Karlsruhe Institute of Technology

ZH production in gluon fusion at NLO QCD

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in collaboration with
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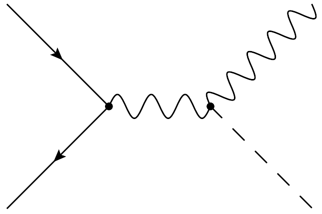
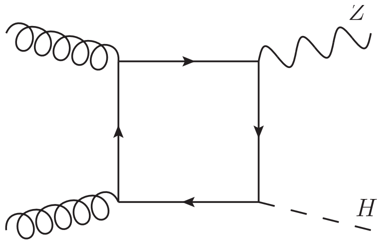
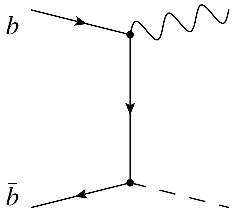
Introduction – ZH Production Modes

ZH production modes



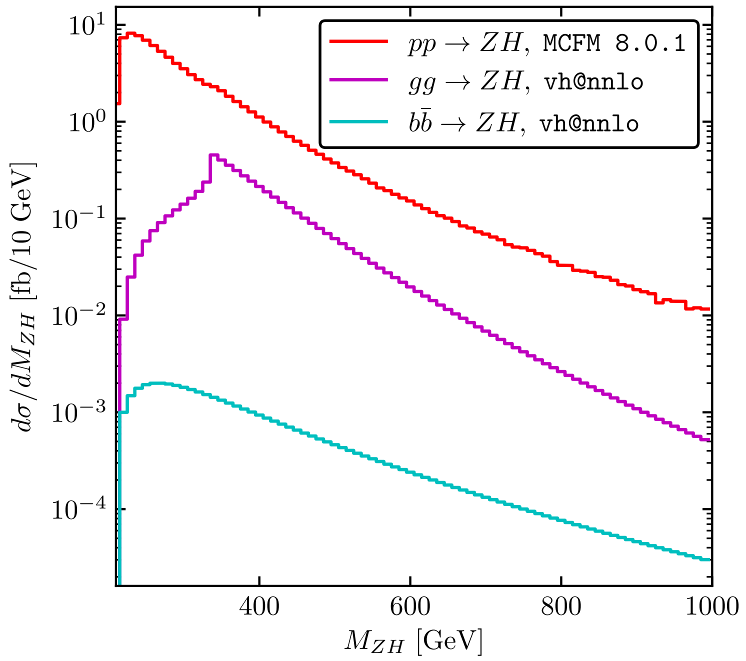
NNLO: Brein, Djouadi, Harlander 03
N^{3}LO: Baglio, Mistlberger, Szafron 22





NNLO
Ahmed, Ajjath, Chen, Dhani, Mukherjee, Ravindran 19

[Harlander, Klappert, Liebler, Simon 18]



Gluon-induced production:

- Contribution to to total cross section ~10%
- Large scale uncertainties

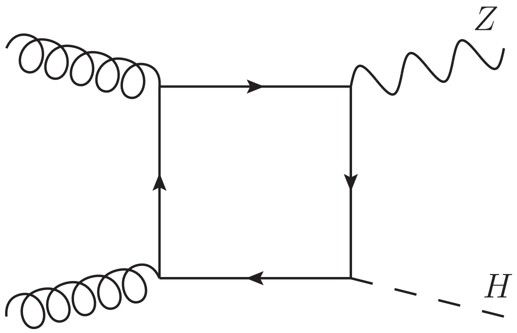
quark-initiated production known with high accuracy:

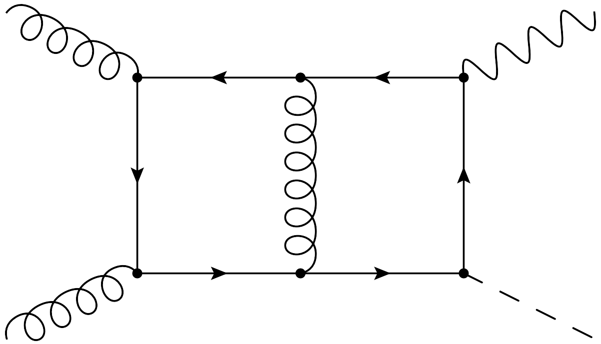
NNLO: Brein, Harlander, Wiesemann, Zirke; Ferrera, Grazzini, Somogyi, Tramontano; Campbell, Ellis, Williams; Gauld, Gehrmann-De Rideer, Glover, Huss, Majer
NLO EW(+QCD): Ciccolini, Denner, Dittmaier, Kallweit, Krämer, Mück; Granata, Lindert, Oleari, Pozzorini; Obul, Dulat, Hou, Tursun, Yulkun
N^{3}LO: Baglio, Mistlberger, Szafron 22

Uncertainties in ZH, WH measurements — ATLAS 2007.02873

	Signal
Cross-section (scale)	0.7% (qq), 25% (gg)
$H \to b\bar{b}$ branching fraction	1.7%
Scale variations in STXS bins	3.0%–3.9% ($qq \to WH$), 6.7%–12% ($qq \to ZH$), 37%–100% ($gg \to ZH$)
PS/UE variations in STXS bins	1%–5% for $qq \to VH$, 5%–20% for $gg \to ZH$
PDF+α_S variations in STXS bins	1.8%–2.2% ($qq \to WH$), 1.4%–1.7% ($qq \to ZH$), 2.9%–3.3% ($gg \to ZH$)
m_{bb} from scale variations	M+S ($qq \to VH$, $gg \to ZH$)
m_{bb} from PS/UE variations	M+S
m_{bb} from PDF+α_S variations	M+S
p_T^V from NLO EW correction	M+S

Introduction – gg → ZH: Calculations at LO and NLO





Leading Order
[Dicus, Kao 88; Kniehl 90]

NLO in $m_t \to \infty$ limit
[Altenkamp, Dittmaier, Harlander, H. Rzehak, Zirke 12]

Virtual corrections with m_t dependence

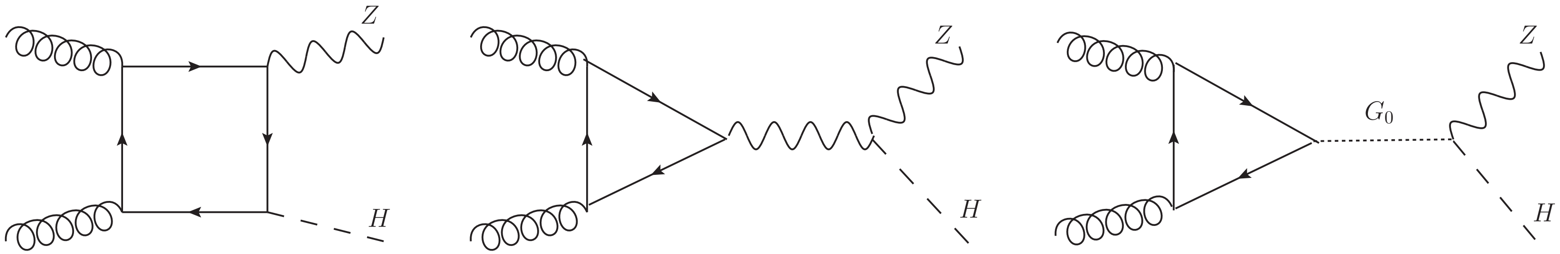
- Expansion in large m_t, up to $1/m_t^8$, improved by Padé approx.
  [Hasselhuhn, Luthe, Steinhauser 17]
- expansion in small and large m_t, up to $1/m_t^{10}, m_t^{32}$ + Padé approx.
  [Davies, Mishima, Steinhauser 20]
- numerical evaluation using pySecDec
  [Chen, Heinrich, Jones, MK, Klappert, Schlenk 20]
- expansion in small p_T up to p_T^4
  [Alasfar, Degrassi, Giardino, Gröber, Vitti 21]
- combine expansions in small p_T and small m_t
  [Bellafronte, Degrassi, Giardino, Gröber, Vitti 22]

