NLO Renormalization of a Standard Model Extension with a Dark Abelian Sector

Jonas Rehberg

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in collaboration with Stefan Dittmaier and Heidi Rzehak

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- Motivation
- The Dark Abelian Sector Model (DASM)
  - Yang-Mills part
  - Higgs part
- Renormalization schemes for the DASM
  - Renormalization of mixing angles
- The W-boson mass in the DASM
- Summary

Why extend the SM?

- SM cannot explain:
  - matter-antimatter asymmetry in the universe
  - dark matter
- top–down approach: more complete theories (SUSY, GUTs)
  - introduce plenty of new parameters and particles
  - often include extensions of the Higgs and gauge sector
- generic approach: analyze influence of generic building blocks
  - Higgs and gauge sector extensions
  - introduction of hidden sectors

    $\rightarrow$ hint towards more complete models
- need for highest possible precision in predictions
  - inclusion of higher-order corrections
  - sophisticated renormalization scheme

The Dark Abelian Sector Model (DASM)

- DASM introduces generic structures of a possible hidden sector [Schabinger, Wells, 2005],[Gopalakrishna, Jung, Wells, 2008]
- hidden sector:
  - not charged under SM gauge group
  - SM particles are not charged under possible gauge structures of the hidden sector
- relevant SM operators as potential portals to a hidden sector
  - field-strength tensor of $U(1)_\mathrm{Y}$
  - SM Higgs mass operator $\Phi(x)^\dagger\Phi(x)$
  - possible right-handed neutrino fields ν_R

Lagrangian of the DASM

- no modifications in QCD part
- EW part can be split up

$$\mathcal{L}_{\text{DASM}}^{\text{EW}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{Fix}} + \mathcal{L}_{\text{FP}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Yukawa}}$$

Yang-Mills part

- gauge group of DASM given by:

$$U(1)_{\mathrm{Y}} \times SU(2)_{\mathrm{W}} \times SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{d}}$$

- Yang-Mills part in DASM modified

$$\mathcal{L}_{\mathrm{YM}} = \mathcal{L}_{\mathrm{YM,SM}} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{a}{2} B_{\mu\nu} C^{\mu\nu}$$

- kinetic mixing of $U(1)$ gauge groups ruled by parameter a
- field redefinition to diagonalize kinetic terms

$$\{B_\mu, C_\mu\} \to \{B'_\mu, C'_\mu\}$$

$$\mathcal{L}_{\mathrm{YM}} = \cdots - \frac{1}{4} C'_{\mu\nu} C'^{\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu}$$

$\to$ $\mathcal{L}_{\mathrm{Fix}}$ and $\mathcal{L}_{\mathrm{FP}}$ modified as well

Higgs part

- extension of the Higgs sector by complex, scalar Higgs field ρ, with non-vanishing vev v_1

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left(h_2 + v_2 + \mathrm{i}\chi_2\right) \end{pmatrix}, \qquad \rho = \frac{1}{\sqrt{2}}\left(h_1 + v_1 + \mathrm{i}\chi_1\right)$$

- h_2, h_1 neutral CP-even fields, χ_1, χ_2 neutral CP-odd fields, ϕ^+ charged field, v_1, v_2 constants
- ρ is singlet under SM gauge group, charged under $U(1)_\mathrm{d}$

  $\rightarrow U(1)_\mathrm{d}$ spontaneously broken
- Higgs part of the DASM is given by

$$\mathcal{L}_\mathrm{Higgs} = \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) + \left(D_{\mathrm{c},\mu}\rho\right)^\dagger \left(D_\mathrm{c}^\mu \rho\right) - \mathrm{V}(\Phi, \rho)$$

Higgs potential

- most general gauge-invariant Higgs potential

$$V(\Phi,\rho) = -\mu_2^2\Phi^\dagger\Phi - 2\mu_1^2\rho^\dagger\rho + \frac{\lambda_2}{4}\left(\Phi^\dagger\Phi\right)^2 + 4\lambda_1\left(\rho^\dagger\rho\right)^2 + 2\lambda_{12}\Phi^\dagger\Phi\rho^\dagger\rho$$

- 3 additional parameters: $\lambda_1, \mu_1^2, \lambda_{12}$
- λ_{12} rules the mixing of the two Higgs fields
- leads to non-diagonal mass matrix for Higgs fields $\{h_1, h_2\}$
- rotation into fields corresponding to mass eigenstates $\{h, H\}$

