Two-loop Yukawa corrections to double Higgs production

JHEP 08 (2022) 259

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High Precision for Hard Processes (HP2)

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21st September, 2022

Higgs self coupling

Standard Model Higgs potential:

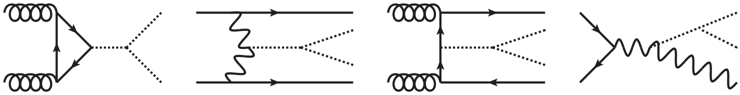
$$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4,$$

where $\lambda = m_H^2/(2v^2) \approx 0.13$.

Want to measure λ, to determine if $V(H)$ is consistent with nature.

- Challenging! Cross-section $\approx 10^{-3} \times H$ prod.
- $-3.3 < \lambda/\lambda_{SM} < 8.5$ [CMS '21]

λ appears in various production channels:



- Gluon fusion – dominant, 10x
- VBF
- $t\bar{t}$ associated production
- H-strahlung

Gluon Fusion

Leading order (1 loop) partonic amplitude:

$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

- $\mathcal{F}_{tri}$ contains the dependence on λ at LO

Form factors:

- LO: known exactly [Glover, van der Bij '88]
- Beyond LO... no fully-exact (analytic) results to date
  - QCD: numerical evaluation, expansion in various kinematic limits
  - EW: first steps: this work (HE) [Davies, Mishima, Schönwald, Steinhauser, Zhang '22]
    - (see also HTL considerations) [Mühlleitner,Schlenk,Spira '22]

$gg \to HH$ Beyond LO

NLO QCD:

- large-m_t [Dawson,Dittmaier,Spira '98] [Grigo,Hoff,Melnikov,Steinhauser '13]
- numeric [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke '16] [Baglio,Campanario,Glaus,Mühlleitner,Spira,Streicher '19]
- large-m_t + threshold exp. Padé [Gröber, Maier, Rauh '17]
- high-energy expansion [Davies, Mishima, Steinhauser, Wellmann '18,'19]
- small-p_T expansion [Bonciani, Degrassi, Giardino, Gröber '18]

NNLO QCD:

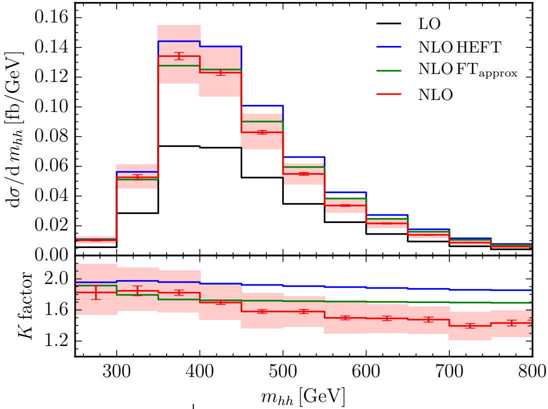
- large-m_t virtuals [de Florian, Mazzitelli '13] [Grigo, Hoff, Steinhauser '15][Davies, Steinhauser '19]
- HTL+numeric real ("FTapprox") [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli '18]
- large-m_t reals [Davies, Herren, Mishima, Steinhauser '19 '21]

N3LO QCD:

- Wilson coefficient C_{HH} [Spira '16][Gerlach, Herren, Steinhauser '18]
- HTL [Chen, Li, Shao, Wang '19]

$gg \to HH$ Beyond LO

[Borowka, Greiner, Heinrich, Jones, Kerner '16]



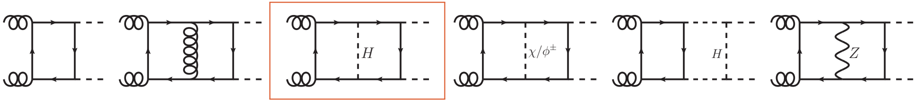
Total XS (14TeV):

