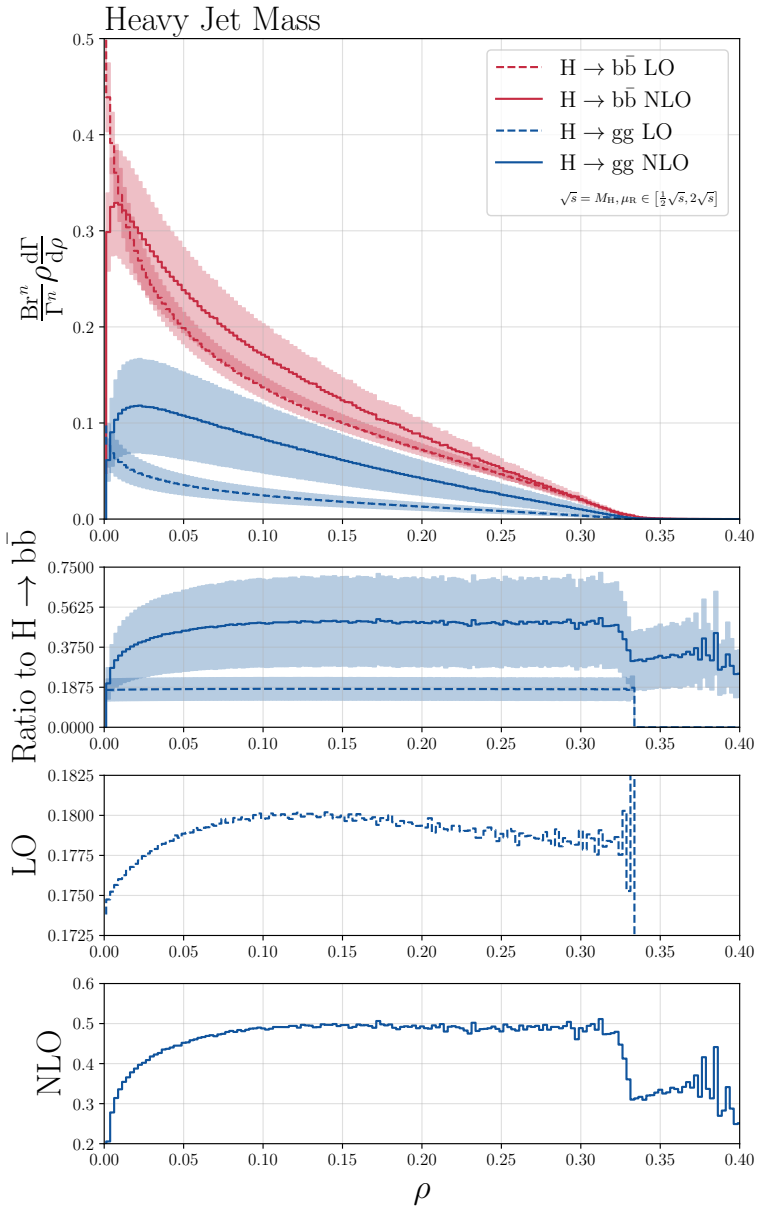


Results – heavy-hemisphere mass

$$\rho \equiv \frac{M_{\mathrm{H}}^2}{s} = \max_{i\in\{1,2\}} \left\{ \frac{1}{E_{\mathrm{vis}}^2} \left(\sum_{j\in H_i} p_j \right)^2 \right\}$$

$\rho \to 0$: 2-jet event
$\rho \leq \frac{1}{3}$ for 3-particle events

Almost constant ratio in multi-jet limit.



Conclusions

3-jet event shapes can be classified into two classes

$Hgg/Hb\bar{b}$ ratio has **plateau in multijet limit** vs. **maximum in transition region**

- first class ("bad discriminators"): C, B_{W}, M_{H}
- second class ("good discriminators"): T, B_{T}, y_{23}^{D}

... but should be enhanced by **well-placed cuts**!

Generally:

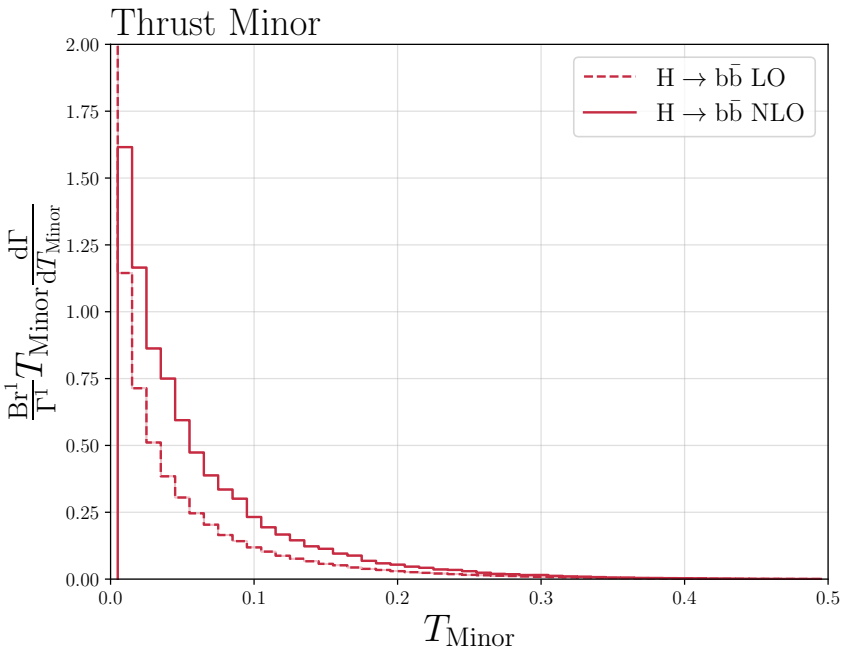
- event-shape distributions **peaked** more towards **multi-jet region** for $H \to gg$
- larger corrections in $H \to gg$ channel (unsurprisingly!)
- sizeable **renormalisation-scale uncertainties**