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_2 \\ h_1 \end{pmatrix}$$

Physical gauge bosons

$$\mathcal{L}_{\mathrm{M_V}} = \frac{1}{2}\left(B'_\mu, W^3_\mu, C'_\mu\right) \mathbf{M}^2_{\mathrm{V}} \begin{pmatrix} B'_\mu \\ W^3_\mu \\ C'_\mu \end{pmatrix} + \mathrm{M}^2_{\mathrm{W}} W^+ W^-$$

- $\mathrm{M}^2_{\mathrm{W}} = \frac{g_2^2}{4} v_2^2$ same parameter dependence as in the SM
- fields for mass eigenstates obtained by diagonalization of $\mathbf{M}^2_{\mathrm{V}}$

$$\begin{pmatrix} B'_\mu \\ W^3_\mu \\ C'_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W c_\gamma & -s_W s_\gamma \\ -s_W & c_W c_\gamma & -c_W s_\gamma \\ 0 & s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

$$s_W = \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \qquad s_\gamma = \sin\gamma, \qquad c_\gamma = \cos\gamma$$

- two massive, neutral bosons Z, Z'
- massless photon as in the SM

Lagrangian of the DASM

$$\mathcal{L}^{\mathrm{EW}}_{\mathrm{DASM}} = \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{Fix}} + \mathcal{L}_{\mathrm{FP}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Fermion}} + \mathcal{L}_{\mathrm{Yukawa}}$$

- $\mathcal{L}_{\mathrm{YM}}$ modified due to additional $U(1)_{\mathrm{d}}$ gauge group
  $\rightarrow$ $\mathcal{L}_{\mathrm{Fix}}$ and $\mathcal{L}_{\mathrm{FP}}$ modified as well
- $\mathcal{L}_{\mathrm{Higgs}}$ modified due to additional Higgs field ρ
- $\mathcal{L}_{\mathrm{Fermion}}$ and $\mathcal{L}_{\mathrm{Yukawa}}$ modified due to presence of $\nu_{\mathrm{R},i}$ and ν_4
- appropriate choice of additional input parameters:

$$\{M_{Z'}, M_H, \gamma, \alpha, \lambda_{12}, \ldots\}$$

$\rightarrow$ sophisticated renormalization scheme needed

Renormalization of the DASM

- renormalization transformations for bare parameters (x_0) and fields (F_0) at NLO

$$x_0 = x + \delta x, \qquad F_{i,0} = F_i + \frac{1}{2}\delta Z_F^{ij} F_j$$

- additional off-diagonal renormalization constants (RCs) present, e.g.:

$$\begin{pmatrix} Z'_0 \\ Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{Z'Z'} & \frac{1}{2}\delta Z_{Z'Z} & \frac{1}{2}\delta Z_{Z'A} \\ \frac{1}{2}\delta Z_{ZZ'} & 1+\frac{1}{2}\delta Z_{ZZ} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ'} & \frac{1}{2}\delta Z_{AZ} & 1+\frac{1}{2}\delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z' \\ Z \\ A \end{pmatrix}$$

- OS renormalization for masses, fields, and electric charge (similar to SM case) [S. Dittmaier, 2021]

How to renormalize the mixing angles $\gamma, \alpha, \ldots$?

$\overline{\text{MS}}$ renormalization scheme:

- ✓ easy to implement
- ✓ process independent
- ✓ easy way of uncertainty estimation via scale variation
- ✗ numerically unstable in certain regions of the parameter space:
  - ✗ degenerate masses of mixing particles ($M_{Z'} \approx M_Z$)
  - ✗ exceptional values of respective mixing angles
- ✗ tadpole scheme dependent:
  - ✗ large tadpole corrections to RCs in Fleischer-Jegerlehner tadpole scheme (FJTS) [J. Fleischer, F. Jegerlehner, 1981]
  - ✗ parameterization of observables is gauge dependent in Parameter-Renormalized Tadpole Scheme (PRTS) [M. Böhm, H. Spiesberger, W. Hollik, 1986]
  - ➡ Solution: Gauge-Invariant Vacuum expectation value Scheme (GIVS) (gauge independent + small tadpole corrections) [S. Dittmaier, H. Rzehak, 2022]

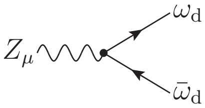
On Shell (OS) renormalization for mixing angle γ