Full NLO results:

- Chen, Davies, Heinrich, Jones, MK, Mishima, Schlenk, Steinhauser 22
- Degrassi, Gröber, Vitti, Zhao 22
- small m_Z, m_H expansion
  Wang, Xu, Xu, Yang 21

Introduction – ZH in Gluon Fusion

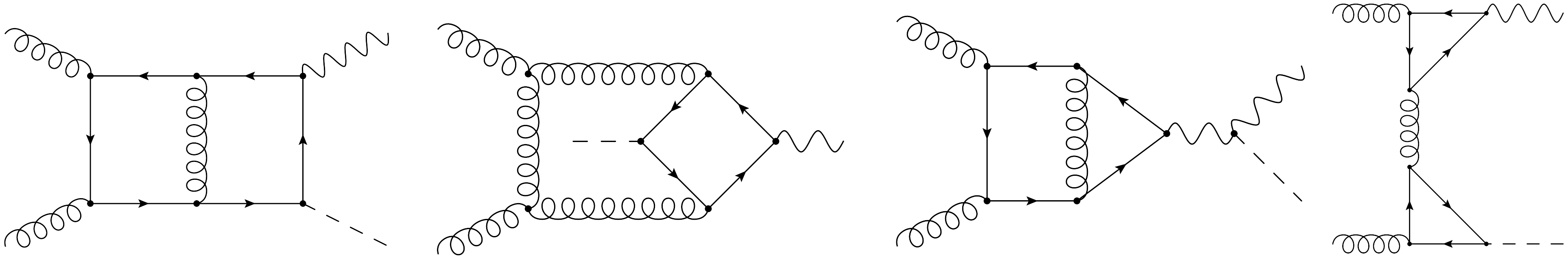
LO



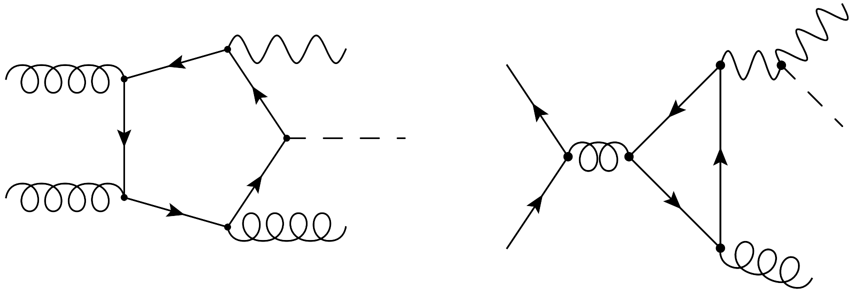
only top-quark contributing

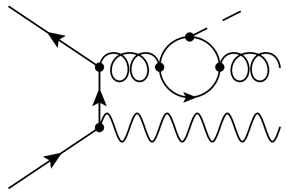
triangle contributions vanish for mass-degenerate quark doublets
→ contributions by b- and t-quark

NLO virtual

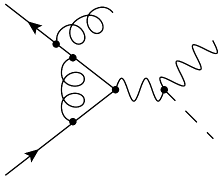


NLO reals





finite & gauge invariant; can be included with DY contribution



RV correction to DY-type

not included here

Overview of Calculation

Virtual Corrections using 2 methods:

Numerical evaluation using pySecDec
[Chen, Heinrich, Jones, MK, Klappert, Schlenk 20]

- ✓ valid for arbitrary kinematics
- evaluation challenging in HE region
- masses fixed during integral reduction
- $\rightarrow$ can only use OS mass

High-energy expansion
[Davies, Mishima, Steinhauser 20]

- only valid in HE region
- ✓ fast evaluation
- ✓ arbitrary masses

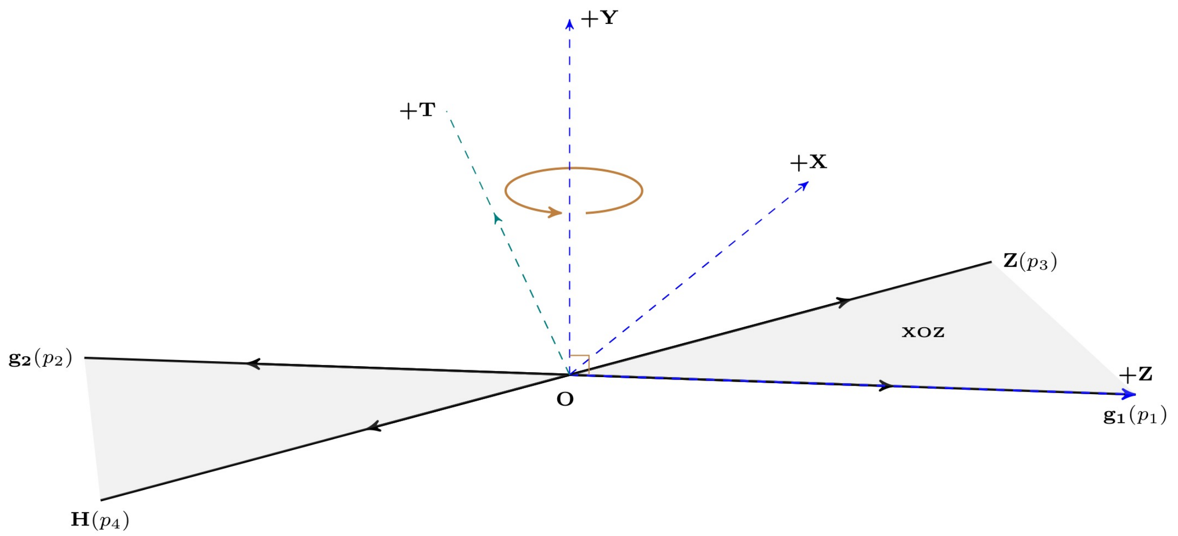
$\rightarrow$ We combine these calculations at histogram level, using $p_T = 200$ GeV as a threshold

Real-radiation amplitudes generated with Gosam [Cullen et.al.]