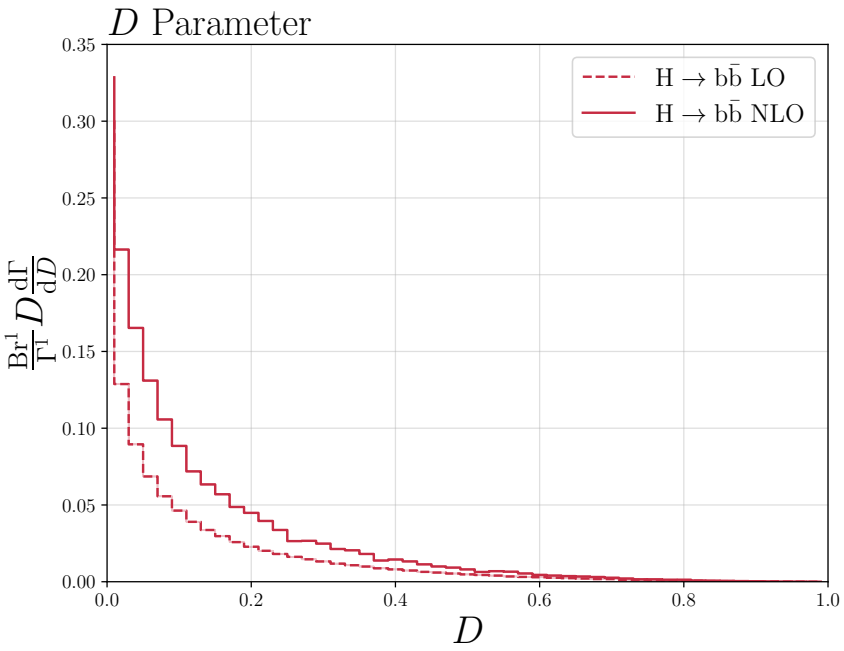
$\Rightarrow$ inclusion of **higher-order** effects **mandatory** (stay tuned!)

Sneak peek: four-jet event shapes

[Gehrmann-De Ridder, Geissbühler, CTP, Williams in preparation]

Similar size of **NLO corrections** for **four-jet event shapes**.





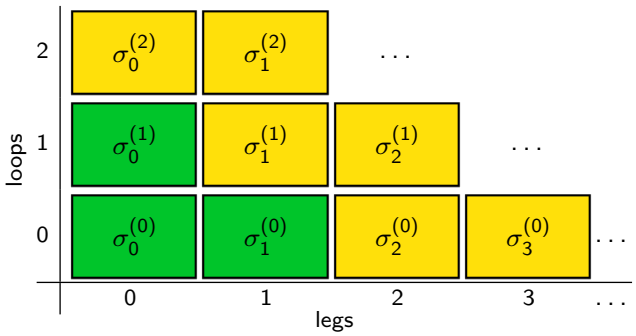
. . . but difficult to model **peak structure** at NLO, so **logarithmic** and **non-perturbative** corrections likely to be more prominent than in three-jet case

Towards fully-differential NNLO+PS matching

Towards fully-differential* NNLO+PS matching

* fully-differential $\equiv$ no auxiliary scales

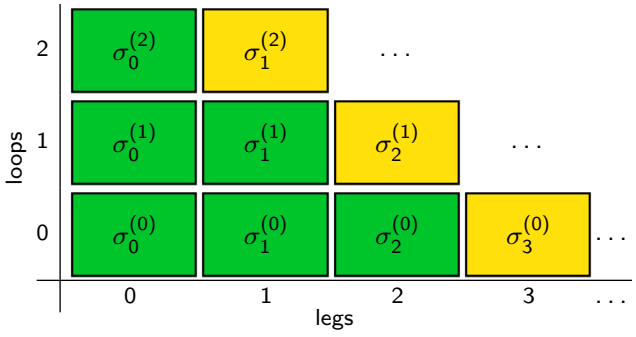
Status of (N)NLO+PS matching



NLO+PS: two **general** approaches

- MC@NLO [Frixione, Webber hep-ph/0204244]
  modified subtraction with shower kernels
- POWHEG [Nason hep-ph/0409146]
  Born-local NLO weight + MEC in shower
- refinements KRKNLO [Jadach et al. 1503.06849]
  and MACNLOPS [Nason, Salam 2111.03553]

+ shower **matches** fixed-order singularity structure

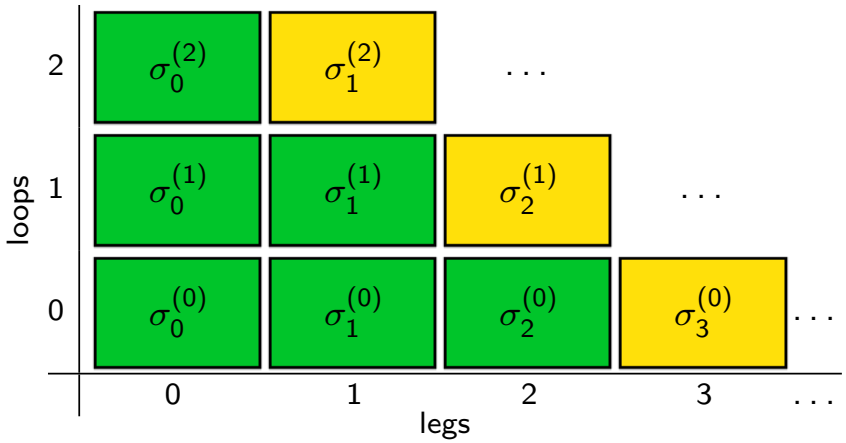


NNLO+PS: first approaches, for some processes

- UN2LOPS [Höche et al. 1405.3607]
  inclusive NNLO + unitary merging
- NNLOPS/MiNNLO$_{\mathrm{PS}}$
  [Hamilton et al. 1212.4504]/[Monni et al. 1908.06987]
  regulated NLO POWHEG $1j$ + NNLO
- GENEVA [Alioli et al. 1211.7049]
  NNLO matched resummation + truncated shower