- perturbatively stable OS renormalization for Singlet/THDM Higgs mixing angles known [A. Denner, S. Dittmaier, J. Lang, 2018]

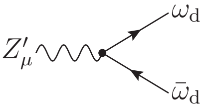
  $\rightarrow$ transfer these ideas to gauge (and fermion) sector
- introduce "fake fermion" ω_{d} (SM singlet, $U(1)_{\mathrm{d}}$ charge $\tilde{q}_\omega$)

$$\mathcal{L}_{\omega_{\mathrm{d}}} = \mathrm{i}\bar{\omega}_{\mathrm{d}} \left[\not{\partial} + \mathrm{i}\tilde{e}\tilde{q}_\omega \left(s_\gamma \not{Z} + c_\gamma \not{Z}' \right) + \mathrm{i}m_{\omega_{\mathrm{d}}} \right] \omega_{\mathrm{d}}$$

- only two new vertices:



$\propto \tilde{q}_\omega s_\gamma,$



$\propto \tilde{q}_\omega c_\gamma$

- for $\tilde{q}_\omega \rightarrow 0$ fake fermion ω_{d} decouples from theory

OS renormalization for gauge-sector mixing angle γ

- two possible decays $Z/Z' \to \bar{\omega}_\mathrm{d}\omega_\mathrm{d}$

$$\mathcal{M}^{Z\to\bar{\omega}_\mathrm{d}\omega_\mathrm{d}} = [\bar{u}_{\omega_\mathrm{d}} \not{\epsilon} v_{\omega_\mathrm{d}}]_Z \mathcal{F}^{Z\bar{\omega}_\mathrm{d}\omega_\mathrm{d}}, \quad \mathcal{M}^{Z'\to\bar{\omega}_\mathrm{d}\omega_\mathrm{d}} = [\bar{u}_{\omega_\mathrm{d}} \not{\epsilon} v_{\omega_\mathrm{d}}]_{Z'} \mathcal{F}^{Z'\bar{\omega}_\mathrm{d}\omega_\mathrm{d}}$$

- use ratio of OS form factors in decoupling limit for OS renormalization condition

$$\lim_{\tilde{q}_\omega \to 0} \frac{\mathcal{F}^{Z\bar{\omega}_\mathrm{d}\omega_\mathrm{d}}_{\mathrm{NLO}}}{\mathcal{F}^{Z'\bar{\omega}_\mathrm{d}\omega_\mathrm{d}}_{\mathrm{NLO}}} \overset{!}{=} \frac{\mathcal{F}^{Z\bar{\omega}_\mathrm{d}\omega_\mathrm{d}}_{\mathrm{LO}}}{\mathcal{F}^{Z'\bar{\omega}_\mathrm{d}\omega_\mathrm{d}}_{\mathrm{LO}}} = \frac{s_\gamma}{c_\gamma}$$

- process dependence and contributions coming from ω_d are $\mathcal{O}\left(\tilde{q}_\omega^2\right)$ and drop out at NLO

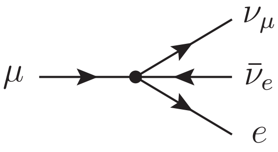
OS renormalization for gauge-sector mixing angle γ

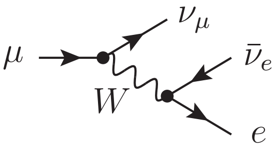
$$\delta\gamma^{\mathrm{OS}} = \frac{1}{2} s_\gamma c_\gamma \left(\delta Z_{Z'Z'} - \delta Z_{ZZ}\right) + \frac{1}{2}\left(s_\gamma^2 \delta Z_{ZZ'} - c_\gamma^2 \delta Z_{Z'Z}\right)$$

- ✓ gauge-independent combination of field RCs
- ✓ numerically stable for:
  - ✓ degenerate masses of mixing particles ($M_{Z'} \approx M_Z$)
  - ✓ exceptional values of γ
- ✓ predictions for observables in OS scheme independent of tadpole treatment
- ✦ process independent
- ✘ higher order uncertainty estimation more difficult
  - ✘ comparison with different schemes needed
  - ✦ $\overline{\mathrm{MS}}$ result easily obtained via $\delta\gamma^{\overline{\mathrm{MS}}} = \delta\gamma^{\mathrm{OS}}\Big|_{\mathrm{UV}}$