Polarized Amplitudes

$$\mathcal{A} = \varepsilon_{\lambda_1}^{\mu_1}(p_1)\,\varepsilon_{\lambda_2}^{\mu_2}(p_2)\,(\varepsilon_{\lambda_3}^{\mu_3}(p_3))^\star\,\mathcal{A}_{\mu_1\mu_2\mu_3}$$

$$g(p_1) + g(p_2) \to Z(p_3) + H(p_4)$$



Polarization vectors can be constructed from external momenta [L. Chen 19]

choose

$$\begin{aligned}
\varepsilon_x^\mu &= \mathcal{N}_x\,\left(-s_{23}p_1^\mu - s_{13}p_2^\mu + s_{12}p_3^\mu\right)\,,\\
\varepsilon_y^\mu &= \mathcal{N}_y\,\left(\epsilon^\mu_{\mu_1\,\mu_2\,\mu_3}\,p_1^{\mu_1}\,p_2^{\mu_2}\,p_3^{\mu_3}\right),\\
\varepsilon_T^\mu &= \mathcal{N}_T\,\left(\left(-s_{23}(s_{13}+s_{23}) + 2m_Z^2 s_{12}\right)p_1^\mu + \left(s_{13}(s_{13}+s_{23}) - 2m_Z^2 s_{12}\right)p_2^\mu \right.\\
&\qquad \left. + s_{12}(-s_{13}+s_{23})\,p_3^\mu\,\right)\,,\\
\varepsilon_l^\mu &= \mathcal{N}_l\,\left(-2m_Z^2\,(p_1^\mu + p_2^\mu) + (s_{13}+s_{23})\,p_3^\mu\right)\,,
\end{aligned}$$

such that

$$\underline{\{\varepsilon_x,\,\varepsilon_y\}}\cdot\{p_1,\,p_2\} = 0, \qquad \underline{\{\varepsilon_y,\,\varepsilon_T,\,\varepsilon_l\}}\cdot p_3 = 0, \qquad \varepsilon_i^2 = -1$$

Can be used as polarization vectors of gluons and Z, respectively

circular polarizations:

$$\varepsilon_\pm^{\mu_1}(p_1) = \frac{1}{\sqrt{2}}\left(\varepsilon_x^{\mu_1} \pm i\varepsilon_y^{\mu_1}\right) \qquad \varepsilon_\pm^{\mu_2}(p_2) = \frac{1}{\sqrt{2}}\left(\varepsilon_x^{\mu_2} \mp i\varepsilon_y^{\mu_2}\right) \qquad \varepsilon_\pm^{\mu_3}(p_3) = \frac{1}{\sqrt{2}}\left(\varepsilon_T^{\mu_3} \pm i\varepsilon_y^{\mu_3}\right)$$

6 polarization configurations:

$$\begin{aligned}
\mathcal{P}_1^{\mu_1\mu_2\mu_3} &= \varepsilon_x^{\mu_1}\,\varepsilon_x^{\mu_2}\,\varepsilon_y^{\mu_3}\,, & \mathcal{P}_2^{\mu_1\mu_2\mu_3} &= \varepsilon_x^{\mu_1}\,\varepsilon_y^{\mu_2}\,\varepsilon_T^{\mu_3}\\
\mathcal{P}_3^{\mu_1\mu_2\mu_3} &= \varepsilon_x^{\mu_1}\,\varepsilon_y^{\mu_2}\,\varepsilon_l^{\mu_3}\,, & \mathcal{P}_4^{\mu_1\mu_2\mu_3} &= \varepsilon_y^{\mu_1}\,\varepsilon_x^{\mu_2}\,\varepsilon_T^{\mu_3}\\
\mathcal{P}_5^{\mu_1\mu_2\mu_3} &= \varepsilon_y^{\mu_1}\,\varepsilon_x^{\mu_2}\,\varepsilon_l^{\mu_3}\,, & \mathcal{P}_6^{\mu_1\mu_2\mu_3} &= \varepsilon_y^{\mu_1}\,\varepsilon_y^{\mu_2}\,\varepsilon_y^{\mu_3}
\end{aligned}$$

Integral Reduction

Use Integration-by-Parts Identities [Chetyrkin, Tkachov; Laporta]
to express appearing 2-loop integrals in terms of master integrals.

$$\int \mathrm{d}^d p_i \frac{\partial}{\partial p_i^\mu} \left[q^\mu \, \mathbf{I}'(p_1, \ldots, p_l; k_1, \ldots, k_m) \right] = 0$$

~13.000 unreduced integrals → 452 masters

Reduction is quite challenging, can be simplified by fixing mass ratios $\frac{m_Z^2}{m_t^2} = \frac{23}{83}, \qquad \frac{m_H^2}{m_t^2} = \frac{12}{23}$

→ Eliminates 2 of the 5 mass scales $s,\, t,\, m_t,\, m_Z,\, m_H$

Obtained using the programs:

– Kira [Klappert, Lange, Maierhöfer, Usovitsch]

– Firefly [Klappert, Klein, Lange]

→ uses finite-field methods to avoid large intermediate expressions

Choice of Master Integrals

- Use a (quasi-)finite basis of master integrals [von Manteuffel, Panzer, Schabinger 14]
  - simplifies numerical evaluation of integrals
  - poles in ε only in coefficients
  - requires integrals in shifted dimensions [Bern, Dixon, Kosower 92; Tarasov 96; Lee 10]

- Further improvements of integral basis to achieve:
  (by trying different basis choices for each sector)
  - d-dependence factorizes from kinematic dependence in denominators of reduction coefficients [Smirnov, Smirnov `20; Usovitsch `20]
    $$\frac{N(s,t,d)}{D_1(d)D_2(s,t)}$$
  - simple denominator factors D_1, D_2
  - avoid poles in coefficients of integrals in top-level sectors as far as possible
  - small file-size of reductions

→ Some spurious poles & cancellations between integrals can be avoided
→ Reduced File sizes of expressions
  - Amplitude: factor of 5 improvement
  - Largest coefficient (double-tadpole): 150 MB → 5 MB

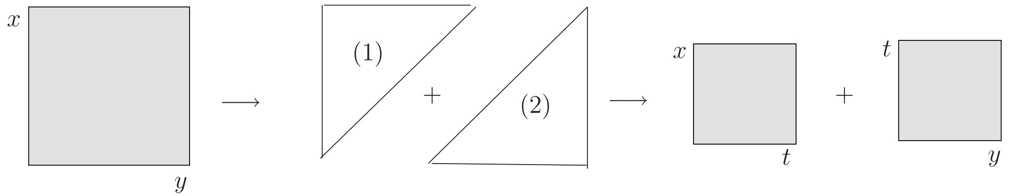
Loop Integrals – Sector Decomposition

Numerical evaluation of loop integrals with pySecDec

[Borowka, Heinrich, Jahn, Jones, MK, Langer, Magerya, Põldaru, Schlenk, Villa]

Available at github.com/gudrunhe/secdec

- Sector decomposition [Binoth, Heinrich 00] factorizes overlapping singularities