− shower **does not** match fixed-order singularity structure

Towards NNLO+PS [Campbell, Höche, Li, CTP, Skands 2108.07133]



Idea: "POWHEG at NNLO" (focus here on colour singlet $\to 2j$)

$$\langle O \rangle^{\text{VINCIA}}_{\text{NNLO+PS}} = \int \mathrm{d}\Phi_2\, \mathrm{B}(\Phi_2)\, \underbrace{\boxed{k_{\text{NNLO}}(\Phi_2)}}_{\text{local } K\text{-factor}}\, \underbrace{\boxed{\mathcal{S}_2(t_0, O)}}_{\text{shower operator}}$$

Need:

1. (Born-local) NNLO K-factors
2. shower filling ordered and unordered regions of 1- and 2-emission phase space
3. tree-level MECs in ordered and unordered shower paths
4. NLO MECs in the first emission

...implemented in VINCIA **sector antenna shower** in PYTHIA 8.3 [Brooks, CTP, Skands 2003.00702]

NNLO+PS with sector showers

Key aspect

up to matched order, include **process-specific** **NLO corrections** into shower evolution:

1. correct first branching to exclusive ($< t'$) NLO rate:

$$\Delta^{\mathrm{NLO}}_{2\mapsto 3}(t_0, t') = \exp\left\{ -\int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\, \mathrm{A}_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \right\}$$

2. correct second branching to LO ME:

$$\Delta^{\mathrm{LO}}_{3\mapsto 4}(t', t) = \exp\left\{ -\int_{t}^{t'} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{3\mapsto 4}(\Phi'_{+1}) w^{\mathrm{LO}}_{3\mapsto 4}(\Phi_3, \Phi'_{+1}) \right\}$$

3. add direct $2 \mapsto 4$ branching and correct it to LO ME:

$$\Delta^{\mathrm{LO}}_{2\mapsto 4}(t_0, t) = \exp\left\{ -\int_{t}^{t_0} \mathrm{d}\Phi^{>}_{+2}\, \mathrm{A}_{2\mapsto 4}(\Phi_{+2}) w^{\mathrm{LO}}_{2\mapsto 4}(\Phi_2, \Phi_{+2}) \right\}$$

$\Rightarrow$ entirely based on **MECs** and **sectorisation**

$\Rightarrow$ **by construction**, expansion of extended shower **matches** NNLO singularity structure

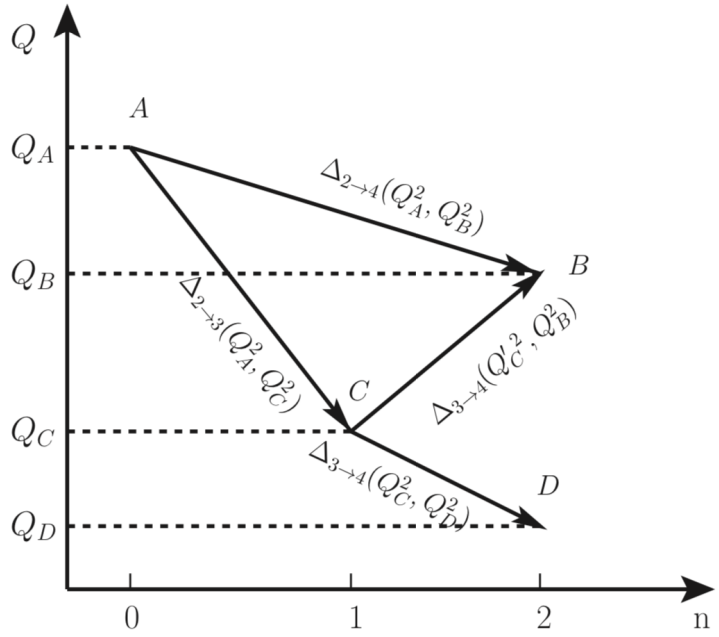
But shower kernels **do not** define **NNLO subtraction terms*** (!)

*This would be required in an "MC@NNLO" scheme, but difficult to realise in antenna showers.

Interleaved single and double branchings

A priori, direct $2 \mapsto 4$ and iterated $2 \mapsto 3$ branchings **overlap** in **ordered** region.

In **sector showers**, iterated $2 \mapsto 3$ branchings are **always strictly ordered**.