NLO M_W prediction in the DASM

- LO Feynman diagrams for muon decay:





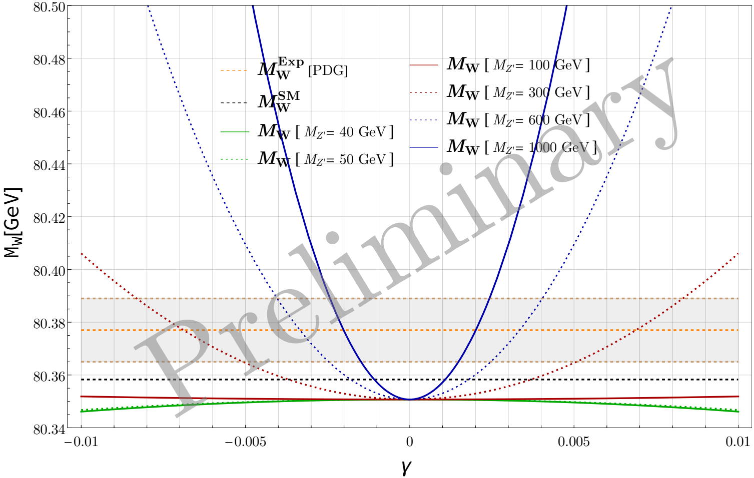
- comparison at NLO

$$\to G_F = \frac{\alpha_{em}\pi}{\sqrt{2} s_w^2 M_W^2}\left(1 + \Delta r\left(\delta\gamma, \ldots\right)\right), \quad s_w^2 = 1 - \frac{M_W^2}{c_\gamma^2 M_Z^2 + s_\gamma^2 M_{Z'}^2}$$

- appearance of various RCs
  $\to$ prediction depends on all additional parameters

NLO M_W prediction in the DASM

SM part taken from M. Awramik, M. Czakon, A. Freitas, G. Weiglein, 2004



$\alpha = 0.2, \quad M_H = 2000\,\text{GeV}, \quad m_{\nu_4} = 10\,\text{GeV}, \quad \theta_r = 0.2$

Summary

- DASM introduces three portals to a possible hidden sector
  - new physical particles H, Z' and ν_4
  - mixing between SM and BSM particles in scalar, gauge and fermion sector

  $\rightarrow$ various changes of coupling constants
- renormalization of the DASM at NLO
  - electric charge, masses, and fields OS renormalized
  - renormalization of mixing angles:
    - $\overline{\text{MS}}$: Easy to implement,
      intrinsic higher-order uncertainty estimation
    - OS: Numerically stable in whole parameter space,
      independent of tadpole treatment
  - underlying idea can be transferred to various
    BSM mixing angles

Backup

Extending the fermion sector

$$\mathcal{L}_{f'_\mathrm{d}} = \bar{f}'_\mathrm{d}\left(\mathrm{i}\not{D}_c - m_{f'_\mathrm{d}}\right) f'_\mathrm{d} - \sum_{k,l=e,\mu,\tau}\left(\bar{L}'^L_k G'^\nu_{kl}\nu'^R_l\tilde{\Phi} + \mathrm{h.c.}\right)$$
$$+ \sum_{j=e,\mu,\tau}\left[\bar{\nu}'_{\mathrm{R},j}\mathrm{i}\not{\partial}\nu'_{\mathrm{R},j} - \left(y_{\rho,j}\rho\bar{f}'_{\mathrm{d,L}}\nu'_{\mathrm{R},j} + \mathrm{h.c.}\right)\right]$$

- kinetic term and gauge interaction for the new fermion

  $\rightarrow$ couples to Z- and Z'-bosons
- non-diagonal 4×4 mass matrix for neutrinos and new fermion
- mass eigenstates are linear combinations of all 4 flavor eigenstates
- left-handed neutrino fields mix with newly introduced fermion

  $\rightarrow$ strongly suppressed for large mass of the new fermion

Extending the fermion sector

- new fermion should be heavy

$$\mathcal{L}_{f'_\mathrm{d}} = \bar{f}'_\mathrm{d} \left(\mathrm{i} \not{D}_c - m_{f'_\mathrm{d}} \right) f'_\mathrm{d} + \sum_{j=e,\mu,\tau} \bar{\nu}'_{\mathrm{R},j} \mathrm{i} \not{\partial} \nu'_{\mathrm{R},j}$$
$$- \left(y_{\rho,\tau} \rho \bar{f}'_{\mathrm{d,L}} \nu'_{\mathrm{R},\tau} + \mathrm{h.c.} \right)$$