- Subtraction of poles & expansion in ε
- Contour deformation [Soper 00; Binoth et.al. 05, Nagy, Soper 06; Borowka et al. 12]

→ Finite integrals at each order in ε

→ Numerical integration possible

New release:

- expansion by regions
- evaluation of linear combinations of integrals, with automated optimization of sampling points per sector
- automated reduction of contour-def. parameter
- automatically adjusts FORM settings

pySecDec integral libraries can be directly linked to amplitude code

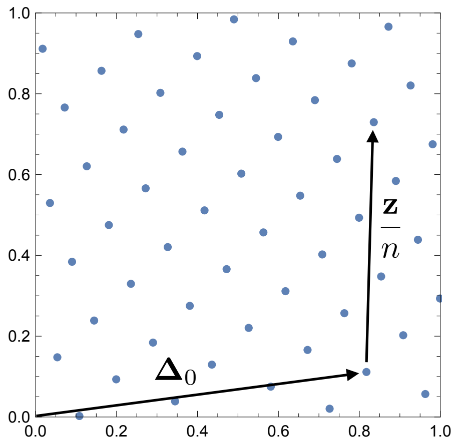
pySecDec – Quasi-Monte Carlo

Review: Dick, Kuo, Sloan 13
First application to loop integrals:
Li, Wang, Yan, Zhao 15

Our preferred integration algorithm is a
Quasi-Monte Carlo using rank-1 shifted lattice rule

Integrator available at github.com/mppmu/qmc
[Borowka, Heinrich, Jahn, Jones, MK, Schlenk]

$$I[f] \approx I_k = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\mathbf{x}_{i,k}), \quad \mathbf{x}_{i,k} = \left\{ \frac{i \cdot \mathbf{z}}{N} + \boldsymbol{\Delta}_k \right\}$$

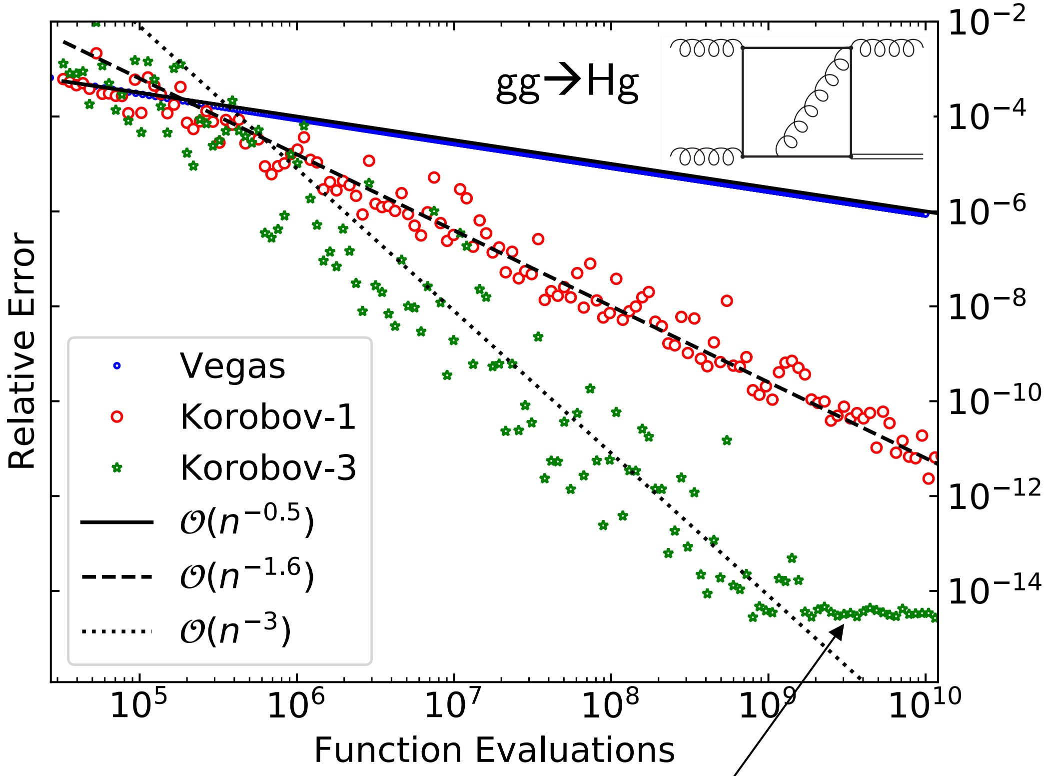


$\{\ldots\} =$ fractional part ($\rightarrow x \in [0;1[$)

$\boldsymbol{\Delta}_k =$ randomized shifts
$\rightarrow$ m different estimates of Integral
$\rightarrow$ error estimate of result

$\mathbf{z} =$ generating vector
constructed component-by-component [Nuyens 07]
minimizing worst-case error

$\rightarrow$ integration error scales as $\mathcal{O}(n^{-1})$ or better



Limited by double precision arithmetic

Evaluation of Amplitude

After sector decomposition and expansion in ε → amplitude written in terms of 19.530 finite integrals

Optimizations to reduce run time:

- dynamically set n for each integral, minimizing

$$T = \sum_{\text{integral } i} t_i + \lambda \left(\sigma^2 - \sum_i \sigma_i^2 \right) \qquad \sigma_i = c_i \cdot t_i^{-e}$$

$\sigma_i =$ error estimate (including coefficients in amplitude)
$\lambda =$ Lagrange multiplier $\qquad \sigma =$ precision goal

- avoid reevaluation of integrals for different orders in ε and form factors

compute once

$$F^a = \sum_i \left[\left(\sum_j C^a_{i,j} \varepsilon^j \right) \cdot \left(\sum_k I_{i,k} \varepsilon^k \right) \right] = \frac{C^a_{1,-2} I_{1,0} + C^a_{1,-1} I_{1,-1} + \ldots}{\varepsilon^2} + \frac{C^a_{1,-1} I_{1,0} + \ldots}{\varepsilon^1} + \ldots$$

- parallelization on GPUs
  typical run-time to obtain virtual amplitude with 0.3% precision:
  2h using 2x Nvidia Tesla V100 GPUs

High-Energy Expansion

Davies, Mishima, Steinhauser 20

Scale hierarchy in high-energy region:

$$m_Z, m_H < m_t \ll s, t$$

1) Use Taylor series expansion in m_Z, m_H
   $\rightarrow$ remaining integrals only depend on m_t, s, t