Divide double-emission phase space into **strongly-ordered** and **unordered** region: [Li, Skands 1611.00013]

$$\mathrm{d}\Phi_{+2} = \underbrace{\mathrm{d}\Phi^{>}_{+2}}_{\text{u.o.}} + \underbrace{\mathrm{d}\Phi^{<}_{+2}}_{\text{s.o.}}$$

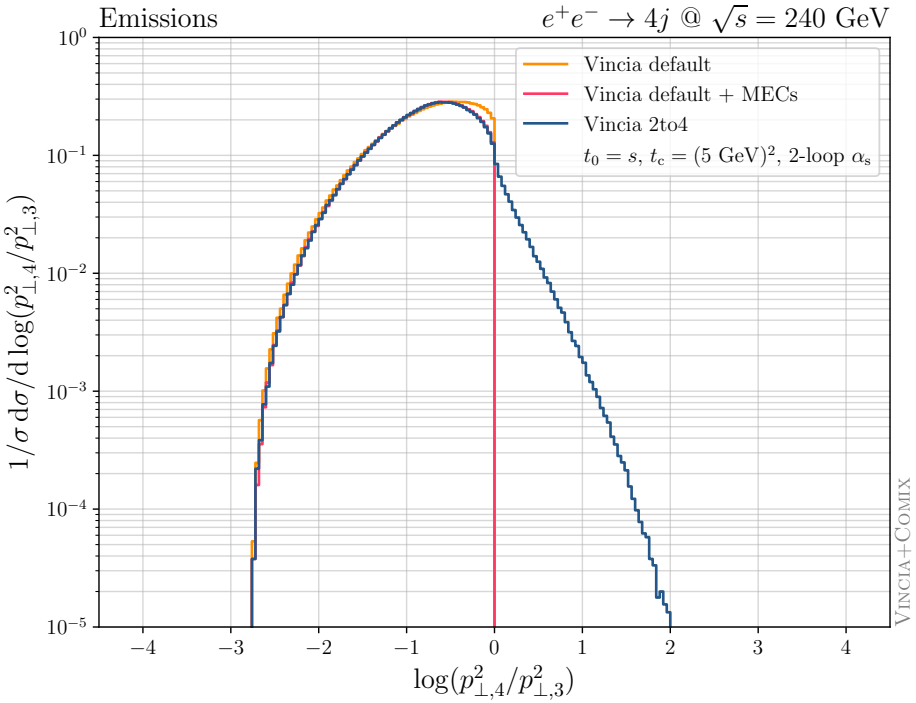
$\mathrm{d}\Phi^{<}_{+2}$: **single-unresolved** limits $\Rightarrow$ iterated $2 \mapsto 3$
$\mathrm{d}\Phi^{>}_{+2}$: **double-unresolved** limits $\Rightarrow$ direct $2 \mapsto 4$

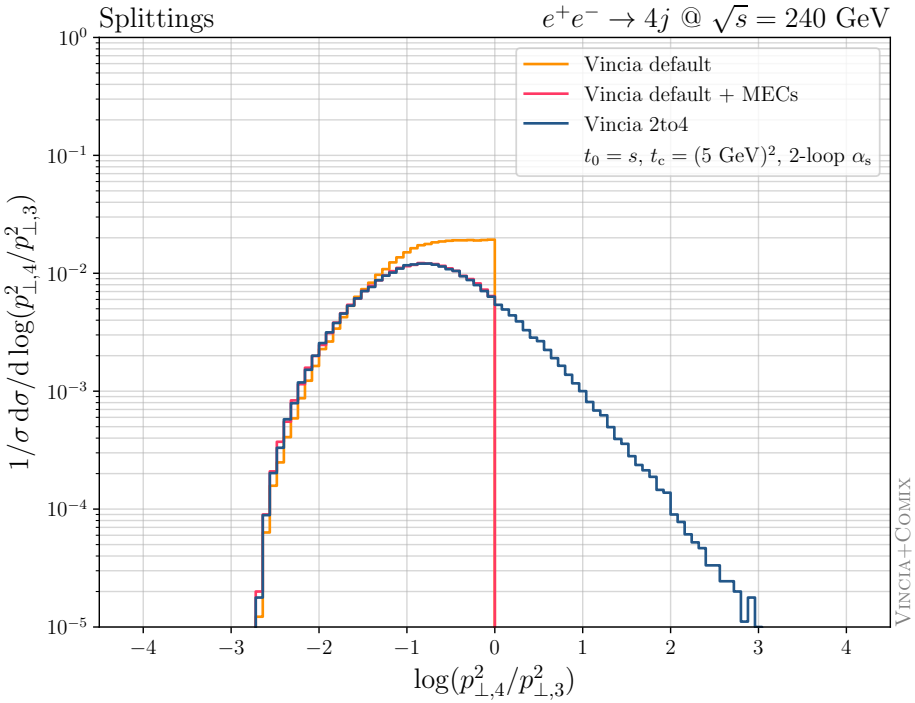
Restriction on double-branching phase space enforced by additional veto:

$$\mathrm{d}\Phi^{>}_{+2} = \sum_j \theta\left(p^2_{\perp,+2} - \hat{p}^2_{\perp,+1}\right) \Theta^{\mathrm{sct}}_{ijk}\, \mathrm{d}\Phi_{+2}$$

Real and double-real corrections

Direct $2 \mapsto 4$ shower component fills **unordered region** of phase space $p^2_{\perp,4} > p^2_{\perp,3}$.