	σ_{LO}	σ_{NLO}	σ_{NNLO}
B-i HTL	–	$38.32^{+18.1\%}_{-14.9\%}$	$39.58^{+1.4\%}_{-4.7\%}$
FTapprox	–	$34.25^{+14.7\%}_{-13.2\%}$	$36.69^{+2.1\%}_{-4.9\%}$
Full	$19.85^{+27.6\%}_{-20.5\%}$	$32.88^{+13.5\%}_{-12.5\%}$	–

EW Corrections

As we investigate NNLO QCD and beyond, we should consider NLO EW:

$$\mathcal{M} \sim \alpha_s \alpha_t \Big(A_1 + \alpha_s A_2 + \alpha_t A_3 + \alpha_{t,\lambda,gauge} A_4 + \mathcal{O}(\alpha_s^2, \alpha_t^2, \ldots) \Big)$$



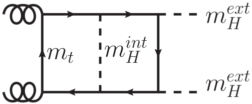
There are more scales to deal with, compared to the QCD contribution,

- start with $\alpha_s \alpha_t^2$ diagrams with internally propagating Higgs
  - expansion parameter $\alpha_t = \alpha m_t^2/(2 s_W^2 m_W^2) \sim \alpha_s/2$
  - only planar integrals in this subset

High-Energy Expansion

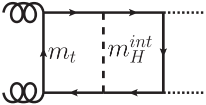
The full diagrams depend on a lot of variables:

- ϵ, s, t, m_t, m_H
- complete analytic solution is out of reach



First, expand around $m_H^{ext} = 0$ (as for QCD):

- expand amplitude integrals with `LiteRed` [Lee '14]



Unlike for QCD the scale "m_H^{int}" remains, from the propagator:

- complicates the IBP reduction
- Master Integrals with this many scales are difficult.

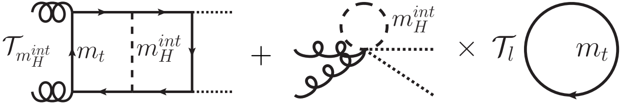
We expand in this scale also, and propose two ways to do it:

- A: $s, |t| \gg m_t^2 \gg {m_H^{int}}^2 \sim {m_H^{ext}}^2$,
- B: $s, |t| \gg m_t^2 \sim {m_H^{int}}^2 \gg {m_H^{ext}}^2$.

High-Energy Expansion “A”

Option A: asymptotic expansion around $m_H^{int} = 0$:

- two sub-graphs:



The two-loop subgraph is a Taylor expansion of the Higgs propagator:

- results in integrals with a massless internal line, scales s, t, m_t.
- IBP reduce with `FIRE` and `Kira` [Smirnov ‘15] [Klappert,Lange,Maierhöfer,Usovitsch ‘21]
- these coincide with the QCD Master Integrals – reuse the old results [Davies,Mishima,Steinhauser,Wellmann ‘18,‘19]

The massive tadpoles are easily computed by `MATAD`. [Steinhauser ‘00]

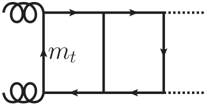
The asymp. expansion procedure is done by `exp` and `FORM` [Harlander,Seidelsticker,Steinhauser ‘97] [Ruijl,Ueda,Vermaseren ‘17]

We expand to quartic order: $(m_H^{int})^a \, (m_H^{ext})^b, \;\; 0 \leq (a+b) \leq 4$.

High-Energy Expansion “B”

Option B: expand around $m_H^{int} \approx m_t$,

- simple Taylor expansion, `exp` not necessary
  - much easier to implement
- IBP reduce resulting integrals, `FIRE+Kira`



Write Higgs propagator as: $\frac{1}{p^2-m_H^2} = \frac{1}{p^2-m_t^2(1-[2-\delta]\delta)}$

- expand around $\delta \to 0$ where $\delta = 1 - m_H/m_t \approx 0.28$.

This yields new integral families compared to the QCD computation:

- all lines have the mass m_t,
- compute the MIs in the high-energy limit: see Kay Schönwald’s talk.

We expand to $(m_H^{ext})^4$ and δ^3.