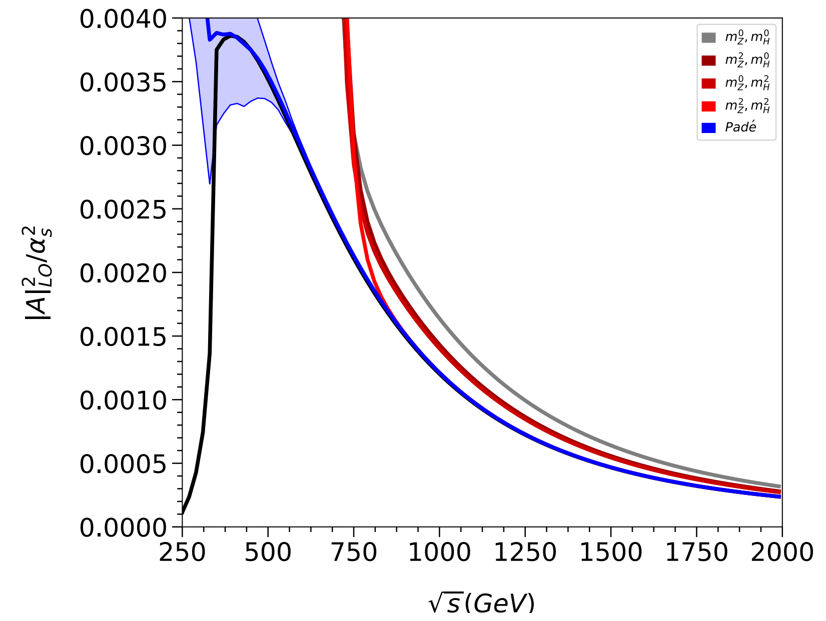
2) Solve differential equations using ansatz

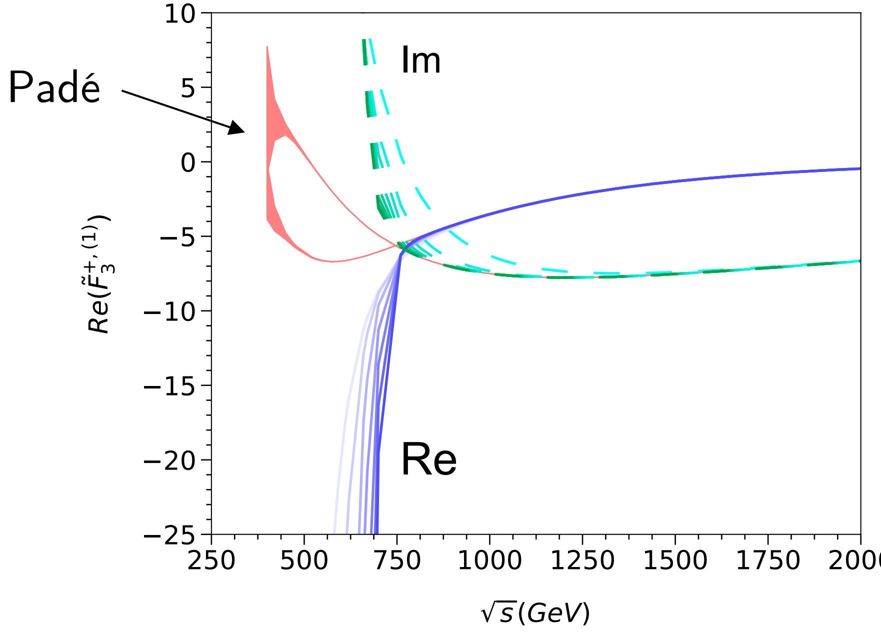
$$I = \sum_{n_1=n_1^{\min}}^{\infty} \sum_{n_2=n_2^{\min}}^{\infty} \sum_{n_3=0}^{2l+n_1} c(I, n_1, n_2, n_3, s, t)\, \epsilon^{n_1} \left(m_t^2\right)^{n_2} \left(\log(m_t^2)\right)^{n_3}$$

3) Boundary conditions using [see Mishima 18]
   - expansion-by-regions [Beneke, Smirnov; Jantzen]
   - Mellin-Barnes techniques

4) Series convergence improved using Padé approximants:

$$\mathcal{V}_{\rm fin}^{N} = \frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m} \equiv [n/m](x)$$



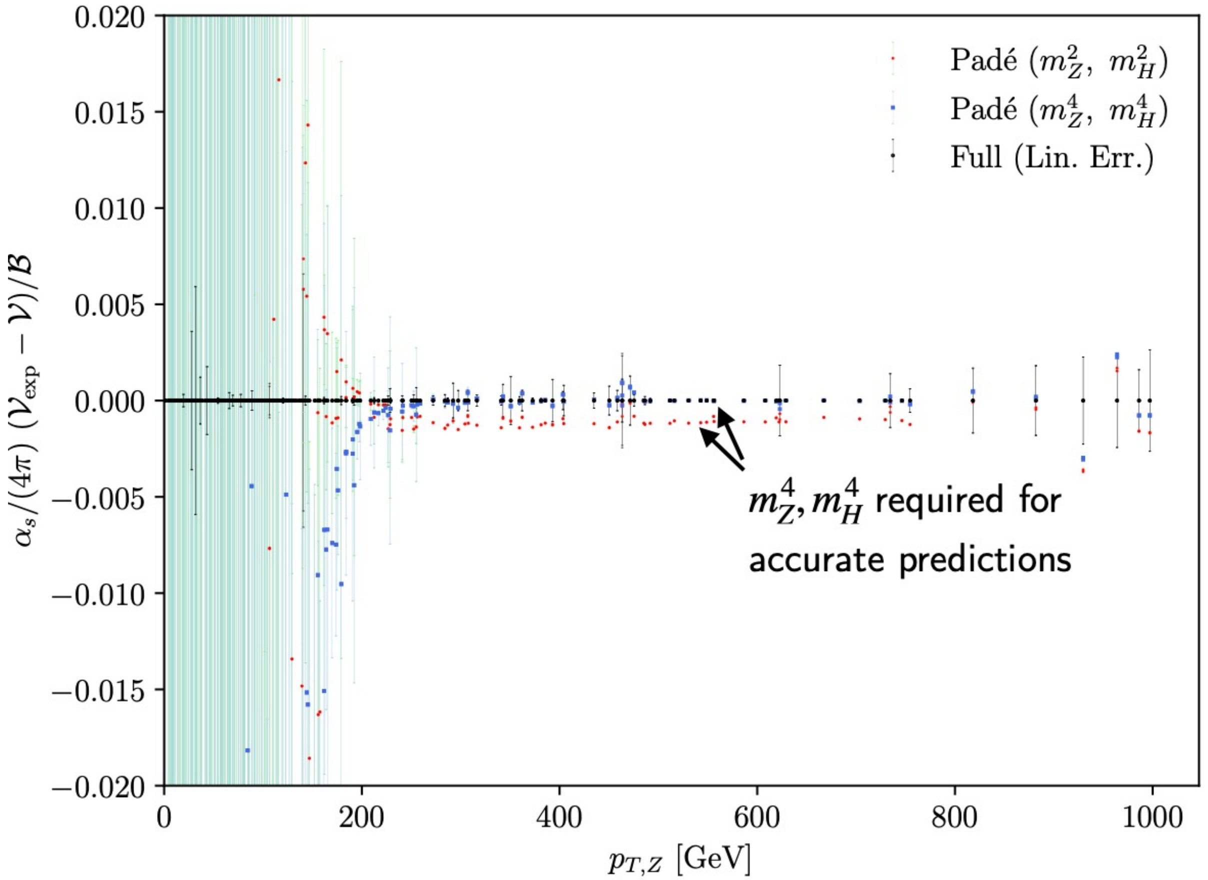


Combination with Expansions

Comparison of numerical results with high-energy expansion

- expansion around small masses up to m_t^{32}, m_Z^4, m_H^4
- agreement at 0.1% level or better for p_T>200 GeV
- m_Z^4, m_H^4 terms required to reach this accuracy