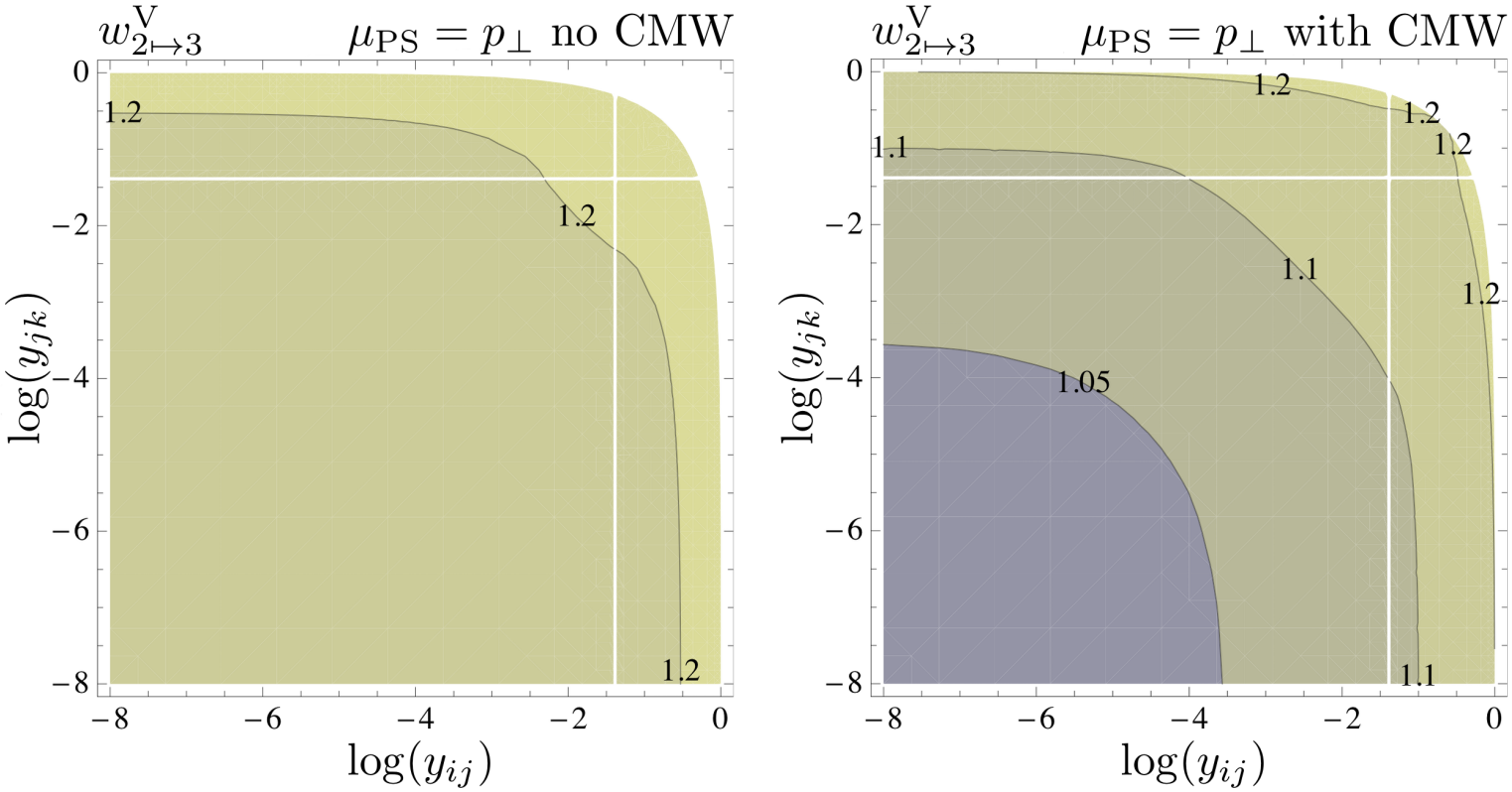
Sectorisation enforces **strict** cutoff at $p^2_{\perp,4} = p^2_{\perp,3}$ in iterated $2 \mapsto 3$ shower. (**No recoil effects**!)

Real-virtual corrections

Real-virtual correction factor

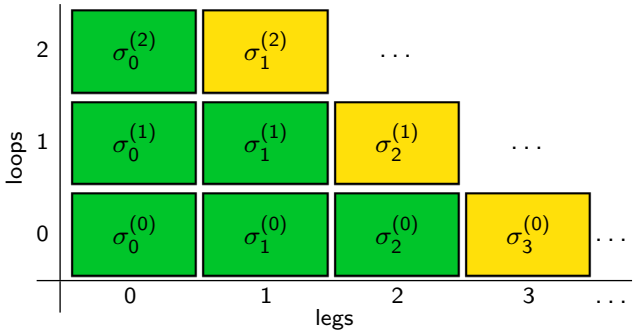
$$w_{2\mapsto 3}^{\mathrm{NLO}} = w_{2\mapsto 3}^{\mathrm{LO}} \left(1 + w_{2\mapsto 3}^{\mathrm{V}}\right)$$

studied **analytically** in detail for $Z \to q\bar{q}$ in [Hartgring, Laenen, Skands 1303.4974]:

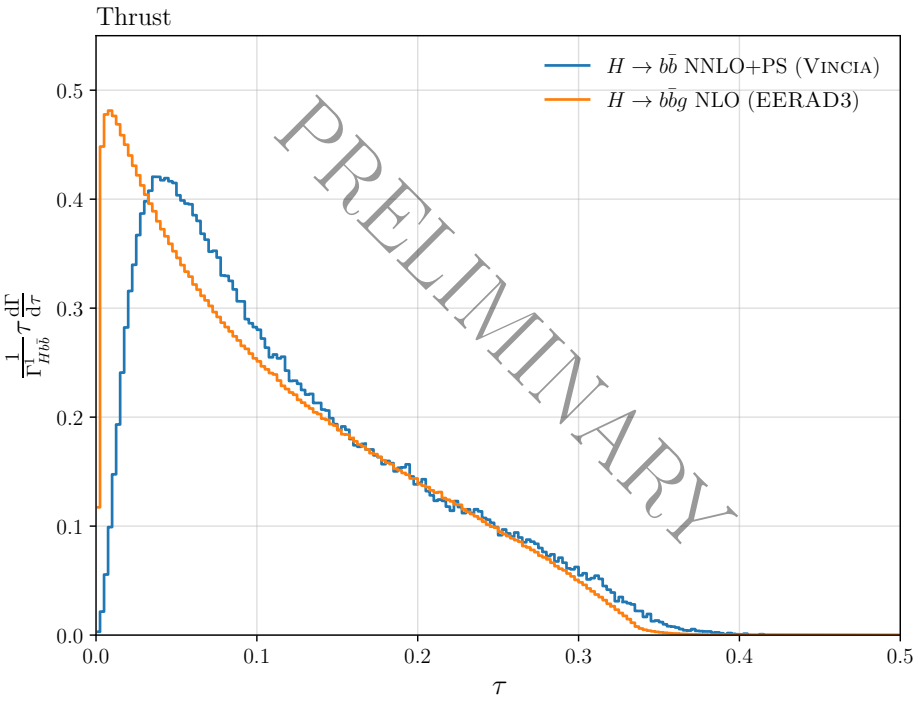


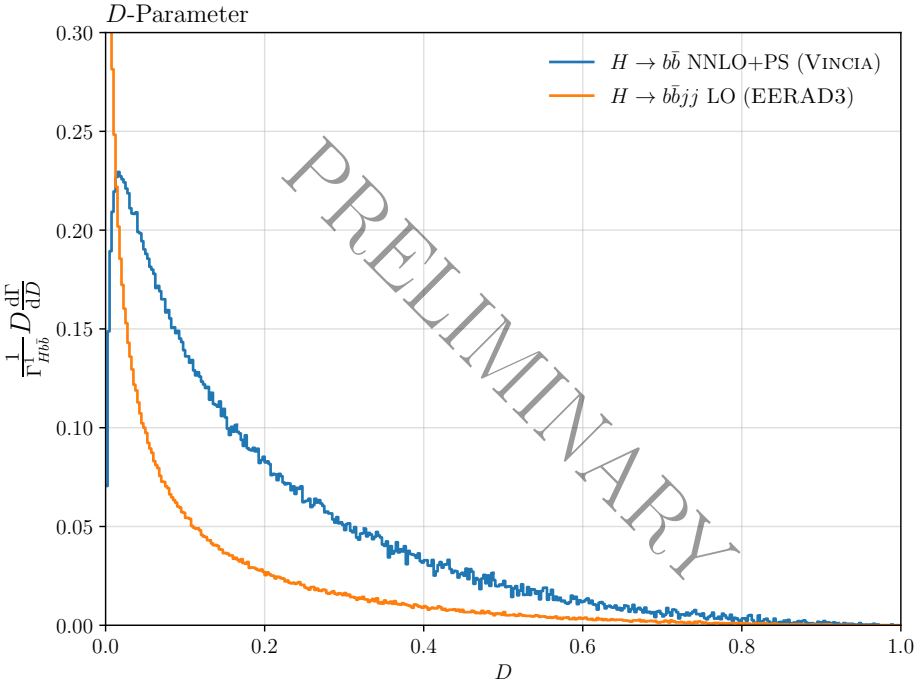
⇒ now: **generalisation** & **(semi-)automation** in VINCIA in form of NLO MECs

NNLO+PS matching in hadronic Higgs decays



NNLO accuracy in $H \to 2j$ implies **NLO correction in first emission** and **LO correction in second emission.**





VinciaNnlo vs other NNLO+PS schemes

A (rough) comparison between Vincia's NNLO+PS matching and...

UN2LOPS: (not applied to hadronic Higgs decays)

- ✓ amend inclusive NNLO calculation by fully-differential NLO $1j$ calculation
- ✗ unitary NLO shower evolution vs unitary NLO merging
- ✗ Powheg $1j$ vs UNLOPS $1j$

NNLOPS/MiNNLO$_{\mathrm{PS}}$: [Bizoń et al. 1912.09982]

- ✓ regulate Powheg $1j$ calculation in $0j$ limit
- ✗ regulation via shower Sudakovs vs analytic Sudakovs
- ✗ exponentiation of Powheg $1j$ calculation

Geneva: [Alioli et al. 2009.13533]

- ✗ shower resummation vs analytic resummation

Generalisations and limitations

The method is **in principle general**.

Addition of **colour singlets** trivial, due to **automation** on the level of **process classes**.
E.g., if $e^+e^- \to 2j$ implemented, also $e^+e^- \to 2j + X$ with any set of colour singlets X.

Addition of **final-state partons** straightforward. In practice, some pitfalls:

- **Born-local** NNLO weight **not available** in general
- **Quark-gluon** double-branching antenna functions develop **spurious singularities**, but
  - No exact knowledge of double-branching kernels required
  - Sector-antenna functions can effectively be replaced by matrix-element ratios

For **hadronic initial states**, the technique remains **structurally the same**. However:

- Interplay of NLO parton evolution and NLO shower evolution needs clarification
- Choice of shower starting scale potentially problematic
  ("power showers" needed to fill full phase space?)

What about higher orders?

$H \to b\bar{b}$ calculated fully-differentially at N3LO [Mondini, Schiavi, Williams 1904.08960], so what about N3LO+PS?

TOMTE *(somewhat similar in spirit to UN2LOPS)* [Prestel, 2106.03206] & [Bertone, Prestel, 2202.01082]

Starts from NNLO+PS matched cross section for X + jet ~ UN2LOPS

Allow jet to become unresolved, regulated by shower Sudakov

Remove unwanted NNLO terms and subtract projected 1-jet bin from 0-jet bin

Include N3LO jet-vetoed zero-jet cross section

Some challenges:

- Large amount of book-keeping ➜ complex code & computational bottlenecks?
- Many counter-events, counter-counter-events, etc ➜ many weight sign flips.
- $\Longrightarrow$ Huge computing resources for relatively slow convergence?

N3LO MECs? (hypothetical extension of VINCIA NNLO MECs)

Method in principle generalises.