Padé-Improved High-Energy Expansion

The MIs for both methods are computed as an expansion in $m_t \ll s, |t|$.

The expansions diverge for $\sqrt{s} \sim 750\text{GeV}$ ("A"), $\sqrt{s} \sim 1000\text{GeV}$ ("B").

The situation can be improved using Padé Approximants:

- approximate a function using a rational polynomial:

$$f(x) \approx [n/m](x) = \frac{a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n}{1 + b_1 x + b_2 x^2 + \cdots + b_m x^m},$$

  where a_i, b_j coefficients are fixed by the series coefficients of $f(x)$.

We compute a set of various Padé Approximants:

- combine to give a central value and error estimates
- a deeper input expansion $\rightarrow$ larger $n + m$ $\rightarrow$ smaller error
- here, m_t^{120} exp. allows for very high-order Padé Approximants

Renormalization

The form factors require UV renormalization (they are IR finite):

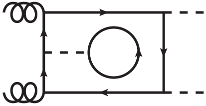
- $\overline{\text{MS}}$ renormalization of the top quark mass,

$$m_t^0 \to \overline{m}_t \left[1 + \frac{\alpha_t}{\pi} \frac{1}{\epsilon} \left(\frac{3}{16} + \frac{N_C}{2} \frac{\overline{m}_t^2}{m_H^2} \right) \right]$$

- LO has no δ expansion, so NLO δ terms must already be finite ✓

The second term in $(\cdots)$ renormalizes the tadpole diagrams,

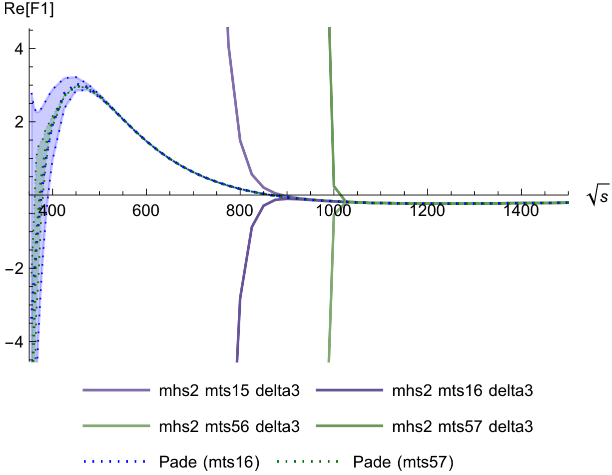
- computed, but not included in the following plots.



High-Energy Expansion and Padé Approximation

Re(F_{box1}), fixed $\cos\theta = 0$, expansion "B" (to $(m_H^2)^2\delta^3(m_t^2)^{\{15,16,56,57\}}$):

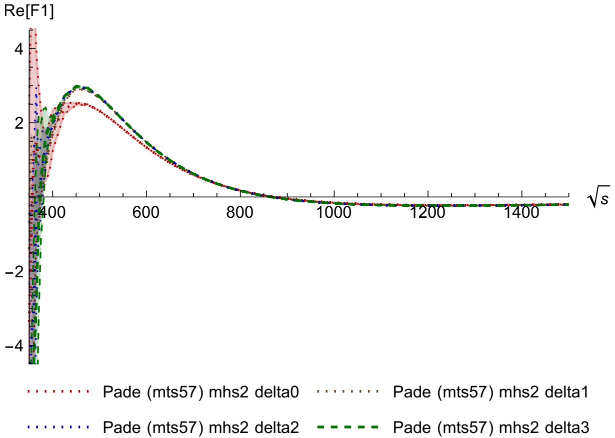
- m_t expansion diverges (strongly) around $\sqrt{s} \sim 1000$GeV



Convergence of delta expansion ("B")

Re(F_{box1}), fixed $\cos\theta = 0$, expansion "B" Padé (to $(m_H^2)^2\delta^{\{0,1,2,3\}}$):