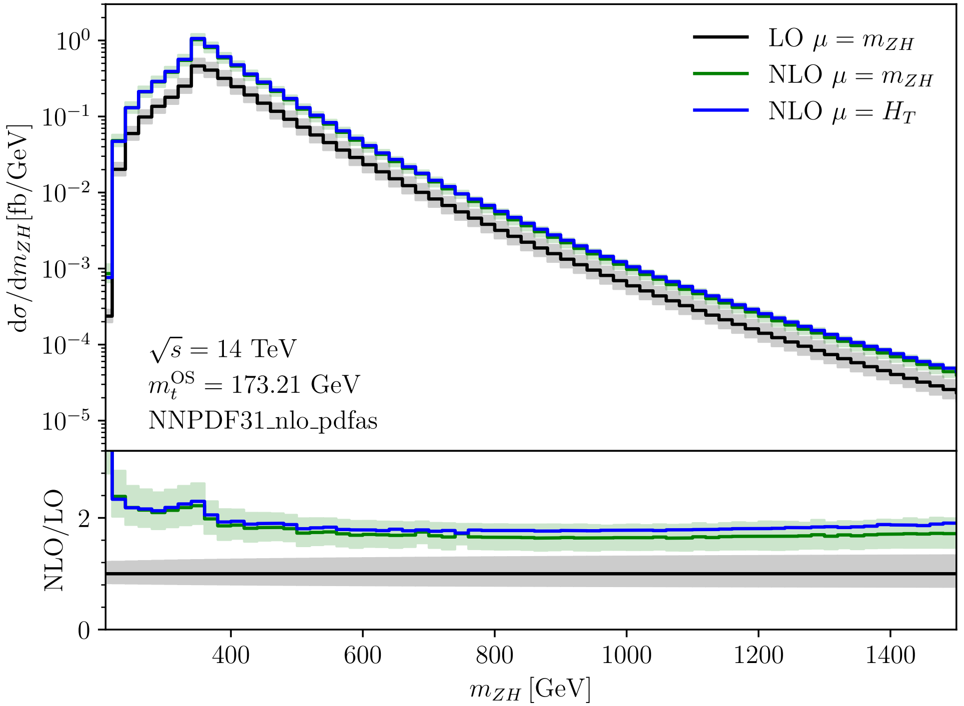
We switch from the numerical calculation
to the expansion at p_T=200



Results – Total Cross Section & Invariant Mass

$\sqrt{s}$	LO [fb]	NLO [fb]
13 TeV	$52.42^{+25.5\%}_{-19.3\%}$	$103.8(3)^{+16.4\%}_{-13.9\%}$
13.6 TeV	$58.06^{+25.1\%}_{-19.0\%}$	$114.7(3)^{+16.2\%}_{-13.7\%}$
14 TeV	$61.96^{+24.9\%}_{-18.9\%}$	$122.2(3)^{+16.1\%}_{-13.6\%}$

- NLO/LO ≈ 2
- NLO scale uncertainty: ±15%
- K-factor relatively flat for $m_{ZH} > 400$GeV
  larger effects at top-pair threshold and below



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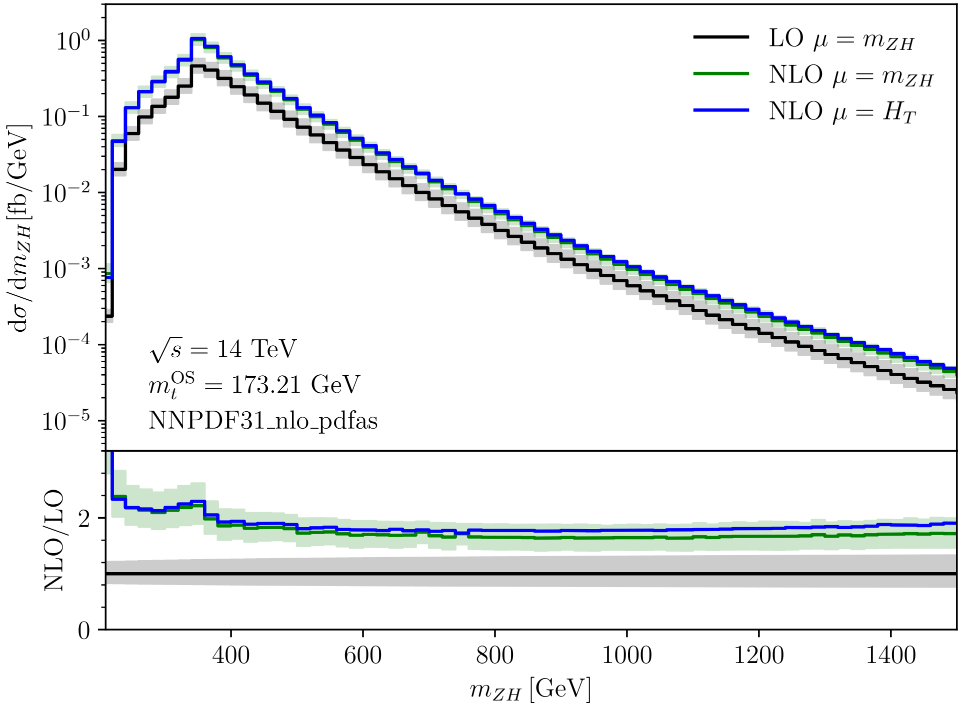
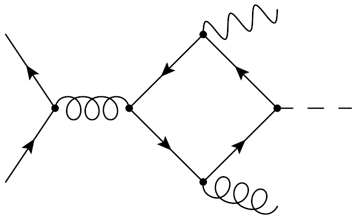
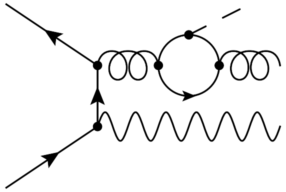