Add direct-triple ($2 \to 5$) branchings to cover all of phase space: in principle **simple**.

Challenging: need local NNLO subtractions for Born + 1.

...

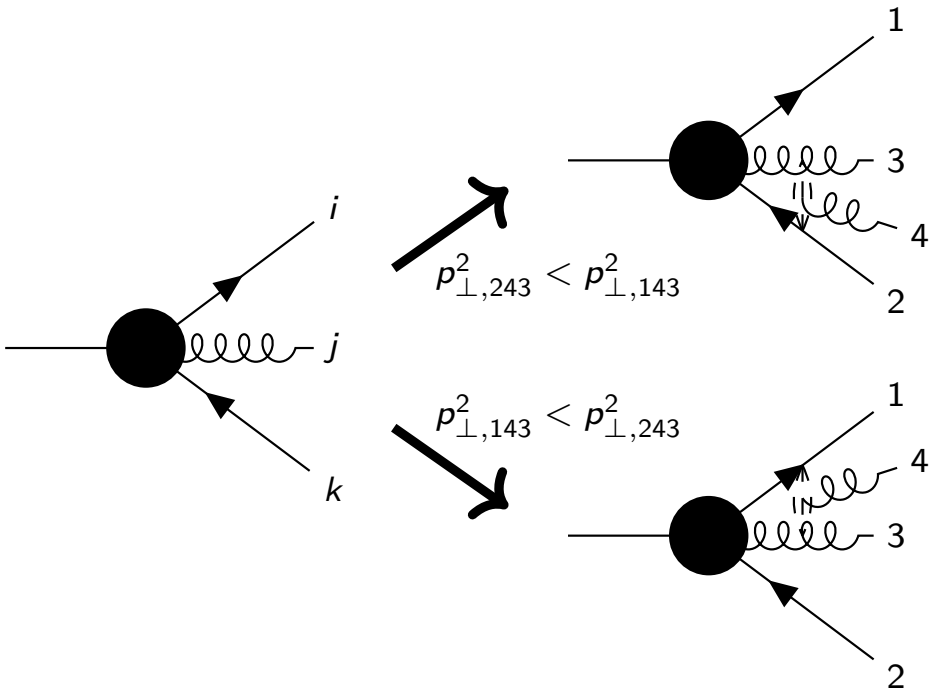
(Adapted from P. Skands)

Backup

Sector showers [Brooks, CTP, Skands 2003.00702; Lopez-Villarejo, Skands 1109.3608]

Idea: combine antenna shower with deterministic jet-clustering algorithm

- let shower only generate emissions that would be clustered by a $(3 \mapsto 2)$ jet algorithm ($\sim$ ARCLUS [Lönnblad Z.Phys.C 58 (1993)])



$$p^2_{\perp,ijk} = \frac{2(p_i \cdot p_j)(p_j \cdot p_k)}{p_i \cdot p_j + p_j \cdot p_k + p_i \cdot p_k}$$

$\Rightarrow$ **softest gluon** always regarded as the emitted one

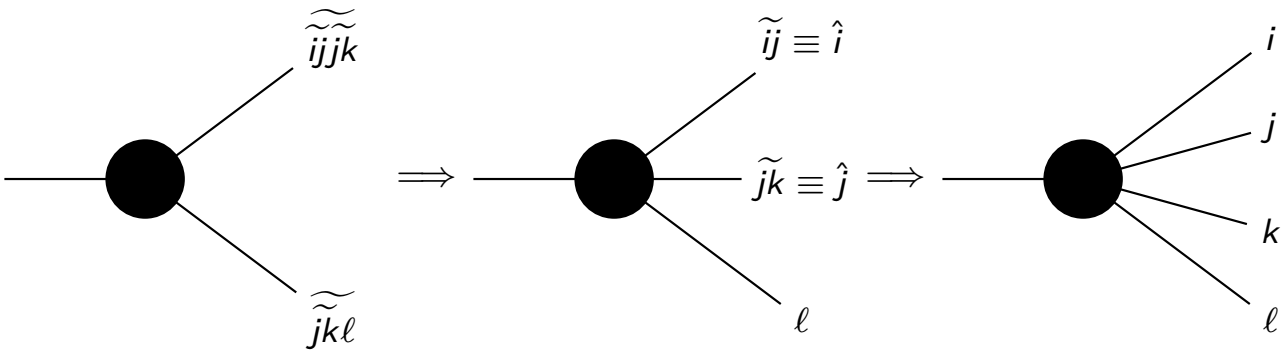
$\Rightarrow$ only **one** (most singular) splitting kernel contributes per phase space point

Since PYTHIA 8.304: **full-fledged*** implementation of sector showers in VINCIA

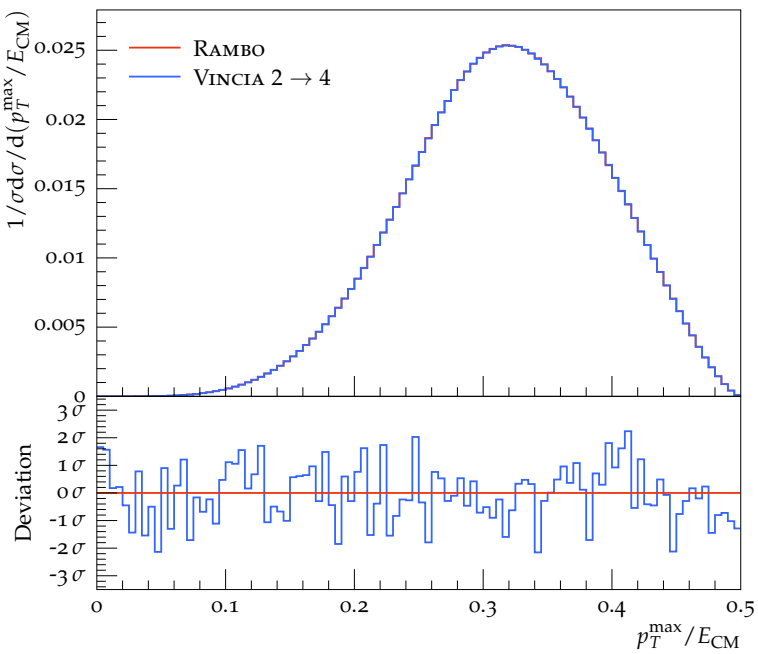
*including FSR, ISR, resonance-decay showers