- no mixing of the left-handed fields
- mass eigenstates $\nu_{1,2,3}$ massless, ν_4 massive
- only $\nu_{\mathrm{R},3/4}$ carry $U(1)_\mathrm{d}$ charge
  $\rightarrow$ influence of fermion sector extension on observables small

OS renormalization for Higgs-sector mixing angle α

- no useful introduction of "fake field" possible
- use the decays $h/H \to \bar{\tau}\tau$

$$\mathcal{M}^{\mathrm{h}\to\bar{\tau}\tau} = [\bar{u}_\tau v_\tau]_h \mathcal{F}^{h\bar{\tau}\tau}, \quad \mathcal{M}^{\mathrm{H}\to\bar{\tau}\tau} = [\bar{u}_\tau v_\tau]_H \mathcal{F}^{H\bar{\tau}\tau}$$

- use ratio of OS form factors for OS renormalization condition

$$\lim_{\tilde{q}_\omega \to 0} \frac{\mathcal{F}^{h\bar{\tau}\tau}_{\mathrm{NLO}}}{\mathcal{F}^{H\bar{\tau}\tau}_{\mathrm{NLO}}} \stackrel{!}{=} \frac{\mathcal{F}^{h\bar{\tau}\tau}_0}{\mathcal{F}^{H\bar{\tau}\tau}_0} = \frac{c_\alpha}{s_\alpha}$$

$$\delta\alpha^{\mathrm{OS}} = \frac{1}{2} c_\alpha s_\alpha \left(\delta Z_{hh} - \delta Z_{HH}\right) + \frac{1}{2}\left(s_\alpha^2 \delta Z_{Hh} - c_\alpha^2 \delta Z_{hH}\right) + c_\alpha s_\alpha \left(\delta^{h\bar{\tau}\tau}_{\mathrm{loop}} - \delta^{H\bar{\tau}\tau}_{\mathrm{loop}}\right)$$

- process dependent loop contributions appear

OS field and mass renormalization

- OS mass and field RCs (similar to SM)

$$\delta M_V^2 = \mathrm{Re}\,\Sigma_{\mathrm{T}}^{V^\dagger V}(M_V^2), \qquad \delta Z_{VV'} = -2\mathrm{Re}\,\frac{\Sigma_{\mathrm{T}}^{V^\dagger V'}(M_{V'}^2)}{M_{V'}^2 - M_V^2},$$

$$\delta Z_{V^\dagger V} = -\mathrm{Re}\,\frac{\partial \Sigma_{\mathrm{T}}^{V^\dagger V}(k^2)}{\partial k^2}\bigg|_{k^2=M_{\mathrm{V}}^2}, \qquad \delta M_S^2 = \mathrm{Re}\,\Sigma^{SS}(M_S^2),$$

$$\delta Z_{SS'} = -2\mathrm{Re}\,\frac{\Sigma^{S'S}(M_{S'}^2)}{M_{S'}^2 - M_S^2}, \qquad \delta Z_{SS} = -\mathrm{Re}\,\frac{\partial \Sigma^{SS}(k^2)}{\partial k^2}\bigg|_{k^2=M_S^2}$$

Smooth limits for degenerated masses

- appearance of off-diagonal field RCs can lead to problems for $M_{Z'} \approx M_Z$
- mixing-angle RC enters Lagrangian at two different places
  1) introduced via parameter replacements in the original $\mathcal{L}$
     $\rightarrow$ always proportional to $M_Z^2 - M_{Z'}^2$
  2) from the field rotation matrix always in the combination

$$-\delta\gamma + \frac{1}{2}\delta Z_{ZZ'}, \qquad \delta\gamma + \frac{1}{2}\delta Z_{Z'Z}$$

  - $\delta\gamma^{\mathrm{OS}}$ chosen such that off-diagonal field RCs only appear in combination

$$\delta Z_{Z'Z} + \delta Z_{ZZ'} = 2\mathrm{Re}\frac{\Sigma_{\mathrm{T}}^{ZZ'}(M_Z^2) - \Sigma_{\mathrm{T}}^{Z'Z}(M_{Z'}^2)}{M_{Z'}^2 - M_Z^2}.$$

$\rightarrow$ non-singular for degenerate masses