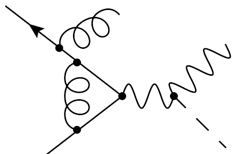
There is some freedom, which diagrams to include in $q\bar{q}$ real radiation



part of gg → ZH



can bet attributed to
gg → ZH or DY



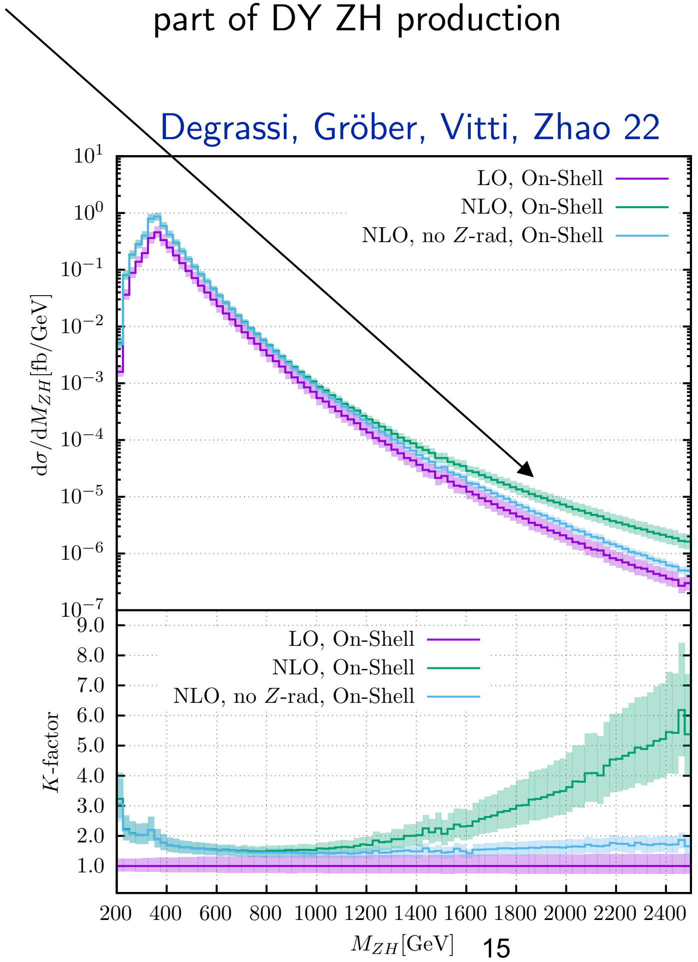
part of DY ZH production

We only include diagrams, where Z is radiated from closed quark loop

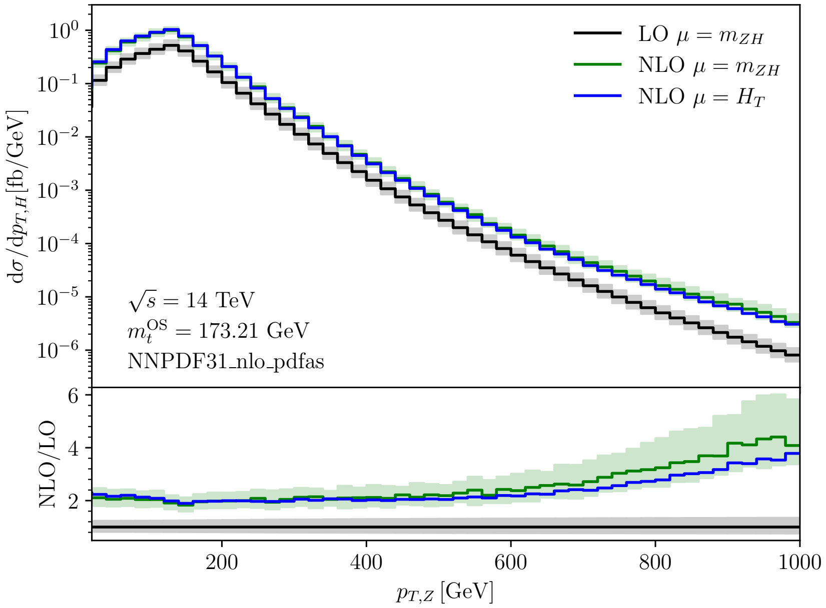
2nd class has been studied as
part of NNLO $q\bar{q} \to$ ZH production
see e.g. Brein, Harlander, Wiesemann, Zirke 12

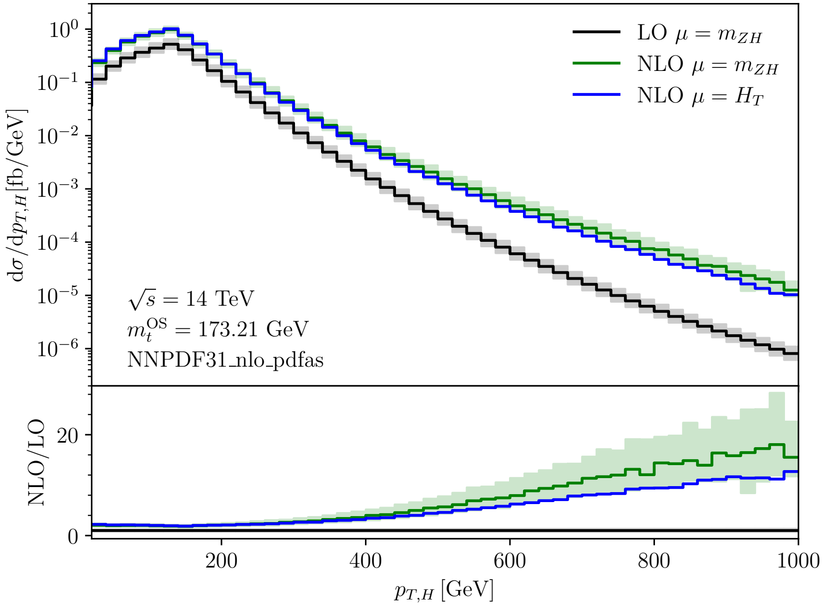
included in independent
calculation of gg → ZH production
(formally N3LO of DY type)
Degrassi, Gröber, Vitti, Zhao 22

Degrassi, Gröber, Vitti, Zhao 22



Results – p_T distributions

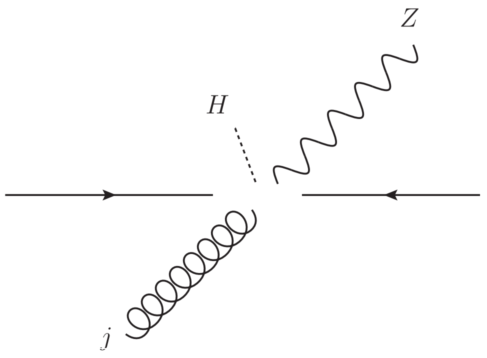


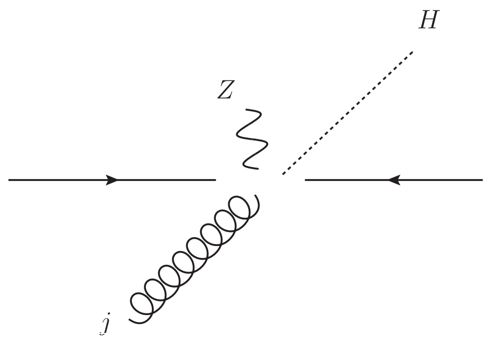


large corrections at high p_T

already observed in ZHj@LO
Hespel, Maltoni, Vryonidou 15; Les Houches 19

caused by new kinematic region in real radiation:





difference of eikonal factors:

soft Z emission: $\frac{p^\mu}{p \cdot p_Z}$

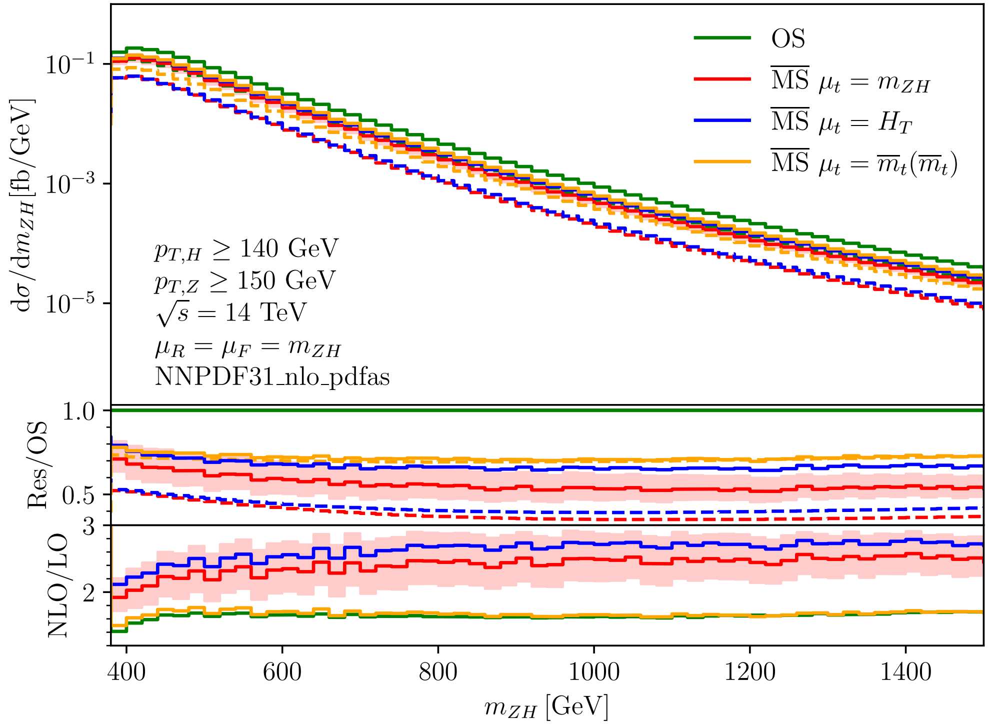
soft H emission: $\frac{m_t}{p \cdot p_H}$

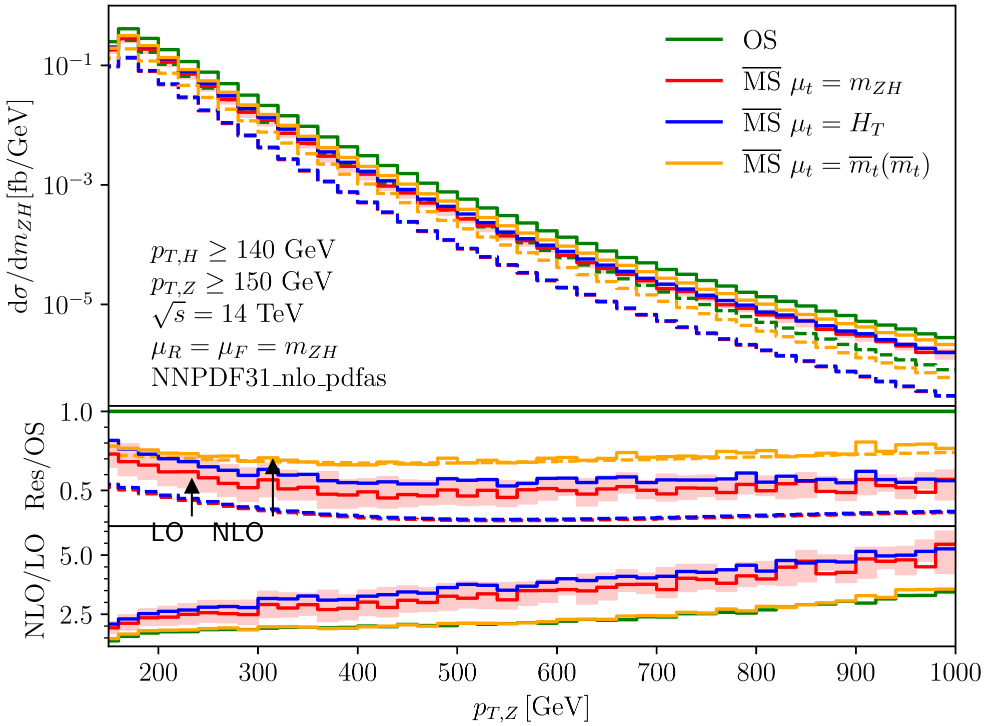
→ larger enhancement of Z emission for large p_T

Mass Scheme Depedence

The results presented so far use OS renormalization of m_t, we can change to $\overline{\text{MS}}$ renormalization
(using the high-energy expansion where m_t is not fixed in reduction)

$$m_t \to \overline{m}_t(\mu_t)\left(1 + \frac{\alpha_s(\mu_R)}{4\pi} C_F \left\{4 + 3\log\left[\frac{\mu_t^2}{\overline{m}_t(\mu_t)^2}\right]\right\}\right)$$





The $\overline{\text{MS}}$ result is significantly smaller than OS result:

LO: ~ factor 2.9
NLO: ~ factor 1.9

at $m_{ZH} = 1$ TeV

If taken as uncertainty, it is much larger than scale dependence

Mass Scheme Dependence

Leading HE contributions in gg $\to$ HH and gg $\to$ ZH production

$$A_i^{\text{fin}} = a_s A_i^{(0),\text{fin}} + a_s^2 A_i^{(1),\text{fin}} + \mathcal{O}(a_s^3)$$

HH

$$A_i^{(0)} \sim m_t^2 f_i(s,t)$$

$$A_i^{(1)} \sim 6 C_F A_i^{(0)} \log\left[\frac{m_t^2}{s}\right]$$

LO: m_t^2 from y_t^2
NLO: leading $\log(m_t^2)$ from mass c.t.
converting to $\overline{MS}$ gives $\log(\mu_t^2/s)$
motivating scale choice of $\mu_t^2 = s$

ZH

$$A_i^{(0)} \sim m_t^2 f_i(s,t) \log^2\left[\frac{m_t^2}{s}\right],$$

$$A_i^{(1)} \sim \frac{(C_A - C_F)}{6} A_i^{(0)} \log^2\left[\frac{m_t^2}{s}\right]$$

LO: one m_t from y_t
NLO: leading $\log(m_t^2)$ not coming from mass c.t.

$\to$ The leading contributions seem to have different origins for the 2 processes

It would be interesting to understand these logarithms in more detail.
(for some recent progress for off-shell H production, see Liu, Modi, Penin 21; Mazzitelli 22)

Conclusion

NLO corrections to ZH production in gluon-fusion

Virtual corrections obtained from combination of 2 calculations

- numeric evaluation using pySecDec
- high-energy expansion

Phenomenological results

- K-factor ≈ 2
- large corrections at high-p_T due to new kin. configurations
- large dependence on top-mass renormalization scheme

Thank you for your attention!