Double-branching kinematics

Iterate $2 \mapsto 3$ kinematics ($\sim$ tripole map):



Can be used for **shower kinematics** and as **forward-branching phase-space generator** (FBPS).



For **shower kinematics**:

- First, generate **second** scale $p^2_{\perp,+2}$ via "P_{imp}" factor

$$\frac{1}{\hat{p}^2_{\perp,+1}} \frac{\hat{p}^2_{\perp,+1}}{\hat{p}^2_{\perp,+1} + p^2_{\perp,+2}} \frac{1}{p^2_{\perp,+2}}$$

- Then, sample intermediate scale $\hat{p}^2_{\perp,+1} \leq p^2_{\perp,+2}$

As **phase-space generator**:

- Sample $\{\hat{s}_{ij}, \hat{s}_{j\ell}, \hat{\phi}\}$ and $\{s_{ij}, s_{jk}, \phi\}$
- Particularly convenient to generate **angularly correlated** points

Tree-level MECs

Separation of double-real integral defines tree-level MECs:

$$\int_t^{t_0} \mathrm{d}\Phi_{+2}\, \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} = \int_t^{t_0} \mathrm{d}\Phi_{+2}^{>}\, \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)} + \int_t^{t_0} \mathrm{d}\Phi_{+2}^{<}\, \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{B}(\Phi_2)}$$

$$= \int_t^{t_0} \mathrm{d}\Phi_{+2}^{>}\, \mathrm{A}_{2\mapsto 4}(\Phi_{+2}) w^{\mathrm{LO}}_{2\mapsto 4}(\Phi_2, \Phi_{+2})$$

$$+ \int_{t'}^{t_0} \mathrm{d}\Phi_{+1}\, \mathrm{A}_{2\mapsto 3}(\Phi_{+1}) w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \int_t^{t'} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{3\mapsto 4}(\Phi'_{+1}) w^{\mathrm{LO}}_{3\mapsto 4}(\Phi_3, \Phi'_{+1})$$

Iterated tree-level MECs in **ordered** region:

$$w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = \frac{\mathrm{R}(\Phi_2, \Phi_{+1})}{\mathrm{A}_{2\mapsto 3}(\Phi_{+1})\mathrm{B}(\Phi_2)}$$

$$w^{\mathrm{LO}}_{3\mapsto 4}(\Phi_3, \Phi'_{+1}) = \frac{\mathrm{RR}(\Phi_3, \Phi'_{+1})}{\mathrm{A}_{3\mapsto 4}(\Phi'_{+1})\mathrm{R}(\Phi_3)}$$

Tree-level MECs in **unordered** region:

$$w^{\mathrm{LO}}_{2\mapsto 4}(\Phi_2, \Phi_{+2}) = \frac{\mathrm{RR}(\Phi_2, \Phi_{+2})}{\mathrm{A}_{2\mapsto 4}(\Phi_{+2})\mathrm{B}(\Phi_2)}$$

NLO MECs

Rewrite **NLO MEC** as product of **LO MEC** and **"Born"-local** K**-factor** $1 + w^{\mathrm{V}}$ ("POWHEG in the exponent"):

$$w^{\mathrm{NLO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) \times (1 + w^{\mathrm{V}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}))$$

Local correction given by **three terms**:

$$\begin{aligned}
w^{\mathrm{V}}_{2\mapsto 3}(\Phi_2, \Phi_{+1}) = \Bigg(& \frac{\mathrm{RV}(\Phi_2, \Phi_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \\
\text{NLO Born+1}j \quad & + \int_0^t \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{RR}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi_{+1}, \Phi'_{+1})}{\mathrm{R}(\Phi_2, \Phi_{+1})} \right] \Bigg) \\
\text{NLO Born} \quad & - \left(\frac{\mathrm{V}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \frac{\mathrm{I}^{\mathrm{NLO}}(\Phi_2)}{\mathrm{B}(\Phi_2)} + \int_0^{t_0} \mathrm{d}\Phi'_{+1} \left[\frac{\mathrm{R}(\Phi_2, \Phi'_{+1})}{\mathrm{B}(\Phi_2)} - \frac{\mathrm{S}^{\mathrm{NLO}}(\Phi_2, \Phi'_{+1})}{\mathrm{B}(\Phi_2)} \right] \right) \\
\text{shower} \quad & + \left(- \frac{\alpha_{\mathrm{S}}}{2\pi} \log \beta_0 \left(\frac{\mu^2_{\mathrm{PS}}}{\kappa^2 p^2_\perp} \right) + \int_t^{t_0} \mathrm{d}\Phi'_{+1}\, \mathrm{A}_{2\mapsto 3}(\Phi'_{+1}) w^{\mathrm{LO}}_{2\mapsto 3}(\Phi_2, \Phi'_{+1}) \right)
\end{aligned}$$

- First and third term from **NLO shower evolution**, second from **NNLO matching**
- Calculation can be **(semi-)automated**, given a suitable NLO subtraction scheme