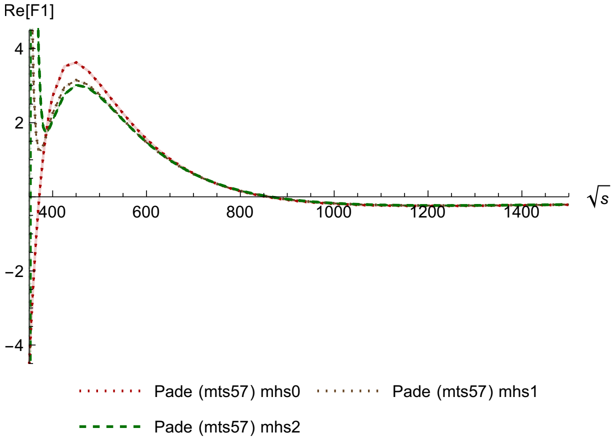
- δ^2 and δ^3 terms differ by at most 0.5% for $\sqrt{s} \geq 400$GeV



Convergence of asymptotic expansion ("A")

Re(F_{box1}), fixed $\cos\theta = 0$, expansion "A" Padé (to $(m_H^2)^{\{0,1,2\}}$):

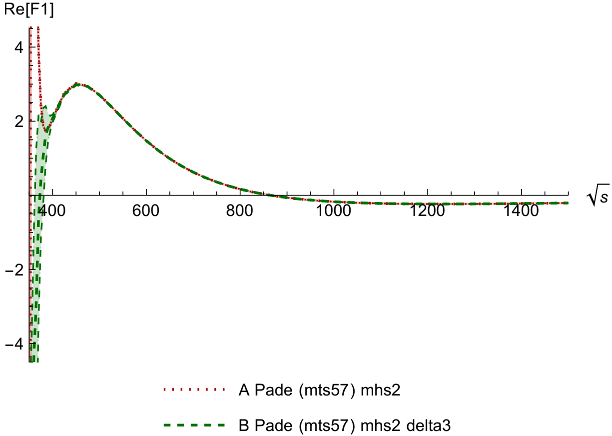
- $(m_H^2)^1$ and $(m_H^2)^2$ terms differ by at most 5% for $\sqrt{s} \geq 400$GeV



Comparison of “A”, “B” expansions

Re(F_{box1}), fixed $\cos\theta = 0$, best “A” and “B” Padé

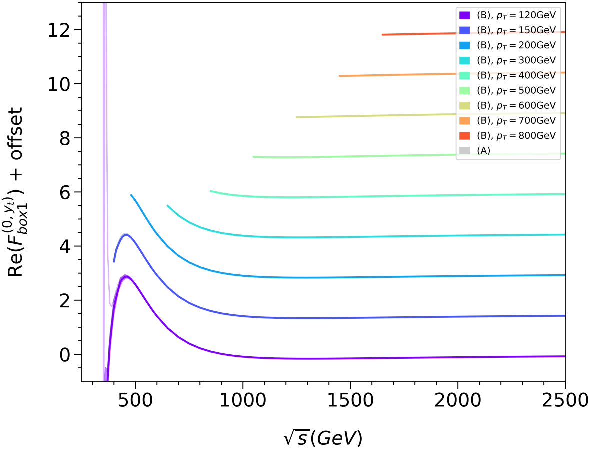
- “A”, “B” differ by at most 2% for $\sqrt{s} \geq 400$GeV,
- 0.1% for $\sqrt{s} \geq 500$GeV



Form factors at fixed p_T

Expansions "A" and "B" agree for p_T values as small as 120 GeV.

- deep expansions of the MIs required, for small Padé errors



Conclusion

First step towards EW corrections to HH production:

- more difficult than the QCD contribution (extra internal scale)
- expansion allows us to compute them

High-energy expansion:

- Padé-based approximation to improve expansion
- good description of (partial) form factors for $p_T \gtrsim 120\mathrm{GeV}$
- two different expansion methods, which give equivalent results
- deeper exp. of MIs compared to QCD papers $\rightarrow$ better Padé

Work in progress:

- compute the remaining sets of diagrams
  - high-energy expansion, new families of MIs to compute
  - combine with other expansions to cover full phase space